Modeling Operating Rate Decisions in the Canadian Forest Industries

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This article examines the problem of characterizing production structures when there is input fixity but fixed inputs can be utilized with varying intensities. Unless the rate of utilization of quasi-fixed factors is adequately measured, primal or dual characterizations of producer behavior common in the empirical literature may not be valid. The problem is overcome by specifying another input, the operating rate, which firms can use in the short run to adjust to unexpected market changes when there is quasi-fixity in production. The model is applied to the Canadian pulp and paper and sawmilling industries. The results do not permit rejection of the hypotheses of quasi-fixity and varying utilization of quasi-fixed factors in the short run. A model of instantaneous adjustment of factor inputs is clearly outperformed by the quasi-fixity model incorporating an operating rate decision.

Key words: cost functions, forest industries, operating rate production functions, quasi-fixity.

Since the early seventies, many studies have utilized flexible functional forms and duality theory to analyze the characteristics of production and to measure producer responses to economic changes. Most of these studies examined the forest industries in Canada and the United States, but a few focused on other countries.[1]

Two critical assumptions in almost all of these studies are that (*a*) production is efficient, i.e., firms are operating along their production possibility frontiers; and (*b*) all or a subset of the inputs can be instantaneously adjusted to their optimum levels without adjustment costs. Assumption (*a*) is required to rationalize the econometric estimation of production functions, while assumptions (*a*) and (*b*) are required if a static dual model is used or if the first-order conditions for optimization are estimated jointly with the production function (the usual procedure with flexible functional forms).

If the above assumptions are inadequate, biases in parameter estimates and test statistics can result due to the misspecification of the econometric model. Furthermore, if the model specifies long-run adjustments with respect to a subset of the inputs (for example, labor) but these inputs are in fact quasi-fixed, the model may be inappropriate for the analysis of short-run policy issues. Quasi-fixity is used in this article to indicate short-term inability to adjust inputs to their optimum levels due to cost constraints.

Several authors of forest industry studies have addressed the problem of assuming instantaneous adjustment by choosing models in which one or more inputs, typically capital, are held fixed (De Borger and Buongiorno; Abt; Constantino and Haley). A drawback of this approach is that the choice of which inputs to hold fixed or assume instantaneously variable is determined by a priori theoretical or em-

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[1] Examples are Stier (1980a, b), Merrifield and Haynes (1983, 1984), Nautiyal and Singh (1983, 1985), Singh and Nautiyal, Banskota, Phillips, and Williamson, Martinello (1985, 1987), Abt, Constantino and Haley in the wood products industries; and Sherif, De Borger and Buongiorno, Stier (1985), and Nautiyal and Singh (1986) in the pulp and paper industries.

pirical considerations and is introduced in the model as a maintained hypothesis. Within the typical observation unit of one year, it is likely that all inputs exhibit some degree of quasi-fixity and undergo only partial adjustment towards their optimal levels. Constraining the input levels to be fixed when in reality there is partial adjustment can limit the usefulness of the elasticities estimated even for short-run policy analysis.

The more recent dynamic models (Berndt, Morrison, and Watkins) which have been applied to the forest industries by Merrifield and Singleton partially overcome this problem by allowing the estimation of the parameters of the adjustment process, but the a priori specification of fixed inputs is still required. This approach requires an assumption of intertemporal optimizing behavior and adds an equation describing the adjustment process, typically investment, to the equations representing the fully adjustable variable factors. Due to the econometric constraints, only one or two inputs are assumed to be fixed, and the remaining inputs are usually treated as variable. Another approach, pioneered by Mohr and applied to the forest sector by Nautiyal and Singh (1986), and Singh and Nautiyal involves estimation of the production or cost function and factor demand equations within a partial adjustment process. Although this approach does not require the a priori choice of fixed inputs and is useful for estimating the long-run demands and production structure, it is restrictive in that the adjustment process is independent of economic variables.

None of the approaches noted above satisfactorily represents quasi-fixity. The problem is that data limitations frequently force researchers to ignore another choice that producers have available when faced with input fixity, viz., varying the rate of utilization of quasi-fixed factors. That is, even if certain inputs are quasi-fixed in the short run, they can, in most situations, be utilized with varying intensity, so that the service flows from those quasi-fixed inputs are in fact variable. If there are no fixed factors and firms in the industry are characterized by instantaneous and costless adjustment, one can expect all inputs to be utilized at constant rates. Long-run cost functions, for example, would be appropriate for modeling the industry in this case. On the other hand, if quasi-fixity is present, the rate of utilization of quasi-fixed factors will vary. The latter case would result in misspecification of the model if quasi-fixed factors were treated as fixed, whether in a static or in a dynamic context, because their service flows would be variable. Lack of data on rates of utilization of quasi-fixed factors presents a serious difficulty. Moreover, in this case the duality results normally used to justify cost or profit functions would not hold, because two different production levels using the same amount of measured inputs but different utilization rates would correspond to the same dual function. Some of these problems have been noted by Cardellichio although he did not identify possible solutions.

In this paper we use a model that can calculate the rate of utilization of quasi-fixed factors and yield estimates of long-run production structures that are consistent with quasi-fixity in the short run but with variable service flows. To do this, we adopt the theory of production developed and tested at the macroeconomic level by Helliwell and Helliwell and Chung. The major innovation is the way short- and long-run producer decisions are integrated in the production function through the specification of an additional input, the operating rate or rate of utilization of quasi-fixed factors, which is one short-run decision instrument firms can use to adjust to temporary or unexpected change. We apply and test this theory at the micro level for the Canadian sawmilling and pulp and paper industries. We test for quasi-fixity of factor inputs and varying service flows and investigate economic factors underlying the choice of a rate of utilization of quasi-fixed factors.

