Machinery Costs and Inflation

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This article addresses (1) the differences in machinery cost estimating techniques, particularly for depreciation and opportunity cost, and (2) the necessary modifications in cost estimating techniques to account for the changing monetary base under inflation. The conditions under which capital budgeting and traditional budgeting differ are examined on a before tax and after tax basis, with and without inflation. The variations in cost estimates depending upon techniques, and with and without inflation, are compared.

The need for accurate estimates of machinery costs are well recognized. This paper is concerned with placing variable and fixed machinery costs (mainly depreciation and opportunity cost) on an annual basis. Estimates of annual costs of agricultural machinery are useful when comparing costs of different machines, when examining alternatives to ownership, in analysis of machinery-labor tradeoffs, in estimating optimum machinery replacement, for making hedging decisions based on cost of production, and where cost of crop production is required for an individual producer or as a representative budget. Annualizing depreciation and opportunity cost under inflationary times is not well understood. Under double digit inflation, incorrect estimation of opportunity cost and depreciation can lead to large cost estimation errors.

Both traditional and capital budgeting are used to estimate annual machinery costs. Machinery related costs can be classified into two broad categories: 1) depreciation and opportunity cost and 2) adjunct costs, defined as fuel, repairs, maintenance, insurance, property taxes, and other cash costs which are paid annually. Traditional budgeting estimates the annual cost of a depreciable asset using straight line depreciation with opportunity cost based on mid-value. Adjunct costs, both fixed and variable, are estimated on an annual basis as a function of purchase price, mid-value, or as an independent estimate. Capital budgeting can also be used to estimate annual costs. In capital budgeting, flows are discounted from the point of occurrence during the machine life. The sum of the discounted flows (net present cost) is placed on an annual basis by amortizing the net present cost over the machine life. It is commonly recognized that the primary advantage of capital budgeting over traditional budgeting is the capacity to include flows which are variable over time.
Furthermore, income tax influences are easily incorporated into a capital budgeting model; however, as will be discussed later, certain income tax influences can also be incorporated into the traditional estimates. Inflation is conceptually easier to incorporate into a capital budgeting model, but adjustment for inflation is also necessary in the traditional model. Even though, as illustrated by Kay, capital budgeting is more accurate, traditional budgeting will continue to be used because of its computational and conceptual simplicity.

The objectives of this paper are to discuss 1) the relationship between traditional and capital budgeting techniques for estimating annual machinery costs, 2) the impact of inflation on annual machinery cost estimates, and 3) procedures for modifying budgeting techniques to account for inflation. In analyzing inflationary impacts on machinery costs it

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is essential to understand how estimation techniques are affected by inflation. Hence, this paper discusses the relationship of traditional budgeting compared to capital budgeting while examining troublesome elements in cost estimation caused by inflation. Inflation requires close examination of the monetary basis of cost flows. This examination is necessary regardless of the cost estimation method employed.

Implicitly the following assumptions are made throughout the remainder of the paper:

1) Discount rate is constant over time.
2) Inflation rate is constant over time.
3) Inflation affects all variables in a similar manner.
4) Marginal tax rate is constant over time.

An in-depth discussion of the discount rate is beyond the scope of this paper; however, Barry, Hopkin and Baker indicate that the discount rate is influenced by time preference, risk, and inflation. For purposes of this paper, the risk component will be ignored. Kay states that the nominal discount/opportunity cost rate "should be the opportunity rate of return which could be obtained for the capital invested in the asset." For purposes of this paper, the nominal discount rate will be defined as equal to the market rate of interest. Under continuous compounding the real discount rate is assumed to be equal to the nominal discount rate less the inflation rate, the nominal after-tax discount rate is equal to the complement of the marginal tax rate (one minus the marginal tax rate) times the nominal discount rate, and the real after-tax discount rate is equal to the complement of the marginal tax rate times the nominal discount rate less the inflation rate. If the market interest rate is 16%, the inflation rate is 10%, and the marginal tax rate is 25% then the nominal discount rate is 16%, the real discount rate is 6%, the nominal after-tax discount rate is 12%, and the real after-tax discount rate is 2%. Obviously the choice of discount rates will have a large influence on annual opportunity cost of machine ownership, particularly when inflation and tax rates are high. Much of this paper will be concerned with which discount rate is appropiate.

In traditional budgeting adjunct costs are treated as a constant over the ownership period. In the capital budgeting approach the adjunct costs can be variable or constant. Capital budgeting estimates annual adjunct costs by amortizing the present value of the adjunct costs or[1]

$$(1)\ A_c = \left[\int_0^n A(i)e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

where:

A_c = annual adjunct costs under capital budgeting
$A(i)$ = adjunct cost at machine age i
r = discount rate
n = ownership life of the machine.

The discounted present value of the adjunct cost is $\int_0^n A(i)e^{-ri}di$ which is annualized by multiplying by the amortization factor which is $\left[\int_0^n e^{-ri}di\right]^{-1}$

If A(i) = A for all i (adjunct costs are constant and not influenced by machine age) then:

$$(2)\ A_c = \left[\int_0^n Ae^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

$$= A\left[\int_0^n e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

$$= A$$

[1] Continuous time and compounding has been used throughout the paper because it is more convenient than discrete time. However footnotes will offer the discrete counterparts. The discrete counterpart of equation 1 is

$$A_c = \left[\sum_{i=1}^{n} A(i)(1+r)^{-i}\right]\left[\sum_{i=1}^{n}(1+r)^{-i}\right]^{-1}$$

The second term in the above equation is the amortization factor which is equal to

$$\frac{r(1+r)^n}{(1+r)^n-1}$$

In other words, if the adjunct costs are constant, traditional and capital budgeting estimates will be identical. Most other scenarios of adjunct costs will not result in the same cost estimate by the traditional and capital budgeting approach. When adjunct costs vary over time, a logical approach to estimating a surrogate constant cost (A_T) under the traditional approach is to take the simple mean of the adjunct costs over time or $A_T = \frac{1}{n} \sum_{i=1}^{n} A(i)$. If A(i) is a decreasing function of i then the capital budgeting estimate (A_c) will be larger than that of the simple mean (A_T). Conversely, if adjunct costs are an increasing function of i, then the capital budgeting will be smaller than a simple mean. Other scenarios can also be investigated but the general idea should be obvious. For purposes of simplicity throughout the remainder of the paper it will be assumed that adjunct costs are constant in real terms over time. Some of the following conclusions do not hold if adjunct costs are variable.

