Potential Pitfalls in Renewable Resource Decision Making That Utilizes Convex Combinations of Discrete Alternatives

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Decision makers in renewable resource planning are often unable to specify their objective function *a priori,* and are presented with a discrete set of alternatives reflecting a range of options that are actually much more continuous. It is common for the decision maker to be interested in some other alternative than those originally developed. An iterative process thus often takes place between decision maker and analyst as they search for a satisfactory alternative. This paper analyzes the economic tenability of simply interpolating (taking convex combinations of) initial alternatives to generate new alternatives in this process. It is shown that convex combinations of outputs will be producible (feasible) with the interpolated input levels, under very common conditions. In fact, the cost estimate resulting from interpolating the cost of two (or more) alternatives will generally be an overestimate. The magnitude of this overestimate is investigated in a test case. It is concluded that this cost overestimate can be rather large, and is not systematically predictable. Only when the output sets in the original alternatives are very similar are the interpolated cost estimates fairly accurate.

Renewable resource management and planning problems often involve an extremely large spectrum of possible choices. The objective function in such problems is generally multidimensional and often includes market and nonmarket goods. When a decision maker cannot specify, *a*

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priori the objective function to be optimized, analysts commonly present him/her with a relatively small number of discrete alternatives reflecting the range of choices available. These alternatives are then subjectively evaluated, and further iterations between analysts and decision makers may be required [Candler and Boehlje, 1971, provide a more comprehensive discussion of this sort of process]. For the purposes of this paper, **it will suffice to characterize "alternatives" as being a set of outputs that the decision maker is interested in producing, with an attached cost of producing the given output set.** One common way to estimate this cost is through mathematical programming models (such as linear programming) designed to determine a **minimum** cost means of producing the output set.

If, out of a discrete set of alternatives, the decision maker cannot select one that is clearly satisfactory, more alternatives might have to be generated. This could be

a very difficult and costly task. For example, suppose that a national director of a renewable resource management agency is considering ten alternatives from each of 100 lower level planning units. Requesting all or a large portion of the lower level units to generate new alternatives could be quite costly. And, of course, any new alternatives cannot be guaranteed to be thoroughly satisfactory either. Many iterations might be required before the decision maker is sufficiently satisfied.

If the decision maker is not completely satisfied with any one alternative from an initial set, it is certainly not uncommon for the decision maker to be interested in some "interpolation" (weighted average) between two or more of the discrete alternatives. If these interpolations are usable, considerable effort and expense could be saved, because fewer alternatives might be required initially, and developing new alternatives would be quite simple. If one alternative output/cost vector is $\tilde{A}_1$ and another is $\tilde{A}_2$, and if a parameter s is defined such that $0 \leq s \leq 1$, then an interpolation of $\tilde{A}_1$ and $\tilde{A}_2$ is:

$$s\,\tilde{A}_1 + (1 - s)\tilde{A}_2 \tag{1}$$

This will be referred to as a convex combination of the two alternatives. Convex combinations of activity variables within a given linear program have been discussed in general linear programming presentations [e.g., Chiang, 1974; Silberberg, 1978] and in specific situations [e.g., Paris, 1981, who analyzed linear programs with multiple optima; and Dantzig and Wolfe, 1961, who analyzed a decomposition approach to solving large linear programs]. This is contrasted with the convex combinations of outputs and costs in different alternatives, discussed here. Each alternative would generally be determined with a different linear programming or other model solution. The different alternatives might be generated with different output prices in budget-constrained profit maximization or with dif-
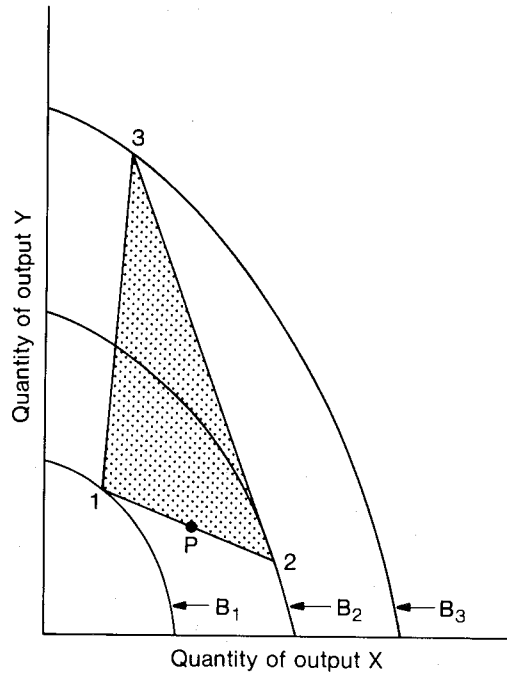
Figure 1. Region of Possible Output Combinations for Convex Combinations of Three Optimal Alternatives.

