Price Forecasting with Time-Series Methods and Nonstationary Data: An Application to Monthly U.S. Cattle Prices

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The forecasting performance of various multivariate as well as univariate ARIMA models is evaluated in the presence of nonstationarity. The results indicate the importance of identifying the characteristics of the time series by testing for types of nonstationarity. Procedures that permit model specifications consistent with the system's dynamics provide the most accurate forecasts.

Key words: Bayesian forecasting, cointegration, nonstationarity, prices, VARs.

The presence of nonstationarity and its treatment complicate the measurement and use of vector autoregressive (VAR) models. Near the unit circle, conventional estimation procedures can underestimate the parameter space. Differencing, a standard approach for reducing nonstationarity, can distort multivariate interactions and cause forecasts to diverge appreciably from actual values (Lütkepohl; Granger; Engle and Granger).

Methods for forecasting with multivariate autoregressive models in the presence of nonstationarity are in their infancy (Stock and Watson). Several approaches have appeared that are applicable to the nonstationarity problem, including estimation in differences, use of Bayesian VARs that shrink the parameter space to the first-differenced framework, and use of error correction models. No single empirical approach for treating the nonstationarity problem has been clearly articulated. However, the literature comparing the forecasting effectiveness of these approaches in the presence of nonstationarity is limited.

The objective of this paper is to evaluate the forecasting performance of various multivariate (VAR with and without differenced data, Bayesian VARs (BVARs), and an error correction model) as well as univariate time-series models in the presence of nonstationarity. Specifically, the accuracy of these approaches for forecasting monthly U.S. prices of slaughter steers (hereafter, cattle price) is examined. Forecast performance is assessed using the root mean-squared error (RMSE), a MSE decomposition, and turning point analysis. Characteristic roots of selected models are calculated to examine the stationarity question in more depth.

Vector Autoregressive and Error Correction Models

Among the class of stationary vector stochastic processes, VAR models are of considerable interest for economic forecasting. The estimable form of a k-dimensional VAR(p) process is

$$(1) \qquad Y(t) = C + \Sigma_j A_j Y(t-j) + e(t), \quad t = 1, 2, \ldots, T,$$

where $Y(t)$ is a vector of stationary time series such that $Y(t) = [1 - B]^d X(t)$; B is the backshift operator; d is the order of integration; $X(t)$ is a vector time series in levels; C is a deterministic component; $A_1, \ldots, A_p$ are $(k \times k)$ ma-

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trices of the unknown parameters, $j = 1, 2, \ldots, p$; and $e(t) = (e_1(t), \ldots, e_k(t))'$ are vectors of stationary disturbances.

The stationarity properties of process (1) require that the roots (μ) of the characteristic equation $[\det(I\mu^p - A_1\mu^{p-1} - \ldots A_p) = 0]$ be equal to one, with roots less than one in absolute value being assumed otherwise (Paulsen). When all the roots in the characteristic equation are less than one, the vector $X(t)$ is integrated of order zero, i.e., $X(t) = Y(t) \sim I(0)$. When $X(t)$ is $I(0)$, the process has a zero mean and finite variance, the effect of an innovation on the value of the process is temporary, the spectrum of $X(t)$ is finite and positive, the expected length of time between crossing of $X(t)$ at the mean is finite, and the autocorrelations decrease steadily in magnitude for large enough lags so that their sum is finite (e.g., Engle and Granger). In the presence of unit roots, however, the order of integration is greater than or equal to one and the vector $X(t)$ is integrated of order d, i.e., $X(t) \sim I(d)$, and the above properties do not hold.

The practical consequences of nonstationarity can be severe. The estimation of VARs generally is performed by applying linear least squares (LS) separately to each equation of (1). The estimated coefficients may be imprecise because conventional procedures appear to underestimate the parameters near the unit circle. Since predictions are conditional on the estimated parameters, the forecasts from a VAR are likely to be suboptimal especially for multistep horizons (Engle and Yoo).

