GARCH Time-Series Models: An Application to Retail Livestock Prices

Satheesh V. Aradhyula and Matthew T. Holt

This article applies recent developments in time-series modeling to analyze the retail prices of beef, pork, and chicken. Specifically, generalized autoregressive conditional heteroscedasticity (GARCH) models were fitted to these data to determine if, unlike more traditional time-series models, the conditional variances of the underlying stochastic processes are nonconstant. The estimation results indicate that the constant conditional variance assumption can be rejected. Furthermore, *ex post* forecast intervals generated from the GARCH processes indicate that the forecasting accuracy of the estimated models has varied widely over time with substantial volatility occurring during the 1970s and early 1980s.

Key words: conditional variance, confidence intervals, forecasts, GARCH models, retail meat prices, time-series models.

In recent years, agricultural economists have made extensive use of time-series analysis to model economic data (Bessler and Brandt; Harris and Leuthold; Shonkwiler and Spreen). Indeed, time-series models, including univariate autoregressive and/or moving-average processes, vector autoregressions, transfer functions, and dynamic regressions, have become fundamental tools of economic analysis. The considerable popularity of the time-series approach can be attributed to a number of reasons. For instance, these models can be used to gain insights into the dynamic properties of complex systems (e.g., Bessler 1984; Brorsen, Chavas, and Grant). In addition, time-series analysis requires less subjective judgment on the part of the analyst; model identification and specification are obtained by exploiting systematic relationships in the data. But perhaps the most important reason for the widespread use of these models is their forecasting accuracy. Often, a parsimoniously specified univariate or multivariate time-series model will yield better forecasts than more complex structural econometric models (Brandt and Bessler).

There are several possible reasons for the enhanced forecasting performance of time-series models, but the most likely is that these processes use past information optimally. To illustrate, consider a standard first-order autoregressive (AR) process

$$y_t = b_0 + b_1 y_{t-1} + \epsilon_t, \tag{1}$$

where y_t is a random variable drawn from a conditional density function $f(y_t \mid y_{t-1})$ and ϵ_t is white noise with mean zero and variance $V(\epsilon_t) = \sigma^2$. The forecast of today's value of y_t, conditioned on past information, is simply $E(y_t \mid y_{t-1}) = b_0 + b_1 y_{t-1}$. Likewise, the unconditional mean of y_t is $b_0/(1 - b_1)$.

The improved forecasting accuracy attributed to many time-series models clearly derives from optimal use of past information. Oddly enough, these optimal forecasting properties have not, until recently, been extended to predictions of the variance.[1] So, for real pro-

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[1] To see this, note that the conditional variance of y_t in (1) is σ^2, whereas the unconditional variance is $\sigma^2/(1 - b_1^2)$. Thus, the conditional variance is constant and does not use information pertaining to past realizations of y_t.

cesses one might expect more accurate forecast intervals if additional information on past observations of y_t were allowed to condition the forecast variance. A more general class of time-series models seems desirable. Realizing this, Engle proposed a class of autoregressive processes better known as ARCH (autoregressive conditional heteroscedasticity) models. The key feature of an ARCH process is that the forecast variance, h_t, is conditioned on past realizations of y_t.[2]

Although ARCH processes have been used successfully to model macroeconomic data by Engle, Engle and Kraft, and Weiss, problems arise because of nonnegativity constraints associated with the parameter vector α in the conditional variance equation. This has resulted in the use of rather arbitrary linear, declining-lag structures in the h_t equation to account for the long memory typically found in empirical work. Recognizing this, Bollerslev (1986) recently introduced a new class of conditional heteroscedastic models known as GARCH (generalized autoregressive conditional heteroscedasticity) processes. A chief advantage of GARCH processes over ARCH processes is that, often, a more flexible and parsimonious lag structure in the conditional variance equation can be obtained.[3]

There are a surprising number of areas in economics where GARCH models could be applied. For instance, portfolio models require information about price variances and GARCH processes are a logical tool for generating proxy variables for risk premiums. Likewise, price and/or output risk variables are often included in aggregate supply equations (Just, Antonovitz and Green; Aradhyula and Holt; Seale and Shonkwiler). Although ARIMA models are frequently used to predict the means included in these equations, ad hoc procedures are often employed to generate variance terms. GARCH models provide a natural framework for generating both conditional means and variances in these situations. There has also been considerable interest in modeling yields as stochastic processes (Bessler 1980). However, the variance associated with standard time-series models is constant and consequently provides only limited information about higher-order moments.

