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Measuring bank capital requirements through Dynamic Factor analysis

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Abstract

In this paper, using industry sector stock returns as proxies of firm asset values, we obtain bank capital requirements (through the cycle). This is achieved by Montecarlo simulation of a bank loan portfolio loss density. We depart from the Basel 2 analytical formula developed by Gordy (2003) for the computation of the economic capital by, first, allowing dynamic heterogeneity in the factor loadings, and, also, by accounting for stochastic dependent recoveries. Dynamic heterogeneity in the factor loadings is introduced by using dynamic forecast of a Dynamic Factor model fitted to a large dataset of macroeconomic credit drivers. The empirical findings show that there is a decrease in the degree of Portfolio Credit Risk, once we move from the Basel 2 analytic formula to the Dynamic Factor model specification.

Keywords*:* Dynamic Factor Model, Forecasting, Stochastic Simulation, Risk Management, Banking

JEL codes: **C32, C53, E17, G21, G33**

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1. Introduction

The Internal Rating Based method, IRB, underlying Pillar 1 of the Basel 2 accord, assigns greater sensitivity of capital requirements to the credit risk inherent in bank loan portfolios. In light of the Basel 2 directives to reform the regulation of bank capital, there has been an extensive research on study of the bank loan portfolio loss density. Particular emphasis is on the measurement of the Value at Risk (VaR). A crucial input of a portfolio credit risk model, PCR, is the appropriate characterisation of default correlations. The first study which provided the theoretical underpinnings of the Basel 2 IRB formula for the determination of the economic capital (through the cycle) is due to Vasicek (2002) (see also Schonbucker, 2000). In this study default correlation is modelled through dependence of firm asset values upon a white noise Gaussian common shock. The other important assumption underlying the study of Vasicek (2002) is the existence of an infinite granular homogeneous portfolio, e.g. a portfolio with homogenous unconditional probability of default, PD, and factor loading and with obligors sharing the same exposure *1/N* (where *N*, a large number, is the number of obligors). The final Basel 2 IRB formula is due to Gordy (2003) which allows for heterogeneity in both the (unconditional) probability of default, *PD*, and in the common factor loadings. Recently the study of Hanson et al. (2007) has found (analytically) that, once the expected loss is controlled for, heterogeneity in the PD is the most important source of credit risk diversification benefits, that is, it contributes the most to obtain measures of economic capital smaller than those obtained from the fully homogeneous IRB model of Vasicek (2002). The studies of Pytkin (2004) and of Cespedes et al. (2006) explore (in terms of closed form solution) the aforementioned benefits of credit risk diversification when multiple common factors underlie default correlation. Furthermore, the study of Hanson et al. (2007) shows, through simulation, that a two factor CAPM model implies a reduction in the economic capital when the benchmark is the Vasicek (2002) model. The aforementioned studies explore credit risk diversification benefits in terms of “static” parameter heterogeneity, given that they rely on static factor models, that is models which produce the same multi-step ahead loss density predictions, regardless of the forecast horizon. In this study we investigate the role played by "dynamic" heterogeneity (only in the factor loadings) to achieve credit diversification benefits, having as a benchmark, the IRB formula of Gordy (2003). For this purpose we use a Dynamic Factor model (see Stock and Watson, 2002, and also Forni et al., 2005) fitted to a large dataset macro variables used as a proxy of the state of the business cycle. To our knowledge, the Dynamic Factor model, as a prediction tool, has been used, so far, for the purpose of point forecast. Our focus is instead on density forecast. More specifically, we are interested in one year ahead density forecasts, using monthly data. For this purpose we employ the dynamic forecasting method (e.g. we roll forward one step ahead predictions) of the Dynamic Factor model to produce multi step ahead projections, and this gives a sufficient degree of heterogeneity in the impulse response of the observables to a single common systemic shock (modelled as a Gaussian random variable). The one year forecast horizon and the dynamic prediction method employed then imply the need of generating twelve Gaussian innovations, one per each interim multiplier characterising the impulse response profile. Therefore, we cannot use the single Gaussian common factor analytic formulas for capital requirements developed by Vasicek (2002) or by Gordy (2003). Specifically, the unconditional Portfolio Density forecast is obtained Montecarlo simulation, and the measurement of the economic capital is obtained by retrieving Value at Risk quantiles of the unconditional Portfolio Loss density.

An additional reason motivating the use of Montecarlo simulation is due to taking into account the role of uncertain recoveries for the determination of Portfolio Credit Risk. The empirical studies of Hu and Perraudin (2002), Altman et al. (2005) show the existence of a negative correlation between probability of default and recovery rate. This finding can, for instance, be explained by observing that both default and recovery are dependent on the state of the macro-economy (see Frye 2000). In particular, given a negative cyclical downturn, collateral values as well as asset firm values would fall, and, as a consequence, there would be an increase in the number of defaults and a decrease in the number of recoveries (given their dependence on the collateral). Acharya et al. (2007) suggest the importance of industry factors in explaining recovery rates. Bruche and Aguado (2007) account for the dependence of default intensities and recovery rates on the business cycle (as well as other controls, such as the seniority of bondholders). Using a time varying beta distribution (conditional upon the business cycle, seniority and industry class), Bruche and Aguado (2007) show that the existence of stochastic dependent recoveries plays a minor role (compared to stochastic defaults) in explaining 99% Credit Portfolio VaR. In our study, based upon stochastic simulation, recoveries and defaults are modelled to be dependent (and inversely related) on specific common systemic shock. In particular, we follow the approach of Altman et al. (2002) and we impose (a conservative) perfect rank correlation between default and loss given default for each of the one million scenarios considered in the Montecarlo simulation.