When the vector of time series in (1) is $I(d)$, the series may be cointegrated. Engle and Granger develop the relationship between cointegration and error correction models suggesting that if each of the components of $X(t)$ first achieves stationarity after differencing, but a linear combination $\alpha'X(t)$ is already stationary, the time series, $X(t)$, are cointegrated with a cointegrating vector, α. Cointegration implies that even when the individual series are nonstationary with an infinite variance, there exists a long-run equilibrium, $\alpha'X(t) = 0$, such that its deviations are stationary with finite variances. When the system is cointegrated and there is a need for an error correction mechanism, the long-run forecasts are tied together regardless of the individual forecast behavior.

The error correction representation of the VAR model in (1) is

$$Y(t) = C + \Sigma_j A^* \, Y(t-j) - gz(t-1) + e(t), \quad t = 1, 2, \ldots, T, \tag{2}$$

where g is a parameter, and $z(t) = \alpha'X(t)$. This representation contains the differences as well as the levels of the data $[X(t)]$ as independent variables. When the series are cointegrated, the levels of the variables are significant and a VAR in differences is misspecified; a VAR estimated on levels of the data omits the cointegration constraints.

Estimation of the error correction model uses a two-step asymptotically efficient procedure (Engle and Granger). First, α is estimated by a cointegration regression which specifies the dependent variables as a function of concurrent explanatory variables. The estimate of α, $\hat{\alpha}$, then is used in estimating (2). The estimation procedure again is LS.

Engle and Granger propose a set of statistics for testing the null hypothesis of noncointegration against the alternative of cointegration. The procedures applied here test for unit roots (Dickey and Fuller) and for the importance of parameters unidentified under the null. Critical values are provided in Engle and Yoo.

The application of the Engle and Granger test procedures is not foolproof for identifying cointegrated models. For example, in small sample sizes (less than 150 observations) they may exhibit a fairly low ability to discriminate between the various hypotheses, particularly with multiple variables (Stock and Watson). Hence, an evaluation of cointegration models should include an assessment of their forecast accuracy.

Bayesian VAR Analysis

The application of Bayesian methods to VARs was introduced to mitigate the VAR overparameterization problem.[1] Through the use of symmetric and/or asymmetric prior information on the variables in (1), the procedure attempts to enhance forecast accuracy. In practice, a search process, based on minimizing out-of-sample prediction errors in a preforecast period, is used to specify the particular characteristics of the prior information. These

[1] Details can be found in Litterman and in Bessler and Kling.

priors are employed to generate subsequent forecasts.

The method begins by specifying a noninformative (flat) prior around a deterministic (intercept) component, the value of which is determined by the data. The estimator imposes the information that a random walk around an unknown deterministic component is a reasonable approximation for the behavior of an economic variable. For the ith equation in a p-order autoregression on the current observation, $X(t)$, this prior is specified as

(3) $$X_i(t) = c_i + X_i(t - 1) + e_i(t).$$

Denoting the lagged values of X (X_{t-j}) as a matrix (W), equation (1) for the ith equation in levels can be written in vector form as,

(4) $$X_i = WA_i + e_i, \qquad i = 1, 2, \ldots, k,$$

where X_i is $(t \times 1)$, W $(t \times (k + p + 1))$, A_i $((k + p + 1) \times 1)$, and e_i $(t \times 1)$.

The prior information is included using stochastic linear restrictions

(5) $$R_iA_i = r_i + v,$$

where the characteristics of R_i describe the tightness of the priors (λ), the decay parameters of the lagged variables (f), and the degree of interaction (w) permitted among the variables in the system. The mean of the A_is is zero except for the first lag on the dependent variable in the ith equation, which is one; this is specified in the column vector, r_i.

Equations (4) and (5) are estimated using the Goldberger-Theil mixed estimator with the prior centered on one. When estimating in levels, the specification permits a restricted nonstationary behavior. The limiting case is where the unit root equals one and the data behave as a pure random walk. As an alternative in the presence of nonstationarity, the individual series can be differenced and the prior centered on zero, the mean of the differenced data. However, as suggested by Lütkepohl and by the cointegration model, differencing of the data can distort the multivariate interaction. The extent to which differencing improves forecasting likely depends on the nature of the underlying series and the ability of the Bayesian priors to reflect the nonstationarity in the data. Tests for the presence of unit roots and for the existence of the cointegration model may provide insight into the effects of differencing in the Bayesian framework.

