Alternative Parameter Specification in E,V Analysis: Implications for Farm Level Decision Making

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This study compares the structure of E,V frontiers under several specifications of the expected income and variance parameters with emphasis on fundamental differences in efficient crop mixes. The results are generated using data from a specific production region and a selected set of cropping activities for Wyoming. The risk-efficient frontiers and underlying crop mixes display sensitivity to alternative parameter definitions and suggest that if researchers intend to use the E,V approach in providing decision information to producers, care should be exercised in the choice of income and risk measures.

The topic of risk continues to generate substantial research interest among agricultural economists. Within those studies of a more empirical nature, one intuitively appealing method or technique for measuring the relationship between risk and return at the farm level has been E,V (expected income-variance) analysis. From an information or decision making standpoint, risk efficient frontiers are suggested as a valuable management aid, as well as a heuristic device, in dealing with risk.

One concern to researchers is the potential sensitivity of such optimal frontiers to data definition and model assumptions used in the analysis. Frankfurter *et. al.* expressed concern over potential divergence in efficient or optimal portfolios due to estimation error in the sample means and variance measures used to estimate E,V frontiers. More recently, Schurle and Erven and Persuad and Mapp have indicated the potential for such divergence in risk efficient frontiers generated in a MOTAD framework, i.e. the E,A frontiers. Specifically, Schurle and Erven examined the divergence in crop plans between optimal plans (those on the frontier) and "near-optimal frontiers." Persuad and Mapp analyzed the effects of alternative measures of dispersion (risk) on the efficient farm plans resulting from E,A frontier estimation.

The issue of solution sensitivity raised by these authors appears to be a significant one and perhaps points out a fundamental limitation of the mean-variance approach to firm-level decision making. In addition to the sources of sensitivity identified by these authors, one would also expect the solution (frontier) and crop mix in an E,V framework to display sensitivity to the values of both the expected income (E) and variance (V) used in the analysis. Potential divergences in underlying risk efficient crop mixes across alternative specifications of the E,V parameters would have implications for prescriptive use of the technique. More specifically, if the resulting efficient crop mixes vary greatly across different specifications of the income and variance measures, then the use of such "efficient" mixes in farm-level decision making must be questioned.

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Providing further support of the need to be cognizant of potential solution sensitivity are suggestions by several authors [Scott and Baker, Lin *et. al.*, Dean, Hazell] that in the absence of appropriate user utility functions, the E,V or efficiency frontiers may simply be presented to producers, allowing the producer to select the underlying crop mix (or income-risk combination) which is subjectively deemed to be "optimal." If alternative parameter specifications alter the structure of the E,V frontier, the usefulness of the technique may depend upon the "correct" specification of parameters, unless *a priori* knowledge is incorporated into the estimation to be consistent with producers' expectations. Any consistency derived by incorporating producers' expectations may only reflect the recent past rather than what a risk-efficient combination may be with respect to future production decisions.

In previous estimation of risk efficient frontiers, income has been represented by (1) the mean income or gross margin over an extended time period (an historical measure of income, such as used by Scott and Baker), or (2) the mean of a relatively short time period designed to portray recent price expectations (a contemporary or adaptive expectations approach to income, as in Halter and Dean). Similarly, variance (the measure of risk) may be either (1) the variance of the total time period under analysis (again, see Scott and Baker), or (2) a variance derived from a detrended version of that same time period, frequently referred to as a "random" variance [Halter and Dean, Schurle and Erven].

The overall purpose of this paper is to examine some alternative definitions of the E,V parameters illustrated with data from a specific agricultural area featuring a relatively diverse set of cropping alternatives. Generation of E,V frontiers under these alternative definitions permits a comparison of crop mixes across these definitions. This study compares the structure of E,V frontiers under several specifications of the expected income and variance parameters with emphasis on fundamental differences in efficient crop mixes. Such a comparison may serve to reveal the sensitivity of farm plans to alternative modifications of data on returns and risk. This information complements the sensitivity analysis performed by Schurle and Erven and Persuad and Mapp, given that the current study employs different assumptions concerning the source of sensitivity and uses the E,V framework rather than the MOTAD-E,A approach to generate risk efficient frontiers.

