

*Klaus Kultti – Hannu Vartiainen*  
**Bargaining with Many Players:  
A Limit Result**

**Aboa Centre for Economics**

Discussion Paper No. 32  
Turku 2008



Copyright © Author(s)

ISSN 1796-3133

Turun kauppakorkeakoulun monistamo  
Turku 2008

*Klaus Kultti – Hannu Vartiainen*  
**Bargaining with Many Players:  
A Limit Result**

**Aboa Centre for Economics**  
Discussion Paper No. 32  
June 2008

**ABSTRACT**

We provide a simple characterization of the stationary subgame perfect equilibrium of an alternating offers bargaining game when the number of players increases without a limit. Core convergence literature is emulated by increasing the number of players by replication. The limit allocation is interpreted in terms of Walrasian market for being the first proposer.

JEL Classification: C72, C78

Keywords: non-cooperative bargaining, stationary equilibrium, replication, Walrasian market

## **Contact information**

Hannu Vartiainen, Turku School of Economics, Department of Economics, Rehtorinpellonkatu 3, FIN-20500 Turku.

E-mail: hannu.vartiainen(a)tse.fi.

Klaus Kultti, University of Helsinki, Department of Economics, Arkadiankatu 7, FIN-00014 University of Helsinki.

## **Acknowledgements**

We thank Hannu Salonen for comments and discussions.

# 1 Introduction

We study an  $n$ -player alternating offers bargaining game where the players try to agree on a division of a cake. Time proceeds in discrete periods to infinity, player 1 starts the game, and the proposer in any period is the player who first rejected the offer of the previous period. We are interested in what happens when the number of players increases. Our way of increasing the population parallels the core convergence literature as we replicate the situation so that while the number of players is increased the size of the cake increases proportionally: each replica of players brings in a new cake to the pool of shareable cakes. This could reflect matters e.g. when similar nations group together as a federation.

Having a large set of players is attractive since in the limit almost all players act as responders; only one player enjoys the first proposer advantage and hence, as the number of replicas becomes large, the solution becomes almost distortion-free. We show that in the limit the unique stationary subgame perfect equilibrium has a simple characterization in terms of a single replica's preferences. Finally, the resulting single replica outcome has an attractive Walrasian interpretation: the unique equilibrium in a market where the first proposer right is sold to a *single* replica of bargainers induces the same outcome.

The primitive of our model are the time preferences á la Fishburn and Rubinstein (1982). This approach does not make assumptions concerning the concavity of utility functions.<sup>1</sup> Under similar assumptions, Kultti and Vartiainen (2007) show that the stationary equilibrium outcome converges to the Nash-bargaining solution when the length of the *time period* goes to zero. Since no additional assumptions are made on the utility representations, this is an extension of Binmore, Rubinstein and Wolinsky (1986). We now show that the limit outcome under replication (but fixed time interval) converges to a well defined solution also, but different from the Nash solution.<sup>2</sup>

## 2 The Model

A cake of size  $X > 0$  is to be divided among the set  $N = \{1, 2, \dots, n\}$  of players. The set of divisions of the cake is

$$S^n(X) = \{x \in \mathbb{R}^n : \sum_{i=1}^n x_i \leq X, x_i \geq 0, \text{ for all } i\}.$$

---

<sup>1</sup>Our assumptions about preferences are weaker than, for instance, in Kirshna and Serrano (1996). As their (unique) equilibrium is stationary, our results can be interpreted as an extension of theirs.

<sup>2</sup>One could also study what happens when the size of the cake is kept fixed and the number of players is increased. Then there is convergence to the Nash-bargaining solution because the utility frontier becomes practically linear. The same reasoning applies when the size of the cake is increased while keeping the number of players fixed.

Let us write  $x = (x_1, \dots, x_n)$  and  $x_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ .

