INFLATION TARGETING AS A SIGNALLING MECHANISM

Bedri Kamil Onur Taş

Department of Economics
TOBB University of Economics and Technology

Working Paper No: 07-01
TOBB University of Economics and Technology
Department of Economics

May 2007

TOBB University of Economics and Technology Department of Economics Working Papers are published without a formal refereeing process, with the sole purpose of generating discussion and feedback.
Inflation Targeting As a Signalling Mechanism

Bedri Kamil Onur Tas†

January 2007

Abstract

This paper theoretically investigates inflation targeting when there is asymmetric information between the Central Bank and the public. The main argument of this study is that the inflation target can be used as a signalling mechanism through which the private-sector learns about the private information of the Central Bank about future inflation and output. Thus, inflation targeting increases transparency and this causes the monetary policy actions (changes in the interest rate) to be more effective. I construct a Kalman filter algorithm to analyze the information and learning dynamics between the Central Bank and a representative private-sector agent. An increase (decrease) in the interest rate and the inflation target signals that the Central Bank has private information that inflation and output will be higher (lower) in the future thus the public expect inflation to be higher (lower) in the future. The main results of the paper are as follows. First, the private-sector agents (public) revise their expectations about future inflation and output after observing the actions of the Central Bank, changes in the interest rate and the inflation target. Second, in the case of inflation targeting, the response of inflation to monetary policy shocks (changes in the interest rate) is higher than it is in the case of no inflation targeting. So, when there is inflation targeting the interest rate tool of the CB is more effective in decreasing inflation.

JEL classification: E31; E52; D82; D83

Keywords: Inflation Target, Asymmetric Information, Monetary Policy, Kalman Filter, Learning

1 Introduction

Since its introduction in New Zealand in 1990, inflation targeting has been adopted by more than 20 countries. As explained in Svensson(1999) inflation targeting has three main characteristics: (i) an explicit quantitative inflation target, (ii) a framework for policy decisions, (iii) a high degree of transparency and

*TOBB-ETU, Economics Department.
†Email: onurtas@etu.edu.tr
accountability. This paper investigates the third characteristic, transparency, in the case of asymmetric information between the Central Bank (CB) and the public (private sector). This paper theoretically investigates inflation targeting when there is asymmetric information between the CB and the public. The analysis shows that interest rate is more effective in decreasing inflation under inflation targeting when there is asymmetric information. The main argument of this study is that the inflation target can be used as a signalling mechanism through which the private-sector learns about the private information of the CB about future inflation and output. Thus, inflation targeting increases transparency which causes the monetary policy actions (changes in the interest rate) to be more effective. The analysis and the results of this paper provides another mechanism to explain the success of inflation targeting. When there is asymmetric information between the CB and the public, inflation target serves as a signal that the public uses to deduce the private information of the CB about future inflation and output. Transparency is higher in the inflation targeting case and the interest rate is more effective in decreasing inflation.

This paper takes a novel theoretical approach and shows that inflation targets affect the expectations of the private-sector when there is asymmetric information between the Central Bank and the public. It has been empirically shown by Romer and Romer (2000) and Sims (2002) that the Federal Reserve Bank has superior information about future inflation and output.1 Starting from these findings, I construct a model of asymmetric information and learning in order to explore the effects of monetary policy on public expectations and future inflation. In this study, monetary policy consists of both determining the interest rate and the inflation target. I use a Kalman filtering algorithm as suggested by Townsend (1983) to analyze the learning dynamics and hierarchical information structure between the Fed and a representative private-sector agent. There are three stages of the learning dynamics. First, the CB receives private signals about inflation and output, constructs its expectations and uses a forward-looking Taylor rule2 to determine the interest rate and uses a forward-looking rule to determine the inflation target. Second, the private-sector agent

1 Faust, Swanson and Wright (2004) find little evidence that Federal Reserve convey superior information about the state of the economy. But, they argue that “... the policy surprise conveys information not about the state of the economy, but rather about the future course of policy, for which the FOMC has a natural informational advantage.” Thus, the central bank can still have private information which the private-sector agent would like to deduce. Since, future actions of the central bank will affect future inflation and output, it will also affect the expectations of the public. In a forward-looking model this will also affect the current inflation and output through the expectations of the public. I believe that findings of Romer and Romer (2000) and Sims (2002) and natural informational advantage of the central bank about its future actions provides the necessary motivation and evidence about asymmetric information between the CB and public.

2 John Taylor (1993) has proposed that U.S. monetary policy in recent years can be described by an interest-rate feedback rule of the form

\[ i_t = 0.04 + 1.5(\pi_t - 0.2) + 0.5(y_t - \bar{y}_t) \]

where \( i_t \) denotes the Fed’s operating target for the federal funds rate, \( \pi_t \) is the inflation rate (measured by the GDP deflator), and \( y_t \) is the log of real GDP.
observes the interest rate and the inflation target and revises her inflation and output expectations.\(^3\) Finally, inflation is affected because of these changes in the private-sector agent’s expectations\(^4\) as well as the change in the interest rate. Thus, introducing asymmetric information and learning allows us to analyze the effects of the CB’s actions on the expectations of the representative private-sector agent. The main idea of this study is that an increase (decrease) in the interest rate and the inflation target signals that the CB has private information that inflation and output will be higher (lower) in the future, so the public expect inflation to be higher (lower) in the future.

The inflation target has two effects in this model. First, it plays a role in the interest rate decision as it is in the monetary policy rule as shown by Svensson (1999). Second, it affects the expectations of the public and increases transparency as it signals the private information of the CB.

There are studies that use information asymmetry between the central bank and the public to investigate the impact of monetary policy surprises. Erceg and Levin (2003) formulate a dynamic general equilibrium model with staggered nominal contracts, in which households and firms use optimal filtering to disentangle persistent and transitory shifts in the monetary policy rule. Gurkaynak, Sack, and Swanson (2003) present a model in which the central bank’s inflation target is not directly observed by the private sector and must be inferred by agents on the basis of the central bank’s actions. They argue that the subsequent adjustment of the private sector’s expectations can explain a significant response of forward rates at long horizons. Svensson and Woodford (2002) constructs a model in which the private sector is assumed to have more information about the state of the economy than the policymaker. They present a general characterization of optimal filtering and control in settings of asymmetric information. Honkapohja and Mitra (2005) shows that when the monetary policy is formulated in terms of a target level, some forms of constant interest rate instrument rules lead to both indeterminacy of equilibria and instability under adaptive learning.

The analysis of this paper differs significantly from the studies above in two ways. First, asymmetric information and learning are examined in the context of inflation targeting. The model displays the significant impact of asymmetric information between the CB and the public on the relation between inflation and monetary policy. Second, the source of asymmetric information in those studies is the central bank’s inflation target. They assume that the inflation target varies over time in order to have learning dynamics in their models. In contrast in this paper, the CB is taken to possess private information about future inflation and output, supported by many empirical studies. Thus, in those studies, asymmetric information plays a different role and the CB’s actions affect public expectations through a different mechanism.

\(^3\)The updating of the expectations is a result of the model with asymmetric information and signal extraction.