Projections are generated using the estimated coefficients according to the "Chain Rule of Forecasting" (e.g., Wold). The forecasting procedure used for BVARs is applicable to VARs and the error correction model.

Evaluation Procedures

The quantitative evaluation of the forecast methods uses the RMSE criterion. A MSE decomposition that separates the sources of forecast error into its bias, regression, and disturbance components also is employed. The bias and the regression components (the systematic errors) measure deviations from the optimal predictor, i.e., they are zero for the optimal predictor. The disturbance component measures the unsystematic deviations in the prediction errors (Granger and Newbold).

As a qualitative measure, a turning point (*TP*) criterion (Naik and Leuthold; Kaylen and Brandt) is used. The measure relies on a (4 × 4) contingency table to distinguish "peak *TP*" from "trough *TP*" and "upward no *TP*" from "downward no *TP*." The two measures of interest are the accurate and worst forecast ratios.

Finally, the characteristic roots for several models are calculated. Stationarity requires that the characteristic roots are less than one in absolute value.

Model Specification

The VAR model specification for cattle prices is based on the econometric model of Garcia et al. The underlying principles used in constructing the VAR model rely on Zellner and Palm who showed that it is possible to derive multiple time-series processes from dynamic econometric specifications by imposing appropriate restrictions. The monthly price of cattle (slaughter steers, \$/cwt., choice, 1,100–1,300 lbs., Omaha) (*PC*); average price of feeder steers (\$/cwt., eight-market average) (*PFS*); and per capita income in dollars (*PCI*) comprise the information set for the trivariate VAR models. The multivariate interaction between these series can be visualized as follows. Feeder steers are a main input in the production process for feedlot operations; thus, their price directly affects production decisions.[2] The relation of *PFS* and *PC* is recursive through an underlying supply equation. Demand forces affecting *PC* are reflected through *PCI*.

Prior to the empirical model specification, the stationarity of the time series was analyzed. Visual inspection and analysis of the autocorrelation functions for the raw (levels) data on *PFS* and *PCI* suggested a nonstationary behavior similar to that of the *PC* series. The Dickey and Fuller test for nonstationarity for each series was applied by regressing the levels of the dependent variable on a lagged level and a lagged change of the dependent variable, i.e., the equation for $PC(t)$ was $PC(t) = f(PC(t-1), \Delta PC(t-1)) + e(t)$, where $\Delta = (1 - B)$. The values of the t-statistics for $PC(t-1)$, $PFS(t-1)$, and $PCI(t-1)$ from each equation were -2.21, -1.93, and 1.15, respectively. When compared with the critical value at the 5% level of -3.17 (see Fuller, table 8.5.2), one fails to reject unit roots, indicating that all series are integrated of order one, $I(1)$.

To test whether taking second differences to induce stationarity is necessary, the second differences of the dependent variables were regressed on lagged first differences and two lags of second differences, i.e., the equation for $PC(t)$ was $\Delta^2 PC(t) = f(\Delta PC(t-1), \Delta^2 PC(t-1), \Delta^2 PC(t-2)) + e(t)$. The t-statistics for the lagged first differences from each equation were -6.26, -9.10, and -8.43 for $PC(t)$, $PFS(t)$, and $PCI(t)$, respectively, indicating that first differences of each series are stationary.

The objective of the analysis is to assess the forecast performance of various multivariate models in the presence of nonstationarity. Consequently, both the raw and first-differenced data are used in most of the subsequent analyses.

The specification process for the various models differs in complexity. The multiple final prediction error (FPE) permitting a maximum lag length of six was used to identify the VAR (Akaike). This approach selected models of order two for both the raw (VAR2R) and differenced (VAR2D) data.

