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Evaluating Opportunities When People are Uncertainty Averse*

RUVIN GEKKER
ASHLEY PIGGINS
National University of Ireland, Galway

Abstract: We consider the problem of ranking sets of alternatives. Standard approaches to this problem regard the addition of an alternative to a set containing one element as enhancing choice. We argue that this monotonicity axiom may not be desirable when an agent is uncertain as to the value of this additional alternative. We replace monotonicity with an uncertainty aversion axiom, and also introduce an axiom that produces lexicographic behaviour. These axioms, in conjunction with an independence axiom, enable us to prove a characterisation theorem. This theorem says that sets are ranked in terms of the number of uncertain elements that they contain, the fewer the better. This is the only ranking rule that satisfies our axioms.

I INTRODUCTION

Economics assumes that individuals have preferences over the elements in a given set of alternatives. Individuals are not required to have preferences over subsets of this set. Of course, the ranking of elements in the superset can induce a ranking of subsets. If the superset is denoted by X, then

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we might think that a subset A is preferred to another B if and only if the most preferred element in A is preferred to the most preferred element in B. This is the indirect utility approach to ranking subsets.

We can think of subsets as alternative "menus" from which an individual must make a choice. Imagine you prefer steak to all other food. According to the indirect utility approach this means that you would prefer the menu (steak) to the menu (monkfish, lamb, risotto, salad). However, this menu gives you no freedom to choose whatsoever; the only choice open to you is to eat steak. The second menu offers more choice, and we would expect individuals to attach some value to this when ranking menus. This is a weakness with the indirect utility approach.

An alternative approach has emerged in which subsets are ranked directly in terms of how much choice they offer an economic agent. The subsets in this literature are called opportunity sets presumably because you have the opportunity to consume anything in the set. The seminal paper by Pattanaik and Xu in 1990 is one of the first axiomatic treatments of this problem. They impose three axioms on the ranking of opportunity sets and demonstrate that these are uniquely satisfied by the cardinality rule. This rule says that opportunity sets are ranked according to the number of elements they contain; the larger the set the more choice it affords. So (monkfish, lamb, risotto, salad) would be ranked above (steak) according to this rule. Their first axiom, Indifference between No-Choice Situations, says that any two singleton sets always generate an equal amount of choice. Their second axiom, Monotonicity (MON), says that any set containing two elements provides more choice than any of its elements. Their third axiom, Independence (IND), says that the ranking of any two opportunity sets A and B is preserved if an element x which is neither in A nor in B is added to them. It turns out that these three axioms (plus transitivity) produce the very surprising result that opportunity sets are ranked according to the cardinality rule and no other rule.

The axiomatic approach has been developed in the literature (see Klemish-Ahlert (1993), Pattanaik and Xu (2000), van Hees (2004), Peragine and Romero-Medina (2006) to name just a few). Sen (1990, 1991, 1993) famously attacked this approach for its neglect of underlying preferences in assessing the value of an opportunity set. His critique of INS and MON is based on this. Sen's influential attack has produced an enormous number of responses (see Arrow (1995); Gravel (1994); Puppe (1996, 1998); Pattanaik and Xu (1998); Sugden (1998); Nehring and Puppe (1999); Romero-Medina (2001); Bavetta and Peragine (2006), for a more complete list of references we refer to Barbera, Bossert and Pattanaik's (2004) survey). Despite Sen's critique of MON, most of the literature continues to use it as an axiom, albeit in some modified form.

The MON axiom seems reasonable if the point of the exercise is to rank sets in terms of how much choice they offer an economic agent. However, if we are interested in an agent's evaluation of an opportunity set then a larger set need not be better. Of course, we can already see this from the perspective of indirect utility. The point we wish to make in this paper, however, is that larger sets need not be better even without appealing to indirect utility. This is our central contribution.

To motivate our approach, imagine that an agent has incomplete information about some elements in the opportunity set. For example, suppose that the agent has never eaten monkfish and so has no idea how it compares with the other food on the menu. Ordinarily this would not matter; the agent could just select one of the remaining foods ignoring the presence of the monkfish. In fact, when evaluating the set (monkfish, lamb, risotto, salad) the agent could regard it as equivalent to (lamb, risotto, salad). However, suppose that the actual choice is determined for the agent at random. More precisely, assume that each element on the menu can be chosen with some probability, but the agent is unaware as to what this probability is (in the literature, this is called an environment of complete uncertainty). All the agent knows is that these probabilities must sum to one. In an environment like this, it is possible to formulate an axiom called Uncertainty Aversion (UA). This axiom says that an uncertainty averse individual will be made worse off, other things equal, if an alternative about which she is uncertain is added to the opportunity set.