The purpose of this article is to develop, estimate, and test GARCH models for the retail prices of beef, pork, and chicken. Retail meat prices seem reasonable to investigate because they were relatively stable during the 1960s but experienced substantial volatility during the 1970s and early 1980s. The working hypothesis, then, is that GARCH models will yield more plausible forecast confidence intervals for these retail meat prices than will traditional time-series models.

The plan of the paper is as follows. First, the key assumptions underlying GARCH processes are reviewed. Next, GARCH models are fitted to real beef, pork, and chicken prices, and the empirical results are evaluated and contrasted with standard autoregressive models. The final section examines the use of GARCH models to estimate conditional variances and reviews implications for future research.

The GARCH(*p*,*q*) Process

Let ϵ_t denote a real valued discrete-time stochastic process and Ω_t the set of all information available through time period t. The GARCH(p,q) process for a normal conditional distribution is then given by

$$(2) \qquad \epsilon_t | \Omega_t \sim N(0, h_t),$$

$$(3) \qquad h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i},$$

where
$p \geq 0, \quad q \geq 0$
$\alpha_0 > 0, \quad \alpha_i \geq 0, \quad i = 1, \ldots, q$, and
$\beta_i \geq 0, \quad i = 1, \ldots, p.$

Note that, for $p = 0$, the process reduces to an ARCH(q) process. Also, for $p = q = 0$ the conditional variance is constant, as in typical time-series models, and the innovation ϵ_t simply reduces to white noise.

In the ARCH(q) process, the conditional variance is specified as a linear function only of the past sample variances. Alternatively, the GARCH(p,q) process allows lagged values of the conditional variance to enter the h_t equation as well. This corresponds to the extension of an AR process to an ARMA process in tra-

[2] For instance, the conditional variance of a first-order ARCH process can be written as $h_t = \alpha_0 + \alpha_1\epsilon_{t-1}^2$. More generally, the variance function can be expressed as $h_t = h(y_{t-1}, \ldots, y_{t-p}; \alpha)$, where p is the order of the ARCH process.

[3] The extension of the ARCH process to a GARCH process bears a striking resemblance to the extension of the standard AR process to a more general ARMA process.

ditional time-series modeling and, consequently, implies some sort of adaptive learning mechanism.

The GARCH(p,q) regression model can be obtained by letting the ϵ_t's be innovations in a linear regression,

$$(4) \qquad \epsilon_t = y_t - x_t'b,$$

where y_t is the dependent variable, x_t is a vector of observations on explanatory variables including past realizations of y_t, and b is a vector of unknown parameters to be estimated. If all roots of $1 - B(p) = 0$ lie outside the unit circle, (3) can be respecified as a distributed lag of past-squared innovations. That is,

$$(5) \quad h_t = \alpha_0(1 - B(1))^{-1} + A(L)(1 - B(L))^{-1}\epsilon_t^2$$
$$= \alpha_0\left(1 - \sum_{i=1}^{p} \beta_i\right)^{-1} + \sum_{i=1}^{\infty} \delta_i\epsilon_{t-i}^2,$$

which, together with (2), implies an infinite-dimensional ARCH(∞) process. The δ_i's can be obtained from a power series expansion of $D(L) = A(L)(1 - B(L))^{-1}$, where

$$(6) \quad \delta_i = \alpha_i + \sum_{j=1}^{n} \beta_j\delta_{i-j}, \quad i = 1, \ldots, q,$$
$$= \sum_{j=1}^{n} \beta_j\delta_{i-j}, \qquad i = q + 1, \ldots,$$

and $n = \min\{p, i - 1\}$. Thus, if $D(1) < 1$, the GARCH(p,q) process can be approximated to any degree of accuracy by a stationary ARCH(q) process with a sufficiently large value of q.

As an ARMA analogue, the GARCH process could be justified through a Wald's decomposition type of argument as a more parsimonious description. Bollerslev (1986) shows that a sufficient condition for the GARCH(p,q) process defined in (2) and (3) to be stationary is that $A(1) + B(1) < 1$. The unconditional mean and variance of the innovation ϵ_t are given by $E(\epsilon_t) = 0$ and $\text{var}(\epsilon_t) = \alpha_0/(1 - A(1) - B(1))$. Thus, in the GARCH($p,q$) process, the unconditional variance is constant while the conditional variance could change over time.

Of practical concern is the identification and diagnostic checking of the appropriate lag structure for the conditional variance equation in a GARCH process. Autocorrelation and partial autocorrelation functions of the innovation series are typically used when identifying and checking the time-series behavior of ARMA models (Box and Jenkins). Bollerslev (1988) shows that these same functions as applied to the squared residual series can be useful for identifying and checking the time-series behavior of the conditional variance equation of the GARCH form.