ferent output constraints in cost minimization. In either case, it is assumed that the purpose of the analysis is to provide the decision maker with a variety of cost-effective alternatives. This should be contrasted with the case discussed by Candler *et al.* [1981], where the decision maker controls only certain variables and the "analyst" is actually a lower level decision maker with an objective function of his/her own.

Consider the two-output product transformation curves shown in Figure 1. Points 1, 2, and 3 represent output sets for three initial alternatives. Associated with each of the three output sets is a minimum cost (B_1, B_2, and B_3). Convex combinations of any or all of the three alternatives could be constructed so as to create any output set in the shaded triangle. To make the discussion more tractable, this paper deals only with convex combinations of two alternatives (e.g., point P).

The principal question involving convex combinations of alternatives as defined here is the tenability of the interpolated cost prediction. In general, the interpolated output set could be unattainable at the predicted cost, or it could be attainable at a lower cost than the predicted cost. The next section will discuss the general theoretical conditions under which each of these outcomes can be expected. The following section will concentrate on production systems that can be modeled with linear programming techniques. A test case (utilizing linear programming) will be presented that evaluates the accuracy of interpolated cost predictions in a particular managed forest.

Before proceeding, it is important to note that convex combinations of discrete alternatives may also be important in "multilevel" optimization models such as those discussed by Bartlett [1974], Wong [1980], Hof [1983], and Hof *et al.* [1983] which seek a global optimum in a multilevel system (this is again contrasted with Candler *et al.* 1981). In these multilevel models, output/cost alternatives are generated by lower level models (linear programs) and, in turn, become zero–one choice variables in the higher level models. The higher level models select alternatives so as to optimize some objective function. Convex combinations of the alternatives generally occur if the higher level model is solved with a continuous variable approach such as linear programming. Integer programming can avoid the convex combinations by treating the alternatives discretely, but with increased solution cost and difficulty. Thus, this paper is relevant to certain multilevel optimization models as well as the more general application discussed heretofore.

Theory

Define a cost function, $C^*(\tilde{W}, \tilde{Y})$ which indicates the **minimum** cost of producing any given output vector $(\tilde{Y})$ with a fixed factor price vector $(\tilde{W})$. Now, in order to analyze convex combinations of two alternatives, define the following:

$\tilde{Y}^0$ = output vector in one alternative
$\tilde{Y}^1$ = output vector in the other alternative
$\tilde{Y}^\gamma$ = convex combination of $\tilde{Y}^0$ and $\tilde{Y}^1$, such that
$\tilde{Y}^\gamma$ = $s\tilde{Y}^0 + (1 - s)\tilde{Y}^1$, from equation (1)

The second order conditions for profit maximization problems are typically summarized as [Henderson and Quandt, 1971; Silberberg, 1978]: "The production function must form a closed, strictly convex point set." Utilizing C^*, the profit function (with, for simplicity, only two outputs) is:

$$\pi = P_1Y_1 + P_2Y_2 - C^*(Y_1, Y_2)$$

The second order conditions for profit maximization[1] can thus be interpreted as "The **cost** function must form a closed, strictly convex point set."