In the inflation free capital budgeting model, annual adjunct costs at a particular machine age are multiplied by the complement of the marginal tax rate to adjust to an after-tax basis. The discount rate is also adjusted to an after-tax basis by multiplying by the complement of marginal tax rate since interest earned is taxable and interest paid is tax deductible. The traditional and capital budgeting estimates of after-tax adjunct costs are identical (given adjunct costs are constant over time) since[2]

$$(3)\quad \bar{A}_c = \left[\int_0^n \bar{A} e^{-\bar{r}i} di\right] \left[\int_0^n e^{-\bar{r}i} di\right]^{-1}$$

$$= A\left[\int_0^n e^{-ri} di\right] \left[\int_0^n e^{-ri} di\right]^{-1}$$

$$= \bar{A}$$

where:

$\bar{A}_c$ = annual after-tax cost annuity estimated by capital budgeting.

$\bar{r}$ = after-tax discount rate = r(1−T)

$\bar{A}$ = annual after-tax adjunct cost at any age = A(1−T)

T = marginal tax rate.

Estimation of annual adjunct costs by capital budgeting under inflation involves discounting adjunct costs to a present value and amortizing over the ownership period. Nominal adjunct costs should be discounted with a nominal discount rate. If, as assumed in this paper, adjunct costs increase at the same rate as the rate of inflation, then real adjunct costs (adjunct costs with a current time value reference point) can be discounted by the real discount rate which is equivalent to discounting nominal adjunct costs by a nominal discount rate. For purposes of many decisions it is desirable to obtain an annual cost estimate whose value is constant in real terms over time. Amortizing by the nominal discount rate results in an annuity of constant nominal amount but of decreasing value due to value erosion caused by inflation. However, if the present value is amortized with a real discount rate then the value of the annuity is constant in real terms even though the nominal amount is changing (increasing) with inflation.[3]

This point is particularly important and an example may help clarify the importance of amortizing by a real discount rate in most cases. Cost-of-production studies estimate annual costs which are compared to current returns. Since the returns are estimated in current dollars, the cost must be estimated using a current dollar basis if costs and returns are to be comparable. Annual cost estimated by amortizing with a real amortization factor is on a real or current dollar basis. In other words, the annual cost annuity has a

[2]Equation 3 written in discrete form is

$$\bar{A}_c = \left\{\sum_{i=1}^{n} \bar{A}[1+(1-T)r]^{-i}\right\}\left\{\sum_{i=1}^{n} [1+(1-T)r]^{-i}\right\}^{-1}$$

$= A.$

[3] The transformation of a nominal to a real annuity and vice versa are shown in Watts and Helmers.

constant value with a current time value reference point if the present value of costs is amortized by a real discount rate. If another time value reference point is desired, then the real annual cost annuity can be inflated to that point in time. On the other hand, if the present value of costs is amortized by a nominal discount rate resulting in a nominal annual cost annuity, then no time value reference point is specified.[4] Since no time value reference point is specified, confusion occurs in choosing the time value reference point for estimating comparable benefits.

Furthermore if the annual cost of two or more machines with differing lives are added together, such as in a cost-of-production study, adding nominal annual costs is incorrect since no time value reference point is specified, while adding real annual costs is acceptable as long as the same time value reference point is used as a base in computing the real annual costs.[5]

The relationship between adjunct cost estimated under capital budgeting and traditional budgeting under an inflationary setting can be illustrated as follows.[6]

Let:

$\tilde{A}_c$ = real annual adjunct cost

A = adjunct costs at machine age 0 assumed to be constant in real terms

Ae^{fi} = nominal adjunct costs at machine age i

f = rate of inflation

r = nominal discount rate

$\tilde{r}$ = real discount rate defined as r-f

then,

$$\text{(4a)}\quad \tilde{A}_c = \left[\int_0^n Ae^{fi}e^{-ri}di\right]\left[\int_0^n e^{-\tilde{r}i}di\right]^{-1}$$

$$\text{(4b)}\quad = \left[\int_0^n Ae^{(f-r)i}di\right]\left[\int_0^n e^{-\tilde{r}i}di\right]^{-1}$$

$$\text{(4c)}\quad = A\left[\int_0^n e^{-\tilde{r}i}di\right]\left[\int_0^n e^{-\tilde{r}i}di\right]^{-1}$$

$$\text{(4d)}\quad = A$$

In equation 4a, $\int_0^n Ae^{fi}e^{-ri}di$ is the discounted present value of the adjunct costs and

$$\left[\int_0^n e^{-\tilde{r}i}di\right]^{-1}$$

[4] Most adjunct costs for machinery are estimated on a real basis (current time reference point). Thus, a nominally based cost expression requires a change in cost basis from real to nominal for those adjunct cost elements.

[5] There may be situations in which amortizing by a nominal discount rate is appropriate. For instance if the costs are to be compared to returns which are expected to be constant in nominal terms but changing in real terms over time, such as a contractual arrangement, then it is appropriate to amortize by a nominal discount rate to estimate a constant nominal annual cost with changing value over time. Another example in which comparisons of nominal annual cost estimates are correct is when the lives of two machines are equal. However, in both of these examples, comparisons (cost and benefits of two machines) have identical lives, in which case comparison of present values would also be correct. The annual cost becomes especially useful when the lives are not equivalent. When the lives are not equivalent only the real annual cost and/or benefit comparison are correct. However, in most cases in agriculture it appears more desirable to estimate cost with a constant value basis (real basis).

[6] The discrete counterpart to Equation 4 is

$$\tilde{A}_c = \left[\sum_{i=1}^{n} A(1+f)^i(1+r)^{-i}\right]\left[\sum_{i=1}^{n}(1+\tilde{r})^{-i}\right]^{-1}$$

$$= \left[\sum_{i=1}^{n} A\left(\frac{1+f}{1+r}\right)^i\right]\left[\sum_{i=1}^{n}(1+\tilde{r})^{-i}\right]^{-1}$$

$$= A\left[\sum_{i=1}^{n}(1+\tilde{r})^{-i}\right]\left[\sum_{i=1}^{n}(1+\tilde{r})^{-i}\right]^{-1}$$

$$= A.$$

where:

$A(1+f)^i$ = nominal cost at machine age

$\tilde{r}$ = real discount rate = $\frac{1+r}{1+f} - 1$, from Stermole.

is the real amortization factor. Equation 4 implies that the capital and traditional estimates are identical as long as both approaches are based upon the same time value reference point. If $\bar{\bar{A}}_c$ is desired in some future dollars, say at time j, then $\bar{\bar{A}}_c e^{fj}$ provides the desired value. It is important to recognize that in equation 4a nominal flows are discounted by a nominal discount rate and real flows are discounted by a real discount rate as in equation 4b and that the discounted present value of the cost is the same for both methods. However, regardless of the discounting approach, the present value is amortized with a real amortization factor to provide a real annual cost estimate.