\(^4\)Many theoretical (Calvo (1983), Svensson (2000)) and empirical (Mehra (2004), Gali and Lopez-Salido (2001)) studies show that expected inflation is a significant determinant of inflation.
The main results of the paper are as follows. First, the private-sector agents (public) revise their expectations about future inflation and output after observing the actions of the CB, changes in the interest rate and the inflation target. In the model, the investors know that the CB possesses private information and that motivates the investors to follow the CB’s actions and revise their expectations. An increase (decrease) in the interest rate means that CB expects inflation and output to be higher (lower) in the future. Second, in the case of inflation targeting, the response of inflation to monetary policy shocks (changes in the interest rate) is higher than it is in the case of no inflation targeting. Thus, when there is inflation targeting the interest rate tool of the CB is more effective in decreasing inflation. The explanation for this difference in the response of stock returns is as follows. In the inflation targeting case, the positive effect of interest rate on the inflation expectation of the public is lower than the no inflation targeting case. This positive effect is caused by the fact that the CB increases the interest rate because it has private information that inflation and output in the future will be higher. The inflation targeting case is more transparent in the sense that there are two signals that the public can use to deduce the private information of the CB. This result indicates that the success of the inflation targeting in fighting inflation can be caused partly by the extra transparency provided by the inflation target.

2 The Model of Asymmetric Information and Learning Dynamics

2.1 Agents and Information Structure

The model features two agents, the Central Bank and a representative private-sector agent.

The information structure is hierarchical since the Central Bank (CB) is assumed to possess private information that the private-sector agent tries to deduce by observing the CB’s actions. The information structure consists of two steps:

- The CB receives private signals about inflation and output and uses a forward-looking Taylor rule to determine the interest rate and a forward-looking rule for the inflation target.
- The representative private-sector agent observes the interest rate and the inflation target and revises her inflation and output expectations.

Similar to the general forward-looking setup in Sesson and Woodford(2002), inflation and output are determined by the following forward-looking VAR(1) process:\footnote{The model can easily be updated to include private-sector expectation of inflation at time $t+1$, $E(\pi_{t+1} | \Omega^P_R t)$ instead of current inflation expectation, $E(\pi_t | \Omega^P_R t)$). This will not change the results since private-sector expectations affect current inflation in both cases.}
\[
\begin{align*}
\pi_{t+1} &= \beta_{11}\pi_t + \beta_{12}x_t + \beta_{13}E\{\pi_t | \Omega_t^{PRI}\} + \beta_{14}E\{x_t | \Omega_t^{PRI}\} + \varepsilon_{t+1}^\pi \\
x_{t+1} &= \beta_{21}\pi_t + \beta_{22}x_t + \beta_{23}E\{\pi_t | \Omega_t^{PRI}\} + \beta_{24}E\{x_t | \Omega_t^{PRI}\} + \beta_{25}r_t + \varepsilon_{t+1}^x
\end{align*}
\] (1)

The inflation and output equations can be displayed in a convenient state-space representation as the following:

\[
\begin{bmatrix}
\pi_{t+1} \\
x_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t
\end{bmatrix} +
\begin{bmatrix}
\beta_{13} & \beta_{14} & 0 \\
\beta_{23} & \beta_{24} & \beta_{25}
\end{bmatrix}
\begin{bmatrix}
E\{\pi_t | \Omega_t^{PRI}\} \\
E\{x_t | \Omega_t^{PRI}\} \\
r_t
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_{t+1}^\pi \\
\varepsilon_{t+1}^x
\end{bmatrix}
\]

\(\pi_{t+1}\) is inflation at time \(t+1\) and \(x_{t+1}\) is output at \(t+1\). \(E\{\pi_t | \Omega_t^{PRI}\}\) is the private-sector expectation of inflation at time \(t\) and \(r_t\) is the interest rate determined by the central bank at time \(t\). The equation for inflation is a Phillips curve equation that relates inflation positively to lagged output, inflation and inflation and output expectations of the private sector. The equation for output relates output to lagged inflation, output, inflation and output expectations of the private sector and the interest rate. \(\beta_{25}\) is assumed to be negative. \(\varepsilon_{t+1}^\pi\) is a shock to \(t+1\) inflation at time \(t+1\) and \(\varepsilon_{t+1}^x\) is a shock to \(t+1\) output at time \(t+1\). \(\varepsilon_{t+1}^\pi\) and \(\varepsilon_{t+1}^x\) are assumed to be the sum of an autoregressive component \(\theta_{t+1}\) and a completely transitory component \(z_{t+1}\) such that,

\[
\begin{align*}
\varepsilon_{t+1}^\pi &= \theta_{t+1}^\pi + z_{t+1}^\pi \\
\varepsilon_{t+1}^x &= \theta_{t+1}^x + z_{t+1}^x
\end{align*}
\] (3)

\(\theta_{t+1}^\pi\) and \(\theta_{t+1}^x\) follow AR(1) processes as:

\[
\begin{align*}
\theta_{t+1}^\pi &= \rho^\pi \theta_t^\pi + c_{t+1}^\pi \\
\theta_{t+1}^x &= \rho^x \theta_t^x + c_{t+1}^x
\end{align*}
\] (4)

where \(c_{t+1}^\pi, c_{t+1}^x, z_{t+1}^\pi,\) and \(z_{t+1}^x\) are jointly normally distributed, independent among themselves and over time, with mean zero and variances \(\pi \sigma_{c^\pi}^2, \sigma_{c^\pi}^2, \sigma_z^2\) and \(\sigma_z^2\) respectively.

The CB receives the following private signals about autoregressive components of shocks to time \(t\) inflation and output at time \(t\).

\[
\begin{align*}
\pi_t \theta_t^\pi &= \theta_t^\pi + u_t^\pi \\
x_t \theta_t^x &= \theta_t^x + u_t^x
\end{align*}
\] (5)

where \(u_t^\pi\) and \(u_t^x\) are jointly normally distributed, independent among themselves and over time, with mean zero and variances \(\pi \sigma_{u^\pi}^2\) and \(\sigma_{u^x}^2\), respectively.
The CB then uses a forward-looking Taylor rule to determine the interest rate and the CB also determines the inflation target using its private information and announces it to the public.\footnote{Clarida, Gali, and Gertler (1999) show that approximate and in some cases exact forms of this rule are optimal for a central bank that has a quadratic loss function in deviations of inflation and output from their respective targets, given a generic macroeconomic model with nominal price inertia. The forward-looking specification allows the central bank to consider a broad array of information (beyond lagged inflation and output) to form beliefs about the future condition of the economy.}

The rule for the interest rate is specified as:

\[
 r_t = \lambda_1^r (E \{ \pi_t | \Omega_t^{CB} \} - \bar{\pi}_t) + \lambda_2^r (E \{ x_t | \Omega_t^{CB} \}) + \lambda_3^r r_{t-1} + \epsilon_t^r \tag{6}
\]

where \( r_t \) is the current period interest rate and \( \bar{\pi}_t \) is the current period inflation target announced by the central bank. \( \Omega_t^{CB} \) is the information set of the CB at time \( t \). The information set of the CB will be explained in detail in section 2.2. \( E \{ \Omega_t^{CB} \} \) denotes the CB’s expectation at time \( t \) about the next period. \( \lambda_1^r \) and \( \lambda_2^r \) determine the response of the interest rate to the inflation and output expectations of the CB. \( \lambda_3^r \) is the interest rate smoothing coefficient.\( \epsilon_t^r \) is the shock to the interest rate, which is normally distributed with mean zero and variance \( \sigma_{\epsilon_t^r}^2 \) and is independent over time.

The rule for the inflation target\footnote{The Greenbooks of the Fed contain forecasts of current inflation (gdp deflator) and current output (gdp) as well as forecasts up to six quarters ahead.} is specified as:

\[
 \bar{\pi}_t = \lambda_1^{\pi} (E \{ \pi_t | \Omega_t^{CB} \} ) + \lambda_2^{\pi} (E \{ x_t | \Omega_t^{CB} \} ) + \lambda_3^{\pi} \bar{\pi}_{t-1} + \varepsilon_t^{\pi} \tag{7}
\]

\( \lambda_1^{\pi} \) and \( \lambda_2^{\pi} \) determine the response of the inflation target to the inflation and output expectations of the CB. \( \varepsilon_t^{\pi} \) is the shock to the inflation target, which is normally distributed with mean zero and variance \( \sigma_{\varepsilon_t^{\pi}}^2 \) and is independent over time.