By definition of a "strictly convex point set" [see, for example, Silberberg, 1978, p. 385], this directly implies that:

$$C^*(\tilde{W}, \tilde{Y}^\gamma) \leq sC^*(\tilde{W}, \tilde{Y}^0) + (1 - s)C^*(\tilde{W}, \tilde{Y}^1) \quad (2)$$

Thus, the typical second order conditions for profit maximization imply that the interpolated cost estimates in convex combinations of alternatives should be greater than or equal to the true cost of the interpolated output levels in that convex combination. This holds even if:

$$C^*(\tilde{W}, \tilde{Y}^0) \neq C^*(\tilde{W}, \tilde{Y}^1)$$

For the special case where:

$$C^*(\tilde{W}, \tilde{Y}^0) = C^*(\tilde{W}, \tilde{Y}^1) = \bar{C}$$

somewhat weaker conditions are sufficient

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[1] The second order conditions are:

$$\frac{\partial^2 C^*}{\partial Y_1 \partial Y_1} > 0, \quad \frac{\partial^2 C^*}{\partial Y_2 \partial Y_2} > 0,$$

$$\frac{\partial^2 C^*}{\partial Y_1 \partial Y_1} \cdot \frac{\partial^2 C^*}{\partial Y_2 \partial Y_2} - \frac{\partial^2 C^*}{\partial Y_2 \partial Y_1} \cdot \frac{\partial^2 C^*}{\partial Y_1 \partial Y_2} > 0$$

for (2) to hold. These conditions are that the given isocost curve (associated with $\bar{C}$) is **quasi-convex**. This can be more precisely stated as:

> If a straight line joins any two points (a chord) on the given level curve of the cost function (the isocost curve), then that chord cuts through only points of lower cost than the two end points.

This condition is the geometric interpretation of the second order conditions of a multiple-output, cost-constrained revenue maximization problem. This condition only restricts the shape of a given level (isocost) curve, and does not restrict the expansion path of (distance between) different level curves [Henderson and Quandt, 1971; Silberberg, 1978].

The quasi-convexity condition on the isocost curve would imply that:

$$C^*(\tilde{W}, \tilde{Y}^\gamma) \leqq \bar{C}$$

Thus:

$$C^*(\tilde{W}, \tilde{Y}^\gamma) \leqq s\bar{C} + (1 - s)\bar{C}$$

which is equivalent to (2), by the definition of $\bar{C}$.

In sum, interpolated cost predictions in convex combinations of discrete alternatives should be larger than true minimum costs if the typical second order conditions for profit maximization hold. In the special case of equal costs between the discrete alterntives, somewhat weaker conditions are sufficient. It is interesting (though not surprising) that the two types of convex combinations of alternatives are distinguished by the different second order conditions required for cost-constrained revenue maximization models as opposed to profit maximization models. Because of this important distinction, the two types of convex combinations will be distinguished in the remainder of this paper.

Linear Programming Feasibility

Production capabilities of renewable resources are often modeled with linear programs. These models generally solve a slightly different problem than the classic cost minimization formulation—they treat the land base as fixed (and costless) and apply cost (budget) coefficients to alternative production activities that could be applied to the land. Thus, a typical linear program (LP) for renewable resource planning could be described with the following set of constraints:

$$a_{11} X_1 + a_{12} X_2 + \ldots + a_{1n} X_n \leqq r_1$$
$$\vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots$$
$$a_{k1} X_1 + a_{k2} X_2 + \ldots + a_{kn} X_n \leqq r_k$$

$$a_{\ell 1} X_1 + a_{\ell 2} X_2 + \ldots + a_{\ell n} X_n \geqq r_\ell$$
$$\vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots$$
$$a_{m1} X_1 + a_{m2} X_2 + \ldots + a_{mn} X_n \geqq r_m$$

where:

$\ell = k + 1$

X_i; $i = 1, n$ are production activities

$r_1, \ldots, r_k$ are limited inputs: land parcels ($r_2, \ldots, r_k$) and budget (r_1)

$r_\ell, \ldots, r_m$ are levels of outputs such as timber and forage

a_{ij}; $i = 1, m$ $j = 1, n$ is an "A matrix"—a matrix of technical coefficients relating production activities to inputs and outputs.

Any one of these constraints (or any linear combination of them) could be maximized or minimized in the LP solution, as an objective function. It will be useful for the time being, however, to treat all of them as constraints.