In the after-tax inflationary model, the discount rate is adjusted to a real after-tax basis by multiplying the nominal discount rate by the complement of the marginal tax rate and subtracting the inflation rate. Adjusting adjunct costs to an after-tax basis in an inflationary setting under capital budgeting is straightforward.[7]

Let

$\bar{\bar{A}}_c$ = after-tax real annual adjunct costs under capital budgeting

$\bar{A}$ = real after-tax adjunct costs

$\bar{A}e^{fi}$ = nominal after-tax adjunct costs at machine age i

$\bar{\bar{r}}$ = real after-tax discount rate $= \bar{r} - f = (1-T)r - f$

then,

$$(5)\quad \bar{\bar{A}}_c = \left[\int_0^n \bar{A}e^{fi}e^{-\bar{r}i}di\right]\left[\int_0^n e^{-\bar{\bar{r}}i}di\right]^{-1}$$

$$= \bar{A}\left[\int_0^n e^{-\bar{\bar{r}}i}di\right]\left[\int_0^n e^{-\bar{\bar{r}}i}di\right]^{-1}$$

$$= \bar{A}$$

[7] The discrete counterpart of Equation 5 is

$$\bar{\bar{A}}_c = \left[\sum_{i=1}^{n} \bar{A}(1+f)^i(1+\bar{r})^{-i}\right]\left[\sum_{i=1}^{n}(1+\bar{\bar{r}})^{-i}\right]^{-1}$$

$$= \bar{A}.$$

where:

$\bar{\bar{r}}$ = real after tax discount rate $= \dfrac{1+\bar{r}}{1+f} - 1$.

If traditional budgeting adjunct costs are estimated using time zero dollars as a base, then the correct adjustment of capital and traditional budgeting adjunct costs estimates to an after-tax basis is multiplication of the annual adjunct costs by the complement of the marginal tax rate.

Depreciation and Opportunity Cost Under Inflation Free Conditions

Traditional and capital budgeting estimates of depreciation and opportunity cost will be discussed and compared under various inflation and tax assumptions, beginning with a simple before-tax inflation free setting and ending with an after-tax inflationary setting including a short discourse on indexing depreciation for tax purposes.

Inflation Free Before-Tax Setting

Depreciation and opportunity cost are handled quite differently in the traditional and capital budgeting approaches. In the traditional model, generally straight line depreciation is assumed and opportunity cost is based upon mid-value. In the capital budgeting approach, the purchase price minus the present value of the selling price is amortized over the machine life. The amortized value is a combined estimate of annual opportunity cost and depreciation. Mathematically, traditional budgeting estimates opportunity cost as[8]

$$(6)\qquad OC_T = \frac{V(0) + V(n)}{2}\,r$$

where:

OC_T = the traditional budgeting estimate of opportunity cost

$V(i)$ = value of the machine at age i

n = selling or replacement age

r = discount rate.

Traditional budgeting estimate of depreciation (D_T) is

[8] Machines are assumed to be purchased new.

$$(7)\qquad D_T = \left(\frac{V(0) - V(n)}{n}\right)$$

Capital budgeting estimates annual depreciation and opportunity cost as[9]

$$(8)\ D_c + OC_c = \left[V(0) - V(n)e^{-rn}\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

where:

D_c = annual depreciation estimated by capital budgeting

OC_c = annual opportunity cost estimated by capital budgeting.

However, the capital budgeting estimate (right hand side of Equation 8) can be separated into opportunity cost and depreciation. Appendix A shows that

$$(9)\ V(0) - V(n)e^{-rn} = \left[\int_0^n D(i)e^{-ri}di + \int_0^n OC(i)e^{-ri}di\right]$$

where:

$D(i)$ = market depreciation at machine age i = $-\frac{\partial V(i)}{\partial i}$

$OC(i)$ = opportunity cost at machine age i = $V(i)r$

The annual depreciation and opportunity cost estimated by capital budgeting can be separated as[10]

$$(10)\ D_c = \left[\int_0^n D(i)e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

$$(11)\ OC_c = \left[\int_0^n OC(i)e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

Equation 10 is the amortized present value of the depreciation and Equation 11 is the amortized present value of the opportunity cost. Now traditional and capital budgeting estimates of depreciation and opportunity cost can be compared. Appendix B shows that depreciation plus opportunity cost estimated by capital budgeting ($D_c + OC_c$) is always greater than depreciation plus opportunity cost estimated by the traditional approach as long as the discount rate is positive. Furthermore, the estimates are equal when the discount rate is equal to 0. Table 1 presents an example to illustrate the difference in estimated depreciation and opportunity cost under traditional and capital budgeting. The difference between the combined estimates is not large (only 2% in the example in

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[9] The discrete counterpart to Equation 8 is

$$D_c + DC_c = \left[V(0) - V(n)(1+r)^{-n}\right]\left[\sum_{i=1}^{n}(1+r)^{-i}\right]^{-1}$$

[10] The discrete counterparts to Equations 10 and 11 are

$$D_c = \left[\sum_{i=1}^{n} V(i-1) - V(i)(1+r)^{-i}\right]\left[\sum_{i=1}^{n}(1+r)^{-i}\right]^{-1}$$

$$OC_c = \left[\sum_{i=1}^{n} rV(i-1)(1+r)^{-i}\right]\left[\sum_{i=1}^{n}(1+r)^{-i}\right]^{-1}$$

Walrath pointed out that the midvalue implied by discrete capital budgeting and straight line depreciation is equal to

$$\frac{V(0) + V(n)}{2} + \frac{D}{2}.$$

However the correction factor, $\frac{D}{2}$, vanishes under continuous time.

Let MV_c^c = the midvalue under continuous time then

$$MV_c^c = \frac{\int_0^n [V(0) - iD]di}{n}$$

$$= \frac{nV(0) - D\left(\frac{n^2}{2}\right)}{n}$$

$$= V(0) - \frac{Dn}{2}$$

However, since $Dn = V(O) - V(n)$, then

$$MV_c^c = \frac{V(0) + V(n)}{2}$$

Table 1). Furthermore, the combined depreciation and opportunity cost is unaffected by the form of the machinery price function or market depreciation but is dependent on the purchase and selling price, which should be obvious from Equation 8. In Table 1, two machinery price functions are featured, both yielding identical combined opportunity cost and depreciation estimates.