The empirical findings show that there is a substantial reduction in the risk associated to the bank loan portfolio once we move from the one (static) factor Portfolio Credit Risk model of Basel 2 to the Dynamic Factor model. These findings hold across the different empirical model specification considered and for both the case of constant and stochastic dependent recovery.

The outline of the paper is as follows. In section 2 and 3 we describe the basic definitions underlying the credit portfolio loss distribution, and the IRB method for the capital requirements advocated by Basel 2, respectively. In section 4 we describe the Dynamic Factor modelling approach and the stochastic simulation exercise; in section 5 we describe the data used together with the empirical results, and, finally, in section 6, we conclude.

2. Credit Portfolio Loss Distribution

The credit portfolio loss L is given by:

$$L = \sum_{j=1}^{N} (D_j * L_j) \qquad (1)$$

where N is the number of counterparts, D_j is a default indicator for obligor j (e.g. it takes value *1* if firm j defaults, *0* otherwise). Furthermore, the loss from counterpart j is given by:

$$L_j = \sum_{h=1}^{H} EAD_{hj} * LGD_{hj} \quad (2)$$

where EAD_{hj} is the exposure at default to the h business unit of obligor j. Finally, LGD_{hj} is the corresponding loss given default (equal to one minus the recovery rate, see below).

Since L is a random variable, it is crucial to retrieve its probability distribution to measure portfolio credit risk. For this purpose, from (1) and (2) we can observe that we need to consider as a random variable, at least one from D_{Jj}, EAD_{hj}, and LGD_{hj}. In this paper, we concentrate on the stochastic nature of D_{j} and LGD_h, treating the exposures as deterministic.

Beyond the expected loss, *EL*, two are the quantiles of the Portfolio Loss density which are of particular interest. The first, associated with the measurement of the economic capital is the unexpected loss, *UL*, measured as the difference between the 99.9% Value aat Risk, VaR, and the expected loss. If the forecast horizon is a year, then the unexpected loss predicts the minimum loss (above the expected one) that can occur once every thousand years. Finally, if such an extreme (rare) event occurs, the loss is predicted by the expected shortfall, *ES*, computed as the mean of the distribution values beyond the 99.9% VaR.

3. The IRB formula for Portfolio Credit Risk analysis

It is customary, in Portfolio Credit Portfolio Risk analysis, to capture default correlation using a common factor model specification for asset returns. In particular, the firm j's asset value, A_j, is given by:

$$A_j = \sqrt{\rho_j} U + \sqrt{1-\rho_j} \nu_j \qquad (3)$$

where U is a systematic risk shock affecting simultaneously every firm (parodying the state of the macro-economy) and v_j is an idiosyncratic (firm specific) risk shock. The parameter $\sqrt{\rho_j}$ measures the loading of the common shock on the firm j asset value. According to Merton (1974), a firm defaults when its asset value index falls below a threshold c_j. Specifically, define A_j as the level of firm j's asset value index, proxied, in line with the studies of Pesaran et al. (2006) and of Hanson et al. (2007) by stock return. Let D_j symbolise the default event of firm j, then we can observe that:

if $A_j < c_j$, then $D_j = 1$; $D_j = 0$ otherwise. (4)

For given values of the (unconditional) probabilities of default PD_j, we can obtain the default boundaries c_j from the unconditional cumulative distribution of the asset return, that is:

$$PD_j = P(A_j < c_j) = \Phi(c_j) \qquad (5)$$

where Φ is the cumulative probability distribution. From eq. (5) it is possible to retrieve the default threshold c_j, which is given by $\Phi^{-1}(PD_j)$. In this paper, under the assumption of a Gaussian white noise common shock with persistence in the propagation mechanism, Φ is obtained through stochastic simulation. Therefore c_j is the simulated quantile corresponding to a given PD_j.

