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ABSTRACT

Imposing a minimum quality standard (MQS) is conventionally regarded as harmful if firms compete in quantities. This, however, ignores dynamic effects. We show that an MQS can hinder collusion, resulting in dynamic welfare gains that reduce and may even outweigh the usual static losses. Verdicts on MQS thus depend even more on the market at hand than has been acknowledged.

JEL Classification: L41, L51, L15, D43

Keywords: minimal quality standard, Cournot competition, collusion

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1 Introduction

Deliberately high or low quality choices are a common strategy used by oligopolistic firms in order to relax competition and raise profits (Shaked and Sutton 1982). As was first demonstrated by Ronnen (1991), regulatory authorities may in such cases increase welfare by imposing a *minimum quality standard* (MQS). In particular, a suitably chosen MQS can simultaneously reduce hedonic prices and lift average quality.

The circumstances under which an MQS actually increases rather than reduces total surplus have been investigated by a number of authors. Broadly speaking, a moderate MQS is predicted to be socially beneficial if firms compete in prices; it is detrimental if firms compete in quantities. This dichotomy is true, however, only in static environments. The danger of collusion on price or quantity possibly changes the picture because an MQS affects the critical degree of patience which allows anti-competitive behavior to be sustained as an equilibrium. It is known, for example, that an MQS can facilitate collusion in a Bertrand setting (Häckner 1994). This questions an MQS's generally positive effect under price competition.

In this paper, we show that the generally negative effect of an MQS under quantity competition (Valletti 2000) is similarly sensitive to the precise market structure at hand. Namely, imposition of a suitable MQS can destabilize collusion in a Cournot setting. The usual static costs of an MQS hence need to be traded off against dynamic benefits. We show that the latter may outweigh the former, i.e., an MQS can actually raise total surplus under Cournot competition. Moreover, this anti-collusive effect of an MQS is fairly robust: in contrast to the case of Bertrand competition, it does not depend on whether quality primarily affects fixed or variable costs.

We will first briefly survey previous investigations of the welfare effect of an MQS. Section 3 then reviews the baseline model of vertical differentiation with an MQS and Section 4 quantifies the static welfare properties of an MQS. Its role in preventing collusion when quality affects fixed costs is investigated in Section 5; the case when quality affects variable costs and alternative collusion scenarios are dealt with in Section 6. Section 7 concludes.

2 Related Literature

Ronnen (1991) was the first to demonstrate how an MQS can (a) raise the qualities provided and (b) reduce the gap between both firms' qualities in a Bertrand duopoly with endogenous qualities. Effect (a) counters the tendency of quality under-provision without regulation (firms cater to their respective marginal customer, not the average one; Spence 1975); and (b) curbs excess differentiation intended to alleviate competition. Both increase total surplus.

The reduction in the equilibrium level of differentiation in Ronnen's model is driven by fixed costs of production which are assumed to be convex and increasing in quality. Price changes due to the introduction of an MQS are thus not caused by changes in marginal costs but simply better substitutability of products. In particular, the ratio of price and quality – the so-called *hedonic price* – falls for both products.

As Crampes and Hollander (1995) have highlighted, hedonic prices need not fall in general, e.g., if quality also affects variable costs. It turns out to be crucial for a positive welfare effect of the MQS that it decreases the quality gap between products. If unit costs fail to rise sufficiently more for the high-quality producer than for the low-quality producer as each one increases its respective unregulated quality, then an MQS may actually enlarge the quality gap and reduce total surplus. In this case, gains to the low-quality producer (for whom an MQS creates valuable commitment under Bertrand competition) are outweighed by losses to the high-quality producer and to consumers with relatively low quality preference (namely, greater cost plus greater differentiation raise prices by more than the respective willingness to pay for extra quality).[1] However, for a great variety of cost functions, aggregate consumer surplus and with it total surplus tends to increase when a moderate MQS is introduced to a Bertrand oligopoly.

The situation is different under quantity competition. As Valletti (2000) has shown, a binding MQS reduces profits for both producers and moreover decreases market coverage. The extra surplus to consumers with high willingness to pay for quality dominates the loss to those who drop out of the market or keep consuming the low-quality good at a higher hedonic price, i.e., aggregate consumer surplus increases. The net welfare effect of the MQS is still negative. The reason is that when firms compete *à la Cournot*, their need to alleviate competition is relatively weak, and hence the unregulated quality gap is not particularly excessive. The intensification of competition identified by Crampes and Hollander (1995) as the key factor behind welfare gains from an MQS is also rather subdued; its effect is dominated by the reduction of profits and of the surplus generated with consumers of low or moderate willingness to pay for quality.

Häckner (1994) pointed to another detrimental effect associated with an MQS: it can increase the stability of collusion. In the market structure considered by Häckner, notably with exogenous qualities affecting only the fixed costs of production, it is easier to sustain collusion the more similar are firms' products. Intuitively, higher competitive profits which accrue to the high-quality firm for a greater level of differentiation make potential gains from collusion less attractive and give it a greater incentive to deviate. What is beneficial from a static perspective can thus be harmful in a dynamic context.

However, details matter – in particular the cost structure. In contrast to Häckner's study, Ecchia and Lambertini (1997) assume that variable costs rise with quality. The

[1] Unlike Ronnen (1991), Crampes and Hollander assume that the market is always fully covered, i.e., consumers either buy the high or the low-quality good. If some do not buy at all, then a higher hedonic price of the low-quality good also diminishes total market coverage and thereby surplus.

	Fixed quality costs		Variable quality costs	
	static	**dynamic**	**static**	**dynamic**
Bertrand case	MQS raises welfare ... (Ronnen 1991)	... but facilitates collusion (Häckner 1994)	MQS typically raises welfare (if it reduces the quality gap) ... (Crampes and Hollander 1995)	... and hinders collusion, too (Ecchia and Lambertini 1997)
	static	**dynamic**	**static**	**dynamic**
Cournot case	MQS reduces welfare ... (Valletti 2000)	... but hinders collusion and can thus raise welfare	MQS reduces welfare ...	... but hinders collusion and can thus raise welfare

Table 1: Effects of a MQS on welfare for different market structures

profit advantage to the high-quality producer is then no longer very pronounced;[2] it is also less sensitive to an MQS. But the MQS makes products closer substitutes and thus creates bigger scope to raise profits by a unilateral deviation. In summary, Ecchia and Lambertini find that an MQS decreases rather than increases the stability of collusion, i.e., it can be beneficial both from a static and a dynamic perspective.[3]

Our own analysis concentrates on the case of quantity competition and completes the three cells at the bottom right of Table 1. While there are static losses, as already identified by Valletti (2000), independently of whether quality affects fixed or variable costs, collusion becomes more difficult to sustain with an MQS: available total collusion profits are reduced by the cost increases induced by the MQS. This – aided by a weakened bargaining position of the critical high-quality firm – makes it relatively more attractive to go it alone.

3 Model

We consider a standard vertically differentiated duopoly.[4] Firm $i \in \{1, 2\}$ produces an indivisible good of quality s_i. Without loss of generality we assume $s_1 \geq s_2 > 0$. A unit mass of consumers obtain utility

$$U(p_i, s_i) = \theta \cdot s_i - p_i \tag{1}$$

[2]See Lehmann-Grube (1997) on the robustness of this high-quality advantage.

[3]Also see Boom (1995) and Bonroy (2003) on the impact of an MQS on international trade. – Recent empirical evaluations of an MQS include Chitpy and Witte (1997) and Hotz and Xiao (2005), both analyzing data on the market for child care.

[4]See Tirole (1988, Section 7.5), Choi and Shin (1992), Motta (1993) and Wauthy (1996).

from buying exactly one unit of quality s_i at price p_i and zero otherwise; θ characterizes the considered consumer's type. It is assumed to be uniformly distributed on $[0, a]$ ($a > 0$). A consumer with type $\theta = \frac{p_1 - p_2}{s_1 - s_2}$ is indifferent between both products; one with $\theta = \frac{p_2}{s_2}$ is indifferent between the low-quality product 2 and no purchase at all. This implies the inverse demand functions

$$\begin{aligned} p_1(x_1, x_2, s_1, s_2) &= s_1\,(a - x_1) - s_2\,x_2, \\ p_2(x_1, x_2, s_1, s_2) &= s_2\,(a - x_1 - x_2) \end{aligned} \tag{2}$$

with $x_i \geq 0$ denoting the respective quantity choice.

Firms have access to the same technology. Their production is initially assumed to involve only fixed costs, which increase in quality and are denoted by $C(s_i)$. In line with most of the literature, we consider the simple quadratic form

$$C(s_i) = \gamma s_i^2 \qquad (\gamma > 0). \tag{3}$$

The timing of interaction is as follows: First, both firms simultaneously choose their respective quality, which then becomes common knowledge. Second, the firms simultaneously decide on their quantities. Finally, the market is cleared at the prices indicated by (2).[5]

Firms' equilibrium quantity choices for given qualities $s_1 \geq s_2$ are

$$\hat{x}_1(s_1, s_2) = \frac{a\,(2\,s_1 - s_2)}{4\,s_1 - s_2} \quad \text{and} \quad \hat{x}_2(s_1, s_2) = \frac{a\,s_1}{4\,s_1 - s_2}. \tag{4}$$

They define the *reduced profit functions*

$$\pi_1(s_1, s_2) = \frac{a^2\,s_1\,(2\,s_1 - s_2)^2}{(4\,s_1 - s_2)^2} - \gamma s_1^2, \tag{5}$$

$$\pi_2(s_1, s_2) = \frac{a^2\,{s_1}^2\,s_2}{(4\,s_1 - s_2)^2} - \gamma s_2^2. \tag{6}$$

The first-order conditions characterizing optimal qualities are then

$$\frac{\partial \pi_1(s_1, s_2)}{\partial s_1} = \frac{a^2\,(16\,{s_1}^3 - 12\,{s_1}^2\,s_2 + 4\,s_1\,{s_2}^2 - {s_2}^3)}{(4\,s_1 - s_2)^3} - 2\gamma s_1 = 0, \tag{7}$$

$$\frac{\partial \pi_2(s_1, s_2)}{\partial s_2} = \frac{a^2\,{s_1}^2\,(4\,s_1 + s_2)}{(4\,s_1 - s_2)^3} - 2\gamma s_2 = 0. \tag{8}$$

These conditions define firms' best response functions $R_i(s_j)$ ($i \neq j \in \{1, 2\}$); closed-form solutions exist but are very unwieldy. The resulting *unregulated equilibrium qualities* can

[5]In the spirit of Kreps and Scheinkmann (1983), one may think of capacity choices and subsequent price competition. In the context of indefinitely repeated interaction, which we will study below, this interpretation requires, however, that firms can regularly revise their capacities (e.g., with each agricultural season or with each generation of computer chips).

be computed as[6]

$$\hat{s}_1 \approx \frac{0.12597\, a^2}{\gamma} \quad \text{and} \quad \hat{s}_2 \approx \frac{0.04511\, a^2}{\gamma}. \tag{9}$$

Now suppose that $\tilde{s}$ is exogenously imposed as an MQS, i.e., firms face the constraint $s_i \geq \tilde{s}$.[7] We will throughout our analysis focus on the case in which the MQS is *not excessive* but *binding*: both firms stay in the market,[8] but firm 2 needs to increase its quality in order to comply with regulation, i.e., $\tilde{s} > \hat{s}_2$. The resulting *regulated equilibrium qualities* will be denoted by $s_1^*(\tilde{s})$ and $s_2^*(\tilde{s})$.