This LP can be written in matrix notation as:[2]

$$a\tilde{X} \leqq \tilde{r} \qquad\qquad (3)$$

Now, define two sets of right-hand sides as $\tilde{r}^0$ and $\tilde{r}^1$. And, define (any) two X vec-

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[2] Note that $\tilde{X}$ and $\tilde{r}$ are defined as "column vectors" to avoid the need for transposes. And, the ($r_\ell, \ldots,$ r_m) and associated a's are now negative, so that the less-than-or-equal-to sign applies throughout. Also, in practice, the land accounting rows may be strict equalities, but are treated as inequalities here for notational simplicity.

tors that are feasible, one for each set of right-hand sides, as $\tilde{X}^0$ and $\tilde{X}^1$. Then, define:

$$\tilde{X}^\gamma = [s\tilde{X}^0 + (1 - s)\tilde{X}^1]$$

and:

$$\tilde{r}^\gamma = [s\tilde{r}^0 + (1 - s)\tilde{r}^1] \qquad (4)$$

Textbook treatments of LP [Chiang, 1974; Silberberg, 1978] commonly prove that a set of constraints such as (3) form a convex set. It is a simple extension of this proof to show that convex combinations of two right-hand side vectors for which there are feasible solutions ($\tilde{X}^0$ and $\tilde{X}^1$), taken with the original A matrix, always yield a set of constraints with at least one feasible activity vector (namely, $\tilde{X}^\gamma$). Since the outputs and inputs are represented as right-hand sides in this model, this implies that there will always be at least one way to produce a convex combination of outputs with the associated convex combination of inputs—land and cost (budget). More formally, given:

$$a\tilde{X}^0 \leqq \tilde{r}^0$$
$$a\tilde{X}^1 \leqq \tilde{r}^1$$

then:

$$a\tilde{X}^\gamma = a[s\tilde{X}^0 + (1 - s)\tilde{X}^1]$$
$$= sa\tilde{X}^0 + (1 - s)a\tilde{X}^1$$

so:

$$a\tilde{X}^\gamma \leqq s\tilde{r}^0 + (1 - s)\tilde{r}^1$$
$$a\tilde{X}^\gamma \leqq \tilde{r}^\gamma \qquad \text{by (4), Q.E.D.}$$

This activity vector ($\tilde{X}^\gamma$) is feasible, but would generally be suboptimal, given any particular objective funtion—even if $\tilde{X}^0$ and $\tilde{X}^1$ are both optimal. $\tilde{X}^\gamma$ would have too many activity variables in the solution basis unless the same variables happened to be nonzero in $\tilde{X}^0$ and $\tilde{X}^1$. Thus, if cost was minimized subject to the other right-hand sides in $\tilde{r}^\gamma$, a value less than or equal to $[sr_1^0 + (1 - s) r_1^1]$ would be obtained (where r_1^0 and r_1^1 are the right-hand sides for the cost constraints in $\tilde{r}^0$ and $\tilde{r}^1$).

A Test Case

The upshot of the previous discussion is that convex combinations of alternatives generated with LP models will always be feasible,[3] but the estimated cost will generally be suboptimal (larger than the minimum cost that could actually be obtained if the LP was resolved). The possible magnitude of error in the predicted cost of convex combinations of alternatives is of interest for two reasons. First, if cost predictions are inaccurate, the decision maker may choose an output set other than that which he/she would have chosen otherwise. Second, resources (especially agency budget) may be misallocated if cost predictions are not close to the actual cost of producing a selected output set. In order to provide some evidence regarding the potential magnitude of this cost overestimate, a test case was analyzed.

Data assembled by the Coconino National Forest (central Arizona) staff in the current USDA Forest Service land management planning effort were used to build an LP model for this analysis. The data consisted of production coefficients and costs by time period for a set of time-scheduled alternative management prescriptions (production activities) that could potentially be applied to land units called

