The Case of a Queer Isoquant: Increasing Marginal Rates of Substitution of Grain for Roughage in Cattle Finishing

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The purpose of this paper is to argue the reality of increasing marginal rates of substitution of grain for roughage in beef production. Most economists and animal scientists have the idea that these marginal rates of technical substitution are decreasing. Brokken et al., Heady et al., and Goodrich et al. have shown increasing marginal rates of substitution of grain for roughage in cattle feeding response functions.[1] The practical implications of such substitution relations are enormously important for both the producers and consumers of beef as well as for economic and technical research in beef production.

Three Empirical Examples

In the three examples cited, grain-roughage isoquants for cattle finishing are found to be concave to the origin over at least part of the physical region of substitution. All three use different methodology in deriving the substitution relations.

Goodrich et al.

Data from 17 midwestern university experiments involving 878 steer calves on rations varying in the proportions of corn silage and corn grain were used. Rate of gain and total dry matter intake per pound of gain were each related to the proportion of corn silage in the ration. From these relationships, the expected quantities of the separate ingredients required to obtain a given gain and the expected time required were calculated [table 1, p. 19]. A plot of the data shows a concave (to the origin) isoquant for corn silage-corn grain substitution.

Heady et al.

Heady et al. analyzed response functions for finishing steers on various proportions of corn and soilage. Among the alternative regression equations was a function allowing increasing and/or decreasing marginal rates of substitution of corn for soilage in the range of diminishing marginal physical product of both soilage and corn.[2] Even though this function had the best statistical fit of the alternatives tried, it was rejected because it ". . . gave sigmoid isoquant contours denoting first increasing marginal rates of substitution and then decreasing marginal rates of substitution . . . " [p.883]. They state [p.918], "Even though the coefficients of determination for this model of the beef-cattle production function were quite high, the model was rejected on the basis of logic." The logic used was not explained.

Brokken et al.

In this study, an appetite function which relates daily voluntary feed intake to the energy concentration (calories/kilogram) of the ration is combined with a function which relates daily feed requirements for maintenance and gain to body weight, rate of gain and ration energy

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[1] Space prevents discussion of the distinction between the neoclassical concept of the production function and the concept of a response function for livestock production [Dillon].

[2] Response function was of the form $G = aF^b e^{cR} + dR^2$ where G is cumulative weight added, F is soilage, e is the base for natural logarithms, R is the soilage to corn ratios and a, b, c and d are parameters to be estimated. This function was fit as a part of a special form of recursive system which consisted of the response function, the ration relation, the gain relation and the consumption function.

concentration. From the combined functions, a relationship showing daily rate of gain as a functain of energy concentration (calories per kilogram) of the ration is derived. Total feed intake for each energy level is obtained by aggregating daily intake over time (by either discrete or continuous summation). The proportion of grain and roughage for each energy level are, respectively, multiplied by total feed required to produce a given output to obtain coordinates for the grain-roughage isoquants.

The appetite function has two phases. In phase I, gut fill limits intake, but daily dry matter intake increases as ration energy concentration increases because the rate of digestion increases as the ration is enriched. This phase extends from rations with all hay up to about 35 to 40% grain. In phase II, further enrichment of the ration causes daily intake to decline and gut fill does not limit intake. However, the rise in energy concentration more than offsets the decline in dry matter intake so that the net energy intake is also increasing in phase II. This study was concerned only with diets in phase II of the appetite function. The system is summarized as follows.

Notation. Let Y represent daily voluntary dry matter intake per unit metabolic weight, i.e., $Y \equiv F/W^{.75}$ where F is daily feed intake, W is body weight ($W^{.75}$ is metabolic weight); i represents the ith animal; X is net energy for gain per kilogram of feed i.e., the energy concentration of the ration; g is rate of gain in Kg/day; t represents the tth day of the feeding period; W_0 is weight on day zero; n is the total number of days in the feeding period; W_n is the weight on the nth day of the feeding period.

System. The appetite functions are:

1) Daily dry matter intake, $Y_i = f_i(X) = A_i + B_iX$,

2) Daily energy intake, $XY_i = Xf_i(X) = A_iX + B_iX^2$.

Daily energy requirements for gain, $g(X, g_i)$:
3) $XY_i = .08089X - .03185X^2 + .05272g_i + .00684g_i^2$.