Assuming an infinitely granular homogeneous portfolio driven only by one common white noise Gaussian shock (without persistence in the propagation mechanism), and assuming constant recovery, Vasicek (2002) provides an analytic formula for the unexpected portfolio loss useful for the determination of the economic capital. The closed form solution formula used by Basel 2 is due to Gordy (2003) and it allows heterogeneity in both the unconditional probability of default and common factor loadings. Specifically, the unexpected loss, UL_j (as a fraction of total exposure) for each obligor is given by (ignoring a maturity adjustment):

$$UL_j = EAD_j \left\{ LGD * \Phi \left[\frac{(\Phi^{-1}(PD_j) + \sqrt{\rho_j} * \Phi^{-1}(0.999))}{\sqrt{1-\rho_j}} \right] - PD_j \right\} \tag{6}$$

where EAD_j is the exposure at default of obligor j (expressed as a percentage of the total exposure); LGD is equal to one minus the constant recovery rate (set by the Basel 2 accord to 0.55); Φ is the standard cumulative Normal distribution; $\Phi^{-1}(.)$ is the inverse of the cumulative Normal distribution and 0.999 is the confidence level. Finally, ρ_j, the asset correlation function, is given by[1]:

$$\rho_j = 0.12 * \left(\frac{1-e^{-50PD_j}}{1-e^{-50}} \right) + 0.24 * \left(1 - \frac{1-e^{-50PD_j}}{1-e^{-50}} \right) \tag{6'}$$

As shown by Gordy (2003), the total economic capital is simply obtained by adding the individual capital charges given by (6). In this paper, we are interested in comparing the Basel 2 formula for determination of the total unexpected loss (hence, the total economic capital) based upon (6) and (6') with the one obtained from different Dynamic Factor model specifications. For this purpose, we need to resort to stochastic simulation since we cannot rely on the conditional independence assumption useful to obtain the analytic closed form solution for the unexpected portfolio loss. Although there is only one common systemic shock underlying the dynamics of several macro-

[1] Given that we consider only a corporate portfolio, the asset correlation function we consider is the one corresponding to corporate borrowers only.

credit drivers of the Portfolio Loss density, the use of a Dynamic Forecasting method leads to multi step ahead conditional projections for the stock returns (see below) dependent upon twelve common innovations each for a different horizon. The use of stochastic simulation is also motivated by the study of the Portfolio Loss density in presence of stochastic dependent recoveries.

4. Dynamic Factor model of Portfolio Credit Risk

Contrary to the Basel 2 formula for the determination of economic capital which is based upon a single white noise Gaussian common shock with no persistence in the propagation mechanism, there are Portfolio Credit Risk studies which introduce autoregressive dynamics. The study of Wilson (1997) allows for an AR(2) in the macro-credit drivers and the study of Pesaran et al. (2006) is based upon a cointegrated Vector Autoregression, *VAR*, model. Once the autoregressive dynamics is introduced, multi step ahead Portfolio Loss density forecasts can be obtained by means of Dynamic Forecasting. This implies that, in case of *VAR* models, the number of common systemic shocks is given by the number of endogenous variables (considered in the empirical model) times the forecast horizon. Therefore, a large dimensional set of common shocks influencing the systemic component of the Portfolio Credit risk model requires the use of a large number of replications to simulate the Portfolio Loss distribution. In this paper, we also use Dynamic Forecasting to produce conditional multi step ahead projection, but we only consider one common systemic shock as the primitive innovation (with persistence in the propagation mechanism) hitting a large number of macroeconomic aggregates. This is achieved by means of Dynamic Factor modelling (see Stock and Watson, 2002, and Forni et al, 2005). Therefore, the computational intensity of a stochastic simulation exercise can be considerably reduced. It is also important to observe that, given that we rely on an unobservable common shock, it is not meaningful to carry stress testing (see, for instance, Pesaran et al., 2006, for a study on the conditional Portfolio Loss density), but we can only focus on modelling the unconditional Portfolio Loss density. Therefore, in this study, we are only interested in measuring capital requirements through the cycle and in comparing the values generated by Dynamic Factor modelling with those obtained using the Basel 2 formula.

Dynamic Factor model

In order to model the persistence in the propagation mechanism of a single white noise Gaussian common shock, underlying the dynamics of a large dataset of macro-variables (used as a proxy of the state of the macro-economy), we follow Stock and Watson (2002), and also Forni et al. (2005), by considering the following specification for x_{nt}, which is the n dimensional dataset of credit drivers (e.g. the macro-variables):

$$x_{nt} = Cf_t + \xi_t \tag{7}$$

the first addend of the r.h.s. of (7) is the common component for each credit driver given by the product of the r dimensional vector of static factors f_t and C, which is the $n \times r$ coefficient matrix of factor loadings. The factor dynamics is modelled as follows (see Forni et al, 2005):

$$f_t = Df_{t-1} + Ru_t \tag{8}$$

where R measures the impact multiplier effect of q common shocks u_t (e.g. dynamic factors) on f_t. In order to estimate the system given by equation (7) and (8), we follow Stock and Watson (2002) who suggest to estimate consistently the space spanned by the factors f_t by retrieving the principal components of the dataset x_t:

$$\hat{f}_t = \frac{1}{\sqrt{n}} W_n^{'} x_{nt} \tag{9}$$

where W_n is the $n \times r$ matrix having on the columns the eigenvectors corresponding to the first r largest eigenvalues of the covariance matrix of x_{nt} As shown by Forni et al. (2005), given that the static factor vector contains current and past values of the common shocks u, the system given by (8) has (at least asymptotically) some equations which are identities. This implies that the covariance matrix of reduced form shocks in (8), e.g. RR', is singular. Therefore, in the second stage of the analysis, as suggested by Forni et al. (2005), we fit an OLS regression to the reduced form VAR(1):

$$\hat{f}_t = \Gamma \hat{f}_{t-1} + \varepsilon_t \quad (10)$$

The structural form impact multiplier matrix R in (8) is given by KMH, where:

