Uncertainty and the Management of Salinity with Irrigation Water

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The impact of uncertain salt balances in irrigated fields is assessed with a hydroeconomic model that incorporates the effects of salinity. Uncertainty in two parameters that jointly determine root zone salinity is investigated and the conclusions prove to depend upon the way in which these parameters enter the mass-balance equation for soil salinity. It is shown that water has a risk reducing marginal effect on output when growers are risk averse and, under certain conditions, when they are risk neutral. The effects of prices, water quality, and crop salt sensitivity on the conclusions are analyzed and an empirical example is employed to illustrate the magnitude of the impacts.

In many portions of the arid and semi-arid west, the productivity of irrigated agriculture is dependent, in part, on the successful management of soil salinity. The fact that excessive accumulations of salt in the root zone will diminish agricultural productivity has been well-documented (e.g., Bernstein). The conventional means for managing salt balances involves the application of quantities of irrigation water in excess of those utilized by the crop in order to leach salts from the root zone. The leaching fractions or titres necessary to avert salt induced yield reductions can be computed from measurements of salt and moisture balances in the root zone. Modern instruments permit such measurements to be made accurately at a given point within a field. Yet, virtually no field is completely uniform and the expense of instrumenting more than a few points is large. This means that the salt and moisture status of an entire field is not known with certainty, a fact that has been documented in numerous studies.

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Kaddah and Rhoades, for example, analyzed annual salt balance measurements made in the Imperial Valley of California over a period of 31 years and concluded that the measurements, though comprehensive and correct, did not provide adequate information about actual changes in root zone salinity over time. Oster and Wood, in an analysis of salt balances in six fields in Arizona and Colorado, demonstrated that both the variability and uncertainty of salt balances explains the poor predictive performance of existing hydro-salinity models. These conclusions have been echoed by government-sponsored panels and task forces (e.g., U.S. Department of Interior, 1979; State of California, 1979).

While it is generally accepted that the effective management of salinity requires more water intensive irrigation regimes, the economic implications of uncertain salt balances in the root zone have not been fully investigated. Feinerman, Yaron, and Bielorai estimate a linear yield response function for soil salinity while Feinerman and Yaron investigate the effects of additional information on the response function as perceived by the risk neutral grower and identify the conditions characteristic of an optimal level of information. The pertinent work on uncertainty
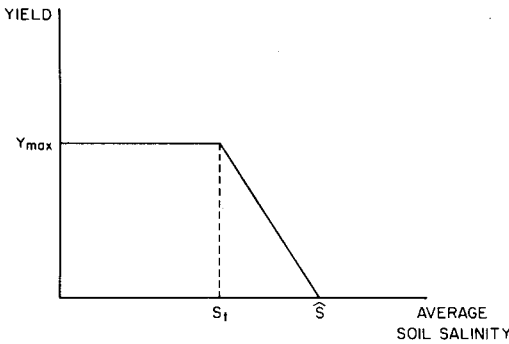
Figure 1. Yield Salinity Response Function.

focuses on uncertainties associated with production technologies. Ratti and Ullah analyze the implications of technical production uncertainty on the demand for labor and capital by competitive firms. They show, that in the presence of uncertainty, the risk neutral firm demands less of both factors that it would under conditions of certainty. The input demands of the risk averse firm for both factors are reduced to levels below those of the risk neutral firm. Feldstein considers the effect of uncertainty in the exponents of a Cobb-Douglas production function and shows that technical uncertainty affects relative factor shares by altering both the absolute and relative demands for capital and labor.

Following Feder (1977), Pope and Kramer argue that most theoretical and empircal inquiries neglect the fact that many factors of production have a risk reducing marginal effect on output. This risk reducing marginal effect leads, in turn, to an increase rather than a decrease in the level of input use. Feder (1979) investigates the case of pesticide use in agriculture. He demonstrates that where growers are either risk neutral or risk averse, an increase in the uncertainty of the size of the pest population and the expected damages associated with those populations will lead unambiguously to an increased level of pesticide applications.

In the latter three works, it is assumed that the parameters in the production

function are known and uncertainty is introduced as an independent multiplicative error. Although the functional relationship between salt balance and yield is understood, growers face uncertainty over the values of various parameters in this relationship. In this paper, we identify the effects of these uncertain parameters on the levels of water applied by risk neutral and risk averse growers and utilize an empirical example to assess the magnitude of those effects.