Identification and diagnostic checking of a GARCH process proceed as follows. Let τ_n denote the nth autocorrelation and ϕ_{kk} the kth partial autocorrelation of ϵ_t^2, obtained by solving the GARCH analogue to the Yule-Walker equations. The usual interpretations apply. For an ARCH(q) process, ϕ_{kk} cuts off after the qth lag. This is identical to the behavior of the partial autocorrelation function of the estimated residuals ϵ_t for an AR(q) process. Likewise the partial autocorrelation function of ϵ_t^2 for a GARCH(p,q) process is in general nonzero and dampens slowly. In this manner, the autocorrelation and partial autocorrelation functions of the ϵ_t^2s can be used for identifying and checking the GARCH form.

Estimation of the GARCH regression model can proceed by using standard maximum likelihood (ML) methods. Let $z'_t = (1, \epsilon_{t-1}^2, \ldots, \epsilon_{t-q}^2; h_{t-1}, \ldots, h_{t-p})$, $w' = (\alpha_0, \alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_p)$, and $\Theta = (b', w')$. The GARCH model in (2), (3), and (4) may then be rewritten as

$$(7) \qquad \begin{aligned} \epsilon_t &= y_t - x'_t b, \\ \epsilon_t | \Omega_t &\sim N(0, h_t), \\ h_t &= z'_t w. \end{aligned}$$

Apart from a constant term, the log likelihood function for a sample of T observations is

$$(8) \qquad L_T = T^{-1} \sum_{t=1}^{T} l_t(\Theta),$$
$$l_t(\Theta) = -0.5 \log h_t - 0.5\, \epsilon_t^2 h_t^{-1}.$$

The first and second derivatives of the log likelihood function in (8) with respect to Θ are outlined in Bollerslev (1986, pp. 315–16).

A convenient feature of the GARCH model is that the off-diagonal blocks of the information matrix associated with the $\partial l_t/\partial b \partial w'$ terms can be shown to be zero. Because of this asymptotic independence, w can be consistently estimated by using initial consistent (OLS) estimates of b. This is a useful property because initial consistent estimates of b and w can be easily obtained for starting the ML iterative estimation. Finally, as with ARMA models, the derivatives of (8) contain recursive

Table 1. Maximum Likelihood Estimates of Autoregressive Models Fitted

Price of Beef (PB_t)

$$(1 - \underset{(0.065)}{0.889}B - \underset{(0.073)}{0.184}B^4 + \underset{(0.065)}{0.236}B^5)PB_t = \underset{(4.825)}{29.737} - \underset{(0.042)}{0.198}t + \epsilon_{1t}$$

$$h_{1t} = \text{var}(\epsilon_{1t}) = \underset{(1.638)}{14.597} \quad R^2 = 0.85 \quad MAPE = 3.03$$

Price of Pork (PP_t)

$$(1 - \underset{(0.047)}{1.088}B + \underset{(0.093)}{0.380}B^3 - \underset{(0.111)}{0.323}B^4 + \underset{(0.076)}{0.153}B^5)PP_t = \underset{(1.045)}{7.991} + \epsilon_{2t}$$

$$h_{2t} = \text{var}(\epsilon_{2t}) = \underset{(1.794)}{15.993} \quad R^2 = 0.85 \quad MAPE = 4.52$$

Price of Chicken (PC_t)

$$(1 - \underset{(0.046)}{0.755}B - \underset{(0.042)}{0.201}B^8)PC_t = \underset{(0.105)}{0.990} + \epsilon_{3t}$$

$$h_{3t} = \text{var}(\epsilon_{3t}) = \underset{(1.242)}{7.818} \quad R^2 = 0.79 \quad MAPE = 4.55$$

Notes: B is a lag operator such that $B^sX_t = X_{t-s}$. Figures in parentheses are approximate standard errors. All prices are real retail prices in cents per pound.

terms. To start the recursion, we need presample estimates for both ϵ_t and h_t, $t \leq 0$. In this paper we use the sample analogue $T^{-1}\ \epsilon'\epsilon$ to obtain consistent estimates for the presample values of ϵ_t and h_t.

Empirical Results

The estimates of GARCH models for three retail price series—beef, pork, and chicken—are reported here, along with the estimates of standard AR models as applied to each series. The retail prices of beef, pork, and chicken were used because they have been associated with varying degrees of volatility over the past twenty years. During the 1960s and early 1970s, meat prices were relatively stable. However, large shocks in the price of feed grains, high inflation rates in the nonfarm economy, price controls, and the subsequent breeding herd liquidations that occurred in the mid- and late-1970s resulted in volatile meat prices during this period. These casual observations would suggest that it is reasonable to believe that the forecast variances associated with these prices would not have remained constant during this period. More specifically, it may be that large forecast errors and small forecast errors tend to be clustered together. Consequently, an improved model specification would allow the conditional variance term to reflect this type of behavior.