The players' preferences over divisions and timing constitute the primitive of the model. The cake can be divided at any point of time  $T = \{0, 1, 2, \dots\}$ . Let division  $\mathbf{0} = (0, \dots, 0)$  serve as the reference point, and let (complete, transitive) preferences over  $S \times T$  satisfy, for all  $x, y \in S$ , for all  $i \in N$ , and for all  $s, t \in T$ , the following properties (Fishburn and Rubinstein, 1982; Osborne and Rubinstein, 1990, Ch. 4):

- A1.  $(x, t) \succeq_i (\mathbf{0}, 0)$ .
- A2.  $(x, t) \succeq_i (y, t)$  if and only if  $x_i \geq y_i$ .
- A3. If  $s > t$ , then  $(x, t) \succeq_i (x, s)$ , with strict preference if  $x_i > 0$ .
- A4. If  $(x^k, t^k) \succeq_i (y^k, s^k)$  for all  $k = 1, \dots$ , with limits  $(x^k, t^k) \rightarrow (x, t)$  and  $(y^k, s^k) \rightarrow (y, s)$ , then  $(x, t) \succeq_i (y, s)$ .
- A5.  $(x, t) \succeq_i (y, t + 1)$  if and only if  $(x, 0) \succeq_i (y, 1)$ , for any  $t \in T$ .

A1-A5 hold throughout the paper. By A2, the Pareto-optimal divisions at any date are given by

$$P^n(X) = \{x \in S^n(X) : \sum_{i=1}^n x_i = X\}.$$

For each  $i$  there is a function  $v_i : [0, X] \rightarrow [0, X]$ , defining the present consumption value of  $x_i$  in date 1:

$$(y, 0) \sim_i (x, 1) \text{ if } v_i(x_i) = y_i, \text{ for all } x, y \in S^n(X). \quad (1)$$

Fishburn and Rubinstein (1982) show that given A1-A5,  $v_i(\cdot)$  is continuous and increasing on  $[0, X]$ .

We assume that the loss of delay increases in the share of the cake.

- A6.  $x_i - v_i(x_i)$  is strictly increasing and differentiable.

That is,

$$\frac{dv_i^{-1}(x_i)}{dx_i} = \frac{1}{v_i'(x_i)} > 1, \text{ for all } x_i \geq 0. \quad (2)$$

This property will be used when we prove the existence of a stationary equilibrium.

### 3 The Game

Given  $N$  and  $X$ , we focus is on a unanimity bargaining game  $\Gamma^N(X)$  defined as follows: At any stage  $t \in \{0, 1, 2, \dots\}$ ,

- Player  $i(t) \in N$  makes an offer  $x \in S^n(X)$ . Players  $j \neq i(t)$  accept or reject the offer in the ascending order of their index.<sup>3</sup>
- If all  $j \neq i(t)$  accept, then  $x$  is implemented. If  $j$  is the first who rejects, then  $j$  becomes  $i(t+1)$ .
- $i(0) = 1$ .

We focus on the stationary subgame perfect equilibria, simply equilibria in the sequel, of the game, where:

1. Each  $i \in N$  makes the same proposal  $x(i)$  whenever he proposes.
2. Each  $i$ 's acceptance decision in period  $t$  depends only on  $x_i$  that is offered to him in that period.

We now characterize equilibria. Let division  $\underline{x} \in S^n(X)$  and  $d > 0$  satisfy

$$\underline{x}_i = v_i(\underline{x}_i + d), \text{ for all } i \in N, \quad (3)$$

$$\sum_{i=1}^n \underline{x}_i = X - d. \quad (4)$$

**Proposition 1**  *$x$  is a stationary equilibrium outcome of  $\Gamma^N(X)$  if and only if  $x = (\underline{x}_1 + d, \underline{x}_2, \dots, \underline{x}_n)$ , for the  $\underline{x}$  and  $d$  that meet (3) and (4).*

**Proof.** Only if: In a stationary SPE the game ends in finite time. Assume that it never ends. Then each player receives zero. This means that in all subgames each player must get zero. Otherwise there would be a subgame where some offer  $y = (y_1, \dots, y_n)$  is accepted. Because of stationarity this offer is accepted in every subgame. In particular, player 1 can deviate in the first period and offer  $y = (y_1, \dots, y_n)$ . This is a profitable deviation and constitutes a contradiction with the assumption that there is a stationary SPE where the game never ends.

Assume next that there is a stationary SPE where an offer  $x(i)$  by some player  $i \in \{1, 2, \dots, n\}$ , is not accepted immediately. Denote by  $z(i)$  the equilibrium outcome in a subgame that starts with an offer  $x(i)$  of player  $i$ . But now player  $i$  could offer  $z(i)$  instead of  $x(i)$ ; everyone else would accept the offer as in the stationary equilibrium acceptance depends only on the offer.

