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Abstract

This paper is about instability of equilibrium real rents in a pure exchange economy. Our main result hinges upon a low degree of substitutability between housing services and other commodities together with a low share of owner-occupied houses. We argue that regions or economies with a low share of owner-occupied houses are more prone to housing bubbles that are driven by locally unstable rent dynamics.

Keywords: Housing bubbles; rents; stability of equilibrium; CES utility; Leontief utility

JEL classification: C62; D11; D51; R21; R31

1 Introduction

Although housing constitutes the biggest asset class, there is a surprisingly low number of general equilibrium asset pricing models that pay specific attention to housing.[1] Apart from being an important asset class, housing differs from many other commodities as its price indices are observed to exhibit boom and bust episodes. Although the term 'bubble' has frequently been used to describe such phenomena, there are no existing theories that approach the issue from the perspective of (in)stability of competitive equilibrium. This paper is a theoretical attempt in that direction.

As for housing services, we focus only on pure dwelling services, i.e. what the rent is paid for, apart from costs of utilities like heating, water, electricity etc. The first natural assumption then is that of a low degree of substitutability between housing services and other commodities like food and energy. But then we know from the pioneering results on the stability of general competitive equilibrium that the key assumption of 'gross substitutability' (Arrow and Hurwicz, 1958; Hahn, 1958) between all commodities is likely to be violated by the presence of housing services. This observation leaves the

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[1] Piazzessi et al. (2007) is a notable exception.

room open for the presence of unstable equilibria in models where housing is modelled separately.

Evidence against a low degree of substitution is poor. Expenditure shares of broadly defined housing services are observed to move in the same direction as real relative rents in Piazzessi et al. (2007). This indicates a low degree of substitution. Likewise, their post-war (WWII) estimates also point to a low elasticity of substitution, although in their paper they assume a high degree of substitution to ensure the uniqueness and stability of their equilibrium. There are no studies to our knowledge that directly estimate the elasticity of substitution of pure dwelling services. Ogaki and Reinhart (1998) report however that when services (including housing services) are included, elasticity estimates between durable and non-durable goods shrink substantially.

As far as housing prices are concerned, assumptions about the present level and future growth prospects of rents are among the key determinants (see Smith and Smith, 2006). That is why any satisfactory model of housing prices needs to focus on the determination and dynamics of rental rates, which is what this paper is about.

The most frequently cited and studied regions for bubble price dynamics in the US are regions like California, Boston, Chicago, Los Angeles County, and Manhattan (Smith and Smith, 2006; Shiller, 1990). These share the common characteristics of being tourist and student destinations as well as having high job turnover, together with legal or physical restrictions on supply of houses. Similar housing price behavior is reported and studied for countries with similar characteristics like Ireland and UK (Kenny, 1999; Black et al., 2006). The natural consequence of these characteristics would be a high share of houses offered for rent in these regions.

Another basic reason for heterogeneity in home ownership is the life cycle. Campbell and Cocco (2007) provide evidence that old and young people tend to be long and short in housing, respectively. Such heterogeneity comes together with a strong income effect for home owners who have a significant degree of rent income. Empirical work trying to identify a long term housing demand function finds a positive price coefficient in the unrestricted regression where income and price are obviously multicollinear in case of high endowment heterogeneity in housing (Kenny, 1999). Here, in this paper, we elaborate on the theoretical possibility of a positively sloped aggregate demand function for housing services when there are strong income effects due to endowment heterogeneity.

The organization of the paper is as follows. Section 2 introduces the model and Section 3 presents our results.

2 Model

We consider a simple exchange economy involving two consumers, indexed by $i = 1, 2$. There are two goods, indexed by $g = b, h$ with b and h denoting bread and housing (services), respectively. Let w_i^g denote the endowment of good g owned by consumer i. We assume that good h is initially endowed by consumer 1, while good b by consumer 2, solely; i.e., $w_1^b = 0$, $w_1^h > 0$, $w_2^b > 0$, and $w_2^h = 0$. Clearly, the aggregate supply of good

b is w_2^b and that of good h is w_1^h.

Let c_i^g denote the consumption of good g by consumer i, who has the utility function of the CES form

$$U_i(c_i^b, c_i^h) = [(c_i^b)^{1-1/\epsilon_i} + (\alpha_i c_i^h)^{1-1/\epsilon_i}]^{1/(1-1/\epsilon_i)},$$

where $\epsilon_i \in [0, 1)$ is the elasticity of substitution and $\alpha_i > 0$.

We normalize the price of good b to 1, and denote by r the relative real rental rate of good h. The choice problem of consumer i is given by

$$\max_{c_i^b, c_i^h} U_i(c_i^b, c_i^h)$$

$$\text{s.t.} \quad c_i^b + rc_i^h = w_i^b + rw_i^h.$$

We denote the solution by $\langle c_i^b(r), c_i^h(r) \rangle$ for a given rental rate r.

Definition 1. Consumers have *endowment bias* if $c_1^b(r)/c_1^h(r) < w_2^b/w_1^h < c_2^b(r)/c_2^h(r)$ for any positive r.