Equating 2 and 3 and solving for g_i obtains
$g_i(X) = G[Xf_i(X), g(X, g_i)]$:
4) $g_i = -3.8538 +$
$$\left[14.85178 + \frac{Xf_i(X) - .08089X + .03185X^2}{.00684} \right]^{1/2}$$

Daily feed intake is $F_{it} = Y_iW_t^{.75} = Y_i(W_0 + gt)^{.75}$. Hence total feed intake over n days (TF) is:

5) $TF = Y_i \int_0^n (W_0 + gt)^{.75} dt =$
$$\frac{Y_i}{g_i} \left[\frac{(W_0 + ng)^{1.75} - W_0^{1.75}}{1.75} \right]$$

Set $W_0 + ng = W_n$ and consider TF for a given span of weight from W_0 to W_n (n being variable). The right hand expression of equation 5 becomes a constant (Γ) and total feed becomes a function of ration energy concentration:

6) $TF = \frac{Y_i}{g_i} \Gamma = \frac{f_i(X)}{g_i(X)} \Gamma$.

Multiplying TF by P_x and $(1 - P_x)$, respectively, the proportions of roughage and concentrates in the ration, obtains the coordinates for plotting the concentrate-roughage isoquant.

The concentrate-roughage isoquant. The shape of the isoquant depends on the appetite function, $f_i(X)$, and on the gain function, $g_i(X)$, both of which appear in equation for total feed intake. The gain function depends, in turn, on the appetite function, $f_i(X)$, and on the requirements function, $g(X, g_i)$. The general shapes of these functions are well established in the literature. For purposes of this argument, the requirements function of the California net energy system is used with alternative appetite functions.

Four appetite functions are shown in figure 1. The assumption of a constant daily feed intake, as in appetite function II, is frequently used in economic analysis. This assumption means that daily energy intake (i.e., X times Y) is increasing linearly as energy concentration is increased and represents a rather extreme bias in favor of high concentrate feeds. Another extreme, represented by appetite function III, is the assumption that the rate of gain would be the same regardless of energy concentration of the ration. In between these two assumptions lies appetite function I which represents the function for an animal of higher than average performance from the experimental sample used in the study by Brokken and Dinius. The fourth function represents the unlikely case in which rate of gain first increases then decreases as ration energy concentration is increased.

The isoquants corresponding to these four appetite functions are shown in figure 2. Proportions

Fig. 1.   Appetite functions used for illustrating shape of concentrate-roughage isoquants.
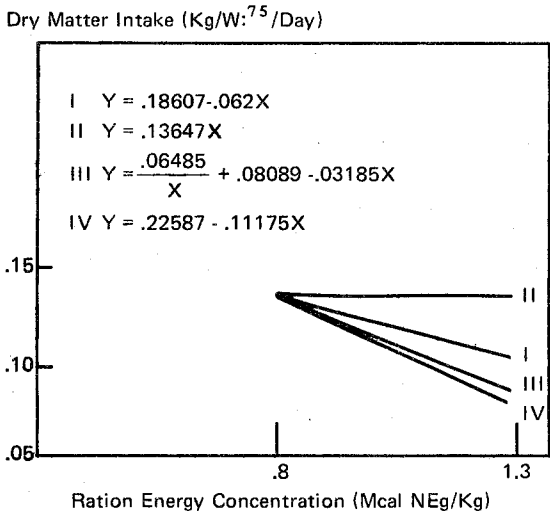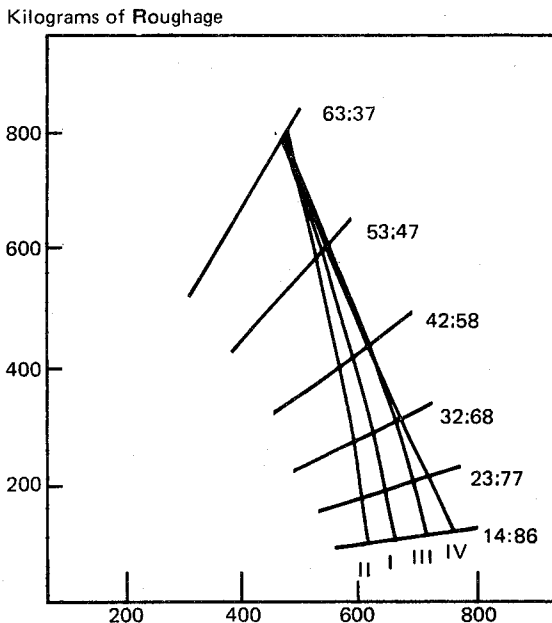
Dry Matter Intake (Kg/W.^{75}/Day)