1) M is the diagonal matrix having on the diagonal the square roots of the q largest eigenvalues of covariance matrix of the residuals ε_{tt}.
2) K is the $r \times q$ matrix whose columns are the eigenvectors corresponding to the q largest eigenvalues of covariance matrix of the residuals ε_{tt}.
3) H is a $q \times q$ rotation matrix

We set q equal to one (therefore, the matrix H is normalised to unity); in other words we consider only one common systemic shock u hitting the whole dataset of macro-credit drivers $x_{n.}$

Multi- step conditional projection of stock returns

Given that the credit drivers used in this paper are observed at monthly frequency and the forecast horizon of a bank is one year, we need to obtain twelve steps ahead projections. Since $\varepsilon_{tt} = KMHu_t$

we can derive the h-step ahead projection of the static factors by rolling forward the VAR(1) in (10):

$$\hat{f}_{t+h} = \left[\Gamma^h \hat{f}_t + \Gamma^{h-1} KMHu_{t+1} + \ldots + KMHu_{t+h} \right] \tag{11}$$

In order to obtain the conditional projection of stock returns, we retrieve the $r \times 1$ vector of sensitivities coefficients β_j, by an OLS regression of the stock return of obligor j on the r estimated static factors. Therefore, the prediction of the systemic component of the stock returns (proxy of firms asset values) is given by:

$$A_{j,t+h} = \beta_j \hat{f}_{t+h} \tag{12}$$

We can observe from (11) and (12) that, in line with multifactor models for asset returns, the systemic component can be split in two parts. The first, described the first addend in the r.h.s of eq. (11), is the predictable component, which is using information on the macro dataset up to and including time t. The remaining addends in (11) capture the unanticipated systemic component, given that they are a function only of future common innovations. Finally, the (partial) unpredictability of A_j is further enhanced by allowing an idiosyncratic (firm specific) disturbance to affect the asset returns. Consequently, plugging (11) in (12), the h step ahead projection of the firm j asset return is given by:

$$A_{j,t+h} = \beta_j \left[\Gamma^h \hat{f}_t + \Gamma^{h-1} KMHu_{t+1} + \ldots + KMHu_{t+h} \right] + \nu_j \tag{13}$$

where ν_{tj} is the idiosyncratic (firm specific) innovation.

For the purpose of Portfolio Credit Risk measurement (e.g. the derivation of the unexpected loss and expected shortfall) what matters is only the unanticipated systemic component of asset returns. Therefore, we need to de-mean the simulated density forecast in (11) by subtracting the point forecast $\Gamma^h \hat{f}_t$. The final specification for the simulated density forecast of the asset return is given by:

$$A_{j,t+h} = \beta_j \left(\Gamma^{h-1} Ru_{t+1} + \ldots + Ru_{t+h} \right) + \nu_j \tag{14}$$

In equation (14) A_j is the un-standardised simulated value of asset returns. Consequently, the default threshold, c_j that we obtain through the stochastic simulation experiment described below is un-standardised as well.

Finally, in the Montecarlo simulation experiment (see below), we consider both the common systemic shock and the idiosyncratic innovation as standardised Gaussian.

Montecarlo Simulation

In addition to the estimated coefficient matrices, β, Γ and R in (14), we use as inputs, for the purpose of generating artificially the scenarios, the exposures at default, *EAD* and the unconditional *PD* (obtained from the internal rating of a specific bank). The Montecarlo simulation experiment can be described as follows. First, we consider 1000 random draws from N(0,1) univariate distribution for each of the twelve common systemic shocks entering in the systemic component of (14)[2]. Therefore a joint set of realisations for these twelve innovations defines a particular macroeconomic scenario. Conditional on each draw for these common shocks, we carry 1000 draws from a *N(0,1)* distribution for each of the 6628 obligors entering in the loan portfolio, describing the realisation of the firm specific scenarios. In total we obtain one million observations and by sorting

[2] We use the Normal pseudo random number generator available from Gauss 6.0.

them in ascending order, we obtain the empirical distribution of the obligor j asset return. The given unconditional PD for obligor j is then used to retrieve the corresponding (simulated) quantile, by picking the value of the simulated density that leaves to its left the aforementioned unconditional PD. This simulated quantile is the unconditional threshold for obligor j, e.g. c_j in equation (5). The comparison between the projection of A_j for a particular scenario (defined by a joint draw for the common and firm specific innovations) with the artificially generated unconditional threshold allows to predict whether default occurs in each scenario. Finally, assuming a constant recovery rate equal to 55%, we are then able to obtain the prediction of the total portfolio loss for that specific scenario. By repeating this exercise for the whole set of one million scenarios, we obtain the unconditional Portfolio Loss density.