The estimated AR and GARCH models were obtained by using quarterly data, from the first quarter of 1967 through the last quarter of 1986, obtained from various published USDA sources. All data were deflated by the CPI so that each price series was expressed in real terms. Deflated prices were used, in conjunction with linear time trends, to ensure stationarity. Maximum likelihood estimates of the model parameters were obtained by following the procedures outlined in the previous section. The parameter estimates were obtained by using the Davidon-Fletcher-Powell (DFP) algorithm with numerical derivatives.[4]

Estimation results for the autoregressive models, along with sample $MAPE$s (mean absolute percent errors) and R^2s, are presented in table 1. Additional summary statistics associated with each estimated AR and GARCH model are reported in table 2. The linear time trend was retained only in the AR model for beef prices because preliminary estimates indicated that linear drift was not present in the estimated AR models for pork and chicken.

[4] The DPF algorithm is a variable-metric algorithm that belongs to the class of quadratically convergent algorithms. The goal of the DFP algorithm is to accumulate information from successive minimizations so that N such minimizations will yield an exact minimum of a quadratic form. The DFP algorithm operates by approximating the objective function locally as a quadratic form. Computation of the objective function gradient is required at each point of successive iterations. This information is used, in turn, to build up iteratively an approximation to the inverse of the hessian matrix. By using the gradient vector, the hessian matrix, and successive function evaluations, the DFP algorithm moves from point to point until an optimum is attained. For further details, see Powell.

Table 2. Summary Statistics

	$y_t - \hat{\mu}_t$			GARCH		
	$Q(10)$	$Q^2(10)$	λ	$Q(10)$	$Q^2(10)$	λ
Price of Beef (PB_t)	8.08	18.32	1.444	8.61	15.99	1.370
Price of Pork (PP_t)	9.91	19.98	1.315	8.10	4.44	1.225
Price of Chicken (PC_t)	6.92	27.41	1.018	9.44	6.81	1.028

Note: $Q(10)$ and $Q^2(10)$ denote the Ljung-Box portmanteau test statistics for serial correlation in the levels and squares, respectively, at ten degrees of freedom. λ is the smallest root (in absolute value) associated with the polynomial of the lag operator for the conditional means of the estimated AR and GARCH models. The value of the χ^2 distribution at 10 degrees of freedom and at the 5% (1%) level of significance is 11.07 (15.09).

The roots of all three estimated AR models are outside the unit circle, thus satisfying the usual stationarity requirements (table 2). The sample *MAPE*s and R^2s indicate that the conditional means of the fitted AR models do a good job of tracking actual levels.

Further information about the validity of the estimated AR models can be obtained by examining the Ljung-Box portmanteau Q-statistic associated with the innovation series ($y_t - \hat{\mu}_t$). In table 2, the Q-statistics are reported for the innovations from each AR model at ten degrees of freedom. In each case, the reported Q-statistic is below the critical value of 18.31 from the asymptotic χ^2_{10} distribution at the 5% level. Thus, the null hypothesis that the residuals from each estimated AR model are white noise cannot be rejected.

A different picture is presented, however, when the squared residual series $(y_t - \hat{\mu}_t)^2$ is examined. As McLeod and Li report, the portmanteau test statistic $Q^2(m)$ associated with the first m-squared innovations will be distributed asymptotically as a χ^2_m distribution. In table 2, Q^2 statistics at ten degrees of freedom are reported for each estimated AR model. In all cases, the $Q^2(10)$ statistic is significant at the 5% level, indicating that second-order serial correlation may be present. As Bollerslev (1987) suggests, this absence of serial correlation in the conditional first moments, coupled with the presence of serial correlation in the conditional second moments, is one of the implications of the GARCH(p,q) model.

As indicated previously, standard Box-Jenkins procedures can be applied to the squared innovations ($y_t - \hat{\mu}_t$) to determine the appropriate orders for p and q; see Bollerslev (1988) and Engle and Bollerslev. In the present case, the autocorrelations and partial autocorrelations of the squared residuals were used only as an overall guide for specifying the appropriate order of the GARCH process. In all instances, there were spikes in the autocorrelation function that exceeded two standard deviations. In addition, the partial autocorrelations were positive and exhibited dampening behavior, suggesting that retail meat prices might be better represented as GARCH processes.