Lemma 1 implies that the equilibrium quality gap between both firms decreases in $\tilde{s}$ if firm 2 adopts the mandated quality, i.e., supposing that $s_2^*(\tilde{s}) = \tilde{s}$:

Lemma 1 *Firm 1 responds to any given increase* Δs_2 *of firm 2's quality by an increase* $\Delta s_1 < \Delta s_2$ *of its own quality. In particular,*

$$0 < \frac{\partial R_1(s_2)}{\partial s_2} < 1.$$

Proof: Substituting $s_2 \equiv t \cdot s_1$ with $t \in (0, 1]$ in (7), the first-order condition for firm 1's quality choice can equivalently be written as

$$s_1 = \frac{a^2(t^3 - 4t^2 + 12t - 16)}{2\gamma(t-4)^3}. \tag{10}$$

Moreover, application of the implicit function theorem to equation (7) and afterwards the substitution $s_2 = t \cdot s_1$ yield

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{4a^2(t-1)t}{4a^2(t-1)t^2 - (t-4)^4 \gamma s_1}. \tag{11}$$

Using the rearranged first-order condition (10) for s_1, this simplifies to

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{8(1-t)t}{t^4 - 16t^3 + 36t^2 - 64t + 64}. \tag{12}$$

Now, recalling the fact that $t \in (0, 1]$, numerical inspection allows to infer that

$$\frac{\partial R_1(s_2)}{\partial s_2} \in (0, 0.05465]. \tag{13}$$

□

Lemma 2 establishes that indeed $s_2^*(\tilde{s}) = \tilde{s}$:

[6]There exists a quality $s_2 \in (0, s_1)$ which is profitable for firm 2 for any given quality s_1, i.e., there is no monopoly in equilibrium. Second-order conditions are satisfied and neither firm has an incentive to "leapfrog". This remains true after an MQS is imposed (see Appendix A). See Motta (1993) for a detailed comparison of $(\hat{s}_1, \hat{s}_2)$ under price vs. quantity competition and fixed vs. variable quality costs.

[7]See Argenton (2006) and Lutz et al. (2000) for analysis of an endogenous MQS. Argenton analyzes bilateral bargaining over an MQS by the duopolists. Lutz et al. allow one of them to influence the MQS by a prior quality commitment.

[8]Firm 2's profit is the smaller one, decreases in $\tilde{s}$, and is zero at $\tilde{s}^c \approx \frac{0.09334\, a^2}{\gamma}$. So "not excessive" means $\tilde{s} \leq \tilde{s}^c$.

Lemma 2 *Given an MQS* $\tilde{s} > \hat{s}_2$ *such that both firms stay in the market, firm 2 selects exactly the mandated quality in equilibrium, i.e.,*

$$s_2^*(\tilde{s}) = \tilde{s}.$$

Proof: Again using the notation $s_2 \equiv t \cdot s_1$ with $t \in (0,1]$, the change of firm 2's profit caused by a marginal increase of s_2 can be written as

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2(4+t) + 2t(t-4)^3 \gamma s_1}{(4-t)^3} \tag{14}$$

Considering a best response by firm 1, i.e., imposing the rearranged first-order condition (10), this becomes

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2(t^4 - 4t^3 + 12t^2 - 15t + 4)}{(4-t)^3}, \tag{15}$$

which is positive (negative) to the left (right) of $t = \hat{s}_2/\hat{s}_1$.

Imposition of $\tilde{s}$ means that s_2 rises by $\Delta s \geq \tilde{s} - \hat{s}_2$. By Lemma 1, s_1 rises by less than Δs. A post-MQS equilibrium quality ratio must hence satisfy $t > \hat{s}_2/\hat{s}_1$. Thus (15) is negative and firm 2 must select the minimum feasible quality $s_2^*(\tilde{s}) = \tilde{s}$ in equilibrium. □

Lemmata 1 and 2 jointly imply that the *regulated equilibrium quality ratio*

$$\alpha(\tilde{s}) \equiv \frac{s_2^*(\tilde{s})}{s_1^*(\tilde{s})} = \frac{\tilde{s}}{R_1(\tilde{s})} \in (\hat{s}_2/\hat{s}_1, 1] \tag{16}$$

is a strictly increasing function of $\tilde{s}$. With slight abuse of notation, one can hence directly consider α, as shorthand for $\alpha(\tilde{s})$, as being the relevant policy variable.

4 Static Welfare Analysis

The static effects of an MQS on profits, consumer surplus and total surplus in case of fixed quality costs and quantity competition have first been analyzed by Valletti (2000). For the sake of completeness, we here include derivations of his two main findings.[9] In contrast to the price competition case, *both* producers are made worse off by the MQS:

Proposition 1 *Both firms' profits decrease in the level of the MQS, i.e.,*

$$\frac{d\pi_i(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0 \quad for \quad i \in \{1, 2\}. \tag{17}$$

[9]Valletti's results apply to more general fixed quality cost functions $C(\cdot)$ with $C'(\cdot), C''(\cdot) > 0$. We investigate the slightly more tedious case of variable quality costs – which is not covered by Valletti – in Section 6.3.

Proof: The marginal profit changes caused by introduction of an MQS are given by

$$\frac{d\pi_1(R_1(\tilde{s}),\tilde{s}))}{d\tilde{s}} = \underbrace{\frac{\partial \pi_1}{\partial s_1}}_{=0}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_1}{\partial s_2} = \frac{\partial \pi_1}{\partial s_2}, \tag{18}$$

$$\frac{d\pi_2(R_1(\tilde{s}),\tilde{s}))}{d\tilde{s}} = \frac{\partial \pi_2}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_2}{\partial s_2}. \tag{19}$$

One can compute

$$\frac{\partial \pi_1}{\partial s_2} = \frac{4\,a^2\,{s_1}^2\,(s_2 - 2\,s_1)}{(4\,s_1 - s_2)^3} < 0. \tag{20}$$

Moreover, we know $\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} > 0$ from Lemma 1 and $\frac{\partial \pi_2}{\partial s_2} < 0$ from the proof of Lemma 2. It thus remains to confirm that

$$\frac{\partial \pi_2}{\partial s_1} = -\frac{2a^2 s_1 s_2^2}{(4\,s_1 - s_2)^3} < 0. \tag{21}$$

□

Now consider the consumer surplus generated by qualities s_1 and s_2,

$$S(s_1,s_2) = \int_{\frac{p_2}{s_2}}^{\frac{p_1-p_2}{s_1-s_2}} (\theta s_2 - p_2)\,d\theta + \int_{\frac{p_1-p_2}{s_1-s_2}}^{a} (\theta s_1 - p_1)\,d\theta \tag{22}$$

$$= \frac{a^2\,s_1\,(4\,{s_1}^2 + s_1\,s_2 - {s_2}^2)}{2\,(4s_1 - s_2)^2}. \tag{23}$$

The change caused by an MQS is given by

$$\frac{dS(s_1^*(\tilde{s}),s_2^*(\tilde{s}))}{d\tilde{s}} = \frac{\partial S}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_2}, \tag{24}$$

where we know that $\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} > 0$ and, using (23), one can check that $\frac{\partial S}{\partial s_i} > 0$ for $i \in \{1,2\}$. So consumer surplus rises in $\tilde{s}$.

Its increase is, however, dominated by the decrease of profits:

Proposition 2 *Total surplus decreases in the level of the MQS, i.e.,*

$$\frac{d\big(\pi_1(\cdot) + \pi_2(\cdot) + S(\cdot)\big)}{d\tilde{s}} < 0. \tag{25}$$

Proof: The change in total surplus due to an MQS is equal to

$$\frac{\partial \pi_1}{\partial s_2} + \underbrace{\frac{\partial \pi_1}{\partial s_1}}_{=0}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial \pi_2}{\partial s_2} + \frac{\partial \pi_2}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_1}\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}} + \frac{\partial S}{\partial s_2} \tag{26}$$

$$= \frac{\partial \pi_1}{\partial s_2} + \underbrace{\frac{\partial \pi_2}{\partial s_2}}_{<0} + \underbrace{\frac{\partial R_1(\tilde{s})}{\partial \tilde{s}}}_{\in(0,0.05465]}\underbrace{\frac{a^2\,(4\,{s_1}^2 - 2\,s_1\,s_2 - {s_2}^2)}{2\,(4\,s_1 - s_2)^2}}_{>0} + \frac{\partial S}{\partial s_2} \tag{27}$$

$$< \frac{\partial \pi_1}{\partial s_2} + \frac{a^2\,(4\,{s_1}^2 - 2\,s_1\,s_2 - {s_2}^2)}{20\,(4\,s_1 - s_2)^2} + \frac{\partial S}{\partial s_2} \tag{28}$$

$$= -\frac{a^2\,(6\,{s_1}^2 + 2\,s_1\,s_2 + {s_2}^2)}{20\,(4\,s_1 - s_2)^2} < 0, \tag{29}$$

where $\frac{\partial \pi_2}{\partial s_2} < 0$ because constraint $s_2 \geq \tilde{s}$ binds.

□

The key difference to Ronnen's (1991) Bertrand setting is that in the Cournot case – in view of relatively low competitive pressure even for undifferentiated goods – the MQS does not decrease hedonic prices. Hence, consumers with relatively low marginal willingness to pay for quality leave the market or switch from the high to the low quality, weighing down the overall increase in consumer surplus. A second distinction is that the MQS tends to give the low-quality firm valuable commitment power under price competition, i.e., it benefits from the constraint $s_2 \geq \tilde{s}$ in equilibrium. Here, both firms suffer (Proposition 1). These differences jointly reverse Ronnen's finding of a welfare increase.