The application of this theory to a micro setting is important. In technologically unsophisticated industries such as sawmilling, one could hypothesize, along the lines of Berndt and Fuss, that there is a fully variable factor, wood or sawlogs, and that service flows of fixed factors will be proportional to wood consumption. In this case the specification of a gross output type production function with wood as a fully variable input would be a correct procedure. However, this is an empirical matter, and in this paper we investigate whether such an assumption leads to a reasonable description of the behavior of the industry.

In the model to be discussed, producers form expectations about future market conditions, such as prices and sales levels, to make long-run production plans and choose profit-maximizing input levels. Changes in market conditions that affect these expectations induce adjustments in factor input levels with the speed of adjustment dependent on the degree of quasi-fixity of each input. Since there are costs associated with the adjustment of quasi-fixed factors, firms may vary their operating rate or adjust inventory levels to hedge against adjustment to what may prove to be temporary market changes. This behavior is commonly observed but not incorporated in the estimation of production technologies. The model does not impose instantaneous adjustment of any of the inputs as a maintained hypothesis, but it can encompass that situation as a special case.

The Theoretical Model

Suppose the technology of a firm can be represented through the following production function:

$$Y = F(M, L, K, E), \tag{1}$$

where Y is output and M, L, K, E are, respectively, the measured input levels of materials, labor, capital, and energy. If all inputs are variable so there are no adjustment costs and the firm is behaving competitively in input markets, factor levels will be chosen to minimize average production costs. The following total cost function can be specified:

$$C^* = C(P_M, P_L, P_K, P_E, Y). \tag{2}$$

The conditional demand for material is:

$$M^* = D(P_M, P_L, P_K, P_E, Y), \tag{3}$$

and similarly for the other inputs. The asterisk indicates cost-minimizing input levels of producing output, Y, given actual input prices, P_M, P_L, P_K, P_E. The production function (1) can then be rewritten as:

$$Y = F(M^*, L^*, K^*, E^*). \tag{4}$$

The four equations above summarize a very popular model of production technology and behavior, which has received wide attention both in the forest products and other industries. There are strong arguments for an alternative hypothesis, however, that most, if not all, of the inputs to the firm are quasi-fixed in the short run. In such a case the above model is inadequate. Quasi-fixity results from costs associated with adjustment of input levels particularly when uncertainty exists about the permanence of the change in market conditions. Production labor is a common example since hiring additional labor has associated costs that will cause a producer to look first at alternatives such as overtime, i.e., increased intensity of utilization of the labor input. If labor is measured as number of workers rather than hours actually worked, variations in the intensity of use of the labor input will not be captured in the data. There are also costs associated with releasing labor when a producer is faced with a decrease in demand. These costs can take the form of severence pay or the present value cost of rehiring and retraining labor if demand rises in the future. These costs may deter the employer from releasing labor and result in lower utilization of employee services if production is reduced.

Capital is another example of a quasi-fixed factor with a high cost of short-run adjustment and is in many cases modeled as fixed in the short run. Producers can vary the rate at which capital is utilized, though, by changing the rate of production or changing the hours of operation.

Because of different adjustment costs, inputs will have varying degrees of quasi-fixity. Capital is likely to have a higher degree of quasi-fixity than labor, and labor is likely to be more quasi-fixed than the materials input. We argue that all factor inputs face adjustment costs, which prevent instantaneous adjustment to some degree. Following this line of reasoning, instantaneous adjustment should not be imposed a priori in the production model but rather tested empirically.

Ignore for now the optimization assumptions implicit in equations (2)–(4), and suppose we were to estimate econometrically equation (1), the production function. An immediate problem with the existence of quasi-fixed factors in production is the measurement of their contribution to the production of output. Rather than the stock of quasi-fixed factors, their services flows should be measured. In other words, if quasi-fixed factors can be used with varying levels of intensity, the amount of output forthcoming can be different for the same measured input quantities. In this case the concept of a production function such as (1) loses meaning. Data on the utilization rates of inputs are usually not available and have to be obtained indirectly. Due to these measurement problems, the firm may appear to be producing an amount different from that indicated by its production function. Instead of Y in (1), it will be producing a different amount, Y^A, which is actual output.

Suppose now that the production model is that specified in equations (2)–(4). In this case we are requiring not only that observed output be explained by the measured levels of the inputs but also that these levels be optimum, i.e., that they minimize the costs of producing output, Y. Following the work of Helliwell, the cost-minimizing output level is called normal output (Y^N), that is, the output forthcoming at minimum average costs and at a normal rate of utilization of quasi-fixed factors.

Now suppose an unforeseen change in demand takes place resulting in an increase in quantity demanded. Assuming inventories do not change, a typical firm will produce an actual output, Y^A, greater than normal output, Y^N. (Y^A is that output which maximizes short-run profits or quasi-rents.) How can Y^A be produced? It will be produced by increasing the amount of instantaneously variable inputs, if any, and by utilizing quasi-fixed inputs more intensively, i.e., by varying the operating rate. But the industry will not be producing under the conditions of the model in equations (2)–(4). That model is now inappropriate because input levels will not be at their long-run, cost-minimizing equilibrium levels, and quasi-fixed factors will be utilized more intensively than they would be at normal output. Even model (1) is not appropriate if one cannot adequately measure the variable flows of services from the quasi-fixed factors.

A consistent way of defining the operating rate (OR), is:

$$OR = Y^A/Y^N. \tag{5}$$

Clearly, if $Y^A = Y^N$, the operating rate will equal one. Given that normal output, Y^N, can be explained by equations (2)–(4), it now remains to explain actual output, Y^A, or equivalently to explain the operating rate. With the above definition of operating rate, equation (5) can be modified to:

$$Y^A = Y^N \cdot OR = F(M, L, K, E) \cdot OR. \tag{6}$$

If the operating rate is an important explanatory variable of output, it should be estimated jointly with the production structure. In other words, if we believe that the observed data were generated by an industry in short-run disequilibrium so that measured input levels are not optimal, it would be incorrect to estimate $Y^N = F(M^*, L^*, K^*, E^*)$ or for that matter $Y = F(M, L, K, E)$. In order to estimate equation (6) it is necessary to explain the choice of the operating rate, i.e., why does actual output differ from normal output.