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[3] This feasibility is ensured, even if the alternatives were generated with linear programs that have different numbers or types of activities (columns) in their A-matrices. Nonbasic columns could hypothetically be added to each linear program so as to make the A-matrices identical. The proof in the previous section would then apply. In analyzing a given land area, the number and type of rows in the linear programs should not, in principal, vary from one alternative to the next. If, for example, a given alternative was developed with no forage output constraint, it effectively reflects a constraint that forage output has a right-hand side greater than or equal to zero. Assuming that the output rows include all constraints (other than cost) whose right-hand sides vary across alternatives, convex combinations of attainable output right-hand sides will be attainable, given the associated convex combination of cost right-hand sides.
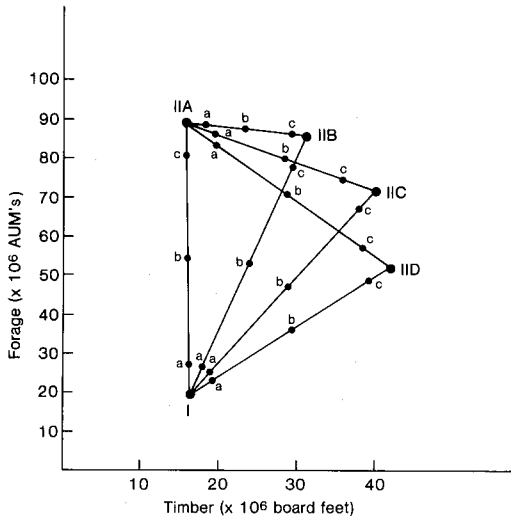
Figure 2. Graphical Depiction of Alternatives and Convex Combinations: a = 90/10 Convex Combinations; b = 50/50 Convex Combinations; c = 10/90 Convex Combinations.

analysis areas, prices for valued outputs, and coefficients for other constraints. The model determines an optimal timber harvest schedule as well as scheduling other management actions and output mixes. The data base included 51 analysis areas. The planning horizon was 200 years, represented by decades for the first 50 years and by three 50-year periods for the remaining 150 years. The number of valued outputs included in the model was limited to only two products: sawtimber (board feet) and forage grazing (AUMs). Every attempt was made to build a realistic model, however, it cannot be taken as an exact representation of the Coconino National Forest planning model.

A third output included in the model was net timber yield, an aggregation of the yields of sawtimber, pulpwood, and fuelwood. This output was not priced and was included in the model for the sole purpose of imposing a nondeclining yield constraint. It was considered to be more in keeping with standard practice to apply this constraint to the total output of

all timber products rather than to sawtimber only. Also in keeping with standard practice, an "ending inventory constraint" was included that constrained the total timber inventory at the end of the planning period to be greater than or equal to an inventory level associated with long-run sustained yield capacity. The LP A-matrix was generated with the Direct Entry FORPLAN [Johnson, 1982] matrix generator, and solved with the UNIVAC FMPS linear programming OPTIMIZE solution procedure. The model is structured similarly to what Johnson and Scheurman [1977] referred to as Model I, with the addition of the forage outputs.

Generation of a Set of Alternatives

For test case purposes, a set of alternatives was desired that spanned the production possibilities with a given budget and also included an alternative associated with a lower budget. This allowed examination of both within- and between-budget convex combinations. To this end, the following steps were taken to generate a set of alternatives with the test case linear program. These alternatives are shown graphically in Figure 2.

Step 1

Maximize present net worth under no budget restrictions (with the output prices included by the Coconino National Forest), and then extract the cost associated with that unfettered maximization. This discounted budget ($308,440,000 for 200 years) was used to construct alternatives IIA, IIB, IIC, and IID in Figure 2.

Step 2

Maximize forage subject to the budget constraint ($308,440,000), and then maximize timber subject to that forage level. This generated point IIA in Figure 2.

TABLE 1. Output Sets and Minimum Costs in Alternatives.

Alterna- tive	Total Timber (Bd. Ft.)	Total Forage (AUMs)	Minimum Cost ($1,000)
IIA	15,333,600	88,991,600	308,440
IIB	30,892,300	85,675,000	308,440
IIC	39,701,200	72,347,800	308,440
IID	41,467,500	52,347,800	308,440
I	16,387,800	19,871,500	205,627

Step 3

Sequentially lower forage output levels and maximize timber subject to those forage levels and subject to the budget constraint ($308,440,000). This procedure generated points, IIB, IIC, and IID in Figure 2. Point IID involves a level of timber production very near the maximum possible level of timber production with a budget of $308,440,000 (determined with a separate run to be 41,472,140 board feet). This approach was taken because it was desired (for consistency) to generate all alternatives at this budget level by maximizing timber subject to budget and forage constraints. Obviously, considerable experimentation was necessary, and points IIB and IIC were selected as intermediary points between IIA and IID.[4]

Step 4

Point I was determined by maximizing present net worth (with output prices as described above) with a budget two-thirds of $308,440,000, or $205,627,000.