Identification of nonstationarity in the series suggests the possibility of a cointegrated system (Engle and Granger). The procedure used here to assess the appropriateness of the cointegration framework consists of three tests based on a series of regressions: the Durbin Watson (DW), the augmented Dickey and Fuller (ADF), and the Dickey and Fuller (DF) (table 1).[3] First, a cointegrating regression for *PC* on concurrent values of *PFS* and *PCI* and a constant was run (table 1, column 2). The t-statistics for these coefficients were 15.97, 5.01, and 6.63, respectively, with an R^2 of .89. However, with a DW statistic of .28, one fails to reject noncointegration at the 5 % level (critical value: .386)

Next, the change in the residuals from the cointegrating equation (ΔEPC) was regressed on past levels of the residuals ($EPC(t-1)$) and lags of their changes ($\Delta EPC(t-1)$, $\Delta EPC(t-2)$). For a cointegrated system the value of past levels of the residuals is significantly different from zero; this is the ADF test. The results from this equation (including additional lagged changes does not alter the results) were -4.20, 2.91, and 2.14, respectively (table 1, column 3), indicating the possibility of cointegration (critical value at 5% level: 3.93). Finally, because the lagged changes were not significant, the change in the residuals from the cointegrating equation were regressed only on their past levels. Finding a significant value of the past levels indicates a cointegrated system; this is the DF test. A t-statistic of -2.79 for the cattle price equation (table 1, column 4) indicates that at the 5% level one can reject cointegration. When the regressions were run with *PFS* and *PCI* as dependent variables, the magnitudes of the statistics changed somewhat, but the conclusion was to reject cointegration.

While the tests suggest that the error correction model is not appropriate, its estimation and forecasting were carried out to assess the robustness of the testing procedure and to ascertain its forecasting accuracy. An unrestricted autoregression of changes in $PC(t)$, i.e., $\Delta PC = Y(t)$, on lagged levels of $PC(t)$, $PFS(t)$, and $PCI(t)$, and two lags of changes of these three variables was estimated (table 1, column 5). All the lagged levels and the first lag of the changes of *PC* were significant. The significance of the lagged levels indicates an error correction term estimated from the cointegration regression along with the first lag of the changes of *PC* needs to be included in developing the final model. Beginning with this error

[2] The price of corn, the main feed ingredient, did not prove significant in any of the estimations. This result is consistent with Garcia et al.

[3] Error correction specifications were estimated for the other two variables. They are not presented for purposes of brevity. Details of these and other specifications can be obtained from the authors.

Table 1. Regressions of Cattle Prices (*PC*) in the Cointegration Analysis

	Dependent Variables[a,c]				
Independent Variables[b]	*PC*	ΔEPC	ΔEPC	ΔPC	ΔPC
PFS	0.60 (15.97)				
PCI	1.39 (5.01)				
$PC(t-1)$				−0.24 (3.54)	
$PFS(t-1)$				0.11 (2.36)	
$PCI(t-1)$				0.48 (2.28)	
$EPC(t-1)$		−0.22 (−4.20)	−0.14 (−2.79)		−0.23 (−3.52)
$\Delta PC(t-1)$				0.36 (2.77)	0.44 (3.02)
$\Delta PC(t-2)$				0.15 (1.14)	
$\Delta PFS(t-1)$				0.06 (0.51)	
$\Delta PFS(t-2)$				−0.01 (−0.10)	
$\Delta PCI(t-1)$				−0.54 (−0.08)	
$\Delta PCI(t-2)$				−4.49 (−0.65)	
$\Delta EPC(t-1)$		0.27 (2.91)			−0.05 (−0.26)
$\Delta EPC(t-2)$		0.21 (2.14)			0.13 (1.03)
Constant	11.48 (6.63)			3.56 (2.53)	0.16 (0.66)
σ	4.02	1.93	2.04	2.51	2.47
DW	0.28	1.89	1.48	1.93	1.95

[a] The regressions are: the cointegration regression, the augmented Dickey-Fuller test, the Dickey-Fuller test, an unrestricted VAR, and the final model, respectively. ΔEPC represents the changes (Δ) in the residuals (E) from the cointegration regression (PC), and ΔPC represents the changes in PC.
[b] *PFS* is the price of feeder steers, *PCI* is per capita income; $t-1$ and $t-2$ are lags 1 and 2, σ is the regression standard error, and DW is the Durbin-Watson statistic.
[c] t-statistics are in parentheses.