The private-sector agent then observes the interest rate and the inflation target set by the CB. The private-sector agent is assumed to receive no signal about inflation and output.\footnote{The Greenbooks of the Fed contain forecasts of current inflation (gdp deflator) and current output (gdp) as well as forecasts up to six quarters ahead.} She then forms her own expectations about future inflation and output. The expectations of the representative investor are derived in section 2.2.\footnote{Rudebusch (1995) provides empirical evidence on the serial correlation of interest rate changes. Goodfriend (1991) names fear of disruption of financial markets as an explanation for interest rate smoothing, and Sack (1997) mentions uncertainty about the effects of interest rate changes.}\footnote{Rudebusch (1995) provides empirical evidence on the serial correlation of interest rate changes. Goodfriend (1991) names fear of disruption of financial markets as an explanation for interest rate smoothing, and Sack (1997) mentions uncertainty about the effects of interest rate changes.}

\footnote{In Gurkaynak, Sack, and Swanson (2003) the inflation target follows an AR(1) process, where the inflation inflation target depends on the previous inflation target.}\footnote{The Greenbooks of the Fed contain forecasts of current inflation (gdp deflator) and current output (gdp) as well as forecasts up to six quarters ahead.} The model can be modified such that investor also receives a private signal that is different from the signal received by the Fed. In that case, we still have asymmetric information since the signals of the Fed and the representative investor are not the same. Assuming that the investor receives a signal does not change the results as the original model has asymmetric information but makes the analysis more complicated. Here I assume that the investor has no private information for the purpose of simplicity.}

\footnote{Rom and Romer (2000) state that “Our estimates suggest that if they had access to the Federal Reserve’s forecast of inflation, commercial forecasters would find it nearly optimal to discard their forecasts and adopt the Federal Reserve’s.”}

6

\[
 r_t = \lambda_1^r (E \{ \pi_t | \Omega_t^{CB} \} - \bar{\pi}_t) + \lambda_2^r (E \{ x_t | \Omega_t^{CB} \}) + \lambda_3^r r_{t-1} + \epsilon_t^r \tag{6}
\]

\[
 \bar{\pi}_t = \lambda_1^{\pi} (E \{ \pi_t | \Omega_t^{CB} \} ) + \lambda_2^{\pi} (E \{ x_t | \Omega_t^{CB} \} ) + \lambda_3^{\pi} \bar{\pi}_{t-1} + \varepsilon_t^{\pi} \tag{7}
\]
2.2 Expectation Dynamics Under Inflation Targeting

This section focuses on a highly stylized structure to illustrate the hierarchical information structure and the interplay of expectations across agents. I construct a learning model using a Kalman filter algorithm to model the information dynamics between the CB and the private-sector agent as suggested by Townsend (1983). The computation of the equilibrium involves three steps: the definition of an appropriate state space, the solution of the filtering problem for each agent, and the derivation of the expectations of the CB and the private-sector agent.

The equations for the expectations of the CB can be found using a Kalman filtering algorithm. The information set of the CB, $\Omega_{CB}^t$, should be described to able to derive the CB’s expectations.

The CB then observes private signals about autoregressive components of shocks to time $t$ inflation and output at time $t$. So, the observation equation is

\[
\begin{bmatrix}
\pi_t \\
x_t \\
\theta_{t+1}^t
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
x_t \\
\theta_{t+1}^t
\end{bmatrix}
+ \begin{bmatrix}
u_t \\
u_t
\end{bmatrix}
\]

or

\[
z_t = H\zeta_t + \omega_t.
\]

The Kalman filtering algorithm indicates that when a system can be represented in a state-space representation form, the forecast equation is

\[
\hat{z}_{t+1/t+1} = A\hat{\zeta}_{t+1} + BY_t + K_t \left( z_t - H \left( A\hat{\zeta}_{t+1} + BY_t \right) \right)
\]

\[
K_t = P_{t-1} H' (HP_{t-1} H' + R)^{-1}
\]
where

\[ \hat{\zeta}_{t+1/t+1} = E \left( \zeta_{t+1} \mid \Omega_{t+1} \right) \]

\[ P_t = E \left( \left( \zeta_t - \hat{\zeta}_{t/t} \right) \left( \zeta_t - \hat{\zeta}_{t/t} \right) \right) . \]

Equations (8) and (10 form a state-space representation. After application of the Kalman filter algorithm and appropriate substitutions and manipulations, the CB’s expectations about one-period ahead inflation and output can be written as:

\[
E \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ \theta_{t+1}^\pi \\ \theta_{t+1}^x \end{bmatrix} | \Omega_{t+1}^{CB} = (A - K_t H A) E \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} | \Omega_t^{CB} + (B - K_t H B) E \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} | \Omega_t^{PRI} + K_t H \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} + H \begin{bmatrix} u_t^\pi \\ u_t^x \end{bmatrix},
\]

\[ (14) \]

\((A - K_t H A)\) provides the relationship between the current and previous expectations of the CB. \((B - K_t H B)\) relates the expectations of the CB to inflation and output expectations of the private-sector agent and the interest rate. \(K_t H\) delivers the connection of the current expectations of the CB with current inflation, output, current shock to inflation, and output. \(A, H, B\) and \(K\) are defined in equations (8), (10), and (13) respectively. Derivations of equation (12) and \((A - K_t H A)\) and \(K_t\) are displayed in Appendix B. By proposition 13.1 in Hamilton (1994) it can be shown that \(P_t\) converges to some constant \(P\). Thus, the time dimension of \(K_t\) drops out and \(K_t\) converges to some constant \(K\).\(^{12}\) Equation (14) indicates that the CB’s inflation, output, autoregressive components of shocks to inflation, and output expectations are linear combinations of expectations made at the previous period, expectations of the private-sector, interest rate, current inflation, current output, and current autoregressive components of shocks to inflation and output.

Using the result above, I now derive the inflation and output expectations of the private-sector agent. The private-sector agent observes the interest rate and the inflation target which are determined by the CB’s private information about future inflation and output. Thus, the private-sector agent sees a filtered version

---

\(^{12}\)Proposition 13.1 in Hamilton (1994) is as follows:

Let \( F \) be (r\times r) matrix whose eigenvalues are all inside the unit circle, let \( H' \) denote an arbitrary (n\times r) matrix, and let \( Q \) and \( R \) be positive semidefinite symmetric (r\times r) and (n\times n) matrices, respectively. Let \( \{ P_{t+1|t} \}_{t=1}^T \) be the sequence of MSE matrices calculated by the Kalman filter. Then \( \{ P_{t+1|t} \}_{t=1}^T \) is a monotonically nonincreasing sequence and converges as \( T \to \infty \) to a steady-state matrix \( P \). Moreover, the steady-state value for the Kalman gain matrix, defined by

\[ K \equiv FPH (H'PH + R)^{-1} \]

has the property that the eigenvalues of \((F - KH')\) all lie on or inside the unit circle.
of the CB’s private information. The private-sector agent establishes her expectations based on her information set \( \Omega^\text{PRI}_t = \{\pi_{t-1}, \ldots, x_{t-1}, \ldots, r_t, r_{t-1}, \ldots\} \). Similarly, we can use a Kalman filter to obtain the private-sector agent’s expectation equations. The state-space representation of the system faced by the private-sector agent is as follows:

State equation:

\[
\begin{bmatrix}
\pi_{t+1} \\
x_{t+1} \\
\theta^\pi_{t+1} \\
\theta^x_{t+1} \\
\end{bmatrix}
= \begin{bmatrix}
E\{\pi_{t+1} | \Omega^CB_t\} \\
E\{x_{t+1} | \Omega^CB_t\} \\
E\{\theta^\pi_t | \Omega^CB_t\} \\
E\{\theta^x_t | \Omega^CB_t\} \\
\end{bmatrix}
+ \begin{bmatrix}
B^\text{PRI} \cdot E\{\pi_t | \Omega^\text{PRI}_t\} \\
B^\text{PRI} \cdot E\{x_t | \Omega^\text{PRI}_t\} \\
E\{\theta^\pi_{t+2} | \Omega^CB_t\} \\
E\{\theta^x_{t+2} | \Omega^CB_t\} \\
\end{bmatrix}
\]

or

\[
G\zeta_{t+1} = A^\text{PRI} \zeta_t + B^\text{PRI} \Upsilon_t + \nu_{t+1}
\]

where \( \alpha^\pi_{t+1}, \alpha^x_{t+1}, \theta \alpha^\pi_{t+1}, \theta \alpha^x_{t+1} \) are the error terms of the Fed’s expectations. They are defined in equation (14).

\[
G, A^\text{PRI} \text{ and } B^\text{PRI} \text{ are displayed in Appendix C.}
\]

\[
\zeta_{t+1} = G^{-1} A^\text{PRI} \zeta_t + G^{-1} B^\text{PRI} \Upsilon_t + G^{-1} \nu_{t+1}
\]  

(16)

Define \( A^\text{PRI}_G = G^{-1} A^\text{PRI} \) and \( B^\text{PRI}_G = G^{-1} B^\text{PRI} \).

The private-sector agent observes the interest rate and the inflation target, so the observation equation is

\[
\begin{bmatrix}
r_t \\
\pi_t \\
\theta^\pi_t \\
\theta^x_t \\
\end{bmatrix}
= H^\text{PRI}
\begin{bmatrix}
\pi_t \\
x_t \\
\theta^\pi_t \\
\theta^x_t \\
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon^\pi_t \\
\varepsilon^x_t \\
\end{bmatrix}
\]

or

\[
z_t = H^\text{PRI} \zeta_t + \omega_t
\]

\( H^\text{PRI} \) displayed in Appendix C.
Applying the Kalman filter to the state-space representation of equations (15) and (17), we have

\[
E[\zeta_{t+1} | \Omega_{t+1}^{PRI}] = A_G^{PRI} E[\zeta_t | \Omega_{t}^{PRI}] + B_G^{PRI} [\Upsilon_t] + K_t^{PRI} ([z_t] - H^{PRI} A_G^{PRI} E[\zeta_t | \Omega_{t}^{PRI}] - H^{PRI} B_G^{PRI} [\Upsilon_t])
\]

(18)

Rearranging the equation above indicates that:

\[
E[\zeta_{t+1} | \Omega_{t+1}^{PRI}] = (A_G^{PRI} - K_t^{PRI} H^{PRI} A_G^{PRI}) E[\zeta_t | \Omega_{t}^{PRI}] + (B_G^{PRI} - K_t^{PRI} H^{PRI} B_G^{PRI}) [\Upsilon_t] + K_t^{PRI} [z_t]
\]

(19)

where \(K_t^{PRI} \equiv P_{t-1} H^{PRI} (H^{PRI} P_{t-1} H^{PRI} + R)^{-1}\). \(K_t^{PRI}, B^{PRI}, C\) and \(H^{PRI}\) are presented in detail in Appendix C. Equation (20) indicates that the representative private-sector agent’s inflation and output expectations are affected by the CB’s actions, i.e., the interest rate, \(r_t\) and \(r_{t-1}\) the inflation target, \(\bar{\pi}_t\) and \(\bar{\pi}_{t-1}\), \((A_G^{PRI} - K_t^{PRI} H^{PRI} A_G^{PRI})\) provides the relationship between the private-sector agent’s current and previous expectations.

In particular, the following equation can be derived from equation (20):

\[
E\{\pi_{t+1} | \Omega_{t+1}^{PRI}\} = \Pi_1 E\begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^\gamma \\ E\{\pi_t | \Omega_{t}^{CB}\} \\ E\{x_t | \Omega_{t}^{CB}\} \\ E\{\theta_t^\pi | \Omega_{t}^{CB}\} \\ E\{\theta_t^\gamma | \Omega_{t}^{CB}\} \\ r_t - 1 \\ \bar{\pi}_{t-1} \end{bmatrix} + \Pi_2 \begin{bmatrix} r_t \\ \bar{\pi}_t \end{bmatrix}
\]

(20)

where \(\Pi_1\) is the first row of \((A_G^{PRI} - K_t^{PRI} H^{PRI} A_G^{PRI})\). \(\Pi_2\) is the first row of \(K_t^{PRI}\). \(\Pi_1\) and \(\Pi_2\) displays the effects of the CB’s current and previous actions on the private-sector agent’s inflation expectations.

---

13 The first two elements of the first row of \(\Pi_1\) includes \((B_G^{PRI} - K_t^{PRI} H^{PRI} B_G^{PRI})\) since \(E\{\pi_t | \Omega_{t}^{PRI}\}\) and \(E\{x_t | \Omega_{t}^{PRI}\}\) affect \(E\{\Omega_{t+1}^{PRI}\}\) through two separate channels.

---

10
\[
E\{x_{t+1} \mid \Omega^I_{t+1}\} = \Pi_3 E\left[
\begin{array}{c}
\pi_t \\
x_t \\
\theta^x_t \\
\theta^\pi_t \\
E\{\pi_t \mid \Omega^CB_t\} \\
E\{x_t \mid \Omega^CB_t\} \\
E\{\theta^x_t \mid \Omega^CB_t\} \\
E\{\theta^\pi_t \mid \Omega^CB_t\} \\
r_{t-1} \\
\bar{\pi}_{t-1}
\end{array}
\right] + \Pi_4 \left[
\begin{array}{c}
r_t \\
\bar{\pi}_t
\end{array}
\right]
\] (21)

where \(\Pi_3\) is the second row of \((A^PRI - K^PRI H^PRI A^PRI)\). \(\Pi_4\) is the second row of \(K^PRI\). \(\Pi_3\) and \(\Pi_4\) displays the effects of the CB’s current and previous actions on the private-sector agent’s output expectations.

Equations (22) and (23) present the relationship between the private-sector agent’s expectations and the actions of the CB. They identify another transmission mechanism for monetary policy. Besides having direct effects, the changes in the interest rate and the inflation target, implementations of monetary policy affect the expectations of the private-sector agent. Thus, in section 2, I showed that because of the asymmetric information between the CB and the investor, the response of the inflation and output expectations of the investor are different than the symmetric information case. The private-sector agent knows that the CB possesses private information about future inflation and output and updates her expectations after observing the CB’s actions. Equations (22) and (23) display how the investor forms her expectations in this hierarchical information structure.

3 Expectation Dynamics Without Inflation Targeting

When the Central Bank does not follow an inflation target then it only determines the interest rate using its private information. The only signal that the private-sector agent receives is the inflation target. Thus, the expectations of the private-sector agent is determined by the following equation after applying the Kalman filter.