5 Dynamic Welfare Analysis

By changing firms' static profits, an MQS also affects their incentives to collude. We will investigate a market environment in which quantities may be repeatedly set in periods $t = 0, 1, 2, \ldots$ Quality choices are made in period $t = -1$ and then become irreversible. We assume that the associated fixed costs of production are incurred in every period (e.g., maintenance of physical or human capital, advertising, licenses). Firms care about their discounted stream of profits

$$\sum_{t=0}^{\infty} \delta^t \pi_{i;t} \tag{30}$$

where $\pi_{i;t}$ denotes firm i's profit in period t. For simplicity we assume that both firms apply the same discount factor $\delta \in (0, 1)$, which may capture pure impatience (determined, e.g., by an interest rate) as well as the likelihood that there is in fact another round of quantity competition between the considered two firms.

It will initially be assumed that firms can transfer profits from one to the other if they decide to collude – e.g., by trading a costless intermediate good at an inflated price. Situations without the possibility of side payments will be dealt with in Section 6.2. In either case, we take collusion to only affect firms' short-term quantity decisions – not the initial choice of quality.[10]

The standard measure of *instability of collusion* for indefinitely repeated interaction is the maximal discount factor such that, for both firms, the short-run gains from a deviation outweigh anticipated long-run losses from consequent punishment. We refer to it as the *critical discount factor*, denoted by r. In line with Häckner (1994) and Ecchia and Lambertini (1997), punishment is taken to be a reversion to the static Cournot-Nash equilibrium (which corresponds to a subgame-perfect equilibrium involving simple trigger strategies),

[10] Collusion on quality would entail a significant hold-up problem for fixed quality costs: total profit is maximized by setting $s_2 = 0$ in $t = -1$, but then firm 2 would be deprived of all punishment opportunity.

even though more severe punishments exist.[11]

Comparison of the anticipation of a *collusion profit* π_i^c in every period $t = 0, 1, \ldots$ and of once receiving the *deviation profit* π_i^d and thereafter the *punishment payoff* π_i^p shows that firm i has an incentive to collude with firm j if and only if

$$\delta > r_i \equiv \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p}. \tag{31}$$

So the critical discount factor is $r = \max\{r_1, r_2\}$.

We will next derive π_i^p, π_i^c, and π_i^d for given quality choices s_1 and s_2, and later replace s_i by the respective regulated equilibrium level $s_i^*(\tilde{s})$ in order to analyze the effect of increases of $\tilde{s}$. For Nash reversion equilibria, the punishment payoff $\pi_i^p(s_1, s_2)$ is simply the reduced profit displayed in equation (5) and (6), respectively. Collusion profits $\pi_i^c(s_1, s_2)$ are assumed to result from, first, firms choosing quantities such that their aggregate per period profit, π_Σ, is maximal and, second, bargaining over the division of π_Σ. Regarding the latter, firms are supposed to "split the difference" between total competitive and total collusive profits equally – in line with Nash's (1950) and, in fact, any symmetric and efficient cooperative bargaining solution (e.g., the proportional solution or the one proposed by Kalai and Smorodinsky 1975).[12] We will consider alternatives in Sections 6.1 and 6.2.

Aggregate profit equals

$$p_1(x_1, x_2, s_1, s_2) \cdot x_1 + p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_1) - C(s_2) \tag{32}$$

and is maximized by $x_1^c = \frac{a}{2}$ and $x_2^c = 0$: the firms will eliminate any competition and produce only the quality s_1, which has a higher margin. We assume that firm 2 incurs fixed costs $C(s_2)$ even if its current output is zero; namely, the firm needs to retain its production capability as a deterrent. The maximal aggregate profit for given qualities is thus

$$\pi_\Sigma(s_1, s_2) \equiv \frac{a^2\, s_1}{4} - \gamma\, {s_1}^2 - \gamma\, {s_2}^2. \tag{33}$$

Splitting the difference

$$\pi_\Delta(s_1, s_2) \equiv \pi_\Sigma(s_1, s_2) - \pi_1^p(s_1, s_2) - \pi_2^p(s_1, s_2) = \frac{a^2\, s_1\, s_2\, (4\, s_1 - 3\, s_2)}{4\,(4 s_1 - s_2)^2} > 0 \tag{34}$$

between both firms equally then implies the collusion profits

$$\pi_1^c(s_1, s_2) = \pi_1^p(s_1, s_2) + \frac{1}{2}\pi_\Delta(s_1, s_2) = \frac{a^2\, s_1\, (8\, s_1 - 5\, s_2)}{8(4\, s_1 - s_2)} - \gamma s_1^2 \tag{35}$$

[11] See Abreu (1986). One might try to replace the discount factor which ensures existence of collusive Cournot-Nash reversion equilibria by one ensuring existence of (symmetric) optimal punishment equilibria with specified minimal collusion payoffs. Another option would be to consider any particular discount factor δ and investigate how maximal collusive payoffs are affected by an MQS. Neither path has, to our knowledge, been pursued in the literature so far.

[12] As demonstrated by Binmore (1987), the Nash solution also approximates non-cooperative alternating-offers bargaining between patient players in single-shot interaction (see Rubinstein 1982). Repeated interaction would support alternative divisions but our results continue to hold for many other division rules (e.g., proportional to competitive Cournot-Nash profits).

and

$$\pi_2^c(s_1,s_2)=\pi_2^p(s_1,s_2)+\frac{1}{2}\pi_\Delta(s_1,s_2)=\frac{3\,a^2\,s_1\,s_2}{8(4\,s_1-s_2)}-\gamma s_2^2. \tag{36}$$

A deviation by firm i involves a best response to firm j's collusive output x_j^c and a refusal to share any part of its profits. While firm 1 is bound by its quantity choice for the current period, we assume that it can immediately react to 2's deviation by refusing to share profits. This implies that firm 2's incentive to collude is actually independent of discount factor δ:

Lemma 3 *Firm 2 always prefers collusion to a deviation. In particular,*

$$\pi_2^c(s_1,s_2)>\pi_2^d(s_1,s_2). \tag{37}$$

Proof: Given $x_1^c=\frac{a}{2}$, firm 2's profit equals

$$p_2(x_1^c,x_2,s_1,s_2)\cdot x_2-C(s_2)=s_2\left(\frac{a}{2}-x_2\right)x_2-\gamma s_2^2 \tag{38}$$

and is maximized by $x_2^d=\frac{a}{4}$. The maximal deviation profit is hence

$$\pi_2^d(s_1,s_2)\quad=\quad\frac{s_2\,(a^2-16\,\gamma\,s_2)}{16} \tag{39}$$

where, however,

$$\pi_2^d(s_1,s_2)-\pi_2^p(s_1,s_2)\quad=\quad-\frac{a^2 s_2^2\,(8s_1-s_2)}{16(4s_1-s_2)^2}<0. \tag{40}$$

So, in particular, $\pi_2^d(s_1,s_2)<\pi_2^p(s_1,s_2)+\frac{1}{2}\pi_\Delta(s_1,s_2)=\pi_2^c(s_1,s_2)$.

□

So only firm 1's incentive to collude or, respectively, to deviate needs to be considered (and only δ_1 would matter if firm-specific discount factors δ_i were applied). In view of $x_2^c=0$, the jointly profit-maximizing quantity $x_1^c=\frac{a}{2}$ in fact maximizes firm 1's profit. A deviation by firm 1 thus boils down to refusing to transfer the designated share of profits to firm 2 after the latter chose not to produce. Hence firm 1's deviation profit is

$$\pi_1^d(s_1,s_2)=\frac{a^2\,s_1}{4}-\gamma\,{s_1}^2. \tag{41}$$

We are now ready to prove our main result:

Proposition 3 *The critical discount factor increases in the level of the MQS, i.e.,*

$$\frac{dr(s_1^*(\tilde{s}),s_2^*(\tilde{s}))}{d\tilde{s}}>0. \tag{42}$$

$\frac{dr}{d\pi_1^p} = \frac{\pi_1^d+\pi_2^p-\pi_\Sigma}{2(\pi_1^p+\pi_1^d)^2} > 0$	$\frac{\partial \pi_1^p(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_2^p} = \frac{1}{2(\pi_1^d-\pi_1^p)} > 0$	$\frac{\partial \pi_2^p(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_1^d} = -\frac{\pi_1^p+\pi_2^p-\pi_\Sigma}{2(\pi_1^p+\pi_1^d)^2} > 0$	$\frac{\partial \pi_1^d(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \downarrow$
$\frac{dr}{d\pi_\Sigma} = \frac{1}{2(\pi_1^p-\pi_1^d)} < 0$	$\frac{\partial \pi_\Sigma(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} < 0$	$r \uparrow$

Table 2: Partial effects of an increase of MQS $\tilde{s}$ on the critical discount factor r

Proof: Using Lemma 3 and inserting the respective expressions for π_1^p, π_1^c, and π_1^d (cf. equations (5), (35), and (41)) into (31) yields

$$r(s_1, s_2) = r_1(s_1, s_2) = \frac{12\, s_1 - 3\, s_2}{16\, s_1 - 6\, s_2}. \tag{43}$$

Replacing s_i by the regulated equilibrium quality $s_i^*(\tilde{s})$ and then using the substitution $\alpha(\tilde{s}) = s_2^*(\tilde{s})/s_1^*(\tilde{s})$, we can write the critical discount factor as a function of this regulated equilibrium quality ratio:

$$r(s_1^*(\tilde{s}), s_2^*(\tilde{s})) = \frac{3}{2} \cdot \frac{\alpha(\tilde{s}) - 4}{3\,\alpha(\tilde{s}) - 8} \equiv \rho(\alpha(\tilde{s})). \tag{44}$$

Using that $\alpha(\tilde{s})$ is strictly increasing in $\tilde{s}$ (see Lemmata 1 and 2), one obtains

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} = \frac{d\rho(\alpha(\tilde{s}))}{d\tilde{s}} = \frac{6}{(3\alpha - 8)^2} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} > 0. \tag{45}$$

□

An MQS can hence be an effective policy to prevent collusion between vertically differentiated Cournot duopolists or to destabilize it. The main reason for the rise of

$$r = \frac{\pi_1^d - \pi_1^c}{\pi_1^d - \pi_1^p} = \frac{\pi_1^d - (\pi_1^p + \frac{1}{2}(\pi_\Sigma - \pi_1^p - \pi_2^p))}{\pi_1^d - \pi_1^p} \tag{46}$$

is the negative impact on firm 1's collusion profit π_1^c caused by a drop of π_Σ. Table 2 decomposes and ranks (by arrow size) the involved partial effects. As we have seen, the MQS induces both firms to produce higher qualities. Therefore *both* incur increased fixed costs, which are partially offset by positive sales of only *one* quality in case of collusion. This reduces the aggregate collusion profit π_Σ which is available for distribution significantly. As a consequence, π_1^c drops and – despite the partially compensating reductions also of π_1^d and π_1^p – the critical discount factor rises on balance.[13]

[13]Interestingly, firm 1's relative share of the diminished total profit π_Σ drops under Nash bargaining. The reason is that its competitive profit (and thus its fallback position) falls by more than that of firm 2. This indirect effect of the MQS on the relative attractiveness of collusion is not crucial for r's upward slope, but explains a greater steepness under Nash bargaining than for situations in which, say, firm 1 can keep the entire difference π_Δ (see Section 6.1).