An implication of quasi-fixity in production is that minimization of short-run costs in (2) will not take place instantaneously but will be based on expected future market conditions. If changes in demand had been foreseen and considered permanent, quasi-fixed factors would have been adjusted or would be moving along an adjustment path to their optimum levels. If unforeseen changes in demand take place—a typical situation in the lumber and pulp and paper industries—a firm can adjust its output level by varying the operating rate and any fully variable factors that may exist. Thus, a variable capturing the effect of unexpected demand changes should be important in explaining the operating rate decision.

Another implication of quasi-fixity of factor inputs is that, in addition to meeting demand changes with changes in production through the operating rate and variable inputs, producers have the option of inventory accumulation or depletion. There are increasing costs associated with the more intensive use of quasi-fixed factors. But there are also costs associated with an "abnormal" inventory level. For example, if existing inventories are very large, interest will be foregone on the capital held in inventory, and it may be optimal to sell from inventory rather than increase the operating rate to produce more. The operating rate will be chosen so that at the margin costs of selling out of inventory are equal to the short-run cost of increasing production. Therefore, a variable accounting for the inventory stocks should be important in explaining the operating rate decision.

For a given level of sales, production may still be increased through the operating rate with the objective of accumulating short-term inventories for future sales. If producers expect that additional profits may be generated in the following year through an increase in sales, they face the trade-off of increasing the operating rate in the current year or deferring increased production until the following year. Thus, a third variable that measures the expected returns-to-inventory accumulation for future sales could be important in explaining the operating rate decision.

Using the discussion above, it is possible to build a two-component model. The first component is a long-run production structure which incorporates cost-minimizing behavior with respect to all inputs, as described by equations (2)–(4). It gives the output Y^N forthcoming under normal operating rates, i.e., the operating rate is equal to one. Intuitively, and in this context, Y^N is a measure of the capacity output where capacity stands for that point where short-run average production costs are equal to long-run average production costs, or, in the case of a constant returns-to-scale industry, where short-run average costs are minimized (Berndt and Morrison).

The second component of the model which explains the deviation of actual or short-run output from long-run output is the producer operating rate. This operating rate is specified as a function of unexpected demand conditions, inventory stocks, and returns to accumulating inventories for future sales:

(7) $OR = G$(unexpected sales, inventory levels, returns-to-inventory accumulation).

The Empirical Model

The Long-Run Production Structure and Normal Output

A translog functional form is utilized to characterize the long-term production structure, i.e., normal output and cost-minimizing input levels. The four-factor translog production function is:

(8) $$\ln Y^N = \alpha_0 + \sum_i \alpha_i \ln X_i + 0.5 \cdot \sum_i \sum_j \delta_{ij} \ln X_i \cdot \ln X_j + \alpha_T \cdot T + 0.5 \cdot \gamma_{TT} \cdot T^2,$$

where, $i, j = L, M, K, E$ are, respectively, quantity measures of labor, materials, capital, and energy inputs. Y^N is normal output and T is a time trend which is interpreted as a proxy for technical progress.[2]

We assume constant returns to scale at the industry level and impose the homogeneity restrictions together with the usual symmetry restrictions.[3] If the production function (1) is homogeneous of degree one and there is profit maximization, then the output elasticities are equal to the cost shares, and we can add the cost-minimizing input cost share equations (Z^*) to the production function:

(9) $$Z_i^* = \alpha_i + \sum_j \delta_{ij} \ln X_j.$$

To increase the efficiency of the parameter estimates, the cost share equations are estimated with the production function as a system of simultaneous equations.

There are conditions on the theoretical properties of the long-run translog production function, and these are well described in the literature (Berndt and Christensen).[4] The production function should be monotonic and quasi-concave in quantities. Monotonicity implies that the predicted cost shares are positive, while quasi-concavity ensures that the isoquants are convex to the origin and also implies that the translog bordered Hessian is negative semidefinite. Useful results generated by the estimation of production technologies are elasticities of substitution, own-price elasticities of demand, and the rate of technical progress.

The Operating Rate Decision

The standard model as specified by equations (2)–(4) requires cost-minimizing behavior to hold at each data point. This requires the industry to be producing normal output, Y^N, for each data point. We argued previously that this assumption may be inappropriate and should not be maintained a priori. The more general model proposed here assumes that the firm is operating at Y^N only on average.

Similar to the concept of normal output, Y^N, we can think of normal sales, S^N. Normal sales are sales levels that are expected and viewed as permanent, so that the quasi-fixed factors are chosen to produce the normal output, Y^N, at minimum costs. Thus we define normal sales (S^N) as a proportion of capacity output (Y^N). The proportionality factor is calculated as the sample mean of the ratio of actual sales, S^A, to normal output, Y^N. Unexpected demand is measured as changes in the ratio of actual sales, S^A, to normal (or expected) sales, S^N. This specification implies that the operating rate will be a function of the logarithmic gap between expected and actual sales. S_t^N is calculated as:

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[2] In this specification of the model we assume Hicks-neutral technical change because of problems collecting for the sawmilling industry a consistent data set which would provide enough degrees of freedom for a biased technical change model.

[3] The assumption of constant returns to scale is for consistency with our definition of variables affecting the operating rate decision to be discussed later. As will be shown, we assume that the industries are in long-run competitive equilibrium at the mean of the data and that there are no quasi-rents being generated.