A Salinity-Yield Model

It has been well documented that the severity of yield reductions attributable to soil salinity is directly related to the average soil salinity in the root zone during the growing season. The effects of soil salinity on crop yields have been specified for a large number of crops by Maas and Hoffman. They demonstrate that there is some threshold level of soil salinity, S_t, beyond which crop yields decline linearly with increasing soil salinity. The general relationship between average soil salinity and yield which they identify is depicted in Figure 1. The basic production function follows this relationship and can be written

$$Y = \begin{cases} a + b\left(\dfrac{S_0 + S_1}{2}\right) & \text{if } \hat{S} \ge \left(\dfrac{S_0 + S_1}{2}\right) > S_t \\[2ex] Y_{max} & \text{if } \left(\dfrac{S_0 + S_1}{2}\right) \le S_t \end{cases} \quad (1)$$

where

Y = yield (per acre)

Y_{max} = maximum yield with no salinity losses (per acre)

S_0 = soil salinity prior to growing season

S_1 = soil salinity at end of growing season

$S = \dfrac{S_0 + S_1}{2}$ average soil salinity

$\hat{S}$ = S where Y = 0

a > 0, b < 0 are known parameters.

In the analysis that follows we focus on the range where $S_t \le S_0 \le S$. Bresler utilized the principle of conservation of mass to derive an equation for estimating S_1, the terminal soil salinity, as a function of irrigation water quantity and quality and the water quantity and quality status of soil moisture in the root zone. Bresler's equation can be written:

$$S_1 = \frac{QC + S_0(V - \frac{1}{2}Q + \frac{1}{2}T)}{V + \frac{1}{2}Q - \frac{1}{2}T} \qquad (2)$$

where

Q = Quantity of irrigation water applied
C = Known salt concentration of the irrigation water
T = Soil moisture deficit (from field capacity) in the root zone
V = Soil moisture content at saturated paste, a specified and known level of saturation that varies with soil properties.

Uncertainty about salt balances can be captured in this equation either in the form of uncertain soil moisture deficit, T, or uncertain initial soil salinity, S_0. In the analysis that follows, uncertain T and uncertain S_0 are treated separately. The case in which both variables are uncertain is not treated because virtually nothing is known about the joint distribution of their values.

Utilizing equations (1) and (2), a profit function may be defined as follows:

$$\pi(Q) = \begin{cases} R f(Q) - P_Q Q & \text{if } \hat{S} \ge \dfrac{S_0 + S_1}{2} > S_t \\ RY_{max} - P_Q Q & \text{if } \dfrac{S_0 + S_1}{2} \le S_t \end{cases} \qquad (3)$$

where $f(Q) = a + b\dfrac{QC + 2S_0V}{2V + Q - T}$

R = income net of nonwater variable cost directly related to yield

P_Q = price of irrigation water ($/acre foot).

This function forms the basis for our evaluation which draws on the works of Horowitz, Sandmo, Ratti and Ullah, and Feder (1977). For purposes of this analysis, we assume that the absolute quantity of soil moisture (matric potential) is not limiting.[1] With ample soil moisture, T becomes limiting only at the threshold level of saline concentration and we restrict our investigation to this case of limiting osmotic potential. Additionally, we note that $C < S_0$ is a necessary condition if applied irrigation water is to have the potential of improving soil moisture quality.

The piecewise linear nature of the salinity-yield relationship requires us to define first, the quantity of water required to reduce expected average soil salinity to the threshold point, S_t. The optimal quantity of water to be applied is then derived, assuming $\dfrac{S_0 + S_1}{2} > S_t$.[2] The optimum water application is the minimum of these two. When the grower is completely certain about the values of T and S_0 and maximizes profits, the first order condition for an optimum is:

$$R f_Q(Q) = P_Q; \qquad \left(f_Q = \frac{\partial f(Q)}{\partial Q}\right). \qquad (4)$$

Let $\bar{Q}$ be the quantity of water that sat-

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[1] It should be noted that empirical applications of this model would require that this assumption be made explicit in the form of a constraint in order to keep the impact of limiting osmotic potential separate from the moisture stressing impacts of limiting matric potential. Such a constraint can be implied by assumption in a theoretical treatment without compromising the results. To keep the analysis as simple and straightforward as possible, we do not adopt such a constraint, however.