For each price series, GARCH(1,1) models were estimated first because they are parsimonious and are often the most likely candidates in applied analysis. After these initial estimates were obtained, several alternative specifications of the conditional variance equation, h_t, were examined. The alternatives were limited to GARCH(2,1), GARCH(1,2), and GARCH(2,2) processes. Each alternative was examined for improvements in model fit and parameter significance relative to the GARCH(1,1) process. Following this identification and selection process, it was determined that a GARCH(1,1) process was adequate for explaining the conditional variances of the beef and pork price series. On the other hand, a GARCH(1,2) process was found to be more suitable for the chicken price series.

The maximum likelihood estimates of the GARCH regression models for beef, pork, and broiler prices are reported in table 3. As indicated in table 2, the stationarity conditions for the conditional mean of each estimated GARCH model are satisfied (i.e., the smallest roots are all outside of the unit circle). Furthermore, the stationarity conditions and nonnegativity requirements for the estimated parameters in the conditional variance equations are satisfied in each instance. The Ljung-Box test statistic for the standardized residuals, $\hat{\epsilon}_t \hat{h}_t^{-1/2}$, and the standardized squared residuals, $\hat{\epsilon}_t \hat{h}_t^{-1}$, from the estimated GARCH models are also reported in table 2. In each case, the estimated values for $Q(10)$ and $Q^2(10)$ are below

Table 3. Maximum Likelihood Estimates of GARCH Models Fitted

Price of Beef (PB_t)

$(1 - 0.908B - 0.251B^4 + 0.282B^5)PB_t = 27.297 - 0.211t + \epsilon_{1t}$
(0.052) (0.081) (0.074) (5.358) (0.044)

$h_{1t} = 0.017 + 0.113\epsilon^2_{1t-1} + 0.862h_{1t-1}$
(0.031) (0.035) (0.030)

$R^2 = 0.85$ $MAPE = 3.05$

Price of Pork (PP_t)

$(1 - 1.137B + 0.462B^3 - 0.417B^4 + 0.219B^5)PP_t = 8.017 + \epsilon_{2t}$
(0.004) (0.016) (0.015) (0.007) (6.144)

$h_{2t} = 1.502 + 0.178\epsilon^2_{2t-1} + 0.743h_{2t-1}$
(0.496) (0.011) (0.072)

$R^2 = 0.85$ $MAPE = 4.38$

Price of Chicken (PC_t)

$(1 - 0.724B - 0.240B^8)PC_t = 1.528 + \epsilon_{3t}$
(0.068) (0.057) (0.818)

$h_{3t} = 2.610 + 0.379\epsilon^2_{3t-1} + 0.028\epsilon^2_{3t-2} + 0.062h_{3t-1}$
(0.717) (0.124) (0.015) (0.016)

$R^2 = 0.79$ $MAPE = 4.40$

Notes: B is a lag operator such that $B^sX_t = X_{t-s}$. Figures in parentheses are approximate standard errors. All prices are real retail prices in cents per pound.

18.31, the critical value of the χ^2_{10} distribution at the 5% level. Thus, no further first- or second-order serial dependence is indicated in the estimated GARCH models. Finally, checks of the estimated GARCH parameters indicate that the fourth-order moment of ϵ_t exists for each model.[5] Hence, the asymptotic properties of the maximum likelihood estimates are established.

The reported *MAPE*s and R^2s in table 3 indicate that the estimated parameters associated with the conditional means of the estimated GARCH models do a good job of explaining historical movements; however, these results do not indicate any improvement in explanatory power relative to the AR models in table 1. The implication is that GARCH processes will not necessarily improve upon the forecast performance of the means of the stochastic process and, indeed, there is no reason to believe that they should. But GARCH models will provide more information about the precision of these forecasts. That is, there will be a tendency for large and small forecast errors to cluster together as indicated by the significant $Q^2(10)$ statistics in column 2 of table 2.

To illustrate, confidence intervals (99%) for the one-period-ahead within-sample forecasts for each of the retail price series were computed.[6] The 99% confidence intervals for beef, along with the actual price series, are shown in figure 1. Similar plots for pork and chicken prices are illustrated in figures 2 and 3, respectively. As indicated previously, retail beef prices were volatile during the mid-1970s, as reflected by the wider confidence intervals associated with the GARCH forecasts during this period. By comparison, the 1960s and early 1970s were characterized by relatively stable real retail beef prices. The results in figure 1 show that the confidence intervals associated with the one-step-ahead forecasts during this period are much smaller relative to those for

[5] For a GARCH(1, 1) model, the fourth-order moment exists if $3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2 < 1$. Likewise, the necessary and sufficient condition for existence of a finite fourth-order moment for the GARCH(1, 2) model is

$$\alpha_2 + 3\alpha_1^2 + 3\alpha_2^2 + \beta_1^2 + 2\alpha_1\beta_1 - 3\alpha_2^3 + 3\alpha_1^2\alpha_2 + 6\alpha_1\alpha_2\beta_1 + \alpha_2\beta_1^2 < 1.$$

See Bollerslev (1986) for further details.