[4] For a detailed discussion of translog production functions in the forest industries, see Merrifield and Haynes (1983, 1984).

$$(10) \qquad S_t^N = (1/n \cdot \sum_t^n (S_t^A/Y_t^N)) \cdot Y_t^N, \quad t = 1, \ldots, n,$$

where n is the number of observations in the data set, and S_t^A equals actual sales in year t.

Abnormal inventories also should be measured relative to some target level of inventories. As with normal sales, we define the target or normal level of inventories, I^N, as a proportion of normal output, Y^N. The proportionality factor is calculated as the sample mean of the ratio of actual or opening inventories, I^A, to normal output, Y^N. Abnormal inventories are measured as the ratio of target or normal inventories, I^N, to opening inventories, I^A. The operating rate will be a function of the logarithmic gap between target and actual inventories. I_t^N is estimated as:

$$(11) \qquad I_t^N = (1/n \cdot \sum_t (I_t^A/Y_t^N)) \cdot Y_t^N,$$

where I_t^A is actual inventory in period t.

To provide a measure of the returns-to-inventory accumulation for future sales, a cost/revenue ratio is used. A measure of expected marginal profitability of accumulating inventories would be the ideal, but as accounting costs and revenues do not fully reflect changes in the operating rate, such a measure is not available and some type of proxy variable is required. We specify a cost/revenue index which is a measure of quasi-rents to act as a proxy:

$$(12) \qquad CR_t = C_t/(Y_t^A \cdot P_t),$$

where C_t equals cost of production including capital costs in year t, Y_t^A is actual output in year t, and P_t is output price in year t.

Following the discussion of the previous section, we expect the operating rate to equal one when there is no unexpected demand, inventories are at their desired levels, and there are no short-run returns to accumulating inventories for future sales. Through the construction of the variables, we assume that the operating rate equals one at the mean of the data, which implies that expectations are realized on average. This condition is not required to hold at each observation, in contrast to most production models available in the literature which implicitly assume $OR = 1$, or equivalently, $Y^A = Y^N$. In order to ensure that the operating rate is one when the short-run variables are at their target or expected levels, a log-linear functional form without a constant term is specified for the operating rate:

$$(13) \qquad OR = Y^A/Y^N = (S^A/S^N)^{\beta_S} \cdot (I^N/I^A)^{\beta_I} \cdot (CR)^{\beta_{CR}}.$$

Taking logs,

$$(14) \qquad \ln Y^A = \ln Y^N + \beta_S(\ln S^A - \ln S^N) + \beta_I(\ln I^N - \ln I^A) + \beta_{CR} \ln CR.$$

Y^N, which enters the definition of S^N and I^N, is obtained from the translog production function. We propose certain prior expectations of the signs and magnitudes of the coefficients of the operating rate equation. β_S is expected to be positive, so that an increase in unexpected sales leads to an increase in operating rate and in output. In the model, a movement of Y^A away from Y^N can only occur through a change in the operating rate, since the effects of changing cost-minimizing input levels on output are measured through Y^N. If an increase in sales results in an instantaneous increase in Y^N, in which case $Y^N = S^N$, β_S will equal zero and the operating rate will not be an important instrument. On the other hand, if all variation in output due to unexpected demand changes is met through variations in the operating rate, inventories will play no role as a buffer between production and sales. These conditions would imply that β_S and $\beta_I = \beta_{CR} = 0$.

β_I should be positive so that for constant levels of sales and profitability, high opening inventories which have accumulated previously because of unexpectedly low sales will lead to a decline in the operating rate. β_{CR} is expected to be negative so that for constant levels of sales, higher short-run profits (smaller CR) would increase the operating rate with additional production used to accumulate inventory.

Econometric Estimation

Given the hypothesis that the observed data were generated under a situation of quasi-fixity of factor inputs and not cost-minimizing equilibrium, it is incorrect to estimate the long-run production structure given by equations (2)–(4) separately from the operating rate decision given in equation (7). The two components of the model are jointly estimated and the econometric model is:

$$(15) \qquad Y^A = \text{translog}\,(X_i) \cdot \text{log-linear}\,(d_j),$$

where "translog (X_i)" describes the long-run production structure at cost-minimizing equilibrium when the inputs, X_i, are optimally chosen given observed input prices, and "log-linear (d_j)" describes the operating rate decision as a function of the operating rate variables, d_j.

Similar to normal output, Y^N, we can think of normal or optimal cost shares, Z^{N*}, which are the cost-minimizing input shares when it is optimal to produce normal output and utilize input levels, X_i.[5] In other words, they are the output elasticities with respect to a change in the measured input level. Observed shares may not be the cost-minimizing shares, i.e., they will differ from output elasticities just as observed output may not be capacity output. To account for these differences, the short-run operating rate variables are logarithmically added to each of the share equations.[6] No theoretical cross-equation restrictions are placed on the coefficients of these variables (d_j) in the share equations. Each share equation, Z_i, can then be estimated as:

$$(16) \qquad Z_i = Z_i^* \cdot OR_i(d_j),$$

or:

$$(17) \quad Z_i = \alpha_i + \sum_j \gamma_{ij} \ln X_j + \beta_{Si}(\ln S^A - \ln S^N) + \beta_{Ii}(\ln I^N - \ln I^A) + \beta_{CRi} \ln CR \qquad i, j = L, K, M, E.$$

The magnitudes of the coefficients on the operating rate variables in the share equations will indicate how observed cost shares are affected by deviations of actual from normal output. They will show the degree of nonneutrality of short-run adjustments in the factor shares. There are no a priori expectations concerning the signs of these coefficients. For example, an increase in unexpected sales may increase the share of sawlogs and decrease the share of labor or vice versa, and there are no theoretical restrictions on the direction of change.

Estimation of the output equation and the cost shares as a system of seemingly unrelated equations with the restrictions imposed by symmetry and constant returns to scale will produce efficient parameter estimates. Stochastic disturbances (ϵ_i) are added to the production function and share equations. The errors are assumed to be normally distributed with zero mean and a positive semidefinite covariance matrix. To avoid singularity of the variance-covariance matrix, one share equation—in this estimation, the energy share—is dropped from the system.