[2] The assumptions $C < S_0$ and $S_0 < \hat{S}$ imply that $\hat{S} > \dfrac{S_0 + S_1}{2}$. In derivations that follow $\hat{S} > \dfrac{S_0 + S_1}{2}$ although we do not explicitly note it in every instance.

isfies (4) and define $\bar{Q}_{max}$ as the quantity of water required to reduce the average soil salinity to the threshold point S_t so that there is no salt related reduction in yield. $\bar{Q}_{max}$ may be characterized as the quantity of irrigation water that satisfies:

$$\frac{S_0 + S_1}{2} = \frac{QC + 2S_0V}{2V + Q - T} \equiv h(Q) = S_t. \qquad (5)$$

It should be noted that $\frac{S_0 + S_1}{2} > 0$ implies $2V + Q - T > 0$ for every T and its associated Q. The optimal quantity of irrigation water applied under certainty will then be $\bar{Q}_{opt}$ which can be written as:

$$\bar{Q}_{opt} = \min(\bar{Q}_{max}, \bar{Q}). \qquad (6)$$

This solution to the certainty case can be used to assess the implications of uncertain T and uncertain S_0 when the grower is either risk neutral or risk averse.

Optimal Irrigation with Uncertain Quantities of Soil Moisture

Uncertainty surrounding the soil moisture deficit, T, is largely attributable to the nonuniformity of soils and irrigation systems coupled with climatic variations. In assessing uncertain T, we assume that variation in T is bounded by upper and lower limits (T^u and T^l, respectively). Our conversations with farm managers and soil researchers suggest that growers tend to rely heavily on past experience and are thus able to identify a reasonable range within which T varies. We assume that growers perceive T as varying randomly between these limits with a known probability distribution.

Risk Neutrality

In the risk neutral case, the grower maximizes expected profits, $E\pi(Q,T)$, where E is the expectation operator. Assuming that $\frac{S_0 + S_1}{2} > S_t$, the first order condition for an optimum is:

$$RE \, f_Q(Q,T) = P_Q; \quad \left(f_Q = \frac{\partial f(Q,T)}{\partial Q}\right). \qquad (7)$$

Q^* and Q^*_{max} can be defined as the quantities of water which respectively satisfy (7) and (8).

$$E\left[\frac{S_0 + S_1}{2}\right] = E\left[\frac{QC + 2S_0V}{2V + Q - T}\right]$$
$$= E \, h(Q,T) = S_t. \qquad (8)$$

In the case of uncertainty with risk neutrality, then, optimal Q will be:

$$Q^*_{opt} = \min[Q^*_{max}, Q^*]. \qquad (9)$$

A comparison of $\bar{Q}_{opt}$ with Q^*_{opt} requires first a comparison of $\bar{Q}$ with Q^*. Following a method first employed by Sandmo, we subtract $Rf_Q(Q,\bar{T})$ from both sides of (7) to obtain:

$$R[Ef_Q(Q,T) - f_Q(Q,\bar{T})] = P_Q - Rf_Q(Q,\bar{T}) \qquad (10)$$

where all partial derivatives are computed at Q^* and $\bar{T} = E(T)$. It can be readily verified that $f_Q(Q,T)$ is a decreasing convex function of Q and an increasing convex function of T. Using Jensen's inequality [e.g., Rao] we can conclude that:

$$f_Q(Q,\bar{T}) < Ef_Q(Q,T). \qquad (11)$$

An evaluation of (10) in light of this result yields:

$$P_Q > Rf_Q(Q,\bar{T}) \qquad (12)$$

which means that:

$$Q^* > \bar{Q} \qquad (13)$$

This conclusion is illustrated in Figure 2.

In a similar fashion, (8) and the fact that $h(\bar{Q},T)$ is a decreasing convex function of Q and an increasing convex function of T can be used to establish that:

$$Q^*_{max} > \bar{Q}_{max}. \qquad (14)$$

Taken together, (13) and (14) permit us to conclude that:

$$Q^*_{opt} > \bar{Q}_{opt}. \qquad (15)$$

This conclusion suggests that where T is uncertain and the grower is risk neutral, the basic relationships between salinity and

yield as defined by Maas and Hoffman and Bresler and embodied in equations (1) and (2), respectively, imply that water inputs are increased in response to uncertainty about the level of soil moisture. This somewhat surprising finding is explained by the fact that the influence of T on salt balance via equation (2) is nonlinear.