Problem Setting and Approach

Whole farm risk planning or portfolio analysis is an extension of the early work by Markowitz on optimal stock portfolio combinations to an agricultural setting.[1] In using the approach, the goal has been to arrive at some efficient or risk-minimizing mix of cropping activities giving rise to a certain level of income, where risk is usually defined as the second moment of the probability distribution of farm net income. The resource and agronomic limitations of the overall farm or enterprise are typically included as constraints on the solution mix.

The study area for this analysis is the Big Horn Basin in northwestern Wyoming. It features a rather wide range of irrigated crops, including alfalfa, malt and feed barley, corn for grain and silage, dry edible beans, and sugar beets. In 1978, approximately 212,000 of the 225,000 irrigated acres in the Basin were planted in these crops.

The Big Horn Basin was selected as the study area based on several criteria. First, the physical environment makes it conducive to economic analysis. It is a closed basin with similar weather patterns and relatively homogeneous soil and water conditions. All producers face the same general set of input

[1]While the tractability of the Markowitz approach is dependent upon the accessibility of quadratic programming algorithms, the MOTAD system of Hazell has been shown to be an acceptable, although slightly less accurate, alternative in the absence of quadratic programming algorithmic capacity.

and output conditions and the same set of decision variables. Second, the Basin is one of the major agricultural areas of the state, producing a significant portion of the state's sugar beets, barley and dry beans, as well as livestock.

The data used to generate the E,V frontiers were derived from producer interviews and secondary data sources. The crops included in the analysis are alfalfa, corn for grain, corn for silage, dry edible beans, malting barley, feed barley and sugar beets. These crops are currently being grown by some or all of the producers surveyed. Incomes, costs, agronomic and cultural practices and farm resources associated with production of these crops were compiled to simulate a hypothetical 575-acre farm. The decision problem faced by the producer is assumed to be one of obtaining crop mixes with minimum levels of risk (variance) at given levels of income.

The crop mixes and resultant E,V frontiers are constrained by restrictions on land availability, rotational considerations and labor availability. These resource and agronomic restraints are derived from the producer interviews and are intended to approximate the economic and physical environment in which producers operate. Specifically, land availability is set at 575 acres. This farm size represents the average land base of the producers interviewed and is close to the average for the Basin. Rotational and contractual constraints are consistent with agronomic practices and allotment quotas found within the study area. These restraints are approximated by minimum and maximum acreage levels on selected crops, i.e., minimum acreages of alfalfa and maximum acreages of sugar beets and malting barley. Labor availability is equated to the average of family and hired labor man-months as reported in the interviews. Machinery capacity was also obtained from the interviews and indicated a degree of overcapitalization in machinery to insure timeliness of field operations. Certain specialized activities, such as beet thinning and malt barley combining, are assumed to be available through custom operators, the prevailing practice within the region. No constraints were placed on irrigation water, given that per acre contractual deliveries within the Basin exceed physical requirements for the included crops.

The expected income or gross margins for each crop are compiled from data series based on county averages for yields and prices obtained from the Wyoming Crop and Livestock Reporting Service. The county average yields approximate those reported by individual producers and, when combined with time series data on prices, provided a returns series of sufficient length to arrive at a long-term or historical income measure.

Initially, the expected income or gross margin and variance-covariance matrices were derived for a 20-year period (1957-76). These expected or mean incomes are identified as the historical mean in the analysis. The variance-covariance measures, derived from actual (non-detrended) data, are identified as *total* variance. The alternative expected income is the simple average for the four-year period 1973-76, designed to reflect producers' recent observations, as argued by Lin *et. al.*, and Brink and McCarl. The gross margin or income measure for each activity was adjusted for inflationary trends through indexing of costs via the Index of Prices Paid by Farmers.[2] Finally, following Chen, an alternative specification of variance or risk was achieved by detrending the data. Specifically, the data were decomposed using Tintner's variate difference method to arrive at the "random" component or random variance.[3] The use of detrended data to derive variance measures increases the num-

[2]Variable costs used to calculate gross margin or expected income for the income measure were derived by adjusting available Cooperative Extension Service cost of production budgets [Agee] by the Index of Prices Paid by Farmers [U.S. Department of Agriculture].