We now consider the case of stochastic dependent recoveries. In line with the study of Altman et al. (2002), we model stochastic dependent recoveries, by imposing a perfect rank correlation between the LGD and the default rate associated with the common shock scenarios. In particular, we sort (in descending order) the number of defaults for each common shock scenario, from the worst case scenario (e.g. the one with the highest number of defaults) to the one with the smallest number of defaults. The stochastic recovery rate is modelled through the beta distribution (seee Gupton et al., 2000, among the others). This distribution, usually employed by rating agencies to model recoveries, depends only on two parameters a and b and it has support [0, 1]. More specifically, the shape of the beta distribution depends on the parameters a and b, linked to the sample mean and to the standard deviation of the recovery rate, μ and σ, respectively, as follows: $b = \{[\mu^* (\mu-1)^2]/\sigma^2+\mu-1\}$; $a=(b^*\mu)/(\mu-1)$. We use the sample mean and variance of the recovery rate for senior unsecured loans (obtained from the study of Altman et al., 2005). The values of these parameters are set to 55% and 28.4%, respectively. In order to retrieve stochastic recovery rates, we assign the lowest probability of recovery to the scenario with the largest number of default and, then, we invert the cumulative beta distribution, and we carry with this type of sorting till we consider the scenario with the smallest number default[3]. Therefore, the recovery rates are sorted in ascending order and they are associated with the corresponding common scenarios sorted in terms of number of defaults. We argue that the perfect rank correlation between PD's and an aggregate recovery rate as specified in the simulation experiment allows is to investigate the most conservative scenario framework, setting an upper bound to the various measures of Portfolio Credit Risk.

Finally, the one million replications for the stochastic simulation experiment imply that the 0.1% probability tail we focus on (in line with Basel 2 suggestions) is made of one thousand observations.

5. Empirical analysis

Data

We consider a corporate portfolio, describing the exposures of an Italian bank towards corporate. The obligors with marginal exposure have been grouped in homogenous clusters in terms of rating and economic sector. This allows us to consider a portfolio with 6628 obligors (with cluster and non-clusters), with the corresponding *EAD* (exposure at default), and unconditional PD's (obtained from the internal rating system of the bank), treated as input in the Portfolio Credit Risk VaR

The sample of observations (monthly frequency) runs from the first month industry sector MIB stock price indices are available, e.g. January 1996, till December 2005. Proxies of the firm asset values are stock returns in line with the studies of Pesaran et al. (2006) and of Hanson et al. (2007) twenty one MIB sector specific and aggregate stock price indices (transformed into log returns) described in Appendix 2. It is important to observe that given the presence of SME in the corporate loan portfolio considered, the heterogeneity in the systematic component of equation (14) occurs only across industry sectors. The dataset for the Italian economy macro-variables is described in Appendix 1. This dataset includes a total of 68 macro time series for prices, output and interest rates. More specifically we consider short term and long term interest rates, consumer prices and producer prices (both aggregate and industry sector specific); real seasonally adjusted indices for

[3] The inverse of the cumulative beta distribution is obtained from the PDF library In Gauss developed by D. Baird.

aggregate and sector specific industrial production; real seasonally adjusted aggregate and sector specific sales and orders. Finally, for the purpose of principal component analysis, each series in the macro-economic variables dataset has been standardised to have zero mean and unit variance.

Portfolio Credit Risk measurement

Standard AIC and BIC criteria to select the number of static factors cannot be employed since they rely on the minimisation of a penalty function only of the time series dimension. Therefore, we employ the method suggested by Bai-Ng (2002), which involves the minimisation of a penalty function depending on both the cross section and time series dimension. Setting to eight the maximum number of factors, the log version of the Bai-Ng statistics suggest the use of four and eight factors. Given the inconclusive evidence of the optimal number of principal components, we have carried the stochastic simulation using various DF model specifications with four, five, six, seven and eight principal components. According to the mean adjusted R^2 obtained by averaging this goodness of fit measure for a set of OLS regressions of each macro time series, the average systematic variability explained by four, five, six, seven and eight principal components is 45%, 49%, 53%, 57%, and 59%, respectively.

Employing the scenario generation described in section 4.3, we obtain the simulated loss distributions. As we can observe (see Exhibits 3-12) the shape of the unconditional loss distribution is asymmetric and highly skewed (with the degree of asymmetry increasing in presence of stochastic dependent recoveries). From the Figures below and Exhibits 1 and 2 we can draw the following conclusions. First, given that we control for the expected loss, the latter is allowed to vary only when we switch from the constant to the stochastic dependent recovery assumption. More specifically, the expected loss for stochastic dependent recovery is 1.543%, nearly twice as much the one associated with constant recovery, which is equal to 0.871%. Second, within a given model specification for the LGD, the shape of portfolio loss density is dependent on the Dynamic Factor model specification. The Portfolio Credit Risk measures are not too sensitive to the different