Risk Aversion

We introduce risk-averse behavior by writing a grower's utility function $u(\pi)$ such that $\dfrac{\partial u}{\partial \pi} \equiv u_\pi(\pi) > 0$ and $\dfrac{\partial^2 u(\pi)}{\partial \pi^2} \equiv u_{\pi\pi}(\pi) < 0$. So long as $\dfrac{S_0 + S_1}{2} > S_t$, the grower's maximization problem can be written as

$$\max_Q E\{u[R\ f(Q,T) - P_Q Q]\} \Rightarrow Q = Q^{**} \quad (16)$$

where Q^{**} = optimal water application of the risk averse grower facing uncertainty.

The first order condition may be derived following the method first used by Horowitz:

$$RE\ u_\pi(\pi)Ef_Q(Q,T)$$
$$+ R\ COV[u_\pi(\pi);\ f_Q(Q,T)] = P_Q E\ u_\pi(\pi). \quad (17)$$

This is divided through by $E\ u_\pi(\pi)$ yielding:

$$RE\ f_Q(Q,T) - P_Q$$
$$= -\frac{R}{E\ u_\pi(\pi)} COV[u_\pi(\pi);\ f_Q(Q,T)]. \quad (18)$$

Now, $\dfrac{\partial u_\pi}{\partial T} \equiv u_{\pi T} = u_{\pi\pi}(\pi)\dfrac{\partial \pi}{\partial T}$. Consequently, $u_{\pi\pi} < 0$ (risk aversion) and $\dfrac{\partial \pi}{\partial T} < 0$ imply that $u_{\pi\pi} > 0$. Additionally, $\dfrac{\partial f_Q(Q,T)}{\partial T} = f_{QT} > 0$. The work of Lehman allows us to conclude that $COV(u_\pi, f_Q)$ is positive. Hence, the right-hand side of (18) is negative, permitting us to write:

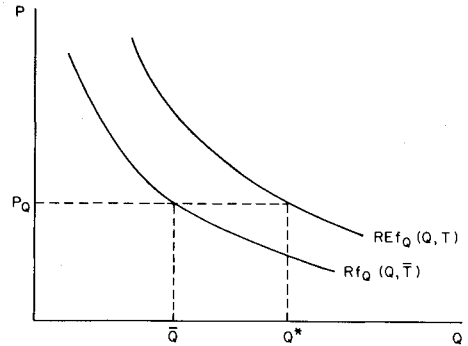$$RE\ f_Q(Q,T) < P_Q. \quad (19)$$

Figure 2. Marginal Value Products of Water with Certainty and Uncertainty.

Thus, the risk averse grower will demand more water than the risk neutral grower in the face of uncertain T. Additionally, the risk averse grower, like his risk neutral counterpart, will select an optimal quantity of water Q^{**}_{max} such that $Q^{**}_{max} = Q^{*}_{max}$ which satisfies $E\ h(Q,T) = S_t$. The optimal quantity of water for the risk averse grower, Q^{**}_{opt} will be:

$$Q^{**}_{opt} = \min(Q^{**}_{max},\ Q^{**}). \quad (20)$$

Water, then, can be characterized as a marginally risk reducing input since risk averse firms utilize larger quantities of it than risk neutral firms when other inputs are fixed (Pope).

Optimum Irrigation with Uncertain Initial Soil Moisture Quality

Uncertainty over the variability and magnitude of salt balances in the soil is attributable to the difficulty and expense of monitoring salt inputs to the root zone. Salt balances are especially difficult to estimate in newly irrigated soils and in areas where there are multiple sources of irrigation water and multiple return flow pathways (Oster and Wood). Accordingly, we consider briefly the implications of uncertainty about initial soil salinity conditions. Following our earlier analysis, we assume that growers confronted with problems of soil salinity know from experience the upper and lower limits

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(S_0^u and S_0^l, respectively) of salinity for their own soils. (S_0^u is assumed to be less than or equal to $\hat{S}$, the salinity level at which yields become zero.) We also assume that S_0 is perceived to vary randomly between these limits with known probability distribution.

For clarity, we distinguish notationally between the results for uncertainty in T and uncertainty in S_0. We substitute Q'_{max}, Q', Q'_{opt} (relative to S_0) for Q^*_{max}, Q^*, Q^*_{opt} (relative to T) for the risk neutral grower and Q''_{max}, Q'', Q''_{opt} (relative to S_0) for Q^{**}_{max}, Q^{**}, Q^{**}_{opt} (relative to T) respectively for the risk averse grower. Following the general procedure used to analyze T, it is easy to verify that $f_Q(Q,S_0)$ and $h(Q,S_0)$ are increasing linear functions of S_0. Jensen's inequality can be employed to show that:

$$f_Q(Q,\bar{S}_0) = E\, f_Q(Q,S_0) \tag{21}$$

and

$$h(Q,\bar{S}_0) = E\, h(Q,S_0) \tag{22}$$

where $\bar{S}_0 = E(S_0)$.