[6] Following Engle and Bollerslev (p. 7), the one-step-ahead forecasts of the conditional mean and conditional variance of y_{t+1}, evaluated at time t, can be expressed as

$$E_t(y_{t+1}) = \sum_{i=0}^{n-1} \phi_i y_{t-i},$$

$$V_t(y_{t+1}) = h_{t+1} = \alpha_0 + \sum_{i=0}^{q-1} \alpha_{i+1}[y_{t-i} - E_{t-i}(y_{t-i})]^2 + \sum_{j=0}^{p-1} \beta_{j+1}h_{t-j}.$$

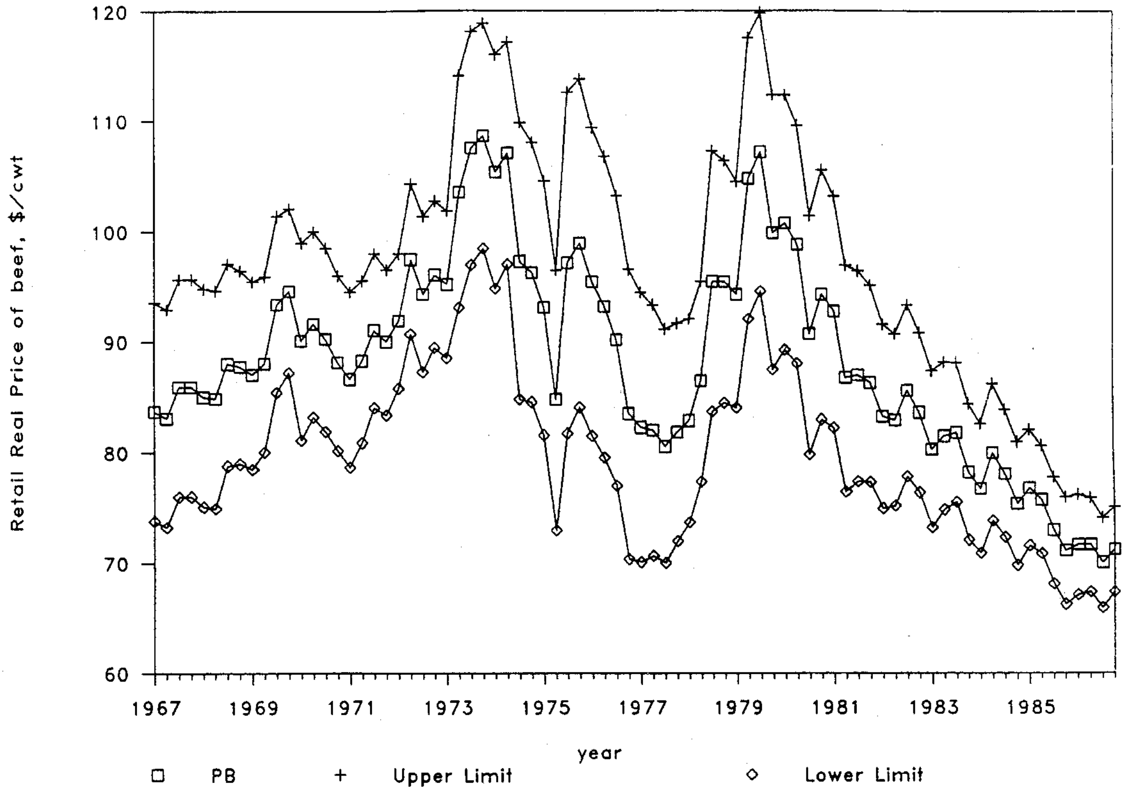

Figure 1. 99% confidence intervals for one-step-ahead forecasts of real retail beef price

the mid-1970s. Traditional time-series models do not give such intuitively appealing results because the width of the confidence interval (i.e., conditional forecast variance) remains constant. Similar results were obtained for the one-step-ahead forecasts of real pork and chicken prices. As with beef, the forecast intervals for pork were widest during the 1970s and were relatively stable during the 1960s and 1980s. That is, there is a tendency for large and small forecast errors to cluster together, which is indicative of the GARCH process. Alternatively, while the confidence intervals for chicken price forecasts, presented in figure 3, do fluctuate, they tend to be more stable relative to the forecast intervals for beef and pork. This, in part, might reflect the relatively constant growth of the poultry industry during the period of analysis.