In particular, the following equation can be derived from equation (20):
The equations above are derived using the methodology in the previous section where the observation equation only contains the interest rate.

\begin{align*}
E\{\pi_{t+1} | \Omega^R_{t+1}\} &= \Pi_1 E \begin{bmatrix}
\pi_t \\
x_t \\
\theta_t \\
\theta_t^r \\
r_{t-1}
\end{bmatrix} | \Omega^R_{t+1} + \Pi_2 [r_t] \tag{22} \\
E\{x_{t+1} | \Omega^{I\text{NV}}_{t+1}\} &= \Pi_3 E \begin{bmatrix}
\pi_t \\
x_t \\
\theta_t \\
\theta_t^r \\
r_{t-1}
\end{bmatrix} | \Omega^{I\text{NV}}_{t+1} + \Pi_4 [r_t] \tag{23}
\end{align*}

The equations above are derived using the methodology in the previous section where the observation equation only contains the interest rate.

\section*{4 Time Series Dynamics Under Inflation Targeting}

In this section, I examine the implications of the theoretical results by representing the economy in a vector autoregression (VAR) format. The VAR format allows us to use impulse response functions. I examine the response of the system from an initial steady state to a positive, one-unit increase in the specified variable’s innovation at time $t$, i.e. the impulse response of the system to a specified shock. The cases with inflation targeting and without inflation targeting are examined and compared. The symmetric information case is also investigated.

\subsection*{4.1 Asymmetric Information Case}

The asymmetric information case is described in section 2. Using equations 1, 2, 3, 4, 6, 7, 14 and 20 the economic dynamics of the asymmetric information
case can be displayed as the following:

\[
\begin{bmatrix}
\pi_{t+1} \\
x_{t+1} \\
\theta^P_{t+1} \\
\theta^V_{t+1} \\
E\{\pi_{t+1} | \Omega^{PRI}_{t+1}\} \\
E\{x_{t+1} | \Omega^{PRI}_{t+1}\} \\
E\{\theta^P_{t+1} | \Omega^{PRI}_{t+1}\} \\
E\{\theta^V_{t+1} | \Omega^{PRI}_{t+1}\} \\
E\{E\{\pi_{t+1} | \Omega^{CB}_{t+1} \} | \Omega^{PRI}_{t+1}\} \\
E\{E\{x_{t+1} | \Omega^{CB}_{t+1} \} | \Omega^{PRI}_{t+1}\} \\
E\{E\{\theta^P_{t+1} | \Omega^{CB}_{t+1} \} | \Omega^{PRI}_{t+1}\} \\
E\{E\{\theta^V_{t+1} | \Omega^{CB}_{t+1} \} | \Omega^{PRI}_{t+1}\} \\
E\{r_t | \Omega^{PRI}_{t+1}\} \\
E\{\pi_t | \Omega^{PRI}_{t}\} \\
E\{x_t | \Omega^{CB}_{t}\} \\
E\{\theta^P_t | \Omega^{CB}_{t}\} \\
E\{\theta^V_t | \Omega^{CB}_{t}\} \\
r_{t-1} \\
\bar{\pi}_{t-1}
\end{bmatrix} = \begin{bmatrix}
\pi_t \\
x_t \\
\theta^P_t \\
\theta^V_t \\
E\{\pi_t | \Omega^{PRI}_{t}\} \\
E\{x_t | \Omega^{PRI}_{t}\} \\
E\{\theta^P_t | \Omega^{PRI}_{t}\} \\
E\{\theta^V_t | \Omega^{PRI}_{t}\} \\
E\{E\{\pi_t | \Omega^{CB}_{t}\} | \Omega^{PRI}_{t}\} \\
E\{E\{x_t | \Omega^{CB}_{t}\} | \Omega^{PRI}_{t}\} \\
E\{E\{\theta^P_t | \Omega^{CB}_{t}\} | \Omega^{PRI}_{t}\} \\
E\{E\{\theta^V_t | \Omega^{CB}_{t}\} | \Omega^{PRI}_{t}\} \\
r_{t-1} \\
\bar{\pi}_{t-1}
\end{bmatrix} + \text{error}_{t+1}
\] (24)

where \(A^{asy}\) is the companion matrix, \(\text{error}_{t+1}\) contains the error terms. \(T^{asy}\) and \(A^{asy}\) are described in Appendix D. Equation (24) presents the hierarchical system of asymmetric information. The recursive manner in which this hierarchical system is determined is the following: the changes in the Fed’s actions affect the expectations of the private-sector agent, and inflation are in turn affected by the Fed’s actions partly because of this change in the private-sector agent’s expectations.

### 4.2 Symmetric Information Case:

In order to examine the effects of the asymmetric information between the CB and the representative investor, we need a benchmark symmetric information case for comparison.

For the symmetric information case, I assume that both the CB and the investor receive exactly the same signal about autoregressive components of shocks to inflation and output at time \(t\).

\[
\begin{align*}
\pi_t \theta^{CB, PRI}_t &= \theta^x_t + u^x_t \\
x_t \theta^{CB, PRI}_t &= \theta^x_t + u^x_t
\end{align*}
\] (25)

\(\pi_t \theta^{CB, PRI}_t \) and \(x_t \theta^{CB, PRI}_t\) are the signals received by both the CB and the investor about autoregressive components of shocks to time \(t\) inflation and output,
respectively. The CB and representative investor receive exactly the same signal, so there is no learning dynamics and the interest rate does not have an effect on the investor’s expectations. Following the algorithm in section 2, the expectations of the CB and the investor can be determined as the following by using the Kalman filtering algorithm:

\[
E \left[ \begin{array}{c}
\pi_{t+1} \\
x_{t+1} \\
\theta_{t+1}^{\pi} \\
\theta_{t+1}^{x}
\end{array} \bigg| \Omega_{t+1}^{CB,PRI} \right] = (A - K_t H A) E \left[ \begin{array}{c}
\pi_t \\
x_t \\
\theta_{t}^{\pi} \\
\theta_{t}^{x}
\end{array} \bigg| \Omega_{t}^{CB,PRI} \right] + (B - K_t H B) E \left[ \begin{array}{c}
\pi_t \\
x_t \\
\theta_{t}^{\pi} \\
\theta_{t}^{x}
\end{array} \bigg| \Omega_{t}^{PRI} \right]
\]

+ \[K_t \left\{ \begin{array}{c}
\pi_t \\
x_t \\
\theta_{t}^{\pi} \\
\theta_{t}^{x}
\end{array} \bigg[ \begin{array}{c}
u_t^\pi \\
u_t^x
\end{array} \right] \bigg]. \tag{26}
\]

The economic dynamics can be displayed using equations 1, 2, 4, 6, and (26) as,

\[
T_{sym} = [A^{sym}] + [error_{t+1}] \tag{27}
\]

where \(A^{sym}\) is the companion matrix, \(error_t\) contains the error terms. \(T_{sym}\) and \(A^{sym}\) are described in Appendix D. Equation (27) presents the system of symmetric information.

5 Time Series Dynamics Without Inflation Targeting

This section represents the economy without inflation targeting. The Central Bank only determines the interest rate using its private information thus this case can be interpreted as less transparent than the case with inflation targeting since there are less signals that the private-sector agent can use to deduce the CB’s private information. This case also can be interpreted as the case of a
constant inflation target. Time series dynamics without inflation targeting is used as the benchmark case.

\[ TW^{asy} \text{ and } AW^{asy} \text{ are described in Appendix D.} \]

6 Impulse Response Functions

As mentioned above, equations 24, 27 and 28 are in the form of a first-order VAR. This VAR format provides a convenient way of examining the response of the system from an initial steady state to a positive, one-standard-deviation impulse in specified economic shocks at date 1. The impulse response functions can be defined as the following:

\[ \frac{\partial z_{t+s}}{\partial w_t} = (A)^s \]

where \( A \) is the companion matrix, \( z_{t+s} \) is \( s \)-period ahead variable, and \( w_t \) is shock to \( z_t \). The derivation and properties of impulse responses are explained in Appendix D.