This implies that the risk neutral grower will apply the same optimum amount of irrigation water irrespective of whether S_0 is certain or uncertain ($Q'_{opt} = \bar{Q}_{opt}$).

The behavior of the risk averse grower can be analyzed by substituting S_0 for T in equation (16) and writing:

$$Q^{max}\, E\{u[R\, f(Q,S_0) - P_Q Q]\} \Rightarrow Q = Q' \tag{23}$$

where Q' = optimal water application of risk averse grower facing uncertainty. The necessary first order condition is:

$$RE\, u_\pi(\pi) E\, f_Q(Q,S_0)$$
$$+ R\, COV[u_\pi(\pi); f_Q(Q,S_0)] = P_Q E\, u_\pi(\pi). \tag{24}$$

Dividing (24) through by $E\, u_\pi(\pi)$ yields

$$RE\, f_Q(Q,S_0) - P_Q$$
$$= -\frac{R}{E\, u_\pi(\pi)} COV[u_\pi(\pi); f_Q(Q,S_0)]. \tag{25}$$

Now, $\dfrac{\partial u_\pi}{\partial S_0} \equiv u_{\pi S_0} = u_{\pi\pi}(\pi)\dfrac{\partial \pi}{\partial S_0}$.

Consequently, $u_{\pi\pi} < 0$, and $\dfrac{\partial \pi}{\partial S_0} < 0$ imply that $U_{\pi S_0} > 0$. Additionally,

$$\frac{\partial f_Q(Q,S_0)}{\partial S_0} \equiv f_{Q S_0} > 0.$$

As a result, $COV[u_\pi(\pi); f_Q(Q,S_0)] > 0$. The right-hand side of (25) is, therefore, negative. This allows us to conclude that:

$$RE\, f_Q(Q,S_0) < P_Q \tag{26}$$

Equation (26), together with the fact that $Q'_{max} = Q''_{max}$, demonstrates that when S_0 is uncertain, the risk averse grower will demand more water than the risk neutral grower ($Q''_{opt} \geq Q'_{opt}$). This confirms that water also has a risk reducing marginal effect on output when S_0 is uncertain. It should be noted that the case of uncertain S_0 is a straight-forward example of a multiplicative random variable which is quite common in the literature (e.g., Feder (1977), Pope and Kramer). This contrasts with the case of uncertain T.

The Role of Water Prices, Crop Sensitivity, Risk, and Water Quality

Analyses of the impact of changes in water price (P_Q), in the sensitivity of crops to soil salinity (b) and in the extent of uncertainty on optimal levels of water use are summarized in Table 1. These results (derivations are omitted here to conserve space and are available from the authors) show that optimal levels of applied water are inversely related to water prices under conditions of certainty and with uncertain T or S_0 when growers are risk neutral. If growers are risk averse, optimal water applications are inversely related to price when assumptions of constant absolute risk aversion for uncertain T and nonincreasing absolute risk aversion for uncertain S_0 obtain.

Crop sensitivity to salinity in the range where yields are affected is a function of the magnitude of the parameter b. Since

TABLE 1. Summary of Sensitivity Analysis.

	Sensitivity of Optimal Water Applications to Parameter Changes When Soil Moisture Deficit (T) is Subject to Uncertainty.			
	Certainty	Risk Neutral	Uncertainty	Risk Averse
Price of Water	$\dfrac{d\bar{Q}_{opt}}{dP_Q} \le 0$	$\dfrac{dQ^*_{opt}}{dP_Q} \le 0$		$\left[\dfrac{dQ^{**}_{opt}}{dP_Q} \le 0\right]$
Salinity Sensitivity	$\dfrac{d\bar{Q}_{opt}}{db} \le 0$	$\dfrac{dQ^*_{opt}}{db} \le 0$		$\dfrac{dQ^{**}_{opt}}{db} \gtrless 0$
Increased Uncertainty	N.A.	$\dfrac{dQ^*_{opt}}{dr} > 0$		$\dfrac{dQ^{**}_{opt}}{dr} \gtrless 0$

[ ]—conclusion dependent on Arrow-Pratt assumption of constant absolute risk aversion.