Dynamic Factor model specifications. In particular, the values of unexpected loss vary between 1.121% and 1.126% for the case of constant recovery and 2.071% and 2.079% for the case of stochastic dependent recovery. Similar findings apply to the values of the expected shortfall. Third, by comparing the last five columns of Exhibits 1 with the first one, we can observe that the Basel 2 measure of the unexpected loss (obtained from the analytic solution described in equation (6), is bigger than the one obtained by stochastic simulation of multifactor models. This finding suggests that, on one hand, the (average) covariance of asset return, given by $\beta_j\left(\Gamma^{11}RR'\Gamma^{11}{}'+\Gamma^{10}RR'\Gamma^{10}{}'+...+RR'\right)\beta_j'$, increases the further ahead is the forecast horizon. This factor tends to increase Portfolio Credit Risk. On the other hand, there is also a considerable degree of (dynamic) heterogeneity in the impulse response coefficients, and this offsets the impact of an increasing innovation uncertainty for a twelve months ahead horizon. Given that both the benchmark model we consider (e.g. Basel 2 analytic formula) and the one based upon Dynamic Factor simulation share the same type of heterogeneity in unconditional PD, our study differs from Hanson et al. (2007) who investigate both the impact of heterogeneity in the unconditional PD and in the factor loadings on Portfolio Credit Risk. We only explore the second type of parameter heterogeneity. While Hanson et al (2007) consider only the case of static heterogeneity, given the static factor models they analyse, we argue that the benefits of credit risk diversification (e.g. a reduction in the unexpected loss relative to Basel 2 analytic formula with heterogeneity in the factor loadings, modelled through 6') are due to dynamic heterogeneity in the factor loadings as described by the conditional projection in equation (14). Furthermore, even though we have imposed in the simulation experiment a perfect rank correlation between PD's and loss given default, we still obtain values of the unexpected loss below the one obtained by using the analytic formula given by equations (6) and (6'). Finally, although the assumption of perfect rank correlation between PD's and loss given default, provides an upper bound to Portfolio Credit Risk, the simulation findings suggest that ignoring stochastic dependent recoveries implies a considerable under-provision of minimum capital requirements.

6. Conclusions

The aim of this paper is to measure bank capital requirements through the cycle. More specifically, we compare the unexpected loss (hence the economic capital) associated with Basel 2 formula due to Gordy (2003) with the one obtained through stochastic simulation of a Dynamic Factor, DF model (see Stock and Watson, 2002, and Forni et al., 2005) fitted to a large dataset of macro-credit drivers. Both models depend on a single Gaussian common shock and they exhibit the same degree of heterogeneity in the unconditional probability of default, PD, but differ in terms of heterogeneity in the factor loadings. In particular, the Basel 2 formula models heterogeneity in the factor loadings in a "static" way allowing the loadings to be inversely related to the PD. The heterogeneity in the loadings obtained from the DF model is of a "dynamic" type, given that it is obtained through Dynamic Forecasting of the Dynamic Factor model. Although there is a considerable degree of innovation uncertainty for a twelve month ahead horizon, the heterogeneity in the impulse response coefficients implies Portfolio Credit Risk measures (in particular, the unexpected loss) below those suggested by the analytic formula used by Basel 2. Furthermore, we also account for stochastic dependent recoveries, by imposing (a conservative) perfect rank correlation between default and loss given default for each of the one million scenarios considered in the simulation. The empirical findings show that, when using the Dynamic Factor model, ignoring stochastic dependent recoveries implies a considerable under-provision of minimum capital requirements.

References

Acharya, V. V., S. T. Bharath, and A. Srinivasan (2007): " Does Industry-wide distress affect defaulted Firms? Evidence from Creditor Recoveries." *Journal of Financial Economics*. Forthcoming.

Altman, E., B. Brady, A. Resti, and A. Sironi (2005): "The Link between Default and Recovery Rates: Theory, Empirical Evidence, and Implications". *Journal of Business,* 78(6):2203–2227.

Bai and Ng (2002): "Determining the number of factors in approximate Factor models", *Econometrica*, 70(1), 191-221.

Bruche, M. and Aguado, C. (2007) "Recovery rates, default probabilities and the credit cycle ", CEMFI working paper.

Cespedes, J.C.G, Herrero, J., Kreinin, A. and D. Rosen (2006) "A Simple Multi-Factor "Factor Adjustment" for the Treatment of Credit Capital Diversification", *Journal of Credit Risk*, forthcoming

Forni, M., Giannone D., Lippi, M., Reichlin L. (2005): "Opening the Black Box: structural factor model versus structural VAR models", *Universite Libre de Bruxelles*, mimeo

Frye, J. (2000): "Depressing recoveries", *Risk* (November)

Gordy, M. (2003): "A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules",' *Journal of Financial Intermediation,* vol. 12 (July 2003), pp. 199-232.

G. Gupton, D. Gates, and L. Carty (2000): " Bank loan losses given default". *Moody's Global Credit Research, Special Comment*.

Hanson, S.G., Pesaran H., and Schuermann T. (2007): "Firm heterogeneity and credit risk diversification", working paper

Hu, Y. T. And Perraudin, W. (2002): "The dependence of recovery rates and defaults", Birbeck College working paper.

Merton, R. C. (1974): "On the pricing of corporate debt: the risk structure of interest rates", *Journal of Finance*, 29, 449-470.