To examine the implications of the model, I need the parameter values to use impulse responses. The model contains both observable (inflation, output, etc.) and unobservable (shocks, private signals, etc.) components. I maintain the parameters of the observables from previous studies\(^{14}\) and I choose values

\(^{14}\)The parameters are taken from Rudebusch(2002), Gurkaynak, Sack, Swanson(2005) and Clarida, Gali and Gertler(2000)
for the parameters of the unobservables to calibrate the impulse response functions. Alternatively, all the parameters of the model could be jointly estimated using maximum likelihood estimation. But I leave that to future research, since the analysis here is intended to illustrate the main findings of the model and present the effects of asymmetric information graphically by comparing impulse responses of asymmetric and symmetric information cases. The table presents the values used to calculate the impulse responses.\textsuperscript{15}\textsuperscript{16}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{11}$</td>
<td>.47</td>
<td>$\beta_{23}$</td>
<td>.09</td>
<td>$\lambda_2^e$</td>
<td>.93</td>
<td>$\pi\sigma_u^e$</td>
<td>.9</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>.13</td>
<td>$\beta_{24}$</td>
<td>.3</td>
<td>$\lambda_3^e$</td>
<td>.1</td>
<td>$x\sigma_u^e$</td>
<td>.9</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>.9</td>
<td>$\beta_{25}$</td>
<td>-.09</td>
<td>$\pi\sigma_z^e$</td>
<td>.8</td>
<td>$r\sigma_u^e$</td>
<td>.8</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>0</td>
<td>$\rho$</td>
<td>.9</td>
<td>$x\sigma_z^e$</td>
<td>.8</td>
<td>$\lambda_1^e$</td>
<td>.3</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>0</td>
<td>$\rho^x$</td>
<td>.9</td>
<td>$\pi\sigma_z^e$</td>
<td>.8</td>
<td>$\lambda_2^e$</td>
<td>.13</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>.8</td>
<td>$\lambda_1^e$</td>
<td>2.15</td>
<td>$x\sigma_z^e$</td>
<td>.8</td>
<td>$\lambda_2^e$</td>
<td>.1</td>
</tr>
</tbody>
</table>

Figure I displays the response of the private-sector agent’s inflation and output expectations to a one-unit shock to the interest rate under inflation targeting and without inflation targeting.

(Figure I about here.)

The first main result of the paper can easily be seen from figure I. In the inflation targeting case which is displayed by the straight line, the positive response of the inflation expectation of the private-sector agent is less than the non-inflation targeting case. After the third period the effect of the interest rate is negative since the signalling effect wades away and the interest rate effects inflation negatively. After the third period, when the effect of the interest rate is negative, inflation expectation of the private-sector agent responds more the a change in the interest rate. Thus, when we consider the asymmetric information between the CB and the public and the learning dynamics between the two, inflation targeting increases the transparency of the CB by revealing more of the CB’s private information. This causes the interest rate to be more effective since the positive effect of the interest rate is lower in the inflation targeting case. As explained before, the positive effect is caused by the fact that the private-sector agents know that the CB possesses private information and that motivates the private-sector agents to follow the CB’s actions and revise her expectations. An increase (decrease) in the interest rate means that CB expects inflation and output to be higher (lower) in the future. The effect of the interest rate on the output expectation of the private-sector agent is similar in inflation targeting and non-inflation targeting cases. The reason for this result is the interest rate immediately and directly affects output. Similar to inflation

\textsuperscript{15}I examined the effects of different parameter choices on the results. Except for the extreme values the main results of the paper are not affected by the choice of parameter values. Thus, the results of the paper are robust to parameter selection. The results with alternative parameters are available from the author upon request.

\textsuperscript{16}Variances of the error terms of the Fed’s expectations in equation (14) are calculated using the values in the table.
expectations, output expectation responds positively since the change in the interest rate signals that the CB possesses private information that output will be higher next period.

(Figure II about here.)

Figure II displays the response of the private-sector agent’s inflation and output expectations to a one-unit shock to the inflation target. Similar to the interest rate, an increase in the inflation target signals that the CB possesses private information that inflation and output will be higher next period. Thus, inflation and output expectations of the private-sector agent first respond positively to a rise in the inflation target. After a certain period, the effect is negative since a higher inflation target means that the CB will increase the interest rate which will decrease inflation and output.

(Figure III about here.)

Figure III displays the main argument and most important result of this paper. Figure III presents the response of inflation to a one-unit shock to the interest rate under inflation targeting and non-inflation targeting. The response of inflation to a one-unit shock to the interest rate is higher in the inflation targeting case. Inflation decreases more under inflation targeting when the interest rate increases. Thus, figure III presents that under inflation targeting, interest rate is much more effective for decreasing inflation than non-inflation targeting.

The different response of inflation can be explained as the following. In the inflation targeting case there are two different signals that the private-sector agent observes and uses to deduce the private information of the CB about future inflation and output. As displayed in figure I, the signalling effect of the interest rate on the inflation expectation of the public is less in the inflation targeting case. Inflation target provides more transparency and this causes the interest rate to be more effective in the inflation targeting case. As a result, this analysis identifies another mechanism for the inflation target, namely, the effect of monetary policy on the expectations of the public and shows that the success of the inflation targeting regimes might partly be caused by the transparency provided by the inflation target when there is asymmetric information between the CB and the public.

(Figure IV about here.)

Figure IV displays the response of inflation to change in the interest rate under asymmetric and symmetric information. Figure IV shows that inflation responds to changes in the interest rate differently under asymmetric information and symmetric information cases. In the asymmetric information case inflation reacts less to an increase in the interest rate, and in the symmetric information case inflation immediately decreases when the interest rate increases.

7 Conclusion

This paper theoretically investigates inflation targeting when there is asymmetric information between the Central Bank and the public. The analysis shows
that interest rate is more effective in decreasing inflation under inflation targeting when there is asymmetric information. The Central Bank is assumed to possess private information about future inflation and output. I construct a model of asymmetric information and learning in order to explore the effects of monetary policy on expectations of the public and inflation. I use a Kalman filtering algorithm as suggested by Townsend (1983) to analyze the learning dynamics and hierarchical information structure between the CB and a representative private-sector agent. The findings of the model show that an increase in the interest rate and inflation target signals that the CB has private information that inflation and output will be higher in the future. Thus, the public expect inflation to be higher in the future. Under inflation targeting transparency is higher in the sense that the private-sector agent observes both the interest rate and inflation target. Interest rate and inflation target are signals that the private-sector agent uses to deduce the CB’s private information.

I calibrate the theoretical model and draw impulse responses for the inflation targeting, non-inflation targeting and symmetric information cases. The comparison of the impulse responses show the following main results of the paper. First, the public revise their expectations about future inflation and output after observing the actions of the CB. In the model, the public know that the CB possesses private information and that motivates the public to follow the Fed’s actions and revise their expectations. In the inflation targeting case, the positive response of the the inflation expectation of the private-sector agent is less than the non-inflation targeting case. Second, the response of inflation is higher in the inflation targeting case to a one-unit shock to the interest rate. Inflation decreases more under inflation targeting when the interest rate increases. Thus, under inflation targeting, interest rate is much more effective for decreasing inflation than non-inflation targeting.

The results of this paper have many policy implications especially for central bank transparency and inflation targeting. These results suggest that the secrecy of the CB might decrease the effectiveness of the monetary policy when the CB’s objective is to decrease inflation. Also, in the framework of this paper, the inflation target of the central bank can be seen as another mechanism of signaling since the central bank would use its private information to determine the inflation target. This paper proposes that the policymaker should take into account the effects of monetary policy on the expectations of the public.