If market depreciation is constant or straight line

$$\left(\frac{\partial D(i)}{\partial i} = \frac{\partial^2 V(i)}{\partial_i^{\,2}} = 0\right)$$

then D(i) becomes a constant (D) and D_c is equal to D. However, since D must equal $\frac{V(0) - V(n)}{n}$ (or total depreciation, nD, must equal the difference between the purchase price and the selling price) then D is also equal to D_T. Therefore, depreciation estimated by capital and traditional budgeting are identical as long as straight line market depreciation is assumed. However, as shown in Appendix C, if depreciation is declining,

$$\left(\frac{\partial D(i)}{\partial i} = -\frac{\partial^2 V(i)}{\partial i^2} < 0\right),$$

then the traditional estimate of depreciation is less than the capital budgeting estimate given a positive discount rate. Table 1 presents an example of depreciation estimated by capital and traditional budgeting. Two machinery price functions are assumed for machinery. Value function $V(i)_A$ assumes straight line depreciation and function $V(i)_B$ assumes declining balance depreciation. The value of both functions are 20,000 at the purchase age 0 and 2000 when i equals selling age of 12 for ease of comparison. Notice that the shape of machinery price function is important and does affect the annual depreciation estimate under capital budgeting. Furthermore, changes in opportunity cost caused by different machinery price functions are offset by changes in depreciation to maintain the same combined cost under capital budgeting. Differences in traditional and capital budgeting estimates of opportunity cost under straight line depreciation are entirely due to amortizing-discounting effects.

Inflation Free After-Tax Setting

The simplest tax adjustments to cost estimates are discussed, in which the used selling price is equal to the salvage value set for tax purposes, the tax depreciable life and useful life are equal, and straight line depreciation is assumed. Both cost estimating approaches can be adjusted to an after-tax basis. The present value of the after-tax cost of depreciation and opportunity cost is implicitly estimated in the capital budgeting approach by subtracting the present value of the depreciation tax benefits from the new

TABLE 1. An Example of Depreciation and Opportunity Cost Estimated Under An Inflation Free Setting

Assumptions

$V(i) = 20{,}000 - 1500i$ Machinery Price Function A

$V(i) = 20{,}000\,(.82540419)^i$ Machinery Price Function B

$r = .05 \quad n = 12$

	Annual Cost Estimates ($)		
	Combined Depreciation and Opportunity Cost	Depreciation	Opportunity Cost
Traditional Budgeting	2050.00	1500.00	550.00
Capital Budgeting			
Machinery Price Function A	2094.73	1500.00	594.73
Machinery Price Function B	2094.73	1661.73	433.00

price reduced by the discounted salvage value or as shown in Appendix D.[11]

$$(12)\qquad V(0) - \frac{V(n)}{e^{\bar{r}n}} - T\int_0^n \frac{D(i)}{e^{\bar{r}i}}di = \int_0^n \frac{D(i) + OC(i)}{e^{\bar{r}i}}(1-T)di$$

where:

T = marginal tax rate

$\bar{r}$ = after-tax discount rate = $r(1-T)$.

Of course, the appropriate discount rate in an after-tax model is the after-tax discount rate ($\bar{r}$). Equations 13 and 14 present the after-tax counterpart to Equations 10 and 11 developed from Equation 12.[12]

$$(13)\ \bar{D}_c = \left[\int_0^n (1-T)D(i)e^{-\bar{r}i}di\right]\left[\int_0^n e^{-\bar{r}i}di\right]^{-1}$$

$$(14)\ \bar{O}\bar{C}_c = \left[\int_0^n (1-T)OC(i)e^{-\bar{r}i}di\right]\left[\int_0^n e^{-\bar{r}i}di\right]^{-1}$$

where:

$\bar{D}_c$ = the annual after-tax depreciation estimated by capital budgeting

$\bar{O}\bar{C}_c$ = the annual after-tax opportunity cost estimated by capital budgeting.

Equations 13 and 14 imply that traditional estimates of opportunity cost and depreciation may be adjusted to an after-tax basis by multiplying the before-tax estimates by the complement of the marginal tax rate. If depreciation is straight line so depreciation is a constant over the machine life and equal to D, then $\bar{D}_c$ equals $(1-T)D$. Therefore, the appropriate adjustment to an after-tax basis for traditional depreciation estimates is multiplication by the complement of the marginal tax rate. Even if depreciation is changing, this adjustment to an after-tax basis incorporates no greater error than in the before-tax model (the error is totally due to the compounding-amortizing influences). The same can be said for the traditional estimate of opportunity cost and depreciation. The traditional estimate of opportunity cost may be placed on an after-basis by multiplying the traditional before-tax estimates by the complement of the marginal tax rate with no increased loss in accuracy. Table 2 presents an example in which the after-tax counterparts of the example in Table 1 are featured. Note that the form of the machinery price function is important and does influence the combined estimate, which should be obvious from the left hand side of Equation 12. The differences in the combined estimates of opportunity cost and depreciation in Table 2 are relatively small, again implying that traditional budgeting estimates the cost, particularly the combined cost, with reasonable accuracy.

Including accelerated depreciation, decreased salvage value, shortened depreciable lives, depreciation recapture, and investment credit in the capital budgeting estimates is relatively straightforward. Chisolm included these influences in capital budgeting estimates used to analyze replacement strategies, which will not be duplicated here due to brevity. There appears to be no easy, straightforward and reasonably accurate method of incorporating these features into the traditional method.

Inflationary Conditions

Inflationary Setting

As explained earlier, for most uses a cost estimate expressed on an annual basis developed to encompass a time period must be

[11] The discrete counterpart of Equation 12 is

$$V(0) - \frac{V(n)}{[1+r(1-T)]^n} - \sum_{i=1}^{n} \frac{T[V(i-1) - V(i)]}{[1+r(1-T)]^i} = \sum_{i=1}^{n} \frac{D(i) + OC(i)}{[1+r(1-T)]^i}(1-T)$$

where:

$D(i) = V(i-1) - V(i)$

$OC(i) = V(i-1)r.$

[12] The discrete counterpart of Equations 13 and 14 are

$$\bar{D}_c = \left\{\sum_{i=1}^{n} (1-T)D(i)[1+r(1-T)]^{-i}\right\}\left\{\sum_{i=1}^{n}[1+r(1-T)]^{-i}\right\}^{-1}$$

$$\bar{O}\bar{C}_c = \left\{\sum_{i=1}^{n} (1-T)\ OC(i)[1+r(1-T)]^{-i}\right\}\left\{\sum_{i=1}^{n}[1+r(1-T)]^{-i}\right\}^{-1}$$

TABLE 2. After Tax Inflation Free Example of Depreciation and Opportunity Cost.