Although the estimated GARCH models result in confidence intervals that are more intuitively appealing than those of the AR models, this is no guarantee that the GARCH process is a statistically valid improvement over the AR process. In other words, it is desirable to have a formal test of the GARCH

Table 4. Results of Likelihood Ratio Tests

	Value of Log Likelihood Function		*LR* Test Statistic	
	AR	GARCH	(χ^2)	Test Result
Price of Beef (PB_t)	−272.39	−265.71	13.36[a]	Reject AR
Price of Pork (PP_t)	−279.13	−267.48	23.30[a]	Reject AR
Price of Chicken (PC_t)	−106.98	−84.81	44.34[b]	Reject AR

Note: The value of the log likelihood function is unique up to an additive constant.
[a] The value of the χ^2 distribution at 2 degrees of freedom and at the 5% (1%) level of significance is 5.99 (9.21).
[b] The value of the χ^2 distribution at 3 degrees of freedom and at the 5% (1%) level of significance is 7.82 (11.34).

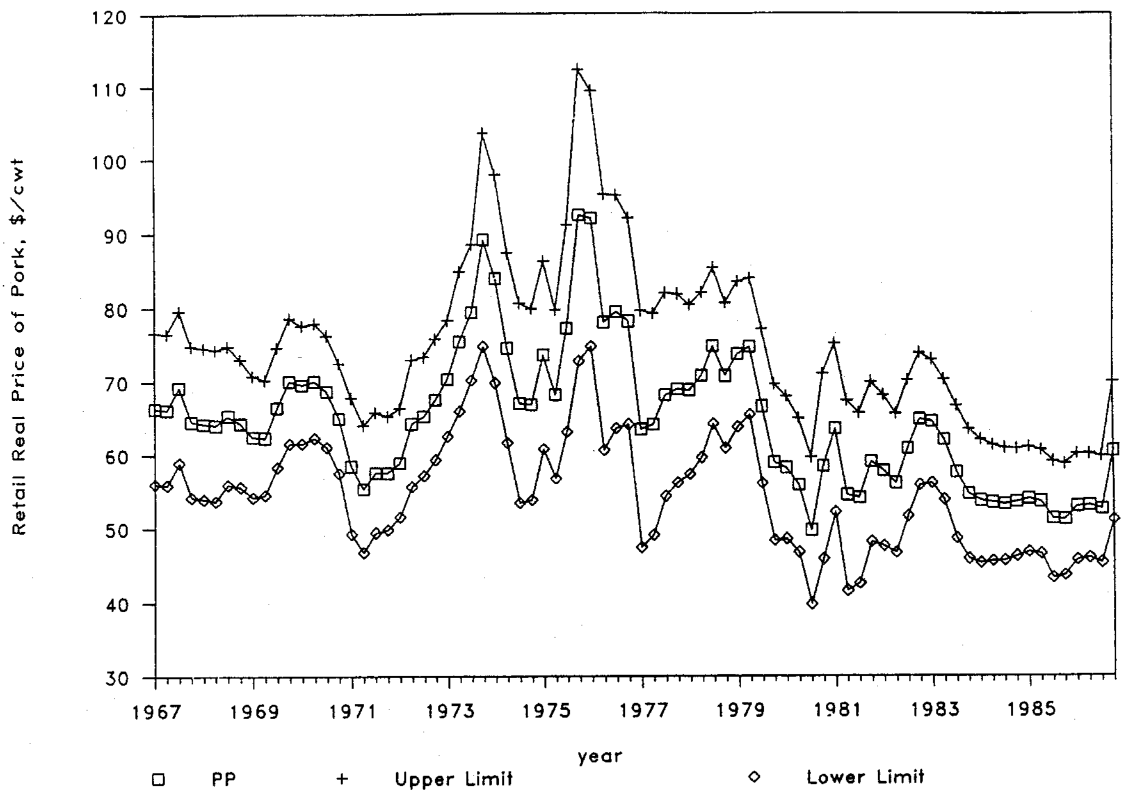

Figure 2. 99% confidence intervals for one-step-ahead forecasts of real retail pork price

hypothesis that conditional forecast variances are nonconstant. This can be accomplished by performing a standard likelihood ratio test in which, under the null hypothesis, the parameters $A(L)$ and $B(L)$ are constrained to zero (the standard AR representation). The alternative hypothesis is that the model follows a GARCH form. The appropriate statistic is twice the difference of the maximized values of the log likelihood functions for the unconstrained and constrained models, respectively, which will have a chi-square distribution with $p + q$ degrees of freedom under the null hypothesis. The results of the likelihood ratio tests are presented in table 4. Importantly, the null hypothesis that the conditional forecast variances are constant could be rejected at all usual levels of significance for all three models. The results in table 4 are encouraging and lend support to our contention that the conditional forecast variances of retail meat prices have been nonstationary during the past twenty years.