As a result, this paper takes a novel theoretical approach to better understand and investigate inflation targeting under asymmetric information. By analyzing the hierarchical information structure between the CB and a representative private-sector agent, I show that the monetary policy is more effective under inflation targeting when there is asymmetric information between the CB and the public.

Appendix

A Overview of the Kalman Filter Algorithm
The Kalman filter is an algorithm for sequentially updating a linear projection for a dynamic system which is expressed in state-space representation. A more complete exposition of the Kalman filter algorithm can be found in Hamilton (1994), chapter 13.

A.1 The State-Space Representation of a Dynamic System

The Kalman filter addresses the general problem of trying to estimate the state $\zeta$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

- **State Equation:**
  \[
  \zeta_{t+1} = A\zeta_t + BY_t + \nu_{t+1}
  \]
  $\zeta_t = (r \times 1)$ state vector  
  $Y_t = (r \times 1)$ optional control input  
  with an observation $z$.

- **Observation Equation:**
  \[
  z_t = H\zeta_t + Cs_t + \omega_t
  \]
  $z_t = (n \times 1)$ vector of variables observed at time $t$.  
  $s_t = (k \times 1)$ vector of exogenous or predetermined variables.

$F, A'$, and $H'$ are matrices of parameters of dimension $(r \times r)$, $(n \times k)$, and $(n \times r)$, respectively. The $(r \times 1)$ vector $\nu_t$ and the $(n \times 1)$ vector $\omega_t$ are vector white noise. $\nu_t$ and $\omega_t$ are assumed to be independent of each other and with normal probability distributions

\[
 p(v) \sim N(0, Q) \\
p(w) \sim N(0, R)
\]

A.2 Forecast Equation of the Kalman Filter

When the system can be expressed in state space representation as in A.1, the Kalman filter delivers the following forecast equation after many calculations and manipulations:
\[ \hat{\zeta}_{t+1/t+1} = A\hat{\zeta}_{t/t} + BY_t + K_t \left( z_t - H \left( A\hat{\zeta}_{t/t} + BT_t \right) \right) \]

\[ K_t \equiv P_t^{-1} H' \left( HQ_{t-1}H' + R \right)^{-1} \]

where

\[ \hat{\zeta}_{t+1/t+1} = E \left( \hat{\zeta}_{t+1}/\Omega_{t+1} \right) \]

\[ P_t = E \left( \hat{\zeta}_t - \hat{\zeta}_{t/t} \right) \left( \hat{\zeta}_t - \hat{\zeta}_{t/t} \right)' \]

### B Derivation of Eq(14)

The equations (7) and (9) form a state-space representation.

Equation (7) is the corresponding state equation

\[
\begin{bmatrix}
\pi_{t+1} \\ x_{t+1} \\ \theta^e_{t+1} \\
\theta^x_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\beta_{11} & \beta_{12} & \rho^x & 0 \\ 
\beta_{21} & \beta_{22} & 0 & \rho^x \\ 
0 & 0 & \rho^x & 0 \\
0 & 0 & 0 & \rho^x
\end{bmatrix}
\begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} +
\begin{bmatrix}
\beta_{13} & \beta_{14} & 0 \\ 
\beta_{23} & \beta_{24} & \beta_{25} \\ 
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
E\{\pi_t | \Omega^{PRI}_t\} \\ E\{x_t | \Omega^{PRI}_t\} \\ r_t
\end{bmatrix} +
\begin{bmatrix}
e^e_{t+1} \\ e^x_{t+1} \\
e^e_{t} \\
e^x_{t}
\end{bmatrix}
\]

(30)

The observation equation (eq. (9)) is

\[
\begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\ 
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} +
\begin{bmatrix}
u^e_t \\ u^x_t
\end{bmatrix}
\]

(31)

The forecast equation, eq. (11) of the Kalman filter, indicates that the Fed’s expectations can be displayed as the following:

\[
\begin{bmatrix}
\pi_{t+1} \\ x_{t+1} \\ \theta^e_{t+1} \\
\theta^x_{t+1}
\end{bmatrix} | \Omega^{FED}_{t+1} = AE
\begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} | \Omega^{FED}_t + B \begin{bmatrix}
E\{\pi_t | \Omega^{PRI}_t\} \\ E\{x_t | \Omega^{PRI}_t\} \\ r_t
\end{bmatrix}
\]

\[ + K_t \left( \begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} - HA \begin{bmatrix}
\pi_t \\ x_t \\ \theta^e_t \\
\theta^x_t
\end{bmatrix} | \Omega^{FED}_t \right) - HB \begin{bmatrix}
E\{\pi_t | \Omega^{PRI}_t\} \\ E\{x_t | \Omega^{PRI}_t\} \\ r_t
\end{bmatrix}
\]

(32)

Substituting (10) into (32) yields
\[ E \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ \theta_{t+1}^\pi \\ \theta_{t+1}^x \end{bmatrix} | \Omega_{t+1}^{FED} = AE \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} | \Omega_t^{FED} + B \begin{bmatrix} E\{\pi_t | \Omega_t^{PRI}\} \\ E\{x_t | \Omega_t^{PRI}\} \end{bmatrix} \]

\[ + K_t \left\{ \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} + \begin{bmatrix} u_t^\pi \\ u_t^x \end{bmatrix} \right\} - HAE \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} | \Omega_t^{FED} - HB \begin{bmatrix} E\{\pi_t | \Omega_t^{PRI}\} \\ E\{x_t | \Omega_t^{PRI}\} \end{bmatrix} \right\} \].

We can arrange this as,

\[ E \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ \theta_{t+1}^\pi \\ \theta_{t+1}^x \end{bmatrix} | \Omega_{t+1}^{FED} = (A - K_t HA) \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} | \Omega_t^{FED} + (B - K_t HB) \begin{bmatrix} E\{\pi_t | \Omega_t^{PRI}\} \\ E\{x_t | \Omega_t^{PRI}\} \end{bmatrix} \]

\[ + K_t \left\{ H \begin{bmatrix} \pi_t \\ x_t \\ \theta_t^\pi \\ \theta_t^x \end{bmatrix} + \begin{bmatrix} u_t^\pi \\ u_t^x \end{bmatrix} \right\} \].

where

\[ K_t = P_{t-1} H' (H P_{t-1} H' + R)^{-1} \]
\[ P_t = E \left( \begin{bmatrix} \zeta_t - \hat{\zeta}_{t/t} \\ \hat{\zeta}_t - \zeta_t \end{bmatrix} \right)^\top \left( \begin{bmatrix} \zeta_t - \hat{\zeta}_{t/t} \\ \hat{\zeta}_t - \zeta_t \end{bmatrix} \right) \]

C \[ A^{PRI}, K^{PRI}, B^{PRI}, H^{PRI} \]

Appendix C displays \( G, A^{PRI}, K^{PRI}, B^{PRI}, H^{PRI} \) and \( H^{PRI} \).

\[ G = \begin{bmatrix} 1 & 0^{1 \times 9} \\ 0 & 1 & 0^{1 \times 6} & -\beta_{25} & 0 \\ 0 & 1^{x 2} & 1 & 0^{1 \times 7} \\ 0 & 1^{x 3} & 1 & 0^{1 \times 6} \\ 0^{4 \times 4} & I^{4 \times 4} & -(B - KHB) & 0^{4 \times 1} \\ 0^{1 \times 8} & 1 & \lambda_1^t \\ 0^{1 \times 9} & 1 \end{bmatrix} \]
\[
A^{INV} = \begin{bmatrix}
A & 0^{4x6} \\
KH (A - KHA) & 0^{4x2} \\
0^{1x4} & \lambda_1^r & \lambda_2^r & 0^{1x2} & \lambda_3^r & 0^{1x2} & 0^{1x3} & \lambda_3^\pi
\end{bmatrix}
\]