Thus, in any equilibrium,  $i(t)$ 's offer  $x(i(t)) = (x_j(i(t)))_{j \in N}$  is accepted at stage  $t \in \{0, 1, 2, \dots\}$ . In stationary equilibrium the time index  $t$  can be relaxed from  $x(i(t))$ . An offer  $x$  by  $i$  is accepted by all  $j \neq i$  if

$$x_j(i) \geq v_j(x_j(j)), \text{ for all } j \neq i. \quad (5)$$

---

<sup>3</sup>The order in which players response to a proposal does not affect the results.

Player  $i$ 's equilibrium offer  $x(i)$  maximizes his payoff with respect to constraint (5) and the resource constraint. By A3, all constraints in (5) and the resource constraint must bind. That is,

$$x_j(i) = v_j(x_j(j)), \text{ for all } j \neq i, \quad (6)$$

and

$$\sum_{i=1}^n x_i(j) = X, \text{ for all } j. \quad (7)$$

Since player  $i$ 's acceptance decision is not dependent on the name of the proposer, there is  $\underline{x}_i > 0$  such that  $x_i(j) = \underline{x}_i$  for all  $j \neq i$ . By (6),  $x_j(i) < x_j(j)$  for all  $j$ . Hence there is  $d > 0$  such that

$$\sum_{i=1}^n \underline{x}_i = X - d. \quad (8)$$

By (6) and (8),  $\underline{x}$  and  $d$  do meet (3) and (4). Since 1 is the first proposer, the resulting outcome is  $x(1) = (\underline{x}_1 + d, \underline{x}_2, \dots, \underline{x}_n)$ .

If: Let  $\underline{x}$  and  $d$  meet (3) and (4). Construct the following stationary strategy: Player  $i$  always offers  $\underline{x}_{-i}$  and does not accept less than  $\underline{x}_i$ . Player  $i$ 's offer  $y$  is accepted by all  $j \neq i$  only if

$$y_j \geq v_j(X - \sum_{k \neq j} \underline{x}_k) = v_j(\underline{x}_j + d), \text{ for all } j \neq i. \quad (9)$$

Since  $v_j$  is increasing, and since

$$\underline{x}_j = v_j(\underline{x}_j + d), \text{ for all } j \neq i,$$

$i$ 's payoff maximizing offer to each  $j$  is  $\underline{x}_j$ . ■

Thus, to find a stationary equilibrium it is sufficient to find  $\underline{x}$  and  $d$  that meet (3) and (4).

By (2),  $v_i^{-1}(x_i) - x_i$  is a continuous and monotonically increasing function. Thus, the function  $e_i(\cdot)$  such that

$$e_i(x_i) := v_i^{-1}(x_i) - x_i, \text{ for any } x_i \geq 0, \quad (10)$$

is continuous and monotonically increasing.

Define  $\bar{e}_i \in (0, \infty]$  by

$$\sup_{x_i \geq 0} e_i(x_i) := \bar{e}_i.$$

Since  $e_i(\cdot)$  is continuous and monotonically increasing, also its inverse

$$x_i(e) := e_i^{-1}(e), \text{ for all } e \in [0, \bar{e}_i],$$

is continuous and monotonically increasing in its domain  $[0, \bar{e}_i]$ . Condition (10) can now be stated in the form

$$x_i(e) = v_i(x_i(e) + e), \text{ for all } e \in [0, \bar{e}_i]. \quad (11)$$



**Proposition 2** *There is a unique stationary equilibrium of  $\Gamma^N(X)$ .*

**Proof.** By A1 and A3,  $x_i(0) = 0$ . Since, for all  $i$ ,  $x_i^{-1}(\cdot)$  is a monotonically increasing function on  $\mathbb{R}_+$  having its supremum at  $\bar{e}$ , it follows that  $\lim_{e \rightarrow \bar{e}_i} x_i(e) = \infty$ . Thus, since  $\sum_{i=1}^n x_i(e) + e$  is a continuous function of  $e$  on  $[0, \bar{e}_i]$  ranging from 0 to  $\infty$ , there is, by the Intermediate Value Theorem, a unique  $d > 0$  such that

$$\sum_{i=1}^n x_i(d) = X - d.$$

By (11), the pair  $(x(d), d)$  meets (3) and (4). ■

## 4 The Limit Result

We increase the size of the problem by replicating a one-cake -  $n$ -player problem  $k$  times. That is, in a  $k$ -replicated problem we allow each *replica* of  $n$  players to bring a cake of size 1 to the pool of shareable cakes, and the resulting set of  $k \cdot n$  players bargain over the resulting cake of size  $k$  according to the procedure specified in the previous section.