Note that prevention of collusion by an MQS involves a trade-off: while competitive quantity decisions create surplus relative to collusive ones, firms' quality choices under an MQS destroy surplus (cf. Proposition 2). There exist an interval of discount factors for which the net effect on surplus is positive:

Proposition 4 *Assume that firms collude whenever this is strictly more profitable than a deviation and subsequent reversion to the Cournot-Nash equilibrium (i.e., for $\delta > r$). Then a welfare-enhancing MQS exists if and only if $\delta \in (\underline{\delta}; \bar{\delta})$ for $\underline{\delta} \approx 0.78878$ and $\bar{\delta} \approx 0.82537$.*

Proof: Collusive behavior in the *unregulated* case generates a total surplus of

$$\hat{W}^{col} = \pi_\Sigma(\hat{s}_1, \hat{s}_2) + \int_{\frac{a}{2}}^{a} \left(\theta \hat{s}_1 - \frac{a}{2}\hat{s}_1\right) d\theta \tag{47}$$

$$= \frac{3a^2}{8}\,\hat{s}_1 - \gamma\,\hat{s}_1^2 - \gamma\,\hat{s}_2^2 \tag{48}$$

per period, which exceeds total surplus under collusion for any regulated equilibrium quality ratio $\alpha > \hat{\alpha}$.

Competitive behavior in the presence of an MQS entails smaller profits but greater consumer welfare. The corresponding total per period surplus amounts to

$$W^{com} = \frac{a^2\, s_1\, (12 s_1^2 - 5 s_1 s_2 + s_2^2)}{2\,(4 s_1 - s_2)^2} - \gamma s_1^2 - \gamma s_2^2. \tag{49}$$

It is illustrated as a function of the regulated equilibrium quality ratio $\alpha = \frac{s_1^*(\tilde{s})}{s_2^*(\tilde{s})}$ in Figure 1 together with the analogous surplus $W^{col}(\alpha)$ for collusive behavior (with $W^{col}(\hat{\alpha}) \equiv \hat{W}^{col}$).

Any regulated quality ratio exceeding $\bar{\alpha}$ defined by $W^{com}(\bar{\alpha}) = \hat{W}^{col}$ lowers welfare independently of the unregulated market conduct. In contrast, a regulated quality ratio $\alpha \in (\hat{\alpha}, \bar{\alpha})$ implies greater surplus *if* it replaces collusive by competitive behavior. By assumption the latter requires the actual discount factor δ to be no greater than the critical one. So letting $\rho(\alpha) = r\big(s_1^*(\tilde{s}), s_2^*(\tilde{s})\big)$ denote the critical discount factor for a regulated quality ratio α (cf. equation (44)), it follows that a welfare-enhancing MQS exists whenever

$$0.78878 \approx \rho(\hat{\alpha}) \equiv \underline{\delta} < \delta < \bar{\delta} \equiv \rho(\bar{\alpha}) \approx 0.82537. \tag{50}$$

□

The range of discount factors δ such that a suitable MQS raises total surplus is small. Even though we only consider a simple model of a market without claim to numerical relevance, this suggests caution if prevention of collusion should be the only motivation for an MQS in practical applications. Still, a large set of, e.g., interest rate and continuation probability combinations will lead to $\delta \in (\underline{\delta}, \bar{\delta})$. In such cases, the realized welfare gain depends on the selected MQS. For example, an MQS slightly below the level $\bar{s} \approx \frac{0.07888 a^2}{\gamma}$ which corresponds to an equilibrium quality ratio $\bar{\alpha}$ will rather robustly prevent collusion

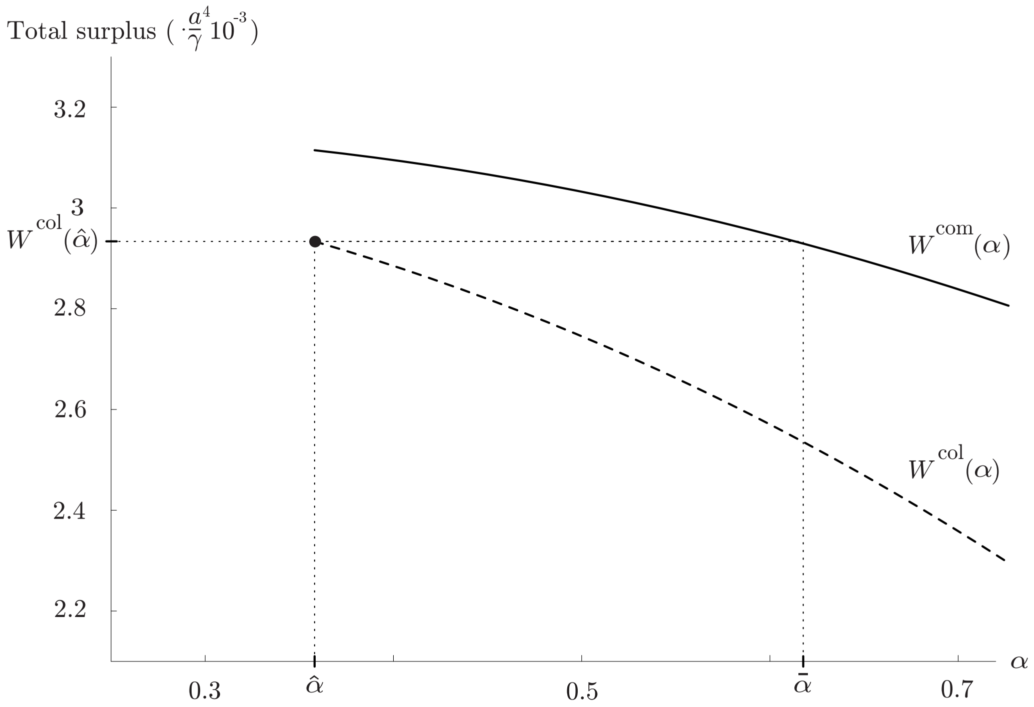

Figure 1: Total surplus under competition and collusion

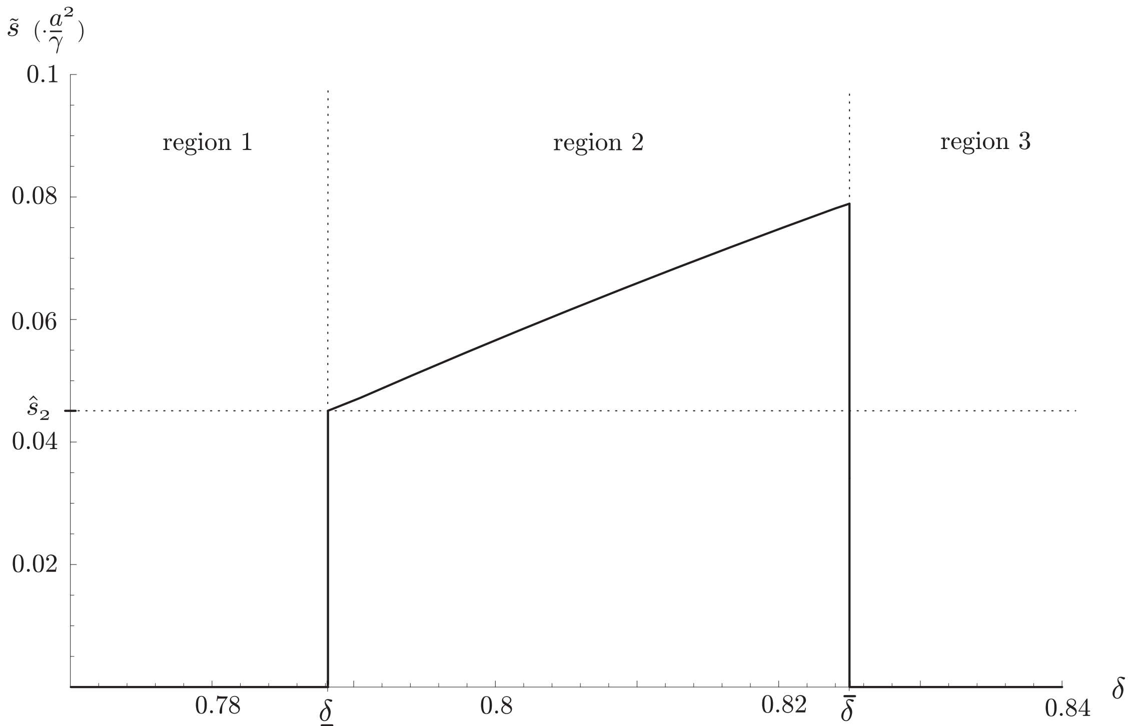

Figure 2: Optimal MQS for given discount factor

but entails only a negligible surplus increase relative to an unregulated market. The *optimal* or *surplus-maximizing MQS* for different discount factors δ is illustrated in Figure 2. It realizes the dynamic benefit of collusion prevention at the smallest static cost:

Proposition 5 *The surplus-maximizing MQS is*

$$\tilde{s}_2^*(\delta) = \frac{a^2\,(3-4\,\delta)(44\,\delta^3 - 120\,\delta^2 + 99\,\delta - 27)}{24\,\gamma\,\delta^3\;(2\,\delta - 1)} \tag{51}$$

for $\delta \in (\underline{\delta}, \bar{\delta})$, and zero or not binding otherwise.