A is the 4x4 matrix from section 2. (A - KHA) and KtH are 4x4 matrices derived in Appendix B. 0^{4x4} is a 4x4 partition matrix with zero as its elements. I^{4x4} is the 4x4 identity matrix.

\[
B^{INV} = \begin{bmatrix}
B(:,1:2) \\
(B - KHB)(;1:2) \\
0^{2x2}
\end{bmatrix}
\]

B(:,1:2) contains all the rows and columns 1 and 2 of B. (B - KHB)(;1:2) contains all the rows and columns 1 and 2 of (B - KHB).

\[
K_{PRI}^{PRI} = P_{t-1}^{PRI} H^{PRI} (H^{PRI} P_{t-1}^{PRI} + R)^{-1}
\]

\[
H^{INV} = \begin{bmatrix}
0^{1x4} & \lambda_1^r & \lambda_2^r & 0^{1x2} & \lambda_3^r & 0^{1x2} & 0^{1x3} & \lambda_3^\pi
\end{bmatrix}
\]

D Time Series Dynamics (T^{asy}, T^{sym}, TW^{sym}, A^{asy}, A^{sym}, AW^{sym})

D.1 Case With Inflation Targeting

In section 4 and 5, I used T^{asy}, T^{sym}, A^{asy} and A^{sym} to derive the impulse responses. T^{asy}, T^{sym}, A^{asy} and A^{sym} are defined as the following:
\[ T_{\text{asy}} = \begin{bmatrix}
1 & 0^{1x19} \\
0 & 1 & 0^{1x16} & -\beta_{25} & 0 \\
0 & 1^{1x2} & 1 & 0^{1x17} \\
0 & 1^{1x3} & 1 & 0^{1x16} \\
0^{10x4} & I^{10x10} & 0^{10x14} & -K_{PRI}^{PRI} \\
0^{4x14} & I^{4x4} & -(B - KHB) (:, 3) & 0^{4x1} \\
0^{1x18} & 1 & 1^{r} \\
0^{1x19} & 1
\end{bmatrix} \]

\[ A_{\text{asy}} = \begin{bmatrix}
A & B (:, 1:2) & 0^{4x14} \\
0^{8x4} & (E (:, 1:2) + M) & E (:, 3:10) & 0^{10x6} \\
0^{1x14} & \lambda_{1}^{r} & \lambda_{2}^{r} & 0^{1x2} & \lambda_{3}^{r} & 0 \\
0^{1x14} & \lambda_{1}^{\pi} & \lambda_{2}^{\pi} & 0^{1x3} & \lambda_{3}^{\pi}
\end{bmatrix} \] (35)

where \( E = A_{PRI}^{PRI} - K_{PRI}^{PRI} H_{PRI}^{PRI} A_{PRI}^{PRI} \) and \( M = B_{PRI}^{PRI} - K_{PRI}^{PRI} H_{PRI}^{PRI} B_{PRI}^{PRI} \) and \( Z = B - KHB \) and \( S = A - KHA \). \( E (:, 3:10) \) is the matrix that contains all the columns from the third to the tenth column of the \( E \) matrix.

\( A_{\text{asy}} \) is the companion matrix of the asymmetric information case. \( A^{PRI}, K^{PRI}, B^{PRI} \) and \( H^{PRI} \) are defined as in Appendix C. \( 0^{nxr} \) is nxr zero matrix.

\[ T_{\text{sym}} = \begin{bmatrix}
1 & 0^{1x12} \\
0 & 1 & 0^{1x10} & -\beta_{25} \\
0 & 1^{1x2} & 1 & 0^{1x10} \\
0 & 1^{1x3} & 1 & 0^{1x9} \\
0^{4x4} & I^{4x4} & 0^{4x4} & -(B - KHB) (:, 3) \\
0^{4x8} & I^{4x4} & -(B - KHB) (:, 3) & 0^{1x12} \\
0^{1x12} & 1
\end{bmatrix} \]
where $E^{sym} = A - KHA$ and $Z = B - KHB$ and $S = A - KHA$. $E(:, ; 3 : 10)$ is the matrix that contains all the columns from the third to the tenth column of the $E$ matrix.

$A^{sym}$ is the companion matrix of the symmetric information case. $A, K, B$ and $H$ are defined as in Section 2. $0^{nxr}$ is $nxr$ zero matrix.

$A^{sym} = \begin{bmatrix}
A & B(:, 1 : 2) & 0^{4x7} \\
KH & (E^{sym}(:, 1 : 2) + Z(:, 1 : 2)) & E^{sym}(:, 3 : 4) & 0^{4x5} \\
KH & Z(:, 1 : 2) & 0^{4x2} & E^{sym} & 0^{4x1} \\
0^{1x8} & \lambda_1^r & \lambda_2^r & 0^{1x2} & \lambda_3^r
\end{bmatrix}$. (36)

$TW^{asy} = \begin{bmatrix}
1 & 0^{1x17} \\
0 & 1 & 0^{1x15} & -\beta_25 \\
0 & 1 & 0^{1x2} & 1 & 0^{1x15} \\
0 & 1 & 0^{1x3} & 1 & 0^{1x14} \\
0^{9x4} & I^{9x9} & 0^{19x4} & -K_G^{PRI} \\
0^{4x13} & I^{4x4} & -(B - KHB)(:, 3) \\
0^{1x17} & 1
\end{bmatrix}$

$AW^{asy} = \begin{bmatrix}
A & B(:, 1 : 2) & 0^{4x12} \\
0^{9x4} & (E(:, 1 : 2) + M) & E(:, 3 : 10) & 0^{9x5} \\
KH & Z(:, 1 : 2) & 0^{4x7} & S & 0^{4x1} \\
0^{1x13} & \lambda_1^r & \lambda_2^r & 0^{1x2} & \lambda_3^r
\end{bmatrix}$

### E Impulse Response Functions

The impulse response functions can be defined as the following. Following the display above the dynamics of this system can be written as a first-order VAR:

$$z_t = Az_{t-1} + w_t$$

It is well known that an AR(1) equation like equations (18) and (20) can be written as an infinite MA process:
\[ z_t = A(L)z_t + w_t \]

\[ [I - A(L)] z_t = w_t \]

\[
\begin{align*}
z_t &= [I - A(L)]^{-1} w_t \\
&= \sum_{i=0}^{\infty} (A(L))^i w_t \\
&= \sum_{i=0}^{\infty} (A)^i w_{t-i} \\
&= w_t + Aw_{t-1} + (A)^2 w_{t-2} \ldots
\end{align*}
\]

Thus, we can derive the impulse response functions, which gives us the consequences of a one-unit increase in one of the variables on another variable. Therefore,

\[
\frac{\partial z_{t+i}}{\partial w_t} = (A)^i .
\]

These properties of the impulse response functions make them an ideal methodology to analyze the results of the Kalman filter algorithm and to analyze the dynamics of our system.

References


Figure I
Response of Inflation and Output Expectations of the Public to a Shock to the Interest Rate
Under Inflation Targeting and Non-Inflation Targeting Cases
Figure II
Response of Inflation and Output Expectations of the Public to a Shock to the Inflation Target
Figure III:
Response of Inflation to a Shock to the Interest Rate Under Inflation Targeting and Non-Inflation Targeting Cases
Figure IV:
Response of Inflation to a Shock to the Interest Rate Under Inflation Targeting and Under Symmetric Information Case