[3]A discussion of the technique in an agricultural setting may be found in Carter and Dean. In general, the use of the variate difference technique to "detrend" seems appropriate, in that in the absence of detailed informa-

ber of negative correlations between crop incomes [Carter and Dean]. Thus, the potential for diversification (and hence more "efficient" crop mixes) is enhanced within the model. As a result of this selection and modification of data, two measures of expected income and two of variance were derived, yielding four possible income-variance pairings.

The rationale for specification of these two time periods was a desire to encompass two distinct income periods as well as to approximate parameter definitions used in past application of this technique. The 20-year period includes both the relatively low but stable crop incomes of the 1960's and the more volatile incomes observed in the mid-1970's. Conversely, the income average of the last four years protrays a period of relatively high crop prices and may more closely portray the subjective probabilities assigned by producers. In an economic or decision making context, the historical income measure may be more appropriate for long-run analysis, such as are typically involved in machinery investments and alfalfa establishment. The contemporary income may be more suitable for short-run decisions such as selecting individual crop acreage within a given crop year. From a methodological standpoint, if the different measures of income alter the relative structure of crop incomes within a period, then one may expect differences in the efficient crop mixes obtained in each E,V solution.

The individual E,V frontiers for the seven individual crops were estimated using the parametric solution of a quadratic programming problem. The program was solved using the Rand QP program developed by Cutler and Pass. The quadratic programming problem solved using the above algorithm is of the general form reported elsewhere [Halter and Dean]. The parametric solution of the quadratic programming problem yields the maximum difference between expected income and variance for each level of expected income associated with different crop mixes. This is equivalent to finding the minimum variance for each level of expected income, i.e., the E,V frontier [Halter and Dean].

tion about production functions, the learning reactions of producers, and other factors, any statistical method not requiring *a priori* specification of rigid functions should be preferable to alternative methods. The variate difference method assumes that the systematic component of time series data can be characterized by any smooth type of function.

Using this solution procedure, frontiers were estimated using the pairings of both the historical (mean) gross margins (1957-1976) and the more contemporary (mean) gross margins (1973-1976) with the random measures of the variance-covariance matrix and total variance-covariance matrix. These pairings resulted in the following E,V frontier solutions: (1) random variance, 1957-1976 expected income; (2) random variance, 1973-1976 expected income; (3) total variance, 1957-1976 expected income; and (4) total variance, 1973-1976 expected income. The frontiers reported in this study represent efficient crop mixes for the seven cash crops in the presence of the restrictions outlined above.

Results and Implications

Given the wide range of assumptions used to derive the expected income and variance parameters, sensitivity in the shape and position of the estimated E,V frontiers would not be surprising. Figure 1 presents the set of possible crop frontiers which result from the four parameter assumptions outlined above. The effects of such specification on the position and shape of the underlying frontiers is indeed dramatic. As expected, the total variance frontiers are "inferior" to random variance (higher risk at same relative income), given that the random variance is by definition less than or equal to the total variance and is characterized by the existence of a larger number of negatively covariant cropping combinations. The same relationship is observed for the historical versus contemporary mean incomes. Thus, four distinct frontiers are derived, each of which is "efficient" within the definition of