Formally, let  $N = \{1, \dots, n\}$  be a set of original agents, and relabel them by  $\{11, 12, \dots, 1n\}$ . Let the  $k$  times replicated - or *k-replicated* - set of agents be  $\{11, \dots, 1n, 21, \dots, 2n, \dots, k1, \dots, kn\}$ . That is, the  $k$ -replicated problem contains  $k$  agents of type  $i \in N$ , each with the preferences of  $i$ . Attaching the player  $li$  the index  $h(li) = n \cdot (l - 1) + i$ , we may order players  $11, \dots, kn$  according to their  $h$ -indices  $\{h(11), \dots, h(kn)\} = \{1, \dots, n \cdot k\}$ . Using this indexation of the players, we specify a game  $\Gamma^{\{1, \dots, n \cdot k\}}(k)$ , for any  $k = 1, 2, \dots$ . Then Propositions 1 and 2 are valid for any  $k$ -replicated problem.<sup>4</sup>

By Proposition 1, the equilibrium of the  $k$ -replicated problem is characterized by  $\underline{x}(k) \in S^{k \cdot n}(k)$  and  $d(k) > 0$  meeting (3) and (4). By symmetry, the following result is immediate:

**Lemma 1**  $\underline{x}_{li}(k) = \underline{x}_{(l+1)i}(k)$ , for all  $i \in N$ , for all  $l \in \{1, \dots, k\}$ , for all  $k = 1, 2, \dots$ .

Because of Lemma 1, it is sufficient to focus on  $\underline{x}_{1 \cdot}(k) = (\underline{x}_{11}(k), \dots, \underline{x}_{1n}(k))$ . We may rewrite (4), for all  $k \in \{1, 2, \dots\}$ ,

$$d(k) = k \left(1 - \sum_{i=1}^n \underline{x}_{1i}(k)\right) \geq 0. \quad (12)$$

Let  $\{\underline{x}(k)\}_{k=1}^{\infty}$  be a sequence of points meeting (3) and (4) for the respective  $k$ -replicated problems, for all  $k$ .

**Lemma 2** *Sequence  $\{\underline{x}_{1 \cdot}(k)\}_{k=1}^{\infty}$  is bounded.*

---

<sup>4</sup>Any indexation of the players would do.

**Proof.** If  $\{\underline{x}_{1\cdot}(k)\}_{k=1}^\infty$  is not bounded, there is a subsequence  $\{\underline{x}_{1\cdot}(k_t)\}_{t=1}^\infty$  and  $j$  such that  $\underline{x}_{1j}(k_t) \rightarrow \infty$ . But given  $\underline{x}_{1i} \geq 0$  for all  $i$ , this would violate the budget constraint (12). ■

**Lemma 3** *Let  $\{\underline{x}_{1\cdot}(k_t)\}_{t=1}^\infty$  be a convergent subsequence of  $\{\underline{x}_{1\cdot}(k)\}_{k=1}^\infty$  such that*

$$\underline{x}_{1j}(k_t) \rightarrow y_j, \text{ for all } j = 1, \dots, n. \quad (13)$$

*Then  $\sum_{i=1}^n y_i = 1$ .*

**Proof.** By (2), and the continuity of  $v_i$

$$\begin{aligned} y_i &= \lim_{k_t} \underline{x}_{1i}(k_t) = \lim_{k_t} v_i(\underline{x}_{1i}(k_t) + d(k_t)) \\ &= v_i\left(\lim_{k_t} \underline{x}_{1i}(k_t) + \lim_k d(k_t)\right) \\ &= v_i\left(y_i + \lim_{k_t} d(k_t)\right). \end{aligned}$$

Since  $v_i$  is an increasing function, there is  $d < \infty$  such that  $d(k_t) \rightarrow d$ . By (12),

$$\sum_{i=1}^n \underline{x}_{1i}(k_t) = 1 - \frac{d(k_t)}{k_t}.$$

Given  $d(k) \rightarrow d$ , we have  $\sum_{i=1}^n \underline{x}_{1i}(k) \rightarrow 1$ . ■

Now we give a characterization of the unique convergence point of  $\underline{x}(k)$  on the Pareto frontier. To do this, identify a property of the preferences of a single replica of players.