Assumptions

Machinery Price Function A	$V(i)_A = 20{,}000 - 1500i$
Machinery Price Function B	$V(i)_B = 20{,}000\ (.82540419)^i$
r = .05 n = 12 T = .20	

	Annual After-Tax Cost Estimate ($)		
	Combined Depreciation and Opportunity Cost	Depreciation	Opportunity Cost
Traditional Budgeting	1640	1200	440
Capital Budgeting			
Machinery Price Function A	1668.69	1200	468.69
Machinery Price Function B	1642.87	1303.27	339.60

expressed on a real basis (or have a value reference point) to allow for meaningful comparisons to other alternatives and/or benefits. Schoney discussed the impact of inflation on used machinery prices and interest rates on capital recovery factors (analogous to annual depreciation and opportunity cost); however, he chose to estimate annual cost (capital recovery factor) on a nominal basis (since he used a nominal discount rate and machinery selling price). If an annual machinery cost estimate is developed on a nominal basis, it becomes largely incomprehensible because of the changing value of those dollar expressions used in forming that estimate. While nominal expressions of costs can be mathematically constructed, a nominal expression is of limited use without a time value reference point. Once a machinery budget has been developed on a real basis it can be readily adjusted to another time value reference point.

To compute an annual real cost estimate of depreciation and opportunity cost under capital budgeting, the sum of the purchase price less the correctly discounted selling price is amortized. If the selling price is in nominal terms then the selling price should be discounted by the nominal discount rate and if the selling price is in real terms the selling price should be discounted by a real discount rate. A nominally estimated selling price requires an estimate of the effect of inflation rates on the projected selling price. The sum of the purchase price less the discounted selling price is amortized with a real discount rate to estimate real annual cost.

More formally, the present value of the nominal used price (using a nominal discount rate) subtracted from the purchase price is equal to the present value of the depreciation plus opportunity cost if the depreciation and opportunity cost are computed using a real salvage value and real discount rate. Appendix E shows that [13]

$$(15)\quad V(0) - \frac{V(n)e^{fn}}{e^{rn}} = \int_0^n \frac{\tilde{D}(i) + \tilde{OC}(i)}{e^{\tilde{r}i}}\,di$$

where:

$V(i)$ = value of machine at age i in a dollar value associated with machine age 0

f = inflation rate

r = nominal discount rate

$\tilde{D}(i)$ = machine depreciation at age i in a dollar value associated with machine age 0 $= \frac{\partial V(i)}{\partial i}$

$\tilde{OC}(i)$ = opportunity cost at machine age i in a dollar value associated with machine age 0 $= V(i)\tilde{r}$

$\tilde{r}$ = real discount rate $= r - f$.

[13] The discrete counterpart of Equation 15 is

$$V(0) - V(n)\frac{(1+f)^n}{(1+r)^n} = \sum_{i=1}^{n} [\tilde{D}(i) + \tilde{OC}(i)]\left(\frac{1+f}{1+r}\right)^i$$

Note that these equations reduce to those in the inflation free setting when f = 0. Furthermore, prediction of future inflation rates is not necessary if the real discount rate is used.

From Equation 15, the following estimates of annual depreciation and opportunity cost under capital budgeting are developed.[14]

$$(16)\quad D_c = \left[\int_0^n \tilde{D}(i)e^{-\bar{r}i}di\right]\left[\int_0^n e^{-\bar{r}i}di\right]^{-1}$$

$$(17)\quad OC_c = \left[\int_0^n \tilde{OC}(i)e^{-\bar{r}i}di\right]\left[\int_0^n e^{-\bar{r}(i)}di\right]^{-1}$$

Equations 16 and 17 imply that the real opportunity cost should be estimated using a real interest rate, which is consistant with Sutherland and Watts and Helmers, and both real opportunity cost and depreciation should be estimated using a real used price for the salvage value under traditional budgeting.[15]

Table 3 presents an example in which depreciation and opportunity cost are estimated correctly, as well as a variety of common errors. Capital budgeting situations 1 and 7 are correct while the remaining situations are incorrect for most uses. Using a nominal discount rate to amortize results in large overestimation of the combined real depreciation and opportunity cost. Other errors presented in the capital budgeting section of Table 3 are less serious.

In Table 3, traditional situation 1 is correct for most purposes. Using a nominal opportunity cost rate under traditional budgeting resulted in large errors, while failure to use a real selling price resulted in smaller errors in estimating combined depreciation and opportunity cost. The individual estimates of depreciation and opportunity cost were more seriously influenced by the basis of the selling price. Modigliani and Cohn report similar results in valuing stocks.

Inflationary After-Tax Setting

Inflation complicates the after-tax analysis because the tax deduction is in nominal versus real terms. From Appendix A, it is obvious that[16]

$$(18)\quad V(0) - \frac{V(n)e^{fn}}{e^{\bar{r}n}} - \int_0^n T\frac{\tilde{D}(i)}{e^{\bar{r}i}}di + TV(n)\left(\frac{e^{fn}-1}{e^{\bar{r}n}}\right)$$
$$= \int_0^n\left[\frac{\tilde{D}(i)+\tilde{OC}(i)}{e^{\bar{\bar{r}}i}} - \frac{T\tilde{D}(i)}{e^{\bar{r}i}}\right]di + TV(n)\left(\frac{e^{fn}-1}{e^{\bar{r}n}}\right)$$

Where:

$\bar{\bar{r}}$ = real after-tax discount rate = $\bar{r} - f = r(1-T) - f$

$\tilde{D}(i)$ = the real depreciation or change in value of the machine = $-V'(i)$

[14] The discrete counterparts of Equations 16 and 17 are

$$D_T = \left[\sum_{i=1}^n \tilde{D}(i)\left(\frac{1+f}{1+r}\right)^i\right]\left[\sum_{i=1}^n\left(\frac{1+f}{1+r}\right)^i\right]^{-1}$$

$$OC_T = \left[\sum_{i=1}^n \tilde{OC}(i)\left(\frac{1+f}{1+r}\right)^i\right]\left[\sum_{i=1}^n\left(\frac{1+f}{1+r}\right)^i\right]^{-1}$$

[15] This formulation satisfies the sinking fund approach if depreciation is inflated. At time i, the nominal depreciation is $D_c e^{fi}$. Inflating the nominal depreciation to time n yields $D_c e^{fi}e^{f(n-i)} = De^{fn}$. However,

$$\int_0^n De^{fn}di = V(0)e^{fn} - V(n)e^{fn}.$$

$$V(O)e^{fn} = nDe^{fn} + V(n)e^{fn}.$$

The above equation implies that the inflated depreciation plus the inflated selling price equals the inflated purchase price which is equal to the purchase price of the subsequent machine if machinery prices are changing at the same rate as inflation.

[16] The discrete counterpart of Equation 18 is

$$V(0) - \frac{V(n)(1+f)^n}{(1+\bar{r})^n} - \sum_{i=1}^n \frac{T\tilde{D}(i)}{(1+\bar{r})^i} + TV(n)\left[\frac{(1+f)^n-1}{(1+\bar{r})^n}\right]$$
$$= \sum_{i=1}^n\left[\frac{\tilde{D}(i)+\tilde{OC}(i)}{(1+\bar{\bar{r}})^i} - \frac{T\tilde{D}(i)}{(1+\bar{r})^i}\right] + \left[TV(n)\left[\frac{(1+f)^n-1}{(1+\bar{r})^n}\right]\right]$$

where: $\bar{\bar{r}} = \frac{1+\bar{r}}{1+f} - 1$

TABLE 3. An Inflationary Example of Depreciation and Opportunity Cost Estimates.