correction formulation, a specification search that examined the effects of adding lagged changes of the residuals from the cointegration on the residuals of the error correction model was performed. Based on diagnostic checks of the autocorrelations and partial autocorrelations of the residuals in the error correction model (Granger and Weiss), the final specification for the cattle price equation was

$$
\begin{aligned}
(6)\qquad \Delta PC = C &+ B_1 * EPC(t-1) \\
&+ B_2 * \Delta EPC(t-1) \\
&+ B_3 * \Delta EPC(t-2) \\
&+ B_4 * \Delta PC(t-1) + e(t),
\end{aligned}
$$

where ΔPC represents the changes in PC, C is a constant, EPC represents the residuals from the cointegrating regression, and ΔEPC denotes the changes in EPC.

Implementation of the Bayesian procedures also is rather complex. The specification of the symmetric prior Bayesian model used the January 1983 to December 1983 (preforecast) period to evaluate the out-of-sample forecast ability of the VAR2R model for values of f, w, and λ over the unit cube. The search was conducted using intervals of size .09 starting with .01 under a geometric lag-decay specification. The three-dimensional symmetric search resulted in the minimum value for the log de-

Table 2. Optimal Weights $w(i, j)$ for the Asymmetric Prior, Bayesian VAR of Order 2 for Raw (ABVAR2R) and First-Differenced (ABVAR2D) Data[a]

	Variables					
	ABVAR2R			ABVAR2D		
Equation	*PC*	*PFS*	*PCI*	*PC*	*PFS*	*PCI*
PC	1.000	0.800	0.001	1.000	0.001	0.010
PFS	0.500	1.000	0.001	0.500	1.000	0.010
PCI	0.001	0.001	1.000	0.001	0.001	1.000

[a] The symmetric weights for ABVAR2R and ABVAR2D models are (f = .55, w = .28, λ = .10) and (f = 1.0, w = 1.0, λ = .01), respectively, where f is the decay parameter of the lagged variables, w measures the degree of interaction among variables in the system, and λ is the tightness parameter. See table 1 for definitions of the variables.

terminant of the out-of-sample (twelve-steps-ahead) forecast error covariance (LNDFE) at (f = .55, w = .28, λ = .10).[4]

Previous research has demonstrated that identifying asymmetric multivariate interactions can result in more accurate forecasts (Bessler and Kling). Here, the asymmetric behavior is formulated as a combination of purely instrumental (data search) and subjective (expected economic relationships and correlation between variables in the VAR model) decisions. The optimal values (λ = .10, f = .55) of the overall tightness and decay parameters previously identified were maintained. The asymmetric tightness parameters for $w(i, j)$ were specified as follows. First, if $i = j$ then w took a value of 1.0; second, for *PCI* a search was conducted for the asymmetric weights, $w(i, j)$, over the interval $.001 < w(i, j) < 1.0$ for the other two variables in an equation. This search was based on the idea that it is reasonable to expect income to affect the level of cattle prices because income is an important determinant of consumer demand, but it is questionable that cattle prices affect the level of income to the same degree. Similarly, the price of feeder steers affects the price of cattle from the production side, suggesting a high degree of interaction between these two series (i.e., the two variables are highly correlated). Again, it does not seem reasonable that *PFS* would have much impact in terms of determining income levels.

The minimum RMSE was used as decision rule to select the optimal weights, $w(i, j)$. The primary reason for using the RMSE is that forecasts for *PC* (rather than all the variables in the system) were of interest.[5] The optimal weights are presented in table 2 and labeled ABVAR2R, asymmetric Bayesian VAR of order two using the raw data. The *PC* and *PFS* equations carry half or more of the weight of their own effect when they appear in the other equation. *PCI* has almost no influence on these variables.[6] For the *PC* equation, tight priors around *PC* and *PFS* were identified.

The Bayesian specification of the first-differenced model (VAR2D) followed the same procedure except that the mean was centered about zero. The three-dimensional symmetric search resulted in the minimum value for LNDFE at (f = 1.0, w = 1.0, λ = .01) under a geometric lag decay specification. The optimal weights are presented in table 2 under ABVAR2D, asymmetric Bayesian of order two using first-differenced data.