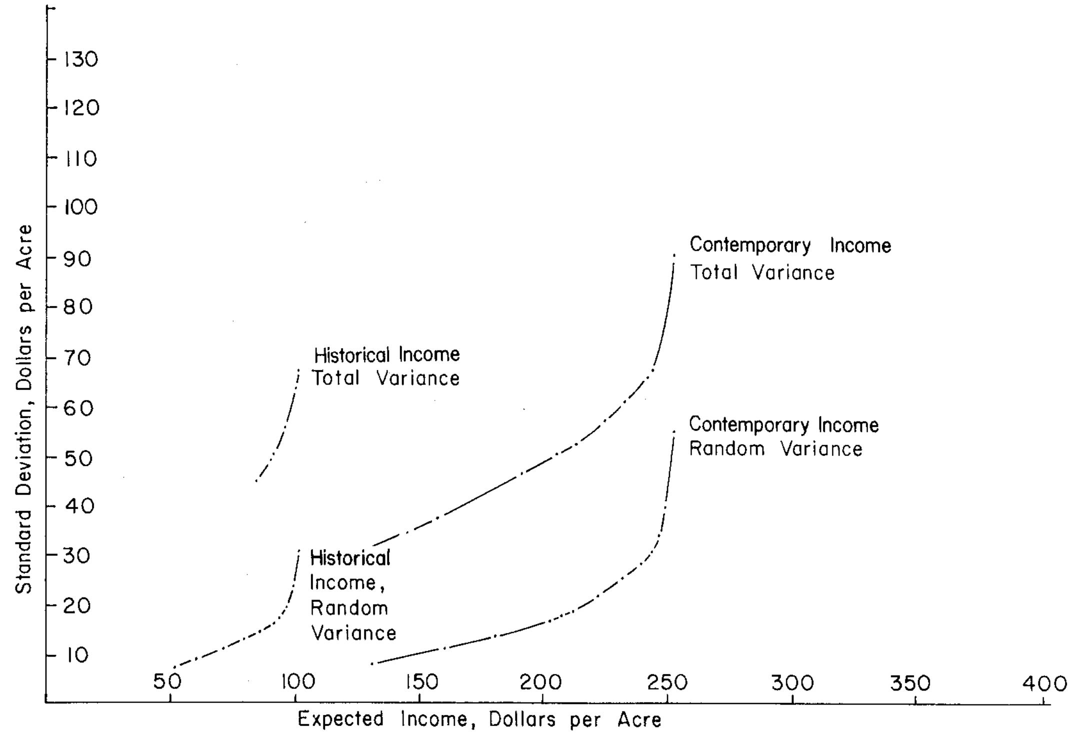

Figure 1. E,V Frontiers for Crop Alternatives, Under Alternative Income and Variance Measures

income and variance used in the estimation procedure.

From the standpoint of producer decision making or prescriptive use, decision makers are not necessarily interested in any variation in shapes and positions of the frontiers but are perhaps interested in the crop mixes along the frontiers. If large differences are observed in the efficient crop mixes across frontiers, then the usefulness of such information to producers will be dependent on the inherent accuracy of each curve and validity of the underlying assumptions giving rise to the respective frontiers. Thus, ideally one needs to know not only what differences, if any, exist across crop mixes, but also if one set of assumptions (and hence frontier) more accurately portrays producer behavior than alternative formulations.

Table 1 presents the incomes, standard deviations and underlying crop mixes at three points on the frontiers for the above four parameters specifications under the restrictions imposed on the model. Substantial differences in crop mixes are observed across the income and variance measures. Given that the model restrictions are the same for all analyses, any divergence must be due to the different values employed for income and variance. Comparing first within expected income and across variance, differences in crop mixes are observed at the lower income levels (initial and intermediate). Crop mixes are identical only at the maximum solution value. Differences are again observed at all levels in the crop mix across incomes and within variance measures. The differences noted here pertain not only to the set of crops in the optimal or efficient mix, but also to their respective proportions. Only in the case of pure profit maximization are the "optimal" crop mixes invariant across variance measures. Even this behavorial assumption yields different results across incomes.

TABLE 1. Summary of the Four E,V Frontiers Estimated for Big Horn Basin Crop Alternatives at Initial, Intermediate and Maximum Income Levels.[a]