Assumed Actual Conditions

$V(i) = 20{,}000 - 1500i$ — Real Machinery Price Function

$V(i)_N = V(i)e^{fi}$ — Nominal Machinery Price Function

$r = .15 \quad f = .10 \quad n = 12$

Capital Budgeting

Situation	Values Used For Computations: Discount Rate Used for Discounting Selling Price	Values Used For Computations: Discount Rate Used for Amortizing	Values Used For Computations: Machinery Price Function	Combined Depreciation and Opportunity Cost
1	.05	.05	Real	2094.73
2	.05	.15	Real	3396.85
3	.15	.05	Real	2179.73
4	.15	.15	Real	3534.69
5	.05	.05	Nominal	1812.52
6	.05	.15	Nominal	2939.21
7	.15	.05	Nominal	2094.73
8	.15	.15	Nominal	3396.85

Traditional Budgeting

Situation	Values Used for Computations: Opportunity Cost Rate	Values Used for Computations: Basis of Machinery Selling Price	Combined Depreciation and Opportunity Cost	Depreciation	Opportunity Cost
1	.05	Real	2050.00	1500.00	550.00
2	.15	Real	3150.00	1500.00	1650.00
3	.05	Nominal	1779.32	1113.31	666.01
4	.15	Nominal	3111.33	1113.31	1998.02

$\widetilde{\overline{OC}}(i)$ = the real after-tax opportunity cost = $V(i)\bar{\bar{r}}$

and $TV(n)\left(\frac{e^{fn}-1}{e^{\bar{r}n}}\right)$ is depreciation recapture.

Note that the depreciation tax benefits and depreciation recapture are discounted by a nominal after-tax discount rate while the rest of the right hand side of the equation is discounted by the real after-tax discount rate. Furthermore, the depreciation recapture is simply a tax on the inflation caused increase in selling price (since the real selling price was correctly anticipated by assumption).

Equation 16 can be separated into depreciation and opportunity cost and modified into annual costs as[17]

$$\bar{D}_c = \int_0^n [\tilde{D}(i)e^{-\bar{\bar{r}}i} - T\tilde{D}(i)e^{-\bar{r}i} + TV(n)(e^{fn}-1)e^{-\bar{r}n}]\,di\,\left[\int_0^n e^{-\bar{\bar{r}}i}di\right]^{-1} \quad (19)$$

$$\bar{\bar{OC}}_c = \left[\int_0^n \widetilde{\overline{OC}}(i)e^{-\bar{\bar{r}}i}di\right]\left[\int_0^n e^{-\bar{\bar{r}}i}di\right]^{-1} \quad (20)$$

The last term in Equations 19 and 20 are the amortization factors using a real after-tax discount rate. Bates et al. developed a capital budgeting model including investment credit under inflationary conditions to analyze replacement decisions which for the sake of brevity will not be duplicated here.

[17] The discrete counterparts of Equations 19 and 20 are

$$\bar{D}_c = \sum_{i=1}^{n}\{\tilde{D}(i)(1+\bar{\bar{r}})^{-i} - T\tilde{D}(i)(1+\bar{r})^{-i} + TV(n)[(1+f)^n - 1](1+\bar{r})^{-n}\}\left[\sum_{i=1}^{n}(1+\bar{\bar{r}})^{-i}\right]^{-1}$$

$$\bar{\bar{OC}}_c = \sum_{i=1}^{n}[\widetilde{OC}(i)(1+\bar{\bar{r}})^{-i}]\left[\sum_{i=1}^{n}(1+\bar{\bar{r}})^{-i}\right]^{-1}$$

Adjustments to the traditional estimates of depreciation and opportunity are more complicated under the inflation-tax setting. Multiplying the real depreciation $\left(\frac{V(0) - V(n)}{n}\right)$ by the complement of the marginal tax rate is less accurate under inflation due to depreciation recapture as well as erosion of depreciation tax benefits as is illustrated in Table 4. Opportunity cost estimates, on the other hand, suffer from the same discounting/amortizing influences which caused differences in the inflation free setting. Furthermore, if the real before-tax discount rate is constant (inflation and the nominal discount rate change in a manner such that the difference between the two is constant), then the real after-tax discount rate is a monotonically decreasing function of both inflation and the marginal tax rate. As a result, the after-tax inflation free discount rate is f_T less than in the after-tax inflation free setting. If, for example, the nominal discount rate is 15%, the inflation rate is 10%, and the marginal tax rate is 20%, then the real after-tax discount rate is only 2%, compared to an inflation free situation in which the discount rate is 5% and, therefore, the after tax discount rate is 4%.

Policy makers may be interested in a scenario in which inflation does not affect the after-tax cost of depreciation and opportunity cost. This can be accomplished by (1) inflation indexing of depreciation for computing both annual depreciation and depreciation recapture, and (2) taxing only real interest. The real after-tax discount rate ($\hat{r}$) would then become $(r-f)(1-T)$ not $r(1-T)-f$. The present value of the depreciation and opportunity cost would then be equal to the new price minus the present value of the salvage value and depreciation benefits or in Appendix F it is shown that

$$(21)\quad V(0) - \frac{V(n)e^{fn}}{e^{\dot{r}n}} - T\int_0^n \frac{D(i)e^{fn}}{e^{\dot{r}n}}\,di = \int_0^n \frac{\tilde{D}(i) + \tilde{OC}(i)}{e^{\hat{r}i}}(1-T)\,di$$

where:

$\dot{r}$ = the nominal before-tax interest rate = $(\dot{r}-f)(1-T)+f = r(1-T)+fT$

$\hat{r}$ = real after-tax interest rate = $(r-f)(1-T)$.

The annual estimates of opportunity cost and depreciation are:

$$(22)\; OC_c = \left[\int_0^n \tilde{OC}(i)e^{-\hat{r}i}di\right]\left[\int_0^n e^{-\hat{r}i}di\right]^{-1}$$

TABLE 4. After-Tax Inflationary Example of Depreciation and Opportunity Cost.