Consumers with endowment bias plan to consume relatively more from the endowed good at each positive rental rate.[2]

3 Results

3.1 The case of no substitution ($\epsilon_i = 0$)

When there is no substitution between bread and housing for the consumers, the utility functions become a Leontief function

$$U_i(c_i^b, c_i^h) = \min\{c_i^b, \alpha_i c_i^h\}$$

for each i.

It is clear that an interior solution must satisfy

$$c_i^b(r) = \alpha_i c_i^h(r) \tag{1}$$

for consumer i.

When $r = 0$, we have corner solutions, requiring $c_1^h(0) \geq 0$ and $c_2^h(0) \geq w_2^b/\alpha_2$.

Using the budget constraints, we find the quantity of good h demanded by consumers 1 and 2 as

$$c_1^h(r) \in \begin{cases} \{\frac{r}{\alpha_1 + r} w_1^h\} & \text{if } r > 0 \\ [0, \infty) & \text{if } r = 0 \end{cases} \tag{2}$$

[2]Endowment bias is used as in the theory of finance, in the sense of attributing a higher value to an asset due to one's ownership of it (Kahneman et al., 1991). An alternative interpretation of endowment bias in our model would be the presence of transaction costs in trade.

and

$$c_2^h(r) \in \begin{cases} \{\frac{1}{\alpha_2+r} w_2^b\} & \text{if } r > 0 \\ [w_2^b/\alpha_2, \infty) & \text{if } r = 0 \end{cases} \tag{3}$$

respectively. The aggregate demand for good h at $r \geq 0$ becomes

$$D^h(r) = c_1^h(r) + c_2^h(r) \in \begin{cases} \{\frac{r}{\alpha_1+r} w_1^h + \frac{1}{\alpha_2+r} w_2^b\} & \text{if } r > 0 \\ [w_2^b/\alpha_2, \infty) & \text{if } r = 0. \end{cases} \tag{4}$$

To characterize the equilibrium of the economy, we restrict ourselves, by Walras Law, to the clearing of the housing market. The aggregate supply of good h is w_1^h whereas the aggregate demand for it is $D^h(r)$ at the rental rate r. Equating the two, we solve for the equilibrium rent.

We immediately observe that $r = 0$ supports equilibrium if and only if $w_2^b/\alpha_2 \leq w_1^h$. Apparently, this is the case if consumers have endowment bias, since with Leontief utility functions condition in Definition 1 reduces to $\alpha_1 < w_2^b/w_1^h < \alpha_2$.

Now, equating $D^h(r^e)$ and w_1^h over $r^e > 0$, we get

$$r^e = \alpha_1 \frac{\alpha_2 w_1^h - w_2^b}{w_2^b - \alpha_1 w_1^h}. \tag{5}$$

Proposition 1. *Equilibrium with a positive rental rate (i) exists if consumers have endowment bias, and (ii) is always unique, whenever it exists.*[3]

Proof. Part (i) follows from (5) and Definition 1, while (ii) is obvious from (5). ∎

Corollary 1. *Assume consumers have endowment bias. Then the positive equilibrium rental rate is increasing in w_1^h/w_2^b.*

Proof. Rewriting (5) as

$$r^e = -\alpha_2 + \frac{\alpha_2 - \alpha_1}{1 - (\alpha_1 w_1^h / w_2^b)}, \tag{6}$$

and using Definition 1, we conclude. ∎

Proposition 2. *The equilibrium with a positive rental rate is unstable if consumers have endowment bias.*

[3] Here, one can prove that equilibrium exists if and only if consumers have either endowment bias or unendowed-good bias (which can be defined by reversing the direction of the inequalities in Definition 1). Above, we omit to state this full characterization, since when consumers have a bias towards unendowed good, equilibrium is observed to be stable.

Proof. Using (4) and (5), we evaluate

$$D^{h\prime}(r^e) \quad = \quad \left(\frac{\alpha_2 - \alpha_1}{1 - (\alpha_1 w_1^h / w_2^b)}\right)^2 \frac{\alpha_1 w_1^h}{w_2^b} \left(1 - \frac{\alpha_1 w_1^h}{w_2^b}\right), \tag{7}$$

which is positive if consumers have endowment bias. ∎

The equilibrium at zero rental rate is stable while the one at positive rental rate is unstable. Rental rates are normally sticky due to the presence of contracts. Therefore, if for any reason the rental rate of housing is above its positive equilibrium level, the excess aggregate demand for housing would inflate the rental rate further up, leading to an unbounded increase in rents. Conversely, when the rental rate of housing is below its positive equilibrium level, the excess aggregate supply of houses would pull the rental rate all the way down to the stable equilibrium at zero.

Given the described dynamics, we can explain by supply shocks the emergence as well as collapse of a housing bubble. Corollary 1 implies for example that starting from an unstable equilibrium, a positive shock to the supply of bread in comparison to the supply of housing would reduce the equilibrium rental rate below its current market rate, initiating the bubble. While the rental rate keeps on rising, the supply of housing (that we do not model explicitly) would increase pushing up the equilibrium rental rate above the market rate; hence the collapse of bubble.