The univariate time-series model used to forecast monthly prices of cattle (*PC*) followed Box and Jenkins; the model provides a basis to evaluate whether more complex vector autoregressions increase the signal that can be extracted about prices of cattle. Because of the nonstationarity, first differences of monthly *PC*, January 1975 to December 1983, were used to identify and estimate alternative structures. Based on analysis of the autocorrelations and partial autocorrelations, an ARIMA(2,1,2) was selected. The estimated equation is

[4] The use of harmonic lag specifications did not improve the forecast accuracy of the Bayesian models. Also, in general, the values for LNDFE revealed a very flat structure; hence, searching about the neighborhood of the optimal weights was not considered necessary.

[5] At this point in the BVAR procedure, where the search combines both instrumental (data search) and subjective decisions, concentrating on the ability of the models to predict cattle prices is a logical criterion, consistent with the objective of the analysis and in keeping with the "spirit" of the search procedure. Hence, the presentation concentrates on the forecast statistics for the cattle price variable. However, see footnote 7.

[6] The optimal weights for the asymmetric models are consistent with the variance decomposition of forecast errors during 1983.

Table 3. Root Mean-Squared Errors (RMSE) and MSE Decomposition at One- to Six-Month Forecast Horizons for Selected Models, U.S. Cattle Prices, 1984-85[a]

Model[b]	Forecast Horizon (months) 1	2	3	4	5	6
ARIMA(2,1,2)	2.44[c]	4.16	5.34	5.75	6.22	6.86
	3.98	7.15	12.37	28.68	46.16	55.12
	5.90	14.02	19.06	11.48	4.07	2.26
	90.12	78.82	68.56	59.83	49.77	42.61
VAR2R	2.47	4.37	5.84	6.72	7.44	8.01
	10.77	18.95	26.57	41.77	53.99	60.81
	3.49	8.28	12.55	9.61	6.37	4.65
	85.74	72.78	60.87	48.62	39.63	34.54
VAR2D	2.27	3.75	4.71	4.62	4.50	4.68
	0.09	0.35	1.07	7.07	20.70	34.33
	9.00	17.40	23.07	16.78	7.34	1.89
	90.91	82.25	75.86	76.14	71.95	63.78
ERR CORR	2.48	4.36	5.91	6.49	6.51	7.35
	8.38	6.82	9.82	21.49	31.26	36.58
	3.51	20.03	30.84	28.29	25.49	25.03
	88.10	73.15	59.34	50.23	43.25	38.39
ABVAR2R	2.46	3.95	4.83	4.88	4.95	5.22
	0.03	0.08	0.79	4.97	12.29	17.78
	2.54	10.14	16.39	12.69	7.91	6.15
	97.43	89.78	82.83	82.34	79.81	76.07
ABVAR2D	2.42	4.06	5.24	5.58	5.96	6.60
	1.01	2.64	5.58	16.65	32.54	44.51
	10.05	21.47	29.58	25.44	17.29	11.89
	88.94	75.88	64.84	57.90	50.17	43.60

[a] The mean and variance of cattle prices during the forecast period were \$63.07/cwt. and \$27.19/cwt., respectively.
[b] The models are, respectively, an autoregressive moving average; vector autoregressions of order two with raw and first-differenced data; an error correction model; and asymmetric Bayesian vector autoregressions with raw and first-differenced data.
[c] The four numbers in each cell are the RMSE; and the bias, regression, and disturbance components of the MSE, respectively.

$$(7)\quad (1 - \underset{[-14.88]}{1.529}B + \underset{[8.23]}{.846}B^2)(1 - B)PC_t = \underset{[1.15]}{.241} + (1 - \underset{[-8.20]}{1.276}B + \underset{[3.53]}{.549}B^2)z_t$$

$$Q = (4.74,\ 12.44,\ 17.61,\ 19.40)$$

where B is the lag operator. The values in brackets are t-ratios, and Q is the Q-statistic (Ljung and Box) at lags 1, 7, 13, and 19, respectively.