Proof: Since the static welfare loss of an MQS is increasing continuously in $\tilde{s}$, the optimal choice of $\tilde{s}$ is such that the resulting regulated equilibrium quality ratio α satisfies

$$\rho(\alpha) = \delta \qquad \Longleftrightarrow \qquad \alpha = \frac{4\;(3-4\,\delta)}{3\;(1-2\,\delta)}. \tag{52}$$

Profit-maximizing behavior by firm 1 entails

$$s_1(\alpha) = \frac{a^2(\alpha^3 - 4\alpha^2 + 12\alpha - 16)}{2\gamma(\alpha-4)^3} \tag{53}$$

(substitute $t = \alpha$ in equation (10)). Recall, moreover, that $s_2^*(\tilde{s}) = \tilde{s}$ for any non-excessive $\tilde{s}$, i.e., firm 2 maximizes profits by selecting the minimum feasible quality (Lemma 2). So any MQS $\tilde{s}$ with corresponding equilibrium quality ratio α satisfies $\tilde{s} = \alpha \cdot s_1(\alpha)$ and, using (52) in order to substitute for α, this implies

$$\tilde{s}_2^*(\delta) = \frac{a^2\,(3-4\,\delta)(44\,\delta^3 - 120\,\delta^2 + 99\,\delta - 27)}{24\,\gamma\,\delta^3\;(2\,\delta - 1)} \tag{54}$$

□

One can distinguish three different regulation regimes: for low discount factors (region 1 in Figure 2), i.e., when firms place great weight on current relative to prospective future profits, collusion would be unstable even without regulation; an MQS could only destroy surplus. For intermediate discount factors (region 2), an MQS would prevent collusion and thereby create gains exceeding the static welfare losses first identified by Valletti (2000). Finally, for high discount factors (region 3), the costs of prevention would either exceed the respective benefits or collusion cannot be prevented by an MQS at all.[14]

6 Extensions

In the baseline collusion scenario, the low quality producer receives half of the difference between total competitive profits and the maximal aggregate profit; then it always prefers collusion to a deviation. With this in mind, consider a situation in which the critical

[14]The latter is the case for $\delta > \delta^c \approx 0.84366$: the implied MQS would be excessive.

discount factor r is slightly above firms' discount factor δ, i.e., collusion as investigated in Section 5 would not be stable. It is at least conceivable – and in our view quite likely – that the low quality producer offers part of its designated equal split to the high quality producer, i.e., the firms settle for a lower side payment, raise firm 1's collusion payoff, and push the critical discount factor below δ. Both firms would thus be better off relative to otherwise unavoidable competition.

This questions the rather standard application of a fixed division rule such as Nash bargaining to the static collusion rent in our model. Its replacement by the (present value of the) dynamic stream of rents is, however, also problematic: it presupposes the very stability of collusion which is being investigated. We therefore suggest to consider the *minimal* critical discount factor which is achievable for *any* conceivable division rule as an alternative indicator of the stability of collusion. We consider its reaction to an MQS in the next subsection. Afterwards, attention will be turned to possible collusion without side payments, and finally the case of variable quality costs.

Obviously, many other extensions or variations are possible. For example, consumers' preferences could be modified in analogy to Kuhn (2007), the number of active firms might be increased as in Scarpa (1998), or the timing of the quality choices could be varied as in Constantatos and Perrakis (1998). We conjecture that such modifications would qualify some statements (e.g., regarding the change of consumer surplus), but not reverse the basic anti-collusive effect of the MQS.

6.1 Minimal critical discount factor

Collusion profits in general amount to

$$\pi_1^c(s_1, s_2) \;=\; \pi_1^p(s_1, s_2) + q \cdot \pi_\Delta(s_1, s_2) \tag{55}$$

and

$$\pi_2^c(s_1, s_2) \;=\; \pi_2^p(s_1, s_2) + (1-q) \cdot \pi_\Delta(s_1, s_2) \tag{56}$$

for some $q \in [0, 1]$ and the given rent $\pi_\Delta(s_1, s_2)$. The critical discount factor of the high quality producer can with that be written as

$$\rho_1(\alpha, q) \;=\; \frac{8 - 4\,q - 3\,\alpha + 3\,q\,\alpha}{8 - 3\,\alpha} \tag{57}$$

for any regulated quality ratio α. Note that $\frac{\partial \rho_1(\alpha,q)}{\partial q} = \frac{4-3\,\alpha}{3\,\alpha-8} < 0$, i.e., raising firm 1's aggregate profits facilitates and stabilizes collusion. The critical discount factor is smallest (and collusion easiest to maintain) if q is chosen to be maximal under the constraint that firm 2's incentive to collude remains unchanged, i.e.,

$$\sum_{t=0}^{\infty} \delta^t \big(\pi_2^p(\alpha) + (1-q) \cdot z(\alpha)\big) \geq \pi_2^d(\alpha) + \sum_{t=1}^{\infty} \delta^t \pi_2^p(\alpha). \tag{58}$$

Inequality (58) happens to be true for *all* $q \in [0,1]$. The *minimal critical discount factor* such that collusion can be maintained by Nash reversion strategies under *any* solution to the bargaining problem between both firms hence evaluates to

$$\rho^*(\alpha) = \rho_1(\alpha, 1) = \frac{4}{8 - 3\,\alpha}. \tag{59}$$

It follows that the minimal critical discount factor increases when a moderate MQS is imposed ($\frac{\partial \rho^*(\alpha)}{\partial \alpha} > 0$). The anti-collusive effect of an MQS therefore does not hinge on the – in our dynamic context somewhat problematic – assumption of Nash bargaining; it is very robust with respect to the division of collusion rents.

6.2 Collusion without side payments

We have so far assumed that firms can collude rather explicitly: they pick the total profit-maximizing production plan and then organize side payments. Especially the latter seems problematic: it would provide antitrust authorities with hard and accessible evidence in legal proceedings. This might make the use of second-best policies against collusion, such as the investigated use of an MQS, unnecessary. It is therefore relevant that an MQS can also prevent tacit collusion *without* side payments.

We maintain the assumption that firms' try to maximize total profits

$$p_1(x_1, x_2, s_1, s_2) \cdot x_1 + p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_1) - C(s_2) \tag{60}$$

but suppose that side payments are replaced by coordinated quantity choices. The collusive quantity choices x_1^c and x_2^c must result in profits which exceed the respective competitive profit $\pi_i^p(s_1, s_2)$ for either firm. As in the previous subsection, we consider the surplus shares that yield the minimal critical discount factor.

In particular, we maximize (60) subject to the constraint

$$\pi_2^c(x_1, x_2, s_1, s_2) \equiv p_2(x_1, x_2, s_1, s_2) \cdot x_2 - C(s_2) = z \cdot \pi_2^p(s_1, s_2) \tag{61}$$

for a given $z \geq 1$. The unwieldy solution determines collusion profits $\pi_i^c(z)$ and – computing the respective best responses – deviation profits $\pi_i^d(z)$. For a given imposed collusion profitability z for firm 2, we thus obtain expressions for $\rho_1(\alpha, z)$ and $\rho_2(\alpha, z)$ such that

$$\rho(\alpha, z) \equiv \max\{\rho_1(\alpha, z), \rho_2(\alpha, z)\} \tag{62}$$

is the critical discount factor. Minimization of $\rho(\alpha, z)$ with respect to $z \geq 1$ then yields the minimal critical discount factor such that collusion can be maintained by Nash reversion equilibria.[15] It is worth noting that the respective minimizer z^* is strictly greater than 1: firm 1's collusion quantity is much smaller than the old $x_1^c = \frac{a}{2}$ in Section 5; firm 2 would deviate to a bigger quantity if it were kept at its competitive profit.

[15] The constraint $\pi_\Sigma(x_1, x_2, s_1, s_2) - z \cdot \pi_2^c(s_1, s_2) \geq \pi_2^p(s_1, s_2)$ is automatically taken care of: any too large share for firm 2 would make a deviation profitable for firm 1.

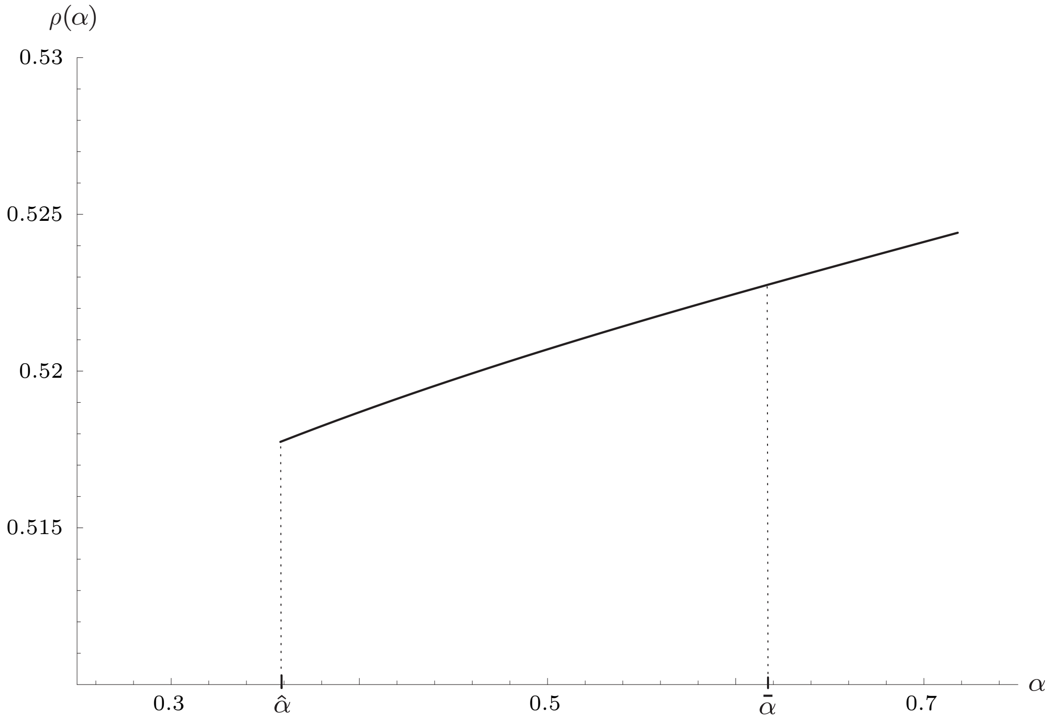

Figure 3: Critical discount factor without side payments

The minimal critical discount factor for given quality ratio, $\rho(\alpha)$, is illustrated in Figure 3. We find that the imposition of an MQS (corresponding to $\alpha > \hat{\alpha}$) robustly makes collusion harder. The indicated beneficial dynamic effect of an MQS is no artifact of a particular form of collusion and remains effective when first-best policies – namely, enforcement of a legal ban – are difficult to implement.