**Lemma 4** *There are unique  $y^* \in S^n(1)$  and  $d^* > 0$  such that the following holds:  $y_i^* = v_i(y_i^* + d^*)$  for all  $i = 1, \dots, n$  and  $\sum_{i=1}^n y_i^* = 1$ .*

**Proof.** Let  $x_i(\cdot)$  be defined as in (10). By Proposition 2, there is a unique  $d^*$  such that  $\sum_{i=1}^n x_i(1 + d^*) = 1$ . By (10),  $x_i(d^*) = v_i(x_i(d^*) + d^*)$ , for all  $i$ . Let  $x(d^*) = y^*$ . ■

Figure 1 depicts how the limit outcome of a single replica is formed in the  $n = 2$  case. For any  $d > 0$ , identify function  $v_1(1 + d - x_2) = x_1$  for all  $x_2 \in [0, 1]$ , and function  $v_2(1 + d - x_1) = x_2$  for all  $x_1 \in [0, 1]$ . The unique intersection  $(y_1, y_2)$  of the two functions satisfies

$$\begin{aligned} v_1(1 + d - y_2) &= y_1, \\ v_2(1 + d - y_1) &= y_2. \end{aligned}$$

Then  $d$  is chosen to be  $d^*$  such that the intersection of the functions,  $(y_1^*, y_2^*)$ , satisfies  $y_1^* + y_2^* = 1$ . Given such  $d^*$ ,

$$\begin{aligned} v_1(1 + d^* - y_2^*) &= v_1(y_1^* + d^*) = y_1^*, \\ v_2(1 + d^* - y_1^*) &= v_2(y_2^* + d^*) = y_2^*. \end{aligned}$$

Thus  $(y_1^*, y_2^*)$  and  $d^*$  satisfy the conditions in (3) and (4) of a two player game with  $X = 1 + d^*$ . The next proposition shows that  $(y_1^*, y_2^*)$  is the converge point of the sharing rule of all generations but the first, and  $(y_1^* + d^*, y_2^*)$  is the convergence point of the first generation.

[FIGURE 1 HERE]

More generally, the efficient  $n$ -vector  $y^*$  specifies how the gains of each generation are distributed among the members of the generation when the economy grows large. This is our main result.

**Proposition 3**  $\underline{x}_1(k)$  converges to  $y^*$  as specified in Lemma 4 when  $k$  tends to infinity.

**Proof.** Since, by Lemma 2, sequence  $\{\underline{x}_1(k)\}_{k=1}^\infty$  is bounded, it suffices to show that every convergent subsequence of it converges to  $y^*$ . Let subsequence  $\{\underline{x}_1(k_t)\}_{t=1}^\infty$  converge to  $y$ . By Lemma 3,  $\sum_{i=1}^n y_i = 1$ . By Propositions 1 and 2, and continuity of  $v_i$ , there is a unique  $d > 0$  such that  $y_i = v_i(y_i + d)$  for all  $i = 1, \dots, n$ . By Lemma 4,  $y = y^*$ . ■

By Lemma 1, all sequences  $\{\underline{x}_l(k)\}$ , for  $l \in \{1, 2, \dots\}$ , converge to  $y^* = (y_1^*, \dots, y_n^*)$ . The main point is that the converge point  $y^*$  is characterized by the data of the original generation of  $n$  players.

## 5 Market for the First-Proposer Right

To conclude, we give a "Walrasian" interpretation to the characterized limit outcome  $y^*$ . Being the first proposer in the bargaining game is valuable. Consider a market where an arbitrator sells the right to be the first proposer in a bargaining game to one of the  $n$  bargainers. The right is sold to the bargainer who makes the highest bid. If many bargainers make the same highest bid, then the winner is chosen according to some rule among those who make the highest bid. Once the winner, say  $i$ , has paid price  $p$  for the right,  $p$  is added to the pool of resources over which bargaining takes

places. That is, given the original size 1 of the cake, player  $i$  becomes the first proposer in the bargaining game  $\Gamma^n(1+p)$ .

We claim that  $d^*$  is the unique Walrasian price for the first-proposer right in these markets, and  $y^*$  is the resulting allocation of the original cake.

Let  $z_i(X)$  be what a receiver  $i$  gets in the game  $\Gamma^n(X)$ . By (3) and (4) and Proposition 2,  $z(X) = (z_1(X), \dots, z_n(X))$  is the unique solution to

$$z_i(X) = v_i(X - \sum_{j \neq i} z_j(X)), \text{ for all } i. \quad (14)$$

By the Implicit Function Theorem,  $z$  is a continuous function.