Assumptions		
V(i) = 20,000 − 1500i	Real Machinery Price Function	
r = .15 f = .10 n = 12 T = .20		
		Annual After-tax Cost Estimate ($)
Capital Budgeting		
Combined Depreciation and Opportunity Cost		1568.99
Depreciation		1341.80
Opportunity Cost		227.19
Traditional Budgeting		
Combined Depreciation and Opportunity Cost		1440.00
Depreciation	$\left[\frac{V(0)-V(n)}{n}(1-T)\right]$	1200.00
Opportunity Cost	$\left[\frac{V(0)+V(n)}{2}\bar{\tilde{r}}\right]$	220.00

$$(23)\ D_c = \left[\int_0^n \tilde{D}(i)e^{-\hat{r}i}di\right]\left[\int_0^n e^{-\hat{r}i}di\right]^{-1}$$

which are identical to Equations 13 and 14. If these adjustments are made, then the annual real after-tax costs are the same under inflation and inflation free setting.

Summary and Conclusions

This paper discussed the differences between traditional and capital budgeting of annual machinery cost estimates. Particular emphasis was plaed on adjustments required under inflation. With the exception of the after-tax inflationary setting, traditional and capital budgeting estimates of depreciation and opportunity cost did not differ greatly. The differences between the estimates were largely due to discounting-amortizing influences. The after-tax inflationary setting resulted in greater differences between the traditional and capital budgeting model due to the erosion of depreciation tax benefits and increased depreciation recapture.

Of course, the correct procedure must be used to compute depreciation and opportunity cost under both budgeting methods. This paper argues that estimating annual costs on a real basis is preferred for most purposes. Under capital budgeting this involves discounting real flows with a real discount rate and/or discounting nominal flows by a nominal discount rate to compute a present value of costs. The present value of cost is amortized over the machinery ownership life with a real discount rate. Furthermore, this paper presents a method by which depreciation and opportunity cost can be separated under capital budgeting as well as presenting after-tax adjustments under inflation free and inflationary settings.

Traditional budgeting estimates of depreciation are estimated in real terms by using a real selling price. Traditional opportunity cost estimates are estimated in real terms by using a real opportunity cost rate and real used price. Failure to make these relatively simple adjustments can result in large errors when estimating cost on a real annual basis. Adjustments to after-tax basis can be accomplished by multiplying depreciation by the complement of the marginal tax rate. A real after-tax opportunity cost rate must be used to compute real after-tax opportunity cost.

It has been demonstrated by an example that wide differences in estimates can result when budgeting machinery costs under inflationary conditions. These differences result primarily because of different assumptions for real vs nominal discount/interest rates and the monetary basis of the salvage value or used selling price. For most purposes a real cost expression is the acceptable form of cost expression rather than a nominal expression. This is achieved in the traditional budgeting method by using 1) a real opportunity cost and 2) a real selling price or salvage value. The comparable adjustments in the capital budgeting method are 1) discounting by a discount rate consistent with the salvage value basis and 2) amortizing with a real amortization rate.

With the extensive use of investment budgeting in agricultural decision making and research, the use of consistent methodology to focus on inflationary affected cost flows is essential. Critical to that process is the recognition of the correct monetary basis (real or nominal) upon which costs should be expressed such that the economic flows for an economic setting are comparable.

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Appendix A

Transformation of Capital Budgeting Fixed Cost Estimates Into Opportunity Cost and Depreciation.

Prove that

$$V(0) - V(n)e^{-rn} = \int_0^n [D(i) + OC(i)]e^{-ri}di$$

Note that

$$\partial V(i)e^{-ri}/\partial i = [V'(i) - V(i)r]e^{-ri}$$

then

$$V(0) - V(n)e^{-rn} = -[V(i)e^{-ri}]_{i=0}^{n}$$

$$= -\int_0^n [\frac{\partial V(i)}{\partial i} - V(i)r]e^{-ri}di$$

Furthermore since

$$-\frac{\partial V(i)}{\partial i} = D(i) \text{ and } V(i)r = OC(i)$$

then

$$V(0) - V(n)e^{-rn} = \int_0^n [D(i) + OC(i)]e^{-ri}di$$

End of Proof

Appendix B

Proof of the Relationship Between the Combined Estimate of Depreciation and Opportunity Cost Under Traditional and Capital Budgeting.

Let

F_c = combined estimate of opportunity cost and depreciation under capital budgeting

$$= \left[V(0) - V(n)e^{-rn}\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

F_T = combined estimate of opportunity cost and depreciation under traditional budgeting

$$= \frac{V(0) - V(n)}{n} + \frac{V(0) + V(n)}{2}r$$

$$F = F_c - F_T$$

If F has a definite sign, then the relationship between F_c and F_T has been established. First note that $F = 0$ when $r = 0$. Furthermore, after some algebraic manipulation

$$F = \left[V(0) - V(n)\right]\left[\frac{r}{1-e^{-rn}} - \frac{1}{n} - \frac{r}{2}\right]$$

and

$$\frac{\partial F}{\partial r} = \left[V(0) - V(n)\right]\left[\frac{1}{1-e^{-rn}} + \frac{r^2e^{-rn}}{[1-e^{-rn}]^2} - \frac{1}{2}\right]$$

$$= \left[V(0) - V(n)\right]\left[\frac{1+e^{-rn}}{2(1-e^{-rn})} + \frac{r^2e^{-rn}}{(1-e^{-rn})^2}\right]$$

Since both bracketed terms are positive (assuming $r > 0$ and $V(0) > V(n)$), then F is a monotonically increasing function of r. Therefore,

$$F_c = F_T \quad \text{if } r = 0$$
$$F_c > F_T \quad \text{if } r > 0$$

End of Proof

Appendix C

Proof of the Relationship Between Depreciation Estimate by Capital Budgeting (D_c) and by Traditional Budgeting (D_T).

$$D_c = \left[\int_0^n - V'(i)e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]$$

where:

$V'(i)$ = rate of market price change at age i or the negative of depreciation or $\frac{\partial V(i)}{\partial i}$

If $r = 0$ then

$$D_c = \left[\int_0^n V'(i)e^{-ri}di\right]\left[\int_0^n e^{-ri}di\right]^{-1}$$

$$= \frac{V(0) - V(n)}{n} = D_T$$

If $\partial D_c / \partial r$ has a definite sign then the relationship between D_T and D_c is established since $\partial D_T / \partial r = 0$. The sign of $\partial D_c / \partial r$ is dependent upon $V''(i)$. Therefore, the numerator of D_c is integrated by parts with respect to i to include information in $V''(i)$.