Pesaran, H., Schuermann, T., Treutler, B.J. and Weiner, S.W. (2006): "Macroeconomics Dynamics and Credit risk: a global perspective", *Journal of Money, Credit and Banking,* 38(5), 1211-1262.
Pykhtin, M. (2004) "Multi-factor adjustment". *Risk*, 17, 85–90.

Schonbucher, P. (2000): "Factor models for portfolio credit risk" Bonn University working paper

Stock and Watson: "Macroeconomic Forecasting using diffusion indices", *Journal of Business and Economic Statistics*, 20(2), 147-162

Vasicek O., (2002), "*Loan Portfolio Value"*, Risk, December, 160-162.

Wilson, T. (1997a): "Portfolio Credit Risk, Part I", *Risk* 10 (9), 111-117

Exhibits 1: Unconditional Portfolio Loss with constant recovery

	Analytic solution	Simulation: 4 factors	Simulation: 5 factors	Simulation: 6 factors	Simulation: 7 factors	Simulation: 8 factors
EL	0.871%	0.871%	0.871%	0.871%	0.871%	0.871%
UL	5.644%	1.125%	1.126%	1.124%	1.126%	1.121%
ES	-	2.145%	2.146%	2.153%	2.144%	2.148%

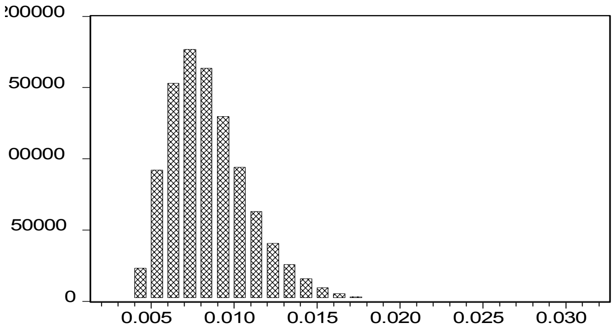
Note: numbers are in percentages of total exposure. EL is the Expected Loss; UL is the unexpected Loss; ES is the expected shortfall

Exhibits 2: Unconditional Portfolio Loss with stochastic dependent recovery

	Simulation: 4 factors	Simulation: 5 factors	Simulation: 6 factors	Simulation: 7 factors	Simulation: 8 factors
EL	0.971%	0.971%	0.971%	0.971%	0.971%
UL	2.071%	2.079%	2.074%	2.074%	2.073%
ES	3.306%	3.314%	3.311%	3.308%	3.305%

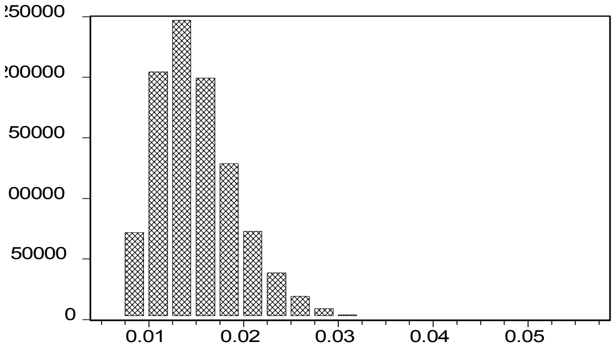
Note: numbers are in percentages of total exposure. EL is the Expected Loss; UL is the unexpected Loss; ES is the expected shortfall

Exhibits 3: Portfolio Loss Density with 4 factors; constant recovery



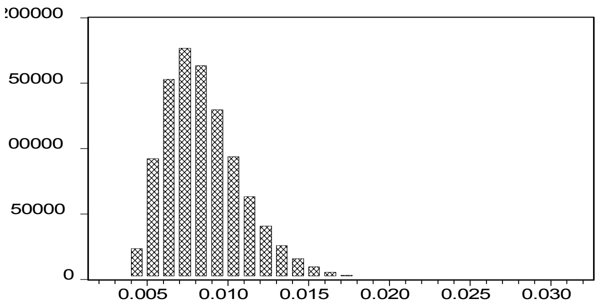
Series: ULOSS4	
Sample 1 1000000	
Observations 1000000	
Mean	0.008712
Median	0.008313
Maximum	0.031251
Minimum	0.002863
Std. Dev.	0.002515
Skewness	0.979452
Kurtosis	4.443072
Jarque-Bera	246656.9
Probability	0.000000

Exhibits 4: Portfolio Loss Density with 4 factors; stochastic dependent recovery



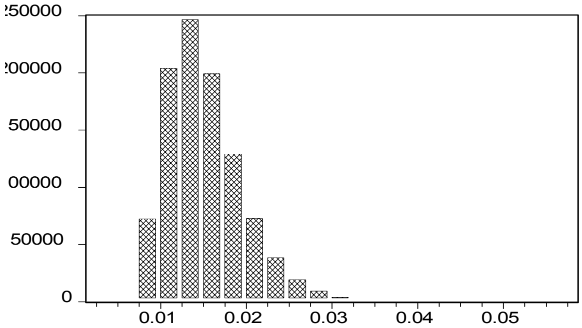
Series: ULOSS4_SR	
Sample 1 1000000	
Observations 1000000	
Mean	0.015440
Median	0.014729
Maximum	0.056759
Minimum	0.005347
Std. Dev.	0.004458
Skewness	0.978689
Kurtosis	4.433119
Jarque-Bera	245215.1
Probability	0.000000