Here is another example. In theories of distributive justice extensive use is made of a device called the "original position". This is a hypothetical situation in which people compare different social states from behind a veil of ignorance. From behind the veil an individual can observe the effects of different social states on different people. For example, if everyone is better off in one state than in another then presumably the observer would place this state above the other in her ranking. What makes the observer's preferences impartial is that she does not know which individual in society she will be should that state of the world actually arise. In the language of our theory, the probability distribution across possible people is unknown to the observer. This, it is argued, gives the observer an incentive to assess the social states from an impartial point of view. It is easy to translate this into our setting. An individual behind the veil would observe a set of possible lives. As social states vary so would this set; individual characteristics vary across states. But, as states vary, so can the size of the population. In some states there may be more people than in others. Imagine that an additional person is added to a state, other things equal. Moreover, imagine that this person's characteristics in this state are hard for the observer to compare with everyone else's characteristics. In other words, the observer cannot tell whether this life is better or worse than any other life in the state. In such circumstances, it seems reasonable to assume a form of uncertainty aversion. In this context, our axiom says that an uncertainty averse observer would be made worse off, other things equal, if a life about which she is uncertain is added to the set of possible lives.

We show that it is possible to express the concept of uncertainty aversion in a purely set-theoretic setting. Using an axiomatic approach reminiscent of that of Pattanaik and Xu, we characterise an extreme form of behaviour called lexicographic uncertainty aversion. This is a rule that ranks opportunity sets according to the number of uncertain elements that they contain, the fewer the better. This means that the number of uncertain elements lexically dominates the number of certain elements, and so the size of the opportunity set itself is irrelevant. This rule is the only rule that satisfies our axioms. Bossert (2000) and Gekker and van Hees (2006) are the only other papers we are aware of that consider information issues when ranking opportunity sets.

Section II introduces the framework and Section III presents the results. We conclude with some brief remarks in Section IV.

II THE FRAMEWORK

Let X denote a non-empty finite set of alternatives. Let $\Pi(X)$ denote the set of all non-empty subsets of X. The elements of $\Pi(X)$ are called opportunity sets and are denoted by A, B, C, … . We assume that an agent is uncertain about some of the elements in X. We denote by X^* the non-empty set of uncertain elements, where X^* is not equal to X. Let $\Pi(X^*)$ denote the set of all non-empty subsets of X^*.

Following van Hees (1998), we are interested in ranking opportunity situations. An opportunity situation is an ordered pair of sets $(A, E) \in \Pi(X) \times \Pi(X^*)$ where $E \subseteq A$. Let T denote the set of all opportunity situations. The interpretation is the following: the agent is faced with the opportunity set A whilst being uncertain (or ignorant) about the elements in E. To illustrate this concept, in our original example the individual's opportunity situation is (monkfish, lamb, risotto, salad), (monkfish).

Let $\succcurlyeq$ be a reflexive and transitive binary relation on T. In this extended framework, $(A, E) \succcurlyeq (B, F)$ should be read as "facing the opportunity set A whilst being uncertain about the elements in E is weakly preferred to facing the opportunity set B whilst being uncertain about the elements in F". As usual we denote by $\succ$ and $\sim$ the asymmetric and symmetric factors of $\succcurlyeq$ respectively.

We now introduce the following axioms.

Indifference between Uncertain Singletons (IUS): For all $(A, \{x\}), (B, \{y\}) \in T$, $(A, \{x\}) \sim (B, \{y\})$.

This is an extremely strong axiom and it may be considered intuitively unattractive. IUS says that situations in which two opportunity sets contain one uncertain element each are regarded as indifferent. In other words, no weight is given to the number of certain elements in an opportunity set when comparing two sets, both of which contain just one uncertain element. This axiom could plausibly describe the behaviour of an extreme uncertainty averter.

Note that this axiom says nothing about how sets containing more than one uncertain element are ranked. Moreover, it says nothing about how sets containing more than one uncertain element are ranked against sets containing one uncertain element.

Independence (IND): For all $(A, E), (B, F) \in T$, and all $x \notin A$ and $y \notin B$, $(A, E) \succcurlyeq (B, F) \Leftrightarrow (A \cup \{x\}, E \cup \{x\}) \succcurlyeq (B \cup \{y\}, F \cup \{y\})$.

IND says that adding new, uncertain elements to a pair of opportunity situations preserves their ranking.

We now introduce our uncertainty aversion axiom.

Uncertainty Aversion (UA): For all $(A, E) \in T$ and for all $x \in X^*$ such that $x \notin A$, $(A, E) \succ (A \cup \{x\}, E \cup \{x\})$.

UA simply says that an agent who is uncertainty averse will be made worse off, other things equal, by the addition of an uncertain alternative to her opportunity situation.

We use these three axioms to characterise a rule for ranking finite opportunity situations in terms of the number of uncertain elements that they contain. This lexicographic uncertainty ordering $\succcurlyeq_U$ on T is defined as follows:

For all $(A, E), (B, F) \in T$, $(A, E) \succcurlyeq_U (B, F) \Leftrightarrow |A \cap E| \leq |B \cap F|$.

III THEOREM

Theorem. $\succcurlyeq$ satisfies IUS, IND and UA if and only if $\succcurlyeq = \succcurlyeq_U$.