	Sensitivity of Optimal Water Applications to Parameter Changes When Initial Soil Salinity (S_0) is Subject to Uncertainty.			
	Certainty	Risk Neutral	Uncertainty	Risk Averse
Price of Water	$\dfrac{d\bar{Q}_{opt}}{dP_Q} \le 0$	$\dfrac{dQ'_{opt}}{dP_Q} \le 0$		$\left\{\dfrac{dQ''_{opt}}{dP_Q} \le 0\right\}$
Salinity Sensitivity	$\dfrac{d\bar{Q}_{opt}}{db} \le 0$	$\dfrac{dQ'_{opt}}{db} \le 0$		$\dfrac{dQ''_{opt}}{db} \gtrless 0$
Increased Uncertainty	N.A.	$\dfrac{dQ^*_{opt}}{dr} = 0$		$\left\{\dfrac{dO''_{opt}}{dr} > 0\right\}$

{ }—conclusion dependent on Arrow-Pratt assumption of non-increasing absolute risk aversion.

b is negatively signed in the salinity yield relationship (Eq. 1), the results are interpreted to show that increasing salt sensitivity leads to an increase in optimum levels of applied water under both certainty and when T or S_0 are uncertain and growers are risk neutral. Where growers are risk averse and either T or S_0 is uncertain, the effect of increased crop sensitivity to salinity on optimal water applications is ambiguous.

The effect of changes in the extent of uncertainty about either soil moisture levels or initial soil salinity levels was analyzed by examining the effects of a mean preserving increase in their distributions, following a procedure first suggested by Feder (1977) which employs a positive parameter, r. An increase in r implies a mean preserving increase in the variance of the random variable under consideration. With uncertain soil moisture (T) optimal water applications for risk neutral growers vary directly with the extent of uncertainty (i.e., information on the status of soil moisture can be substituted for water). If growers are risk averse, the conclusions are ambiguous. For uncertain soil salinity, increasing the degree of uncertainty has no effect on optimal levels of applied water if growers are risk neutral. With risk aversity, increases in the extent of uncertainty result in higher levels of applied water so long as absolute risk aversion is nonincreasing. The difference in the conclusions for uncertain T and uncertain S_0 is attributable to the fact that uncertain S_0 is a multiplicative random variable while the effect of uncertain T is neither multiplicative nor linear (see Eq. 2).

Table 2 summarizes the impact of differing levels of water quality (C) on optimal levels of applied water. As shown, when all values are certain, the response of optimum water levels to changes in

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TABLE 2. Summary of Sensitivity Analysis.

	Sensitivity of Optimal Water Applications to Changes in Water Quality when Soil Moisture Deficit (T) is Subject to Uncertainty.		
Certainty	Risk Neutral	Uncertainty	Risk Averse
$\bar{Q} < \bar{Q}_{max} \Rightarrow \dfrac{d\bar{Q}_{opt}}{dc} \leq 0$	$Q^* < Q^*_{max} \Rightarrow \dfrac{dQ^*_{opt}}{dc} \leq 0$		$Q^{**} < Q^{**}_{max} \Rightarrow \dfrac{dQ^{**}_{opt}}{dc} \lesseqgtr 0$
$\bar{Q} > \bar{Q}_{max} \Rightarrow \dfrac{d\bar{Q}_{opt}}{dc} > 0$	$Q^* > Q^*_{max} \Rightarrow \dfrac{dQ^*_{opt}}{dc} > 0$		$Q^{**} > Q^{**}_{max} \Rightarrow \dfrac{dQ^{**}_{opt}}{dc} > 0$
	Sensitivity of Optimal Water Applications to Changes in Water Quality when Initial Soil Salinity (S_0) is Subject to Uncertainty.		
Certainty	Risk Neutral	Uncertainty	Risk Averse
$\bar{Q} < \bar{Q}_{max} \Rightarrow \dfrac{d\bar{Q}_{opt}}{dc} \leq 0$	$Q' < Q'_{max} \Rightarrow \dfrac{dQ'_{opt}}{dc} \leq 0$		$\left[Q'' < Q''_{max} \Rightarrow \dfrac{dQ''_{opt}}{dc} \leq 0 \right]$
$\bar{Q} > \bar{Q}_{max} \Rightarrow \dfrac{d\bar{Q}_{opt}}{dc} > 0$	$Q' > Q'_{max} \Rightarrow \dfrac{dQ'_{opt}}{dc} > 0$		$Q'' > Q''_{max} \Rightarrow \dfrac{dQ''_{opt}}{dc} > 0$