**Lemma 5**  $z_i(X)$  is strictly increasing in  $X$ , for all  $i$ .

**Proof.** Rewrite condition (14) as

$$v_i^{-1}(z_i(X)) - z_i(X) = X - \sum_{j=1}^n z_j(X).$$

By (2), and since  $z_i$  is a continuous function,  $z_i$  is strictly increasing if  $X - \sum_{j=1}^n z_j(X)$  is. Since this applies to all  $i$ ,  $\sum_{j=1}^n z_j(X)$  is strictly increasing if  $X - \sum_{j=1}^n z_j(X)$  is. But then, since  $\sum_{j=1}^n z_j(X)$  being weakly decreasing means that  $X - \sum_{j=1}^n z_j(X)$  is strictly increasing, it cannot be the case that  $\sum_{j=1}^n z_j(X)$  is not strictly increasing. Thus  $\sum_{j=1}^n z_j(X)$  is strictly increasing and hence  $z_i$  is strictly increasing. ■

**Proposition 4**  $d^*$  is the unique market price for the first-proposer right and  $y^*$  is the resulting allocation of the cake, for  $d^*$  and  $y^*$  as specified in Lemma 4.

**Proof.** Only if: Suppose that there is a single highest bid. Then buying the proposing right with price  $p$  must be at least profitable as the opportunity cost of lowering the bid by small  $\varepsilon > 0$ :

$$\left[1 + p - \sum_{j \neq i} z_j(1+p)\right] - p \geq \left[1 + p - \varepsilon - \sum_{j \neq i} z_j(1+p - \varepsilon)\right] - (p - \varepsilon).$$

That is

$$0 \geq \sum_{j \neq i} [z_j(1+p) - z_j(1+p - \varepsilon)].$$

But by Lemma 5 this cannot hold.

Thus at least two bidders bid the winning bid  $p$ . Then buying the proposing right under  $p$  must be at least profitable as the opportunity cost of letting the other highest bidder win with price  $p$ :

$$\left[1 + p - \sum_{j \neq i} z_j(1+p)\right] - p \geq z_i(1+p). \quad (15)$$

Since increasing ones bid is not profitable for the losing bargainer  $j$  that bids  $p$ ,

$$\left[1 + p + \varepsilon - \sum_{k \neq j} z_k(1 + p + \varepsilon)\right] - (p + \varepsilon) \leq z_j(1 + p), \text{ for all } \varepsilon > 0. \quad (16)$$

Since  $z_k$  is continuous and (16) holds for all  $\varepsilon > 0$ , it follows that

$$\left[1 + p - \sum_{k \neq j} z_k(1 + p)\right] - p \leq z_j(1 + p). \quad (17)$$

Combining (15) and (17) gives

$$1 = \sum_{i=1}^n z_i(1 + p).$$

Thus by (14),

$$z_i(1 + p) = v_i(z_i(1 + p) + p), \text{ for all } i = 1, \dots, n.$$

By Lemma 4, this yields  $z_i(1 + p) = y_i^*$  for all  $i$ , and  $p = d^*$ .

If: Let all  $n$  bargainers bid  $p = d^*$ . By construction,  $z_i(1 + d^*) = y_i^*$  for all  $i$ . We show this does constitute an equilibrium. Since  $n > 1$  and

$$1 = \sum_{i=1}^n z_i(1 + d^*), \quad (18)$$

it follows that

$$\left[1 + d^* - \sum_{j \neq i} z_j(1 + d^*)\right] - d^* = z_i(1 + d^*).$$

Thus *decreasing* one's bid does not have payoff consequences. *Increasing* one's bid by  $\varepsilon > 0$  is strictly profitable if

$$\left[1 + d^* + \varepsilon - \sum_{j \neq i} z_j(1 + d^* + \varepsilon)\right] - (d^* + \varepsilon) > z_i(1 + d^*).$$

That is, by (18),

$$1 - \sum_{j \neq i} z_j(1 + d^* + \varepsilon) > 1 - \sum_{j \neq i} z_j(1 + d^*),$$

which is in conflict with Lemma 5. Thus all players bidding  $d^*$  does constitute an equilibrium. ■

By Proposition 3, the unique outcome  $y^*$  of the market game for the first-proposing right can be thought as the expected outcome of bargaining when the number of bargainers grows large and the probability of a particular player having the right be the first proposer (bargaining power) becomes negligible. Having a large set of players is attractive since the resulting bargaining outcome reflects strong *average* fairness: all but one generation distribute their resources without a player with first mover advantage. Thus the simple market game with small number of players can be used to simulate the outcome that fairly represents what one should expect in a bargaining situation with many players.