Income Level	Historical (Mean) Expected Income 1957-76		Contemporary (Mean) Expected Income 1973-76	
	Random Variance	Total Variance	Random Variance	Total Variance
Initial Expected Income	$ 90.55	$ 90.09	$130.19	$128.57
Standard Deviation	16.38	49.58	8.39	31.05
Crop Mix:				
Alfalfa	20%	25%	31%	35%
Corn (grain and silage)	25%	35%	25%	35%
Dry beans	--	--	--	--
Malting barley	39%	39%	14%	b
Feed barley	15%	--	30%	16%
Sugar beets	b	--	--	b
Coefficient of Variation	.181	.545	.064	.242
Intermediate Expected Income	$ 95.71	$ 95.68	$204.95	$204.59
Standard Deviation	19.92	56.23	17.22	50.55
Crop Mix:				
Alfalfa	20%	20%	20%	20%
Corn (grain and silage)	--	21%	25%	36%
Dry beans	11%	19%	6%	5%
Malting barley	40%	40%	37%	33%
Feed barley	29%	--	10%	--
Sugar beets	--	--	b	7%
Coefficient of variation	.208	.588	.084	.247
Maximum Expected Income	$100.83	$100.83	$251.43	$251.43
Standard Deviation	30.57	67.33	54.07	90.28
Crop Mix:				
Alfalfa	20%	20%	20%	20%
Corn (grain and silage)	--	--	--	--
Dry beans	40%	40%	--	--
Malting barley	40%	40%	40%	40%
Feed barley	--	--	--	--
Sugar beets	--	--	40%	40%
Coefficient of Variation	.303	.667	.215	.359

[a]All values on a per acre basis.

[b]Crops which comprise less than 2 percent of total acreage.

The E,V approach to decision making is essentially a prescriptive tool in the absence of individual risk preference functions.[4] Thus, it is difficult to translate the results of Table 1 into any predictive test of the normative E,V frontiers obtained in the analysis. What one observes in this analysis is that the contemporary-detrended frontier suggests a higher proportion of cash crops, particularly malt barley, than other formulations, consistent with the general trend in cropping patterns realized in the study area. Such information on trends may be of use to producers in short-run decision making.

The determination of which E,V formulation is "best" is beyond the scope of this analysis. Further, the manner in which the results will be used should affect the modification of data on income and variance; e.g., long run investment decisions versus short-run or annual cropping decisions. The E,V approach is a useful management tool in that it displays the generally recognized benefits of diversification (i.e., lower coefficient of variation with a more diverse crop mix) and does provide a demonstration of the relationship or trade-off between return and risk for various cropping combinations. However, the informational content of these frontiers for farm level decision making appears to be less secure, given the observed sensitivity of the efficient crop mixes across alternative definitions of income and variance.[5] If the mean-variance approach is to be used for short-run decision making at the farm level, researchers may need to incorporate individual producers' expectations into the definition of income and variance, as suggested by Lin *et. al.*

Summary and Conclusions

This paper has addressed alternative specifications of return and risk parameters frequently employed in deriving efficient farm plans using E,V analysis. The results generated in this study are based on a specific production region and a selected set of cropping activities and suggest that the resulting efficient crop mixes display sensitivity to the parameter specification. These results add support to the sensitivity concerns raised by other researchers, although the source of the sensitivity differs between this study and those cited earlier. The implication, however, remains the same: if researchers intend to use the E,V approach in providing decision making information to producers, care should be exercised in the choice of income and risk measures used.

In general, the results obtained from the shorter time series on income expectations appear to more closely portray the trends in cropping patterns observed in recent years. In addition, all income and risk parameter formulations display the general benefits of diversification as portrayed by the shape of the resultant frontier. These results may be useful in any prescriptive use of this technique. Balanced against the positive aspects of this approach are the divergences in optimal farm plans observed for this production region, which make specific farm level recommendations tenuous.

The set of observations on income and variance is generated from one specific production region and hence may not be extendable to other regions. However, the results and attendant caveats are generally consistent with those recorded by Schurle and Erven, Persuad and Mapp and Frankfurter *et. al.* Further, results presented here constitute a test of different sources of sensitivity and are cast in an E,V rather than E,A framework. While not providing a definitive

[4]As noted by Hazell, the profession has not had great success in obtaining estimates of producer utility. Bernoullian utility functions were estimated for several producers in this study using a modified von Neumann-Morgenstern technique. While the results were less than encouraging, some degree of risk aversion was recorded across all producers interviewed.

[5]Sensitivity of solution values is not unique to E,V analysis. Other research methodologies, such as conventional linear programming, simulation or budgeting would also display changes in optimal or "most efficient" decision strategies if income or other parameters are substantially altered.

study of E,V frontier solution sensitivity, the results do provide an additional examination of the expected income-risk approach to farm level decision making and limitations attendant to its use. Further research on data definition and assumptions would appear worthwhile.

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