In order to compute the variables S^A/S^N and I^N/I^A, it is necessary to use normal output, Y^N, which is the predicted output from "translog (X_i)." This is accomplished by first estimating the translog independent of the operating rate. In this first round, a biased measure of normal output is obtained from the translog. The predicted normal output, defined as $\hat{Y}_1^N$, is used to construct the sales and inventory variables. The system is then reestimated jointly with the operating rate to yield a new predicted output from the translog and a new measure, $\hat{Y}_2^N$. The procedure is continued until convergence, i.e., until Y_i^N stops changing.[7] At this point the sum of squared errors is minimized.

All of the right-hand-side variables in (15) are endogenous to the firm or industry and will be correlated with the error term resulting in biased and inconsistent parameter estimates. Instrumental variables were utilized to deal with the problem.

Empirical Results

Two models were specified and estimated for the sawmilling industry and the pulp and paper industry. The first specification, the instantaneous adjustment model (IA), is the translog production function with the assumption of instantaneously variable inputs with zero adjustment costs. The second model is the jointly estimated translog production function and operating rate or quasi-fixity model (QF).

[5] Note that because we are estimating a production function, the endogenous variables in the share equations are the prices (quantities would be endogenous with a cost or profit function). When prices are P^*, it will be optimum to produce Y^N utilizing X_i. Because prices in general will be $P^A \neq P^*$, the X_i will not be optimum, given observed input prices. In this case, CR does not equal one and the operating rate also will differ from one.

[6] Note that we are only restricting the share equations to be consistent with optimizing behavior in the long-run component of the model. Our short-run model is not a production function in a neoclassical sense. At the start of this article we questioned the short-run empirical relevance of the concept of a production function when quasi-fixed factors can be utilized with varying intensities and these cannot be observed. Given that we are modeling disequilibrium and not a short-run equilibrium, the usual neoclassical relationships between short-run factor demands and production or cost functions do not apply.

[7] The statistical model contains a right-hand-side nonobserved variable, Y^N. Given that there are enough identifying restrictions on Y^N and the model converges, our iterative procedure leads to estimates of the parameters of the operating rate variables that are consistent with the estimated parameters of the long-run production function. See Judge et al. for a discussion of unobservable variable models.

Table 1. Comparison of Summary Statistics for the Instantaneous Adjustment (IA) and Quasi-Fixity (QF) Models

Model[a]	Sawmilling		Pulp and Paper	
	IA	QF	IA	QF
Statistic				
LLF	278.1	331.1	263.9	301.0
R_Y^2	0.965	0.995	0.946	0.979
R_L^2	0.482	0.475	0.569	0.417
R_K^2	0.550	0.971	0.707	0.800
R_M^2	0.343	0.792	0.685	0.863
SEE_Y	0.0491	0.0195	0.0439	0.0278
SEE_L	0.0099	0.0104	0.0061	0.0071
SEE_K	0.0182	0.0047	0.0135	0.0085
SEE_M	0.0245	0.0138	0.0162	0.0107
DW_Y	0.993	2.624	2.208	2.399
DW_L	1.087	1.236	0.781	0.608
DW_K	0.384	2.481	1.394	2.123
DW_M	0.578	1.742	1.324	1.717

[a] LLF—log of likelihood function; SEE—standard error of the estimate; DW—Durbin-Watson statistic; *Y* is output; *L, K, M* are, respectively, the measured input levels of labor, capital, and materials.

In table 1, summary statistics for the two models of each industry are presented. The statistics indicate that the QF model is superior to the IA model. The standard error of the estimate for the production function in the QF model is approximately one-half that of the IA model, and values of the log likelihood function and R^2 improve as well.

The well-known likelihood ratio test can be used to test the hypothesis of quasi-fixed inputs, that is, the operating rate is one. If λ_1 is the value of the log likelihood function of the restricted model—in this case the IA model—and λ_0 is the value of the log likelihood function of the unrestricted model, then the test statistic $\lambda^* = 2(\lambda_0 - \lambda_1)$ is distributed chi-square with degrees of freedom equal to the number of independent restrictions. From the estimation, $\lambda^* = 106.50$ for the sawmilling industry and $\lambda^* = 74.26$ for the pulp and paper industry. The critical values of χ^2 at 95% and 99% confidence levels are 21.03 and 26.22, respectively. Clearly, the null hypothesis of instantaneously variable inputs is rejected, implying that the operating rate is an important short-run decision instrument.

The Long-Run Production Structure

The two alternative theories (IA and QF) can be evaluated by comparing the empirical results with the theoretical assumptions. In both industries assuming instantaneous and costless factor adjustment leads to violation of the curvature conditions of the production function at each data point. In the sawmilling industry factor demands for capital and materials are upward sloping at the mean of the data. However, when quasi-fixity of factor inputs is allowed, the model is well behaved at every observation. In table 2, parameter estimates and their standard errors are presented for comparison. These parameters are difficult to interpret, and the model performance can be more clearly evaluated by examining the long-run elasticities. In the elasticities shown in tables 3 and 4, wrong signs on own-price elasticities of demand for the IA model in both industries are observed. This problem is corrected with the addition of the operating rate in the QF model. Elasticities of substitution also change for both industries between IA and QF models. In sawmilling, for example, capital and energy switch from highly elastic substitutes to complements, and capital and materials change from highly elastic complements to zero substitution.

Clearly, in terms of goodness-of-fit and theoretical constraints, the model incorporating varying rates of factor utilization through specification of an operating rate performs significantly better than the model assuming instantaneous adjustment of inputs. The superiority of the QF model and the importance of the operating rate suggest that elasticities generated by the QF model are more accurate than elasticities generated by models that do not incorporate the operating rate.