[ ]—conclusion dependent on Arrow-Pratt assumption of constant absolute risk aversion.

water quality depends upon whether the optimal salinity level is in the range to the left of the threshold point where yield is unaffected or in the range where optimal salinity levels entail some yield reduction. In the former instance, changes in water quality are inversely related to optimal water levels. That is, growers are induced to preserve maximum yields by utilizing more water in order to mitigate the effects of increased salinity. Where yields are affected, the opposite is true since increases (decreases) in salinity reduce (increase) the capacity of irrigation water to dilute existing soil salinity. Decreasing (increasing) water quality causes the marginal physical product curve for water to shift inward (outward) and growers thus respond to decreased (increased) water quality by applying less (more) water. (It should be noted that the derivative of the pertinent Q with respect to C is undefined at the single point where the pertinent Q equals the pertinent Q_{max}).

These general conclusions also hold when either T or S_0 are uncertain and the grower is risk neutral and when S_0 is uncertain and growers are risk averse. In this latter instance, the results are contingent upon the assumption of constant absolute risk aversion in the range where yields are affected. When T is uncertain and growers are risk averse, the analytical results are ambiguous.

An Empirical Example

The theoretical derivations of the previous sections provide insight into the impacts on irrigation water use attributable to uncertainties about the magnitudes of salt and water balances. The derivations do not yield conclusions as to whether the magnitude of these effects is significant, however. In order to assess the magnitude of changes in optimal water applications with uncertain T and S_0, an empirical case was investigated.

The case selected for analysis involves the production of citrus in the southern San Joaquin Valley of California. Citrus is relatively sensitive to soil salinity and thus its production requires careful management of water when saline conditions exist. The empirical values used for the base run were obtained from a variety of sources and are displayed in Table 3. Following Buccola and Farnsworth and Mof-

TABLE 3. Base Run Values for Production, Salinity, and Cost Parameters for Citrus in Tulare County, California.

Parameter	Value	Source
S_t	19 meq/l	Maas and Hoffman
$\hat{S}$	98 meq/l	Maas and Hoffman
a	41.31 Mg/acre	Maas and Hoffman
b	−0.4215/(Mg/acre)/(meq/l)	Maas and Hoffman
Y_{max}	33.3 Mg/acre	Cooperative Extension, University of California, Cost Analysis Worksheet, Tulare County, CA 1980
R	$40.1/ton	Cooperative Extension, University of California, Cost Analysis Worksheet, Tulare County, CA 1980
P_Q	$3.0/acre cm	Cooperative Extension, University of California, Cost Analysis Worksheet, Tulare County, CA 1980
T^{ℓ}	58 cm	Bresler, E., 1983 (Personal communication)
T^u	118 cm	Bresler, E., 1983 (Personal communication)
E(T)	88 cm	Vaux, H. J., Jr., 1983 (Unpublished data)
V(t)	300 cm	Computed
S_0^{ℓ}	15 meq/l	Bresler, E., 1983 (Personal communication)
S_0^u	45 meq/l	Bresler, E., 1983 (Personal communication)
$E(S_0)$	30 meq/l	Vaux, H. J., Jr., 1983 (Unpublished data)
$V(S_0)$	75 meq/l	Computed
C	15 meq/l	Vaux, H. J., Jr., 1983 (Unpublished data)
V	61	Vaux, H. J., Jr., 1983 (Unpublished data)

[a] $T \sim U(T^{\ell}, T^u)$.
[b] $S_0 \sim U(S_0^{\ell}, S_0^u)$.

fitt, a utility function was selected of the form $U = -e^{-\gamma\pi}$, a form which embodies constant absolute risk aversion. The risk aversion parameter, γ, was varied parametrically within a range of 0.001 to 0.1. In addition to the base run, sensitivity analyses were conducted by varying the values of P_Q, C, b, and r. The results of the base runs and the sensitivity analyses are summarized in Table 4, for uncertain T, and Table 5, for uncertain S_0.

When T is uncertain, water applications will increase by 5 percent when the grower is risk neutral. If the grower is risk averse, increases will range from 7 to 37 percent. Thus, optimal water applications are quite sensitive to the degree of risk aversity and remain significant even when the grower is risk neutral. The sensitivity analyses show that optimal water appli-

cations are relatively sensitive to the price of water (P_Q) and to the degree of crop sensitivity to salinity (b). Water applications are only moderately sensitive to changes in the variance in soil moisture and relatively insensitive to changes in the quality of the irrigation water (C).