6.3 Variable quality costs

We finally consider the case in which quality affects variable costs – for instance, because greater quality requires more expensive raw materials, specialized labor, more time, etc. Appendix B contains formal derivations of the reported results.

In line with the related literature we assume quality-dependent unit costs

$$c(s_i) = \gamma s_i^2 \tag{63}$$

without fixed costs. For given qualities $s_1 \geq s_2$, the equilibrium quantities are

$$\hat{x}_1(s_1, s_2) = \frac{2\,a\,s_1 - 2\,\gamma\,{s_1}^2 - a\,s_2 + \gamma\,{s_2}^2}{4\,s_1 - s_2}, \tag{64}$$

$$\hat{x}_2(s_1, s_2) = \frac{s_1\,(a + \gamma\,s_1 - 2\,\gamma\,s_2)}{4\,s_1 - s_2} \tag{65}$$

and result in the reduced profits

$$\begin{aligned} \pi_1(s_1, s_2) &= \frac{s_1 \left(2\, s_1 \left(\gamma\, s_1 - 2\, \alpha\right) + s_2\, \left(a - \gamma\, s_2\right)\right)^2}{\left(s_2 - 4\, s_1\right)^2}, & (66) \\ \pi_2(s_1, s_2) &= \frac{{s_1}^2\, s_2 \left(a + \gamma\, s_1 - 2\, \gamma\, s_2\right)^2}{\left(s_2 - 4\, s_1\right)^2}. & (67) \end{aligned}$$

The implied *unregulated equilibrium qualities* can be computed as

$$\hat{s}_1 \approx \frac{0.36905\, a}{\gamma} \quad \text{and} \quad \hat{s}_2 \approx \frac{0.29279\, a}{\gamma}. \tag{68}$$

As in the case of fixed quality costs, firm 2 chooses $s_2^*(\tilde{s}) = \tilde{s}$ if the constraint $s_i \geq \tilde{s}$ is imposed, and again the *regulated equilibrium quality ratio*

$$\alpha(\tilde{s}) \equiv \frac{s_2^*(\tilde{s})}{s_1^*(\tilde{s})} = \frac{\tilde{s}}{R_1(\tilde{s})} \in (\hat{s}_2/\hat{s}_1, 1] \tag{69}$$

is strictly increasing in $\tilde{s}$. Firms' profits can be shown to satisfy

$$\frac{d\pi_1(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0, \tag{70}$$

$$\frac{d\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} \begin{cases} > 0; & \tilde{s} < \tilde{s}^l \\ < 0; & \tilde{s} > \tilde{s}^l \end{cases} \tag{71}$$

with $\tilde{s}^l \approx \frac{0.29443\, a}{\gamma}$. In contrast to the case of fixed quality costs, the low quality producer may now benefit from an MQS: the latter may create a valuable commitment to offering a relatively high quality with greater margins dominating the implied quantity reduction (as for Bertrand competition).

The effect of an MQS on total consumer surplus can now be negative, namely

$$\frac{\partial S(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{\partial \tilde{s}} \begin{cases} > 0; & \tilde{s} < \tilde{s}^m \\ < 0; & \tilde{s} > \tilde{s}^m \end{cases} \tag{72}$$

with $\tilde{s}^m \approx \frac{0.29887\, a}{\gamma}$. Whereas the MQS had only an indirect demand effect on prices in the fixed cost case, higher qualities also raise variable costs here and thus potentially reduce total consumer surplus. It is hence not surprising that, as before, total surplus decreases in the level of the MQS, i.e.,

$$\frac{d(\pi_1(\cdot) + \pi_2(\cdot) + S(\cdot))}{d\tilde{s}} < 0. \tag{73}$$

So the static net effect of an MQS on surplus is again disadvantageous. We are, however, interested mainly in the potential long-run effects of an MQS. Retaining the Nash bargaining assumption of Section 5, one can derive

$$\begin{aligned} \pi_1^c(s_1, s_2) &= \frac{s_1 \left(8\, \gamma^2\, {s_1}^3 + \gamma\, {s_1}^2\, (-16\, a + 3\, \gamma\, s_2)\right)}{32\, s_1 - 8\, s_2} \\ &+ \frac{s_1 \left(s_1\, \left(8\, a^2 + 2\, a\, \gamma\, s_2 - 5\, \gamma^2\, {s_2}^2\right) + s_2\, \left(-5\, a^2 + 8\, a\, \gamma\, s_2 - 3\, \gamma^2\, {s_2}^2\right)\right)}{32\, s_1 - 8\, s_2} \end{aligned} \tag{74}$$

and

$$\pi_2^c(s_1, s_2) = \frac{s_1\, s_2\, \left(3\, a^2 + 3\, \gamma^2\, {s_1}^2 - 8\, a\, \gamma\, s_2 + 5\, \gamma^2\, {s_2}^2 + \gamma\, s_1\, \left(2\, a - 5\, \gamma\, s_2\right)\right)}{32\, s_1 - 8\, s_2} \tag{75}$$

with, in contrast to the baseline situation of Section 5, *both* products on offer. It also turns out that both firms face a short-term temptation to cheat. The corresponding deviation profits are given by

$$\pi_1^d(s_1, s_2) = \frac{s_1 \left(-2\, a + 2\, \gamma\, s_1 + \gamma\, s_2\right)^2}{16}, \tag{76}$$

$$\pi_2^d(s_1, s_2) = \frac{s_2 \left(a + \gamma\, s_1 - \gamma\, s_2\right)^2}{16}. \tag{77}$$

We show in Appendix B that the critical discount factor associated with Nash bargaining decreases in the level of the MQS if $\tilde{s}$ is small and increases if $\tilde{s}$ is high:

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{ds} \begin{cases} < 0; & \tilde{s} < \tilde{s}^b \\ > 0; & \tilde{s} > \tilde{s}^b \end{cases} \tag{78}$$

with $\tilde{s}^b \approx \frac{0.298687\, a}{\gamma} > \tilde{s}$.

Again a trade off between static costs and dynamic benefits of an MQS exists: on the one hand, the MQS destroys surplus by distorting quality choices, but on the other hand it can prevent or destabilize collusion. In contrast to the benchmark case with fixed quality costs, any MQS which prevents collusion automatically raises total welfare (i.e., whenever $r(\tilde{s}) > \delta > r(0)$). The corresponding interval of discount factors δ is therefore larger; namely, a welfare-enhancing MQS exists for the case of variable quality costs if $\delta \in (\underline{\delta}; \bar{\delta})$ for $\underline{\delta} \approx 0.543367$ and $\bar{\delta} \approx 0.695238$. The optimal MQS is again the respective lowest one which prevents collusion.

7 Concluding remarks

We have shown that, contrary to received knowledge, an MQS can be welfare-increasing also if firms compete in quantities. While it robustly lowers the generated total surplus in single-shot interaction, the quality distortions induced by the MQS reduce the attractiveness of collusion relative to competitive behavior when firms interact repeatedly. The MQS can thereby prevent or destabilize collusion, and this creates dynamic benefits. They outweigh the static costs for a non-negligible range of parameters and the MQS raises total surplus.

This main finding is surprisingly robust. In particular, we have considered side payments as well as pure quantity coordination, fixed quality costs as well as variable quality costs, the standard measure of collusion stability as well as an alternative that provides more bargaining flexibility. Having more than two active firms as in Scarpa (1998) is unlikely to make any significant difference either:[16] The static effects of the MQS were

[16] We have confirmed this for an example with three firms.

already negative for a Cournot duopoly. The investigated dynamic effects stay beneficial, though the available range of non-excessive MQS is bound to shrink. So we conjecture that welfare increases remain a generic possibility also in more general oligopoly settings.

Of course, we do not expect our results to hold for *all* reasonable specifications of firms' costs or consumers' utility (see Kuhn 2007). And, more relevantly, other policy measures exist which may well be more effective and economical than an MQS in preventing anticompetitive behavior. In general, deterrence by purely legal means seems preferable. It offers authorities a number of decision alternatives to the distorting introduction of a new MQS or the tightening of an existing one – e.g., lower evidence requirements in antitrust cases, stiffer penalties, greater investment in detection, or immunity for whistle-blowers. But, first, it should not be taken for granted that no or smaller economic distortions are induced by these (think of lobbying, efforts of concealment and deception, bribery). And, second, there may also be other good reasons for the introduction of an MQS – for instance technological spillovers in mobile telecommunication, consumer protection related to child safety or public health concerns, strategic trade policy, etc. When the related benefits of an MQS are compared to the direct welfare losses studied by Valletti (2000), the dynamic gains identified here should be taken into account. They might tip the balance in a number of 'marginal' market environments.

The conventional wisdom concerning the merits of an MQS regularly needed updating in the past. The early investigations emphasized how competition and cost structure matter; several more recent studies indicate how the baseline results depend on consumers' preferences, the number of active firms, or the timing of decisions. This paper highlights the role of the time horizon and the associated market conduct. In particular, we have found that quality regulation under quantity competition can make sense from a dynamic perspective. Tempting dichotomous verdicts deserve further qualification.

Appendix A

The second-order condition for firm 1's quality $s_1^*(\tilde{s})$ in the baseline model

$$\frac{\partial^2 \pi_1(s_1, s_2)}{\partial s_1^2} = \frac{8\,a^2\,{s_2}^2\,\left(s_2 - s_1\right)}{\left(4\,s_1 - s_2\right)^4} - 2\,\gamma < 0 \tag{79}$$

is satisfied for all $s_1 > s_2$. Further conditions for the boundary point maximum $s_2^*(\tilde{s}) = \tilde{s}$ need not be checked.