Table 2. Production Function Parameter Estimates for the Quasi-Fixity Model

Model	Sawmilling Industry		Pulp and Paper Industry	
Parameter				
α_Y	−4.096	(0.0763)**	0.964	(0.191)**
α_L	0.129	(0.0241)**	0.316	(0.0637)**
α_K	0.0695	(0.0387)	0.344	(0.0302)**
α_M	0.998	(0.0739)**	0.486	(0.0414)**
α_E	−0.197	(0.0361)**	−0.146	(0.0571)**
α_T	−0.0106	(0.00223)**	0.0173	(0.00391)**
δ_{LL}	0.0436	(0.0136)**	−0.00365	(0.0187)
δ_{LK}	−0.0717	(0.00665)**	−0.0277	(0.0183)
δ_{LM}	0.0534	(0.0192)**	−0.0265	(0.0201)*
δ_{LE}	−0.0253	(0.00437)**	0.0579	(0.0160)**
δ_{KK}	−0.0461	(0.0111)**	0.110	(0.0244)**
δ_{KM}	0.106	(0.0204)**	−0.111	(0.0196)**
δ_{KE}	0.0118	(0.00852)**	0.0292	(0.0161)*
δ_{MM}	−0.241	(0.0438)**	0.0574	(0.0487)
δ_{ME}	0.0818	(0.0149)**	0.0802	(0.0321)**
δ_{EE}	−0.0683	(0.0113)**	−0.167	(0.0384)**
γ_{TT}	0.00131	(0.00178)**	−0.00149	(0.000431)*

Note: Standard errors in parentheses; ** denotes significant at the 99% level; * denotes significant at the 95% level. Critical values are $t_{.025} = 1.960$ and $t_{.005} = 2.576$. Variables: Y is output; L, K, M, are, respectively, the measured input levels of labor, capital, and materials; T is a time trend.

The Operating Rate

The operating rate parameters are presented in table 5. The operating rate equation for each industry is reproduced below with the standard errors in parenthesis (two asterisks indicate significant at the 99% level):

Sawmilling

$$\text{(18)}\quad \ln Y^A = \ln Y^N + \underset{(.0965)^{**}}{.557}(\ln S^A - \ln S^N) - \underset{(.0298)}{.0392}(\ln I^N - \ln I^A) - \underset{(.0611)^{**}}{.233}(\ln CR),$$

Pulp and Paper

$$\text{(19)}\quad \ln Y^A = \ln Y^N + \underset{(.0143)^{**}}{.713}(\ln S^A - \ln S^N) + \underset{(.0303)}{.0077}(\ln I^N - \ln I^A) - \underset{(.0876)}{.167}(\ln CR).$$

The sales and profitability variables for the sawmilling industry are significant at the 99% level and have the a priori expected sign. The sales variable for the pulp and paper industry is also significant at the 99% level. Inventory variables for both industries, on the other hand,

Table 3. Comparison of Own-Price Elasticities of Demand for the Instantaneous Adjustment (IA) and Quasi-Fixity (QF) Models

Elasticity[a]	Sawmilling		Pulp and Paper	
	IA	QF	IA	QF
η_L	−0.90	−1.14	−0.73	−0.82
η_K	0.54	−0.83	−9.24	−2.00
η_M	0.18	−0.30	−6.36	−1.17
η_E	−0.35	−0.30	−0.15	−0.31

[a] Evaluated at the mean of the data.
Note: L, K, M, and E are, respectively, the measured input levels of labor, capital, materials, and energy.

Table 4. Comparison of Allen-Uzawa Partial Elasticities of Substitution for Instantaneous Adjustment (IA) and Quasi-Fixity (QF) Models

Elasticity[a]	Sawmilling		Pulp and Paper	
	IA	QF	IA	QF
σ_{LK}	2.48	3.11	3.39	1.72
σ_{LM}	0.92	1.12	−0.33	0.90
σ_{LE}	1.63	2.74	−0.15	0.04
σ_{KM}	−2.61	0.00	−24.15	3.66
σ_{KE}	8.67	−1.28	0.34	1.00
σ_{ME}	−2.41	−0.49	0.23	0.10

[a] At the mean of the data.
Note: L, K, M, and E are, respectively, the measured input levels of labor, capital, materials, and energy.

Table 5. Operating Rate Parameter Estimates in the Quasi-Fixity (QF) Model

Parameter	Sawmilling	Pulp and Paper
β_S	0.557 (0.0965)**	0.713 (0.143)**
β_I	−0.0392 (0.0298)	0.0077 (0.0302)
β_{CR}	−0.233 (0.0611)**	−0.167 (0.0876)
β_{SL}	−0.280 (0.0454)	0.00930 (0.0330)
β_{IL}	−0.0373 (0.0157)*	0.00252 (0.00484)
β_{CRL}	−0.0473 (0.0325)	0.0399 (0.0242)
β_{SK}	−0.240 (0.0296)**	0.0534 (0.0396)
β_{IK}	−0.0340 (0.0715)**	0.00267 (0.00603)
β_{CRK}	−0.110 (0.0147)**	0.00853 (0.0290)*
β_{SM}	−0.607 (0.0699)**	−0.123 (0.0482)*
β_{IM}	0.813 (0.211)**	0.0190 (0.00575)**
β_{CRM}	−0.0647 (0.0432)	−0.111 (0.0349)**

Note: Standard errors in parentheses; ** denotes significant at the 99% level; * denotes significant at the 95% level. Critical values are $t_{.025} = 1.960$ and $t_{.005} = 2.576$. Variables: S = sales; I = inventory; CR = cost/revenue ratio; L = labor; K = capital; and M = materials.