The alternatives are summarized in Table 1.

Determination of Convex Combinations

As noted earlier, convex combinations of the output sets in Figure 2 can produce

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[4] The original maximization of present net worth solution was not used as an alternative because it was not a good intermediary point between IIA and IID.

any output set bounded linearly by I, IIA, IIB, IIC, and IID. To test within-budget convex combinations, interpolations were calculated between IIA and IIB, between IIA and IIC, and between IIA and IID. To test between-budget convex combinations, interpolations were calculated between I and IIA, between I and IIB, between I and IIC, and between I and IID.

In an attempt to generate convex combinations with the maximum error possible, 50/50 (s = 0.5) convex combinations were tested. The maximum error will not necessarily occur with equal proportions of initial alternatives, but without prior knowledge, this was expected to yield a reasonable approximation. To determine if error is relatively low near initial (optimal) alternatives, 10/90 (s = 0.1) and 90/10 (s = 0.9) convex combinations were also tested. The convex combinations are described in Table 2 and they are also indicated on Figure 2.

Cost Analysis

The results of the cost analysis for the within-budget convex combinations are presented in Table 3. The predicted cost is the interpolation of the (minimum) costs from the alternatives in each convex combination. The actual cost of each convex combination output set was determined by constraining the timber and forage outputs to be greater than or equal to the appropriate levels, and minimizing discounted cost with the LP. It should be noted that this analysis determined minimum total time period (200 years) cost, subject to constraints on total time period output levels. In actual planning and management situations, shorter time periods may be of interest, but the results should be analogous. Another possibility would be that the implied schedule in the convex combination could be retained in determining actual cost. If this analysis had been constrained to meet the exact schedule of the convex combination, then the

TABLE 2. Output Sets in Convex Combinations of Alternatives.

Alterna-tives	Timber	50/50		90/10		10/90
		Forage	Timber	Forage	Timber	Forage
IIA, IIB	23,112,970	87,333,315	16,889,474	88,659,967	29,336,446	86,006,663
IIA, IIC	27,517,405	80,669,735	17,770,361	87,327,251	37,264,449	74,012,219
IIA, IID	28,400,530	70,669,735	17,946,986	85,327,251	38,854,074	56,012,219
I, IIA	15,860,700	54,431,580	16,282,380	26,783,540	15,439,020	82,079,620
I, IIB	23,640,070	52,773,265	17,838,254	26,451,877	29,441,886	79,094,653
I, IIC	28,044,505	46,109,685	18,719,141	25,119,161	37,369,869	67,100,209
I, IID	28,927,630	36,109,685	18,895,766	23,119,161	38,959,494	49,100,209

resulting costs would necessarily be closer to the predicted costs—the addition of constraints always reduces (or does not change) the level of objective function attainment in solution. It was felt that constraining the actual cost analysis to the rather arbitrary schedule of the convex combination is inconsistent with the logic of a minimum cost function; and the approach taken seems to be the most generally applicable.

In Table 3, the errors in the predicted costs of the convex combinations are quite variable. They are very small (less than one percent) for all of the convex combinations between points IIA and IIB. This would suggest that convex combinations of alternatives that have the same cost and are not terribly "far" from each other (in the sense of Euclidean distance) might actually cost only slightly less than the predicted cost. For a given type (50/50, 90/10, or 10/90) of convex combination of

alternatives that are on the same product transformation curve, it also appears that the "farther apart" the alternatives are, the larger the errors in the predicted cost.

The errors in the predicted costs of the 90/10 combinations are not similar to those of the 10/90 combinations. For example, compare the 90/10 and 10/90 convex combinations of alternatives IIA and IID. The cost error in the former is just under four percent, while the cost error in the latter is just over 19 percent. In general, the closer the convex combination is to alternative IIA, the lower the error in predicted cost. The production system used in this test case is indicated to be highly nonhomothetic (the product transformation curves are not close to being parallel). In such a production system, it would be very difficult for the decision maker to predict the error in a given convex combination's predicted cost without extensive knowledge of the production system

TABLE 3. Convex Combination Cost Tests—Within Budget.