I Y = .18607-.062X

II Y = .13647X

III Y = $\dfrac{.06485}{X}$ + .08089 -.03185X

IV Y = .22587 - .11175X

Ration Energy Concentration (Mcal NEg/Kg)

Fig. 2.   Isoquants corresponding to appetite functions in figure 1.

Kilograms of Roughage

of roughage and concentrates, corresponding to each energy concentration are shown in table 1 and the rates of gain and total feed requirements for each ration for the four appetite functions are shown in table 2. Isoquants corresponding to appetite functions I, II, and III are all concave to

the origin. To obtain an isoquant convex to the origin, it was necessary to have rate of gain first increasing then decreasing as the proportion of concentrates in the diet are increased (appetite function IV and the corresponding isoquant IV). This is not always a sufficient condition; a minimal change in gain would not obtain this behavior. Another behavior resulting in a convex (to the origin) isoquant is to have an appetite function such that the rate of gain is continuously

Table 1. Roughage-concentrate proportions by diet

Item	Ration Concentration, Mcal NE$_g$/Kg					
	.8	.9	1.0	1.1	1.2	1.3
Roughage %	63	53	42	32	23	14
Concentrates %	37	47	58	68	77	86

Table 2. Rate of gain and total feed requirements by diet and by appetite function[a]

Appetite. Functions	Item	Ration Concentration, MCal NE$_g$/Kg					
		.8	.9	1.0	1.1	1.2	1.3
I	Gain (Kg/day)	1.08	1.16	1.23	1.29	1.34	1.38
	Total Feed (Kg)	1265	1125	1011	915	834	764
II	Gain (Kg/day)	1.08	1.24	1.40	1.57	1.74	1.92
	Total Feed (Kg)	1265	1101	973	869	784	712
III	Gain (Kg/day)	1.08	1.08	1.08	1.08	1.08	1.08
	Total Feed (Kg)	1265	1152	1055	971	896	828
IV	Gain (Kg/day)	1.08	1.09	1.08	1.05	.99	.91
	Total Feed (Kg)	1265	1147	1054	982	927	889

[a]Total feed required for a Γ of 10^4 (eg. for growth from 320 Kg to 437 Kg).

decreasing. This also is not a sufficient condition unless the decline in gain is large, which is very unlikely.

Summary and Conclusions

The examples offered do not constitute a proof that grain-roughage isoquants will be concave to the origin for all types of cattle under all growing conditions and all types of grain and roughage. However, they do constitute a strong argument that isoquants are concave in many, if not most, cattle finishing programs. Only two kinds of appetite functions will give isoquants that are convex to the origin. One is an appetite function that obtains a substantially decreasing rate of gain as the proportion of grain in the diet is increased. The other is an appetite function that obtains first an increasing rate of gain and then a decreasing rate of gain as the proportion of grain in the diet is increased. The author has never encountered either of these patterns of performance relative to varying proportions of hay and grain in the diet.

This idea of concavity also has intuitive appeal. Total energy required for a given gain decreases as rate of gain increases, because fewer days of maintenance are required. As the proportion of grain in the ration is increased, replacing roughage, the rate of gain is expected to increase. Hence, grain substitutes for roughage with increasing efficiency. This means the isoquant curve is concave to the origin.

Is this a queer isoquant? It would seem so from the point of view of our economic training and practice. However, the reality has always been independent of our assumptions about it. On careful examination, the concave isoquant should not have seemed so unusual. It is not inconsistent with plain intuition and has been observed when the assumed functional forms did not preclude its observation. What does this tell us about our economic training and practice? It tells us that we can easily become entrapped in the confines of our paradigms. This disconcerting condition challenges us to open up and examine our assumptions, even those related to the most elementary concepts. Where else can increasing marginal rates of technical substitution between factors of production be found?

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