Numerator of D_c =

$$-\left[\frac{V'(i)e^{-ri}}{-r} - \int_0^n \frac{V''(i)e^{-ri}}{-r}di\right]_{i=0}^{n}$$

$$= -\frac{1}{r}\left[V'(0) - V(n)e^{-rn} + \int_0^n V''(i)e^{-ri}di\right]$$

Furthermore the denominator of D_c is

$$= \frac{1 - e^{-rn}}{r}$$

then

$$D_c = \frac{V'(n)}{e^{rn}-1} - \frac{V'(0)}{1-e^{-rn}} - \int_0^n \frac{V''(i)}{e^{ri} - e^{r(i-n)}}di$$

Taking the derivatives of each term with regard to r separately

$$\frac{\partial}{\partial r}\left[\frac{V(n)}{e^{rn}-1}\right] = -\frac{V(n)e^{rn}n}{(e^{rn}-1)^2}$$

$$= -\frac{nV(n)e^{-rn}}{(1-e^{-rn})^2}$$

$$\frac{\partial}{\partial r}\left[\frac{V(0)}{1-e^{-rn}}\right] = -\frac{V(0)(-e^{-rn})(-n)}{(1-e^{-rn})}$$

$$= -\frac{nV(0)e^{-rn}}{(1-e^{-rn})^2}$$

(The above derivatives are equal because they are parallel functions.)

$$\frac{\partial}{\partial r}\int_0^n \frac{V''(i)}{e^{ri} - e^{r(i-n)}}di$$

$$= -\int_0^n \frac{V''(i)[ie^{ri} + (n-i)e^{r(i-n)}]}{[e^{ri} - e^{r(i-n)}]^2}di$$

$$= \int_0^n \frac{V''(i)[ie^{-ri} + (n-i)e^{-r(i+n)}]}{(1-e^{-rn})^2}di$$

therefore

$$\frac{\partial D_c}{\partial r} = [V'(0) - V'(n)]\left[\frac{ne^{-rn}}{(1-e^{-rn})^2}\right]$$

$$+ \int_0^n \frac{V''(i)[ie^{-ri} + (n-i)e^{-r(i+n)}]}{(1-e^{-rn})^2}di$$

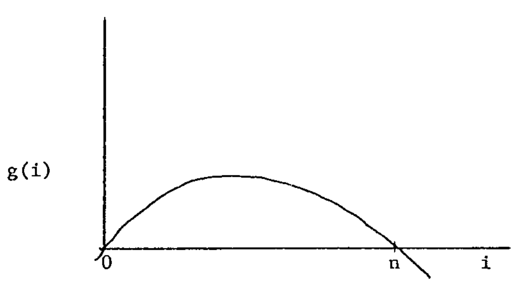
Since

$$V'(0) - V'(n) = \int_0^n V''(i)\,di$$

then

$$\frac{\partial D_c}{\partial r} = \int_0^n \frac{V''(i)[ie^{-ri} + (n-i)e^{-r(i+n)} - ne^{-rn}]}{(1-e^{-rn})^2}di$$

The sign of $\partial D_c / \partial r$ depends upon the sign of the bracketed term and $V''(i)$ since the denominator is obviously positive as long as $r > 0$. The bracketed term is positive as long as $n > 0$ which will now be proven by induction. The bracketed term can be multiplied by $e^{r(i+n)}$ [the result of which is referred to as g(i)] without affecting the sign of the bracketed term at any value of i. Note that $g(i) = 0$ when $i = 0$ or n. Furthermore $\partial g^2(i)/\partial i^2 = -r^2ne^{ri} < 0$. Therefore g(i) must have the following graphical form implying that the sign of $\partial D_c/\partial r$ is the same as $V''(i)$.



Under straight line depreciation $V''(i) = 0$ so $D_c = D_T$. However if $V''(i) \neq 0$ then using straight line depreciation as is used in the traditional method results in a definite bias. If $V''(i) > 0$ or $D'(i) < 0$ (depreciation decreasing as the machine ages as implied by the sum of years digits, or declining balance depreciation) implies that use of straight line depreciation underestimates annual depreciation.

Conversely if $V''(i) < 0$, implying that $D(i) > 0$, then straight line depreciation overestimates annual depreciation.

End of Proof

Appendix D

Separating After-Tax Capital Budgeting Depreciation and Opportunity Cost

Prove that

$$V(0) - \frac{V(n)}{e^{ri}} - T\int_0^n \frac{D(i)}{e^{ri}}di$$

$$= \int_0^n \frac{D(i) + OC(i)}{e^{ri}}(1-T)di$$

From Appendix A, it should be obvious that

$$V(0) - \frac{V(n)}{e^{rn}} = \int_0^n \left[-\frac{\partial V(i)}{\partial i} + V(i)r\right]e^{-ri}di$$

$$= \int_0^n \frac{D(i) + OC(i)(1-T)}{e^{ri}}di$$

$$V(0) - \frac{V(n)}{e^{rn}} - T\int_0^n \frac{D(i)}{e^{ri}}$$

$$= \int_0^n \frac{D(i) + OC(i)(1-T)}{e^{ri}}di - T\int_0^n \frac{D(i)}{e^{ri}}di$$

$$= \int_0^n \frac{D(i) + OC(i)}{e^{ri}}(1-T)\,di$$

End of Proof

Appendix E

Separation of Capital Budgeting Estimated Opportunity Cost Depreciation Under Inflation

Show that

$$V(0) - \frac{V(n)\,e^{fn}}{e^{rn}} = \int_0^n \frac{\tilde{D}(i) + \tilde{OC}(i)}{e^{\tilde{r}i}}\,di$$

However

$$V(0) - \frac{V(n)e^{fn}}{e^{rn}} = V(0) - \frac{V(n)}{e^{\tilde{r}n}}$$

From Appendix A, it should be obvious that

$$V(0) - \frac{V(n)}{e^{\tilde{r}n}} = \int_0^n \left[- \frac{\partial V(i)}{\partial i} + V(i)\tilde{r} \right] e^{-\tilde{r}i} di$$

$$= \int_0^n \frac{\tilde{D}(i) + \tilde{OC}(i)}{e^{\tilde{r}i}}\,di$$

End of Proof

Appendix F

Effects of Indexing Depreciation and Using Real Interest Rates For Computing Tax Deductions

Prove that

$$V(0) - \frac{V(n)e^{fn}}{e^{\tilde{r}n}} - T \int_0^n \frac{D(i)e^{fn}}{e^{\tilde{r}i}}$$

$$= \int_0^n \frac{\tilde{D}(i) + \tilde{OC}(i)}{e^{\tilde{r}i}} (1-T)\,di$$

Beginning of Proof

$$V(0) - \frac{V(n)e^{fn}}{e^{\tilde{r}n}} - T \int_0^n \frac{D(i)e^{fn}}{e^{\tilde{r}i}}\,di$$

$$= V(0) - \frac{V(n)}{e^{\tilde{r}n}} - T \int_0^n \frac{D(i)}{e^{\tilde{r}i}}\,di$$

From Appendix C, it is obvious that

$$V(0) - \frac{V(n)}{e^{\tilde{r}n}} - T \int_0^n \frac{D(i)}{e^{\tilde{r}i}}\,di$$

$$= \int_0^n \frac{D(i) + OC(i)}{e^{\tilde{r}i}} (1-T)\,di$$

End of Proof