The low quality producer has no incentive to leapfrog, i.e., it cannot gain by a deviation $s_2' \geq s_1^*(\tilde{s})$: if the low quality producer chooses $s_2' \geq s_1^*(\tilde{s})$, its profit is $\pi_2^L(s_1^*(\tilde{s}), s_2')) \equiv \pi_1(s_2', s_1^*(\tilde{s}))$, and decreases in the now lower quality $s_1^*(\tilde{s})$; namely

$$\frac{\partial \pi_1(s_1, s_2)}{\partial s_2} = \frac{4\,a^2\,{s_1}^2\,\left(s_2 - 2\,s_1\right)}{\left(4\,s_1 - s_2\right)^3} < 0. \tag{80}$$

By Lemma 1, $s_1^*(\tilde{s}) \geq \hat{s}_1$ and therefore

$$\pi_2^L(s_1^*(\tilde{s}), s_2')) \leq \pi_1(s_2', \hat{s}_1) \approx \frac{s_2' \left(0.01587\, a^6 - 0.51975\, a^4\, \gamma\, s_2' + 5.00777\, a^2\, \gamma^2\, {s_2'}^2 - 16\, \gamma^3\, {s_2'}^3\right)}{\left(4\, \gamma\, s_2' - 0.12597\, a^2\right)^2}. \tag{81}$$

The latter term is maximal at $s_2' = \frac{0.12961\, a^2}{\gamma}$ and bounded above by $\frac{-0.00186\, a^4}{\gamma}$. So firm 2 cannot attain a positive profit by leapfrogging. Firm 1 cannot leapfrog firm 2 because $s_2^*(\tilde{s}) = \tilde{s}$.

Appendix B

This appendix considers the case of *variable quality costs* (without fixed costs), namely unit costs are $c(s_i) = \gamma s_i^2$. The reduced profit functions (66) and (67) yield the first-order conditions

$$\frac{\partial \pi_1(s_1, s_2)}{\partial s_1} = \frac{s_1 \left(-2\, a\, s_1 + 2\, \gamma\, {s_1}^2 + s_2\, \left(a - \gamma\, s_2\right)\right)^2}{\left(s_2 - 4\, s_1\right)^2} = 0 \tag{82}$$

$$\frac{\partial \pi_2(s_1, s_2)}{\partial s_2} = \frac{{s_1}^2\, s_2 \left(a + \gamma\, s_1 - 2\, \gamma\, s_2\right)^2}{\left(s_2 - 4\, s_1\right)^2} = 0, \tag{83}$$

from which the indicated unregulated qualities $(\hat{s}_1, \hat{s}_2)$ can be deduced.

As in the baseline case of fixed quality costs, the quality gap decreases in $\tilde{s}$:

Lemma 4 $\frac{\partial R_1(s_2)}{\partial s_2} < 1$.

Proof: Substituting $s_2 \equiv t \cdot s_1$ with $t \in (0, 1]$ in (82), the first-order condition for firm 1's quality choice can equivalently be written as

$$s_1 = \frac{a\, \left(t^2 - 2\, t + 8\right)}{\left(t^3 + 4\, t^2 - 10\, t + 24\right)\, \gamma}. \tag{84}$$

The second-order condition is satisfied: using the rearranged first-order condition (84) and $s_2 \equiv t \cdot s_1$ with $t \in (0, 1]$, one obtains

$$\frac{\partial^2 \pi_1}{\partial s_1^2} = -\frac{24\, a\, \left(t^5 - 4\, t^4 + 8\, t^3 - 16\, t^2 + 20\, t - 16\right)\, \gamma}{\left(t^3 - 6\, t^2 + 16\, t - 32\right)\, \left(t^3 + 4\, t^2 - 10\, t + 24\right)} < 0. \tag{85}$$

The implicit function theorem applied to (82) and (84) yields

$$\frac{\partial R_1(s_2)}{\partial s_2} = \frac{t^4 - 4\, t^3 + 26\, t^2 + 16\, t - 32}{6t^4 - 36\, t^3 + 60\, t^2 - 96\, t + 192}. \tag{86}$$

Numerical inspection then allows to infer

$$\frac{\partial R_1(s_2)}{\partial s_2} \in \left[-\frac{1}{16}, \frac{1}{6}\right). \tag{87}$$

□

Lemma 5 $s_2^*(\tilde{s}) = \tilde{s}$.

Proof: Using $s_2 = t \cdot s_1$ and (84), we have

$$\frac{\partial \pi_2}{\partial s_2} = \frac{a^2\,(3\,t^5 - 22\,t^4 + 103\,t^3 - 308\,t^2 + 512\,t - 256)}{(t-4)\,(t^3 + 4\,t^2 - 10\,t + 24)^2}, \tag{88}$$

which is positive (negative) to the left (right) of $t = \hat{s}_2/\hat{s}_1$. By Lemma 4, we must have $t > \hat{s}_2/\hat{s}_1$ in equilibrium, i.e., (88) is negative and $s_2^*(\tilde{s}) = \tilde{s}$ becomes a boundary point maximum.

Firm 2 has no incentive to leapfrog, i.e., to choose $s_2' \geq s_1^*(\tilde{s})$. Its profit would then be $\pi_2^L(s_1^*(\tilde{s}), s_2') \equiv \pi_1(s_2', s_1^*(\tilde{s}))$. This decreases in the now lower quality $s_1^*(\tilde{s})$; namely, with (84) and $t \in [0,1]$ we obtain

$$\left.\frac{\partial \pi_1}{\partial s_2}\right|_{s_1 = R_1(s_2)} = -\frac{8\,a^2\,(t^5 - 6\,t^4 + 28\,t^3 - 60\,t^2 + 68\,t - 32)}{(t-4)\,(t^3 + 4\,t^2 - 10\,t + 24)^2} < 0. \tag{89}$$

$s_1^*(\tilde{s})$ satisfies (84); hence $s_1^*(\tilde{s}) \geq \frac{7a}{19\gamma} \equiv s_1^{min}$ and

$$\pi_2^L(s_1^*(\tilde{s}), s_2') \leq \pi_1(s_2', s_1^{min}) = \frac{4\,s_2'\left(42\,a^2 - 361\,a\,\gamma\,s_2' + 361\,\gamma^2\,{s_2'}^2\right)^2}{361\,(7\,a - 76\,\gamma\,s_2')^2}. \tag{90}$$

The latter term is *maximized* by $s_2' = \frac{7\,a}{19\,\gamma} = s_1^{min}$, corresponding to a quality ratio $\alpha = 1$. In contrast, the reference profit $\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))$ (see (67)), written as a function of the equilibrium quality ratio $\alpha \in (\hat{s}_2/\hat{s}_1, 1]$

$$\Pi_2(\alpha) \quad = \quad \frac{a^3\,\alpha\,(\alpha^2 - 5\,\alpha + 8)\,(\alpha^4 - 7\,\alpha^3 + 26\,\alpha^2 - 56\,\alpha + 64)}{(\alpha^3 + 4\,\alpha^2 - 10\,\alpha + 24)^3\,\gamma}, \tag{91}$$

is *minimized* by $\alpha = 1$ and then equal to $\pi_2^L(s_1^{min}, s_1^{min})$. So $\pi_2^L(s_1^*(\tilde{s}), s_2') \leq \pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))$.
□

Proposition 6

$$\frac{d\pi_1(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} < 0 \quad \textit{and} \quad \frac{d\pi_2(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{d\tilde{s}} \begin{cases} > 0; & \tilde{s} \leq \tilde{s}^l \approx \frac{0.294426\,a}{\gamma} \\ < 0; & \tilde{s} > \tilde{s}^l. \end{cases} \tag{92}$$

Proof: Firm 1's profits (see (66)) can be written as a function of the regulated equilibrium quality ratio, using (84), as follows:

$$\Pi_1(\alpha) \quad = \quad \frac{16\,a^3\,(\alpha^2 - 2\,\alpha + 2)\,(\alpha^4 - 4\,\alpha^3 + 14\,\alpha^2 - 20\,\alpha + 16)}{(\alpha^3 + 4\,\alpha^2 - 10\,\alpha + 24)^3\,\gamma}. \tag{93}$$

Firm 2's profit is shown in (91). Changes due to the MQS are

$$\frac{d\Pi_1(\alpha)}{d\tilde{s}} \quad = \quad \frac{\partial \Pi_1(\alpha)}{\partial \alpha} \cdot \underbrace{\frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}}}_{>0} < 0, \tag{94}$$

with

$$\text{sign}\left(\frac{\partial \Pi_1(\alpha(\tilde{s}))}{\partial \alpha}\right) = \text{sign}(\alpha^3 - 4\,\alpha^2 + 18\,\alpha - 16) < 0 \tag{95}$$

using $\frac{\hat{s}_2}{\hat{s}_1} \leq \alpha \leq 1$, and

$$\frac{d\Pi_2(\alpha)}{d\tilde{s}} = \frac{\partial \Pi_2(\alpha)}{\partial \alpha} \cdot \underbrace{\frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}}}_{>0} \tag{96}$$

with

$$\text{sign}\left(\frac{\partial \Pi_2(\alpha(\tilde{s}))}{\partial \alpha}\right) = \text{sign}(-\alpha^7 + 15\,\alpha^6 - 75\,\alpha^5 + 236\,\alpha^4 - 462\,\alpha^3 + +528\,\alpha^2 \tag{97}$$

$$-1184\,\alpha + 768) \tag{98}$$

where the latter is positive for $\alpha < \alpha^l$ and negative for $\alpha \geq \alpha^l$ with $\alpha^l \approx 0.79769$. Finally, $\alpha = \alpha^l$ is equivalent to $\tilde{s} = \tilde{s}^l$.

□

The consumer surplus for given qualities s_1 and s_2 is

$$S(s_1, s_2) = \int_{\frac{p_2}{s_2}}^{\frac{p_1-p_2}{s_1-s_2}} (\theta s_2 - p_2)\, d\theta + \int_{\frac{p_1-p_2}{s_1-s_2}}^{a} (\theta s_1 - p_1)\, d\theta \tag{99}$$

which becomes

$$\Sigma(\alpha) = \frac{a^3\left(9\,\alpha^7 - 68\,\alpha^6 + 309\,\alpha^5 - 834\,\alpha^4 + 1480\,\alpha^3 - 1344\,\alpha^2 + 384\,\alpha + 512\right)}{2\,(24 - 10\,\alpha + 4\,\alpha^2 + \alpha^3)^3\,\gamma}. \tag{100}$$

expressed in terms of the regulated equilibrium quality ratio. We then have

$$\frac{d\Sigma(\alpha)}{d\tilde{s}} = \frac{\partial \Sigma(\alpha)}{\partial \alpha} \cdot \frac{\partial \alpha(\tilde{s})}{\partial \tilde{s}} \tag{101}$$

with

$$\begin{aligned}\text{sign}\left(\tfrac{\partial \Sigma(\alpha)}{\partial \alpha}\right) = \; & \text{sign}(-3\,\alpha^9 + 40\,\alpha^8 - 266\,\alpha^7 + 1081\,\alpha^6 - 3030\,\alpha^5 + 6178\,\alpha^4 \\ & -10272\,\alpha^3 + 13472\,\alpha^2 - 11520\,\alpha + 4096).\end{aligned} \tag{102}$$

Numerical inspection shows that $\frac{\partial \Sigma(\alpha)}{\partial \alpha}$ is positive for $\alpha < \alpha^m$ and negative for $\alpha > \alpha^m$ with $\alpha^m \approx 0.80944$, where the latter corresponds to $\tilde{s}^m \approx \frac{0.29887\,a}{\gamma}$.