With uncertain S_0, optimal quantities of irrigation water are unchanged from the case of certainty so long as growers are risk neutral. When growers are risk averse, the increase in optimal applications ranges from 3 to 33 percent, depending upon the degree of risk aversity. Optimal levels of water application are relatively sensitive to water prices and crop sensitivity to salinity but relatively insensitive to both changes in the variance of initial soil moisture and water quality.

These results suggest that the effects of

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TABLE 4. Empirical Results for Citrus with Uncertain T.

	$\bar{Q}$ (cm)	Q^* (cm)	Q^{**} (cm) $\gamma =$			
			0.001	0.01	0.05	0.1
Base Run	99	104	105.5	114	133	136
$P_Q = 3.5$	89.5	94.5	96.5	109	123	125.5
$C = 20$	95.5	101	103	112.5	132	135.5
$b = -0.50$ ($\hat{S} = 85.6$, $a = 42.8$)	111	115	117.5	130.5	146	148.5
$T^l = 48$; $T^u = 128$ ($E(T) = 88$; $V(t) = 533$)	99	107.5	110.5	128.5	146	148.5

uncertainty on the magnitude of applied irrigation water can be significant when growers are risk averse and crops are sensitive to salinity. In addition, when T is uncertain, the derived demand for irrigation water will increase significantly even when growers are risk neutral. This conclusion results from the fashion in which T enters the basic salinity relationship (Eq. 2). In irrigated agriculture, salinization is ultimately inevitable. Its effects can be offset only by applying additional quantities of irrigation water to leach and dilute salts in the root zone. The empirical results presented here show that uncertainty about salinity parameters will increase the demand for irrigation water for salinity management purposes. This finding may be specially significant in view of the increasing competition for scarce water supplies throughout the semiarid western United States.

Summary and Conclusions

The principal conclusions of this study are two. First, where soil salinity param-

eters are uncertain and growers are risk averse, the derived demand for irrigation water may be increased, sometimes substantially depending upon the degree of risk aversity. The second conclusion is that even when growers are risk neutral, uncertain T will lead to increases in the demand for irrigation water. These findings suggest that research focused on the development of inexpensive means to measure soil moisture and salinity in the field has the potential to reduce the demand for water. Whether that potential can be realized economically depends, of course, on the cost of the research.

The conclusions of previous work on risk production have been based on generalized (e.g., Pope and Kramer) or stylized (e.g., Feder, 1979) formulations of production relationships. These conclusions provide important insights in circumstances where relationships are not understood with complete precision. Our work suggests that where agricultural production relationships do not fit these general formulations, the implications of uncertainty in different parameters in those re-

TABLE 5. Empirical Results for Citrus with Uncertain S_0.

	$\bar{Q}$ (cm)	Q' (cm)	Q'' (cm) $\gamma =$			
			0.001	0.01	0.05	0.1
Base Run	99	99	102	119	130.5	132
$P_Q = 3.5$	89.5	89.5	92.5	108.5	118.5	120
$C = 20$	95.5	95.5	98.5	116.0	127.5	129
$b = -0.50$ ($\hat{S} = 85.6$, $a = 42.8$)	111	111	114.5	133.5	145	147
$S_0^l = 10$; $S_0^u = 50$ ($E(S_0) = 30$; $V(S_0) = 133.3$)	99	99	104.5	128.0	140.5	142.0

lationships may be ambiguous. In particular, the value of information about salinity levels in irrigated fields may be especially dependent upon whether irrigators are risk neutral or risk averse and may have significant implications for the amelioration of water scarcity in arid regions.

The analyses presented here represent only the first step in a comprehensive assessment of the economic implications of uncertain salt and moisture balances. Clearly, the analysis can be improved by examining the case where T and S_0 are simultaneously uncertain. Such an examination requires data on the joint distribution of the values of T and S_0, data which are not currently available. Beyond this, the introduction of additional control variables such as the acreage devoted to irrigated agriculture (where land has an opportunity cost) and the management of salt concentrations in irrigation water through blending from sources with differing qualities are obvious extensions. The impact of uncertainty in the long run, where accretion of salts in the soil profile may be a critical factor, is also deserving of further investigation. Such an extension might focus on the expected value of additional information about uncertain parameters and lead to conclusions about optimal levels of soil salinity and moisture content required to maintain productivity in the long run. The work reported here represents only the beginning of an extended analysis.

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