3.2 The case of low substitution elasticity ($\epsilon_i \in (0, 1)$)

Here, we consider the cases where the elasticity of substitution between bread and housing services is so low that the two commodities remain to be imperfect complements, i.e. $\epsilon_1, \epsilon_2 \in (0, 1)$. Unfortunately, for these cases, the closed-form equilibrium solutions are not available. So, we simulate below an artificial economy.

Let $\alpha_1 = 0.25$, $\alpha_2 = 4$, $w_1^h / w_2^b = 1$, and $\epsilon_1 = \epsilon_2$. For different values of ϵ_i, the calculated equilibrium rental rates are reported in Table 1. (The last row corresponds to the already analyzed Leontief case).

Table 1
Equilibrium Rental Rates under CES Utilities

ϵ_i	r_1^e	r_2^e	r_3^e
1/2	n.a.	1 (stable)	n.a.
1/3	5.5728×10^{-2} (stable)	1 (unstable)	1.7944×10^{1}(stable)
1/6	2.4966×10^{-4} (stable)	1 (unstable)	4.0055×10^{3} (stable)
1/11	2.3843×10^{-7} (stable)	1 (unstable)	4.1941×10^{6} (stable)
1/51	2.8585×10^{-13} (stable)	1 (unstable)	5.0706×10^{30} (stable)
1/101	1.3468×10^{-13} (stable)	1 (unstable)	6.4278×10^{60} (stable)
0	0 (stable)	1 (unstable)	∞ (stable)

In the above table, when the elasticity of substitution is sufficiently high ($\epsilon_i = 1/2$), there exists a unique and stable equilibrium. For sufficiently low elasticities of substitution, we obtain three equilibria, as illustrated in Figure 1, where $S^h(r)$ denotes the aggregate supply of housing which is always equal to w_1^h.

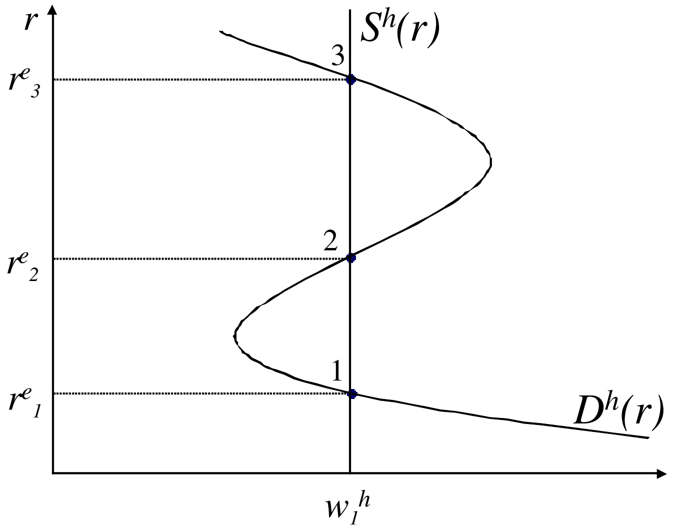

Figure 1: Housing Equilibria

Table 1 shows that as the utility functions converge to a Leontief function, the lowest of the equilibrium rental rates converges to zero while the highest one goes to infinity. Moreover, the lowest and highest of the equilibrium rental rates are found to be stable while the one in between is unstable. Here, we can characterize the set of values of the parameters ϵ_i, for which the middle of these three equilibrium rental rates is unstable. Under CES utilities with positive elasticities of substitution, the aggregate demand for housing is given by

$$D^h(r) = \frac{r^{1-\epsilon}}{\alpha_1 + r^{1-\epsilon}} w_1^h + \frac{r^{-\epsilon}}{\alpha_2 + r^{1-\epsilon}} w_2^b. \tag{8}$$

Assume that the equality

$$w_1^h / w_2^b = \frac{1 + (\alpha_1)^{-1}}{1 + \alpha_2} \tag{9}$$

is satisfied. Then $D^h(1) = w_1^h$, implying $r = 1$ is an equilibrium rental rate. Now evaluating the derivative of (8) at the rental rate $r = 1$, we conclude that $D^{h\prime}(1) > 1$ if and only if

$$\frac{1 - \epsilon_1}{1 + \alpha_1} + \frac{1 - \epsilon_2}{1 + (\alpha_2)^{-1}} > 1. \tag{10}$$

One can check that for the economy simulated in Table 1, $r = 1$ is an unstable equilibrium if and only if $\epsilon_1 + \epsilon_2 < 0.75$. In the extreme case where individuals have full endowment bias, i.e. $\alpha_1 = 1/\alpha_2 = 0$, condition (10) reduces to $\epsilon_1 + \epsilon_2 < 1$.

Finally, we note that in the case of multiple equilibria, the amplitude of a possible bubble is determined by the range between the two stable equilibria which in turn are mainly determined by the elasticity of substitution and ownership parameters.

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