Concluding Remarks

Traditional time-series models assume a constant one-period-ahead forecast variance. In recent years, the implausibility of this assumption has been recognized, and several new classes of stochastic processes have been postulated. These include the ARCH process (Engle) and GARCH process (Bollerslev 1986). These are mean zero, serially uncorrelated processes with nonconstant variances, which are conditioned on past information. The GARCH and ARCH processes represent an important advance in time-series modeling because much of the forecasting accuracy associated with traditional time-series models derives from their optimal use of past information. These same optimality conditions can now be used to generate time-varying predictions of the conditional forecast variance.

In this article, GARCH processes were applied to retail meat prices. The estimated

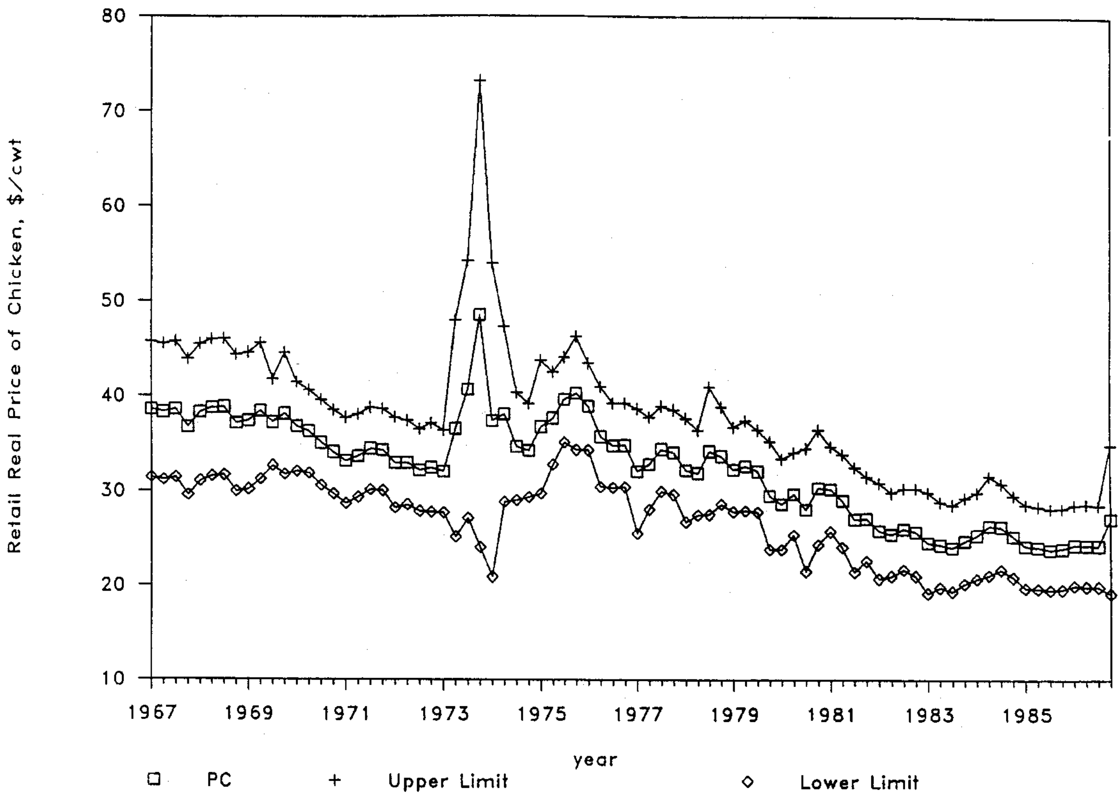

Figure 3. 99% confidence intervals for one-step-ahead forecasts of real retail chicken price

models replicated historical movements in these price series adequately, and confidence intervals, derived from the conditional forecast variances, changed substantially over the sample period. This highlights the potential importance of the GARCH process. A formal test of the joint significance of the $A(L)$ and $B(L)$ parameters in the conditional variance equations in the GARCH models revealed that the constant variance assumption associated with the estimated AR models could be rejected.

The results of this study indicate that recent advances in the econometrics literature may be fruitfully applied to agricultural data. There are many instances where additional knowledge pertaining to forecast variances derived from a GARCH process could be beneficial. In addition, the normality assumption associated with the conditional distribution does not present a limitation; other distributions could be used as well (Bollerslev 1987). The empirical examples presented here should encourage a wider acceptance of GARCH models in applied time-series modeling.

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