are close to zero with low significance. Asymptotic 95% confidence intervals for β_I in sawmilling and pulp include zero. Insignificance of the inventory variables indicates that during the sample period opening inventories at the production plant were not a significant determinant of current output. Total inventories of product stocks, including customer as well as mill inventories, are likely to have more impact on producer demand expectations than opening mill inventories.[8]

A 1% increase in unexpected sales increases production by .56% in sawmilling and by .71% in pulp and paper. The 95% confidence intervals for these elasticities are respectively [.368, .746] and [.433, .993]. An elasticity of one would indicate that all unexpected demand changes are met by increases in production. An elasticity smaller than one indicates that the optimal response to meet unexpected demand changes is jointly to deplete inventories and increase production. Thus, although opening inventories do not affect output decisions, inventories respond to changes in unexpected sales. Inventories are an important buffer between demand and production. On the other hand, an elasticity of zero would imply that production does not respond to unexpected demand, or in other words that the operating rate is not a short-run instrument. This could result if costs rose quickly with nonnormal operating rates on one hand, or if there was no quasi-fixity in production on the other hand.

[8] Considerable product inventories are held by agents and customers for both the sawmilling and pulp and paper industries. Because of data limitations, we were constrained to use mill inventories.

In the latter case, producers would be able to meet demand changes by adjusting normal output, Y^N.

A 1% increase in unit costs of production, or decline in output price, decreases production by .23% in sawmilling and by .17% in pulp and paper. For a given level of sales, this decline in production must be compensated by sales from inventory. The coefficient on profitability reflects the incentives for inventory accumulation or depletion in the industry. In sawmilling, the 95% confidence interval for this elasticity is [−.351, −.113], and in pulp and paper the interval is [−.339, .004].

Since the operating rate equals one when $Y^A = Y^N$, actual output below (above) normal output implies an operating rate less (greater) than one. This result is shown in figure 1 for both industries. The operating rate clearly tracks business cycles. In the sawmilling industry, overutilization of capacity is observed through the sixties and early seventies, followed by general underutilization from 1974 into the eighties.

When the industry is underutilizing its capacity, less output is produced from the same amount of measured inputs, and so productivity is lower than in situations of overutilization of capacity. This fact can explain the lagging productivity performance of sawmilling after the 1970s, verified and discussed by Denny and Fuss and Constantino and Haley.

In the pulp and paper industry, the operating rate does not follow the pattern observed in sawmilling. High demand in the early sixties resulted in overutilization, followed by fluctuations above and below 100% until the peak

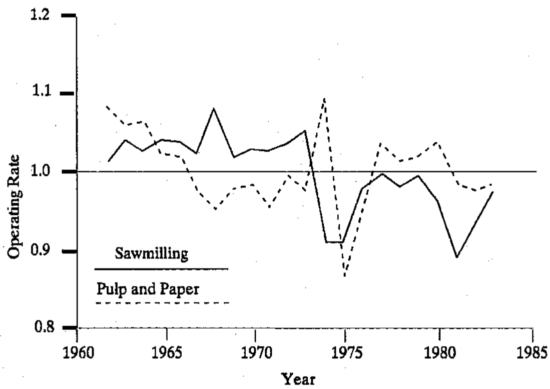

Figure 1. Estimated operating rates for the Canadian sawmilling and pulp and paper industries: 1961–83

year of 1974, when high demand again resulted in overutilization.

The quasi-fixity model accounts for variations in the rate of utilization of quasi-fixed factors and allows the measurement of technical progress as the shift of the production function over time. Given the functional form utilized, technical progress is the increase in output per year with constant inputs and constant operating rate. As a result, technical progress is not affected by the short-run and cyclical variables influencing the operating rate. In sawmilling, the estimated rate of technical progress at the mean of the data for the IA model was −.000155 per year, and .000435 per year for the QF model. In pulp and paper, the technical progress rate for the IA and QF models were, respectively, −.00105 per year and .00120. The rate of technical progress as measured by the QF model has been positive but very low for both industries over the sample period.[9]

Conclusions

The objectives of this analysis were to model disequilibrium in the Canadian sawmilling and pulp and paper industries, to test for quasi-fixity of inputs by investigating variable utilization, and to investigate factors affecting the short-run output decision. If evidence of variable factor utilization could be found, then quasi-fixity of inputs could be confirmed. Failure to reject this hypothesis implies rejection of traditional specifications that assume instantaneous and costless adjustment of factor inputs. More than that, it suggests that assuming constant rates of utilization is not appropriate, even in dynamic and imperfect adjustment models.

A short-run decision instrument, the operating rate, was hypothesized to be a function of unexpected sales, inventory levels, and profitability. For each industry, a long-run translog production function and input cost share equations were jointly estimated with an operating rate function. This model was compared with a production model that assumed costless and instantaneous factor adjustment.

The empirical results justify the inclusion of the operating rate as a short-run instrument in models of production technology, permit us to reject the hypothesis that inputs are utilized at constant rates, and support the hypothesis of quasi-fixed inputs. The quasi-fixity model is theoretically consistent and outperforms the instantaneous model in all goodness-of-fit statistics. The instantaneous adjustment model produces results that are inconsistent with economic theory; for example, upward sloping demand curves are indicated for capital and material inputs to the sawmilling industry. These results indicate that the assumption of instantaneous adjustment or, equivalently, a constant operating rate equal to one found in most econometric estimations of production, cost, and profit functions would be invalid for the data utilized in this study.

Inclusion of an operating rate reduces estimation bias by improving model specification and generates more accurate estimates, such as elasticities of substitution, for use in other applications. Incorporating the operating rate provides a framework for analysis of both long- and short-run policy issues. For example, productivity analysis can take into account short-run output fluctuations in association with long-run factor substitution and technical progress.

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[9] Positive technical change is a significantly different result to that of Martinello (1985) who also found negative technical change in both sawmilling and pulp and paper using an instantaneous adjustment translog cost function.