Proposition 7

$$\frac{d(\pi_1(\cdot) + \pi_2(\cdot) + S(\cdot))}{d\tilde{s}} < 0. \tag{103}$$

Proof: Total surplus expressed in terms of quality ratio α is

$$\Gamma(\alpha) \equiv \Pi_1(\alpha) + \Pi_2(\alpha) + \Sigma(\alpha) \tag{104}$$

$$= \frac{a^3\left(11\,\alpha^7 - 60\,\alpha^6 + 255\,\alpha^5 - 550\,\alpha^4 + 792\,\alpha^3 - 192\,\alpha^2 - 896\,\alpha + 1536\right)}{2\,(24 - 10\,\alpha + 4\,\alpha^2 + \alpha^3)^3\,\gamma}. \tag{105}$$

The change in total surplus is equal to

$$\frac{d\Gamma(\alpha(\tilde{s}))}{d\tilde{s}} = \frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha}\cdot\underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} < 0 \qquad (106)$$

with

$$\text{sign}\left(\frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha}\right) = \text{sign}(-11\,\alpha^9 + 112\,\alpha^8 - 730\,\alpha^7 + 2689\,\alpha^6 - 7046\,\alpha^5 + 13970\,\alpha^4 -21280\,\alpha^3 + 29600\,\alpha^2 - 32000\,\alpha + 12288). \qquad (107)$$

Numerical inspection shows that $\frac{\partial\Gamma(\alpha(\tilde{s}))}{\partial\alpha} < 0$ for all $\alpha \in \left(\frac{\hat{s}_2}{\hat{s}_1}, 1\right]$.

□

Total profit $(p_1(x_1, x_2, s_1, s_2) - c(s_1)) \cdot x_1 + (p_2(x_1, x_2, s_1, s_2) - c(s_2)) \cdot x_2$ is maximized by

$$x_1^c(s_1, s_2) = \frac{a - \gamma s_1 - \gamma s_2}{2} \quad \text{and} \quad x_2^c(s_1, s_2) = \frac{\gamma s_1}{2}. \qquad (108)$$

Nash bargaining over aggregate collusion profits then yields (74) and (75).

An optimal deviation from (x_1^c, x_2^c) for given qualities (s_1, s_2) respectively amounts to

$$x_1^d(s_1, s_2) = \frac{2\,a - 2\,\gamma\,s_1 - \gamma\,s_2}{4} \quad \text{and} \quad x_2^d(s_1, s_2) = \frac{a + \gamma\,s_1 - \gamma\,s_2}{4}, \qquad (109)$$

and implies the profits in (76) and (77). Punishment profits $\pi_i^p(s_1, s_2)$ are given by equations (66) and (67).

Proposition 8

$$\frac{dr(s_1^*(\tilde{s}), s_2^*(\tilde{s}))}{ds} \begin{cases} < 0; & \tilde{s} < \tilde{s}^b \approx \frac{0.29869\,a}{\gamma} \\ > 0; & \tilde{s} > \tilde{s}^b. \end{cases} \qquad (110)$$

Proof: Inserting the respective expressions for π_i^p, π_i^c and π_i^d into $r_i \equiv \frac{\pi^d - _i\pi_i^c}{\pi_i^d - \pi_i^p}$ and using (84), $r_1(s_1, s_2)$ and $r_2(s_1, s_2)$ can be written as functions of the regulated equilibrium quality ratio α:

$$\rho_1(\alpha) \equiv \frac{\alpha^5 + 2\,\alpha^4 - 130\,\alpha^3 + 448\,\alpha^2 - 704\,\alpha + 384}{\alpha^5 + 16\,\alpha^4 - 240\,\alpha^3 + 704\,\alpha^2 - 960\,\alpha + 512}, \qquad (111)$$

$$\rho_2(\alpha) \equiv \frac{19\,\alpha^4 - 10\,\alpha^3 - 64\,\alpha^2 + 256\,\alpha - 128}{3\,\alpha^2\,(11\,\alpha^2 - 40\,\alpha + 64)}. \qquad (112)$$

As illustrated in Figure 4, the functions intersect at $\alpha = \alpha^b \approx 0.80896$, which corresponds to MQS $\tilde{s}^b \approx \frac{0.29869\,a}{\gamma}$. If $\alpha \leq \alpha^b$ ($\alpha > \alpha^b$) firm 1's (firm 2's) temptation to deviate is critical. It is easy to see that $\frac{\partial\rho_1(\alpha)}{\partial\alpha} < 0$ and $\frac{\partial\rho_2(\alpha)}{\partial\alpha} > 0$. Using that $\alpha(\tilde{s})$ is strictly increasing in $\tilde{s}$ (see Lemmata 4 and 5), one obtains

$$\frac{d\rho(\alpha(\tilde{s}))}{d\tilde{s}} = \begin{cases} \frac{d\rho_1(\alpha))}{d\tilde{s}} = \underbrace{\frac{\partial\rho_1(\alpha(\tilde{s}))}{\partial\alpha}}_{<0}\cdot\underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} < 0; & \tilde{s} < \tilde{s}^b \approx \frac{0.29869\,a}{\gamma} \\ \frac{d\rho_2(\alpha)}{d\tilde{s}} = \underbrace{\frac{\partial\rho_2(\alpha(\tilde{s}))}{\partial\alpha}}_{>0}\cdot\underbrace{\frac{\partial\alpha(\tilde{s})}{\partial\tilde{s}}}_{>0} > 0; & \tilde{s} > \tilde{s}^b. \end{cases} \qquad (113)$$

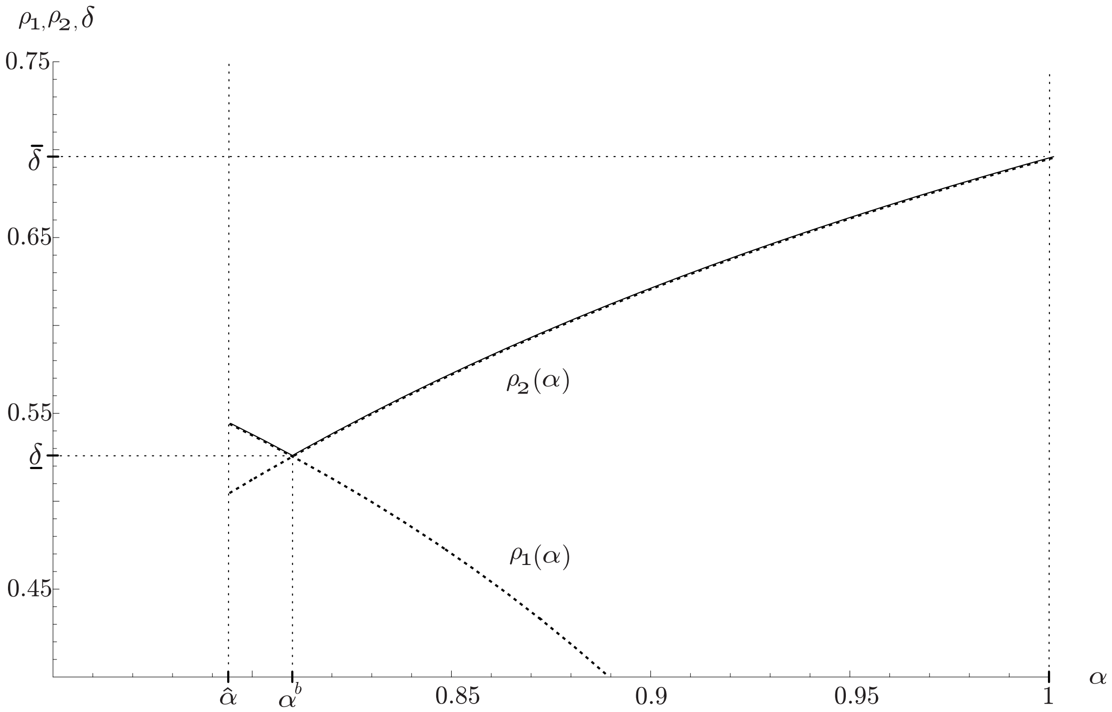

Figure 4: Critical discount factor for given equilibrium quality ratio

□

Proposition 9 *A welfare-enhancing MQS exists if* $\delta \in (\underline{\delta}; \overline{\delta})$ *for* $\underline{\delta} \approx 0.54337$ *and* $\overline{\delta} \approx 0.69524$.

Proof: For given a $\delta > r(\hat{s}_1, \hat{s}_2)$ which makes collusion sustainable in an unregulated equilibrium, an MQS $\tilde{s}$ induces competitive behavior whenever it implies an equilibrium quality ratio α such that $\rho(\alpha) > \delta$. Such an MQS therefore exists for any δ satisfying

$$\underline{\delta} \equiv \min_{\alpha \in \left[\frac{\hat{s}_2}{\hat{s}_1}, 1\right]} \rho(\alpha) < \delta < \max_{\alpha \in \left[\frac{\hat{s}_2}{\hat{s}_1}, 1\right]} \rho(\alpha) \equiv \overline{\delta}, \tag{114}$$

where one obtains $\underline{\delta} \approx 0.54337$ and $\overline{\delta} \approx 0.69524$.

Total surplus rises relative to collusion with unregulated qualities for all regulated equilibrium levels $\alpha(\tilde{s})$: In analogy to the fixed cost case, we obtain

$$W^{com}(\alpha) - W^{col}(\hat{\alpha}) = \frac{a^3 \left(\alpha^2 - 2\alpha + 8\right)\left(11\alpha^5 - 38\alpha^4 + 91\alpha^3 - 64\alpha^2 - 64\alpha + 192\right)}{2\left(\alpha^3 + 4\alpha^2 - 10\alpha + 24\right)^3 \gamma} - b \tag{115}$$

with $b \approx 0.0581843$. Numerical inspection reveals that this is always positive.

□

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