Alterna-tives	Proportions	Predicted Cost	Actual Cost	Error	% Error
IIA, IIB	50/50	$308,440,000	$306,324,300	$ 2,115,700	0.69
IIA, IIC	50/50	$308,440,000	$288,172,100	$20,267,900	7.03
IIA, IID	50/50	$308,440,000	$259,827,600	$48,612,400	18.71
IIA, IIB	90/10	$308,440,000	$307,402,400	$ 1,037,600	0.34
IIA, IIC	90/10	$308,440,000	$303,162,200	$ 5,277,800	1.74
IIA, IID	90/10	$308,440,000	$296,600,600	$11,839,400	3.99
IIA, IIB	10/90	$308,440,000	$306,203,300	$ 2,236,700	0.73
IIA, IIC	10/90	$308,440,000	$289,023,000	$19,417,000	6.72
IIA, IID	10/90	$308,440,000	$259,033,800	$49,406,200	19.07

TABLE 4. Convex Combination Cost Tests—Between Budget.

Alternatives	Proportions	Predicted Cost	Actual Cost	Error	% Error
I, IIA	50/50	$257,033,500	$219,862,100	$37,171,400	16.91
I, IIB	50/50	$257,033,500	$223,446,700	$33,586,800	15.03
I, IIC	50/50	$257,033,500	$221,203,500	$35,830,000	16.20
I, IID	50/50	$257,033,500	$213,869,100	$43,164,400	20.18
I, IIA	90/10	$215,908,300	$196,165,600	$19,742,700	10.06
I, IIB	90/10	$215,908,300	$196,939,300	$18,969,000	9.63
I, IIC	90/10	$215,908,300	$196,503,000	$19,405,300	9.87
I, IID	90/10	$215,908,300	$195,282,800	$20,625,500	10.56
I, IIA	10/90	$298,158,700	$285,082,800	$13,075,900	4.59
I, IIB	10/90	$298,158,700	$284,656,700	$13,502,000	4.74
I, IIC	10/90	$298,158,700	$271,923,700	$26,235,000	9.65
I, IID	10/90	$298,158,700	$250,987,300	$47,171,400	18.79

involved—which is precisely the situation creating the potential use of convex combinations in the first place.

Table 4 presents a similar cost analysis on convex combinations of alternatives that have different costs (they are on different product transformation curves). The percentage errors in predicted cost relative to actual cost are roughly between five and 20 percent. The even 50/50 convex combinations generally show the largest errors. The largest error (over 20 percent) reported in Table 4 is for the 50/50 convex combination of alternatives I and IID. As in Table 3, the 90/10 and 10/90 convex combinations do not exhibit similar errors.

Conclusion

The overall magnitude of errors in predicted costs in the case study varied from less than one percent to more than 20 percent. Whether or not these magnitudes are acceptable depends on the particular planning or management situation. And, of course, these magnitudes apply only to this test case. These results suggest, however, that highly variable and unpredictable errors may be encountered in the predicted cost of convex combinations of alternatives, and that the farther apart the alternatives are, the stronger the potential for substantial error.

It has been shown that alternatives developed with linear programs can be taken in convex combinations and the resulting output set will always be feasible in terms of the linear program. Another important consideration is the decision-making setting within which the analysis takes place. If an analyst or a line officer provides a decision maker with a set of alternatives, this set may have passed feasibility tests that are outside the model (political, biological, operational, etc.). In that case, the convex combination may be feasible only in the sense of the linear program—not in the sense of pragmatic implementability. For example, a timber-intensive alternative may produce enough timber to make a timber sale operational and a forage-intensive alternative may produce enough forage to make a grazing contract operational, but a convex combination of the two alternatives might not produce enough of either output to support a timber sale or a grazing contract. In such a situation, the cost analysis above would only indicate part of the danger in attempting to implement a convex combination of alternatives. In general, this study indicates that convex combinations of discrete alternatives should be used sparingly and with caution.

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