Evaluation

The out-of-sample RMSEs for selected models at forecast horizons of one through six months for the period January 1984 to December 1985 are provided in table 3. All forecasts were generated based on monthly updatings of the estimated models. The symmetric Bayesian models' results are not presented for purposes of brevity but produced higher RMSEs and larger biases than their asymmetric counterparts. This relative forecast improvement corroborates previous research that suggests the usefulness of the fine-tuned priors (Bessler and Kling). The use of asymmetric priors appears to be a rather comprehensive forecasting approach, permitting the researcher to identify the specific set of weights that minimize the forecast error. As suggested by Bessler and Kling, their use is most likely to improve the forecast performance in those cases where economic logic and differences of the correlations among the variables suggest the likelihood of differential or asymmetric behavior.

The results indicate that the ARIMA specification provides relatively accurate forecasts in the short term that tend to deteriorate at longer horizons. In general, and in particular at longer horizons, the VAR2D and the ABVAR2R are the most accurate forecasters, with the VAR2D performing the best. The VAR2D model reduces the RMSE relative to the ARIMA model by 6.97%, 9.86%, 11.80%, 19.65%, 27.65%, and 31.78% for one to six

Table 4. Turning Point Evaluation of the One-Month-Ahead Forecasts for Selected Models, U.S. Cattle Prices, 1984-85

Turning Point Element[b]	Model[a]					
	ARIMA	VAR2R	VAR2D	ERR CORR	ABVAR2R	ABVAR2D
$F11$	1	0	0	0	3	0
$F12$	0	0	0	0	0	0
$F13$	3	4	4	4	1	4
$F14$	0	0	0	0	0	0
$F21$	0	0	0	0	0	0
$F22$	3	2	0	2	1	2
$F23$	0	0	0	0	0	0
$F24$	1	2	4	1	3	2
$F31$	0	0	0	0	0	0
$F32$	0	0	0	0	0	0
$F33$	5	5	5	5	5	5
$F34$	0	0	0	0	0	0
$F41$	0	0	0	0	0	0
$F42$	6	6	1	7	4	1
$F43$	0	0	0	0	0	0
$F44$	5	5	10	4	7	9
RAF	.58	.50	.62	.48	.67	.70
RWF	.00	.00	.00	.00	.00	.00

[a] See table 3 for a description of the models.
[b] F_{ij}, for $i, j = 1, 2, 3, 4$ represents the ith row and jth column on a 4 × 4 contingency table that distinguishes the "peak turning point (*TP*)" from "trough *TP*" and "upward no *TP*" from "downward no *TP*." *RAF* is a ratio of the accurate forecasts to the total, and *RWF* is a ratio of the worst forecasts to the total.

months ahead, respectively. The VAR2R, in the raw data, demonstrated the worst forecasting ability but was closely followed by the error correction model.[7]

An assessment of the MSE decomposition provides somewhat of a similar pattern in the forecast performance. The ARIMA model becomes increasingly biased as the forecast horizons lengthen. For the VAR2D and the ABVAR2R models, the bias component is close to zero at the one- to three-month horizons. At longer horizons, the ABVAR2R model, which registers the smallest bias component, manifests about one-third of the bias associated with the ARIMA specification. Again, the VAR2R and the error correction models perform poorly relative to the other forecasting procedures.

The turning point evaluation (the one-month horizon results are shown in table 4), in general terms, demonstrates a similar pattern. The VAR2D and the two asymmetric Bayesian models followed the actual movements in the data most closely. However, the accuracy of all models deteriorated significantly as the forecast horizon increased. For example, the maximum ratio of the worst forecasts to the total forecasts for the six-month horizon was .33 for the models in table 4.

To further examine the nonstationarity of these models, the characteristic roots (CR) of the VAR2R, VAR2D, and the ABVAR2R were calculated (table 5). The application of least squares to nonstationary data (VAR2R), which leads to the worst forecasting performance, is associated with unstable parameter estimates (several CR greater than one). The estimation of the VAR2D that provides the best forecaster in terms of RMSE and is consistent with the stationarity test results is associated with stable parameter estimates. Interestingly, the Bayesian procedures applied to the nonstationary data, which forecast well in terms of RMSE and minimize the bias proportion of the prediction error, almost eliminate the instability in the estimated parameters (i.e., all the CR are less than or equal one in modules) in this application.[8]

[7] The forecast accuracy of the various VARs for *PFS* and *PCI* produces a similar ranking of the models. At the one-month and six-month horizons, average RMSEs for *PFS* and *PCI* were 2.07, 5.45, and .07, .23, respectively. Their mean and variances for the forecast period were: *PFS*—$61.65/cwt., $13.20/cwt.; *PCI*—$13,250 per capita, $324,962.18 per capita.