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Appendix

Data Definitions and Sources

All data are annual for the period 1961–83. The main data source for the sawmilling industry was *Statistics Canada, Sawmills and Planing Mills and Shingle Mills, Catalogue 35-204 Annual.* The main data source for the pulp and paper industry was *Statistics Canada, Pulp and Paper Mills, Catalogue 36-204 Annual.* Unless otherwise indicated, the following data were obtained from these sources.

Long-Run Production Structure Data. To estimate the translog production function and share equations, price and quantity data are required for industry output and the four inputs labor, capital, materials, and energy.

Sawmilling Industry. Industry output: quantity of output is lumber produced in MMfbm. Value of production is quantity of production times the average lumber price in CD$/Mfbm. Labor: labor quantity is an implicit quantity index derived from the total expenditure on labor where the price index is a Tornqvist index of dollars per workerhour paid of production and nonproduction workers. Capital: capital stock is an implicit quantity index of structures and equipment. The capital stock data are unpublished tabulations from *Statistics Canada* (1961–83). The price index is a Tornqvist index of the rental prices per real 1971 dollars of structures and equipment capital. Rental prices were constructed according to Hall and Jorgenson. Wood: wood expenditures with materials except maintenance and repair expenditures. Wood quantity is an implicit quantity index where the price index is a Tornqvist index of dollars per cubic meter of softwood and hardwood sawlogs. Energy: energy quantity is an implicit quantity index where the price index is a Tornqvist index of dollars per gallon of gasoline, kerosene and petroleum, dollars per kilowatt hour of electricity, and dollars per thousand cubic feet of natural gas.

Pulp and Paper Industry. Industry output: the total value of production is computed as the cost of materials and energy plus value added by manufacturing. The quantity of output is an implicit quantity index derived as the ratio of total value of production to the output price index. The output price index is a Laspeyres index of all pulp and paper products from *Statistics Canada, Industry Price Indices, Catalogue 62-011 Monthly.* Labor: the price of labor is a Tornqvist price index constructed from the number and cost per person of production and nonproduction workers. The quantity of labor is an implicit quantity index derived as the ratio of total labor expenditure to the Tornqvist labor price index. Capital: capital stock is an implicit quantity index of the stocks of structures and equipment. The capital stock data are unpublished tabulations from *Statistics Canada* (1961–83). The price index is a Tornqvist index of the rental prices per real 1971 dollars of structures and equipment capital calculated as in Hall and Jorgenson. Materials: a Tornqvist chemical price index is generated with prices and quantities of the major chemicals used by the industry. An implicit quantity of chemicals is derived as the ratio of total expenditure on chemicals to the chemical price index. A price index of materials is then generated using prices and quantities of pulpwood, pulp chips, other wood residue, pulp used in paper production, and the chemical price index, and implicit quantity of chemicals used. An implicit quantity of materials is derived as the ratio of total expenditure on materials to the materials price index. Energy: a Tornqvist price index of energy is calculated using prices and quantities of the major energy sources consumed. An implicit quantity of energy is derived as the ratio of total expenditure on energy to the energy price index.

Operating Rate Data. To estimate the operating rate parameters, data for inventories, total sales, total revenues, and total costs are required.

Sawmilling Industry. Inventories: volume of opening inventories of lumber in MMfbm. Sales and total revenues: volume of sales is the volume of shipments of lumber in MMfbm. Value of sales is taken to be the volume of lumber shipments times the lumber average price. Total costs: are the sum of expenditures on labor, capital, materials, and energy as developed for the translog data.

Pulp and Paper Industry. Inventories: inventories are the value of opening inventories of goods in process and finished goods. The implicit quantity of inventories is the value of inventories deflated by the *Statistics Canada* output price index. Sales and total revenues: the value of sales is taken to be the value of pulp and paper shipments reported by *Statistics Canada.* Volume of sales is the value of sales deflated by the *Statistics Canada* output price index. Total revenues are taken to be equivalent to the value of total sales. Total costs: total costs are the sum of expenditures on labor, capital, materials, and energy as derived for the translog estimation procedure.

Instrumental Variables Estimation Procedure. The right-hand-side variables of the production function for each industry, with the exception of the time trend and opening inventories, were treated as endogenous. The sales instrument was recomputed for each round of the joint estimation, because the variable changes with each reestimation of Y^N.

Sawmilling Industry. The same exogenous variables used for labor, capital, sawlog, and energy quantities in the translog estimation are used for the sales and profitability variables in the operating rate. These are Canadian GDP, U.S. GDP, U.S. housing starts, U.S. mortage rate, lagged real energy price in sawmilling, lagged real rental price of sawmilling capital, lagged sawmilling capital stock, lagged sawmilling output, opening sawmill inventories of materials and finished goods, and a time trend.

Pulp and Paper Industry. The exogenous variables used for labor, capital, sawlog, and energy quantities in the translog estimation are Canadian GDP, U.S. GDP, U.S. housing starts, U.S. advertising expenditures in newspapers and magazines, lagged prices of energy and capital for pulp and paper, lagged pulp and paper capital stock, lagged pulp and paper output, opening pulp and paper industry inventories, and a time trend. The exogenous variables for the sales variable in the operating rate are Canadian per capita GDP, U.S. per capita GDP, lagged U.S. housing starts, U.S. advertising expenditures in newspapers and magazines, U.S. mortgage rate, lagged prices of energy and capital for pulp and paper, lagged pulp and paper capital stock, U.S.wholesale trade inventory-sales ratio, and an index of world trade. The exogenous variables used for the cost-revenue variable are Canadian GDP, U.S. GDP, U.S. housing starts, U.S. advertising expenditures in newspapers and magazines, an index of U.S. hourly wages in manufacturing, lagged rental price of pulp and paper capital, lagged pulp and paper capital stock, lagged pulp and paper output, opening pulp and paper industry inventories, and a time trend.