## References

- [1] BINMORE, K., A. RUBINSTEIN AND A. WOLINSKY 1986, The Nash bargaining solution in economic modelling, *Rand Journal of Economics* 17, 176-188.
- [2] FISHBURN, P. AND RUBINSTEIN, A. 1982, Time preference, *International Economic Review* 23, 677-95.
- [3] KRISHNA V. AND R. SERRANO 1996, Multilateral bargaining, *Review of Economic Studies* 63, 61-80.
- [4] KULTTI, K. AND H. VARTAINEN 2007, Von Neumann-Morgenstern stable set bridges time-preferences to the Nash solution , *B.E. Journal of Theoretical Economics* 7:1 (Contributions), Article 41.

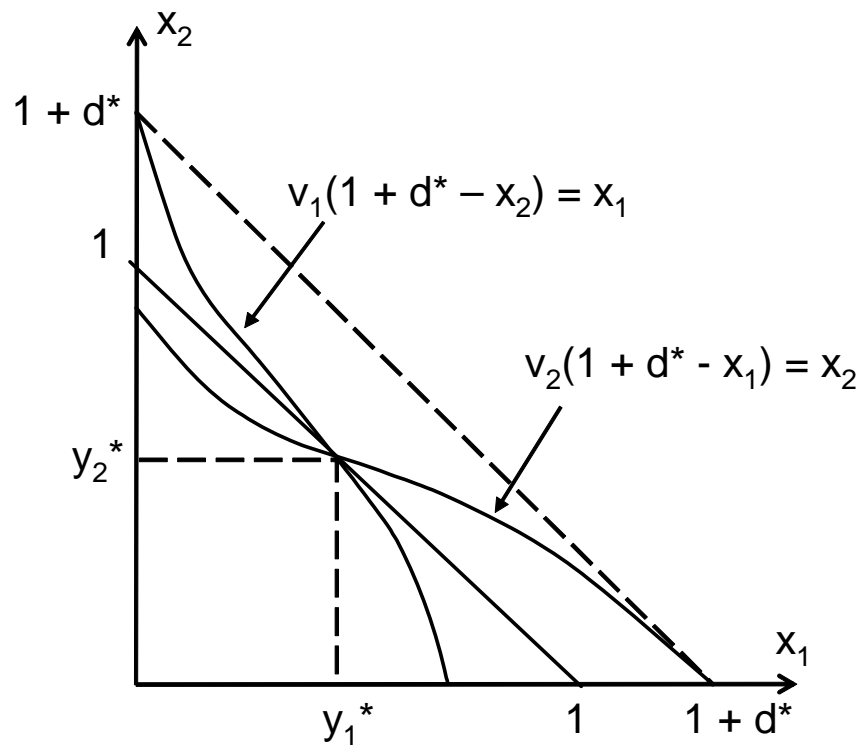


Figure 1

**Aboa Centre for Economics (ACE)** was founded in 1998 by the departments of economics at the Turku School of Economics, Åbo Akademi University and University of Turku. The aim of the Centre is to coordinate research and education related to economics in the three universities.

Contact information: Aboa Centre for Economics, Turku School of Economics, Rehtorinpellonkatu 3, 20500 Turku, Finland.

**Aboa Centre for Economics (ACE)** on Turun kolmen yliopiston vuonna 1998 perustama yhteistyöelin. Sen osapuolet ovat Turun kauppakorkeakoulun kansantaloustieteen oppiaine, Åbo Akademin nationalekonomi-oppiaine ja Turun yliopiston taloustieteen laitos. ACEn toiminta-ajatuksena on koordinoida kansantaloustieteen tutkimusta ja opetusta Turun kolmessa yliopistossa.

Yhteystiedot: Aboa Centre for Economics, Kansantaloustiede, Turun kauppakorkeakoulu, 20500 Turku.

[www.ace-economics.fi](http://www.ace-economics.fi)

ISSN 1796-3133