[8] As pointed out by an anonymous reviewer, these results cannot be generalized to all Bayesian estimations.

Table 5. Characteristic Roots for Selected Models,[a] 1975–83

	VAR2R		ABVAR2R		VAR2D	
Root	Real	Imaginary	Real	Imaginary	Real	Imaginary
1	1.97	0.00	1.00	0.00	0.39	0.18
2	1.04	0.41	0.94	0.13	0.39	−0.18
3	1.04	−0.41	0.94	−0.13	0.32	0.00
4	−0.07	0.11	0.10	0.00	−0.39	0.00
5	−0.07	−0.11	0.03	0.00	−0.03	0.13
6	0.11	0.00	0.02	0.00	−0.13	−0.13

[a] See table 3 for a description of the models.

Summary

Except in the very short run, VARs and BVARs provide more accurate forecasts than the simpler ARIMA specification—a finding consistent with prior research (Bessler and Kling). However, the accurate forecasting performance of the VAR using differenced data (VAR2D) is somewhat surprising. Traditionally, VARs have not performed well relative to other techniques. Proper identification of the order of the model and the consistency of the differencing procedure with the stationarity and cointegration tests may, in part, explain this result. Also, the limited model size (three variables with an order of two) may have minimized the overparameterization problem often associated with VAR estimation.

The poor performances of the VAR in the raw data (VAR2R) and the error correction models particularly at distant horizons verify the importance of stability in the parameter estimates and appropriate model specification. Least squares estimation in the presence of nonstationary behavior leads to unstable parameter estimates and inaccurate forecasts. The forecast performance of the error correction model corroborates the incompatibility of this specification with the data, provides confidence in the discriminating ability of the cointegration procedures, and indicates here that the differenced framework is not inconsistent with the dynamics of the system.

Finally, the results indicate the usefulness of asymmetric priors in Bayesian analysis. For the Bayesian models, the asymmetric specifications always resulted in lower forecast errors. Interestingly, the relatively accurate performance of the Bayesian VAR with asymmetric priors in levels suggests its usefulness even in the presence of the nonstationarity identified in the testing procedures. In all likelihood, the imposition of the asymmetric Bayesian priors on the levels performed reasonably well by permitting the estimated model to approximate the differenced specification.[9] This formulation eliminates the need for filtering the data and, therefore, transforming the forecasts. It also avoids the possible distortion of the multivariate interaction caused by differencing. Hence, the application of asymmetric priors in a Bayesian framework appears useful for improving forecast performance through the search process that can identify multivariate interactions and permit parameter estimates that are fairly stable as prior information is introduced.

Concluding Remarks

Several points emerge from the study. In the presence of nonstationarity, appropriate identification of the characteristics of the data series is critical particularly when forecasting at distant horizons. Testing for forms of nonstationarity and the existence of cointegration models provides insight into the explicit nature of the series. Straightforward model specifications consistent with the dynamic characteristics of the systems can provide stable parameter estimates and accurate forecasts.

Regarding the Bayesian models, the improved performance of the asymmetric specifications indicates their usefulness even in the presence of nonstationarity. Estimation of the asymmetric Bayesian model in levels, which also has been found to produce improved forecast performance in other contexts, appears to have avoided the possible distortions of the multivariate interactions often induced by dif-

[9] We would like to thank an anonymous reviewer for suggesting this explanation.

ferencing. Here, the improved forecast performance was related to the identification of the multivariate interactions in a framework that permitted the estimated model to approximate the differenced specification.

Finally, further empirical research needs to be performed addressing the issues related to forecasting with multivariate models in the presence of nonstationarity. These efforts will permit a better understanding of the relationship between the underlying characteristics of the series and the relative forecasting effectiveness of the techniques and procedures identified here.

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