Proof:
We first prove ($\Leftarrow$).
Assume that $\succcurlyeq = \succcurlyeq_U$ but that $\succcurlyeq$ does not satisfy IUS. Therefore, there exists an $(A, \{x\}), (B, \{y\}) \in T$ such that $\neg(A, \{x\}) \sim (B, \{y\})$. But then it is false that $|A \cap \{x\}| \leq |B \cap \{y\}|$ which is a contradiction.

Assume that $\succcurlyeq \, = \, \succcurlyeq_U$ but that $\succcurlyeq$ does not satisfy IND. Therefore, there exists (A, E), (B, F) $\in$ T with $x \notin A$ and $y \notin B$, such that $(A, E) \succcurlyeq (B, F)$ and $\neg(A \cup \{x\}, E \cup \{x\}) \succcurlyeq (B \cup \{y\}, F \cup \{y\})$. But then it is false that $|\{A \cup \{x\}\} \cap \{E \cup \{x\}| \leq |\{B \cup \{y\}\} \cap \{F \cup \{y\}\}|$. However, this contradicts the fact that $|A \cap E| \leq |B \cap F|$.

Assume that $\succcurlyeq \, = \, \succcurlyeq_U$ but that $\succcurlyeq$ does not satisfy UA. Therefore, there exists $(A, E) \in T$ and an $x \in X^*$ such that $x \notin A$ and $\neg(A, E) \succ (A \cup \{x\}, E \cup \{x\})$. But then it is false that $|A \cap E| \leq |\{A \cup \{x\}\} \cap \{E \cap \{x\}\}|$ which contradicts the fact that $x \notin A$.

We now prove ($\Rightarrow$).

Suppose that $\succcurlyeq$ satisfies IUS, IND and UA. Since $\succcurlyeq_U$ is complete, it is sufficient to prove that for all (A, E), (B, F) $\in$ T,

$$|A \cap E| = |B \cap F| \Rightarrow (A, E) \sim (B, F) \tag{1}$$

and

$$|A \cap E| < |B \cap F| \Rightarrow (A, E) \succ (B, F) \tag{2}$$

We prove (1) by induction on the number of elements in $A \cap E$ and in $B \cap F$. If $|A \cap E| = |B \cap F| = 1$ then $(A, E) \sim (B, F)$ by IUS. Now suppose that (1) is true for all (A, E), (B, F) such that $|A \cap E| = |B \cap F| = n$. We want to establish (1) for all (A', E'), (B', F') such that $|A' \cap E'| = |B' \cap F'| = n + 1$.

Let $E \subset E'$ with $|E'| = n + 1$. Therefore, there exists an $x = E' - E$.

We examine two cases:

(i) $x \in \{B' \cap F'\}$. Let $B = B' - \{x\}$ and $F = F' - \{x\}$.

Let A* be an arbitrary set such that $E \subseteq A^*$ and $x \notin A^*$. By the inductive hypothesis we have $(A^*, E) \sim (B, F)$. Independence implies that $(A^* \cup \{x\}, E \cup \{x\}) \sim (B \cup \{x\}, F \cup \{x\})$ which is what we wanted to prove.

(ii) $x \notin \{B' \cap F'\}$. Since $|A' \cap E'| = |B' \cap F'|$ and $E' \subseteq A'$ and $F' \subseteq B'$ we have $|E'| = |F'|$. Since $x \in E'$ but $x \notin F'$ then there exists $y \in F' - E'$. Let $F = F' - \{y\}$ and let B* be an arbitrary set such that $F \subseteq B^*$ and $y \notin B^*$. By construction $|B^* \cap F| = n$ and so the inductive hypothesis implies that $(A^*, E) \sim (B^*, F)$. Independence implies that $(A^* \cup \{x\}, E \cup \{x\}) \sim (B^* \cup \{y\}, F \cup \{y\})$ which is what we wanted to prove.

We now prove (2).

Let (A, E), (B, F) $\in$ T be such that $|A \cap E| < |B \cap F|$. Let (B**, F*) be an opportunity situation with $|B^{**} \cap F^*| = |A \cap E|$. Moreover, $F^* \subset F$ with $B^{**} = B - \{F - F^*\}$. By (1) we have $(A, E) \sim (B^{**}, F^*)$. Repeated application of

UA implies that (B**, F*) $\succ$ (B** $\cup$ {x}, F* $\cup$ {x}) where x $\in$ {F – F*}. Transitivity implies that (A, E) $\succ$ (B, F).

IV CONCLUSIONS

In this paper we have assumed that an individual is uncertain about some options in her opportunity set and have proposed a set-theoretic definition of uncertainty aversion. Using an approach reminiscent of that of Pattanaik and Xu, we proposed a set of axioms that characterise what we call lexicographic uncertainty aversion. This is a rule that ranks opportunity sets according to the number of uncertain elements that they contain, the fewer the better. This means that the number of uncertain elements lexically dominates the number of certain elements, and so the size of the opportunity set itself is irrelevant.

Perhaps it should be noted that we do not take into account various degrees of uncertainty that could be measured in a more sophisticated approach. We postpone this consideration until the next occasion.

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