Exhibits 5: Portfolio Loss Density with 5 factors; constant recovery



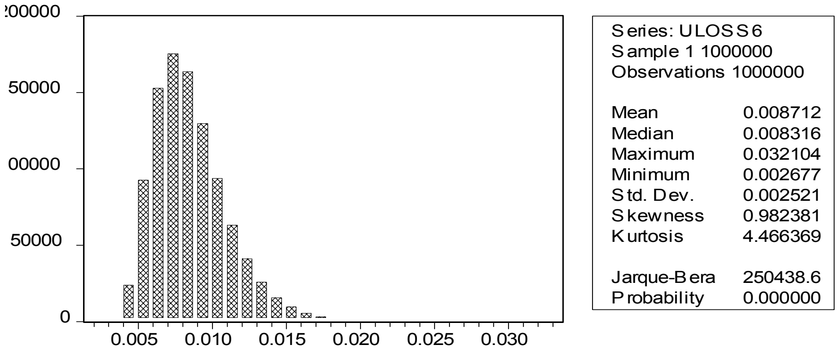
Series: ULOSS5	
Sample 1 1000000	
Observations 1000000	
Mean	0.008712
Median	0.008312
Maximum	0.031285
Minimum	0.002887
Std. Dev.	0.002517
Skewness	0.977968
Kurtosis	4.437833
Jarque-Bera	245543.6
Probability	0.000000

Exhibits 6: Portfolio Loss Density with 5 factors; stochastic dependent recovery

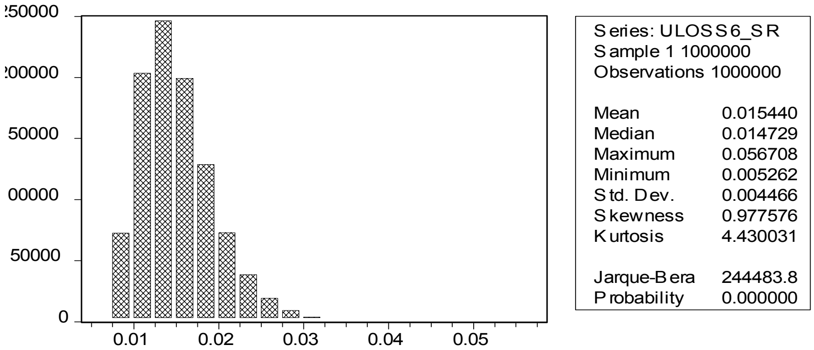


Series: ULOSS5_SR	
Sample 1 1000000	
Observations 1000000	
Mean	0.015440
Median	0.014729
Maximum	0.056781
Minimum	0.004855
Std. Dev.	0.004464
Skewness	0.979013
Kurtosis	4.436951
Jarque-Bera	245779.0
Probability	0.000000

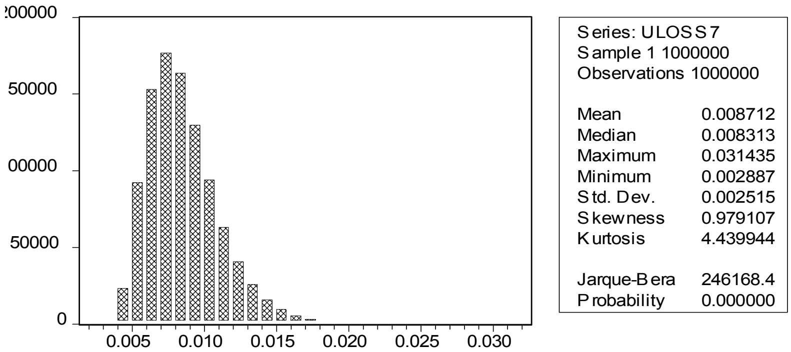
Exhibits 7: Portfolio Loss Density with 6 factors; constant recovery



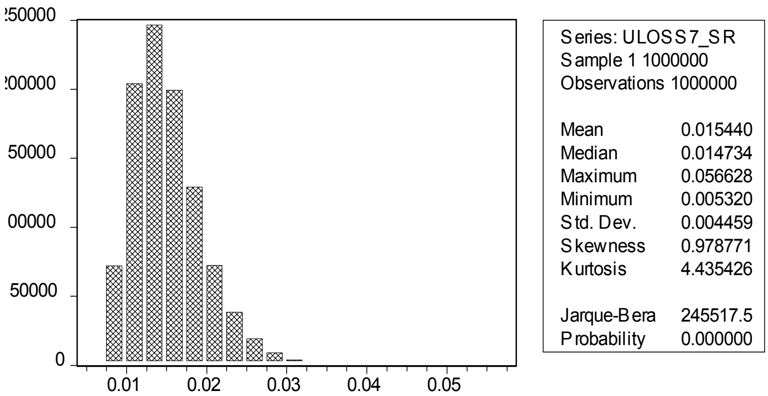
Exhibits 8: Portfolio Loss Density with 6 factors; stochastic dependent recovery



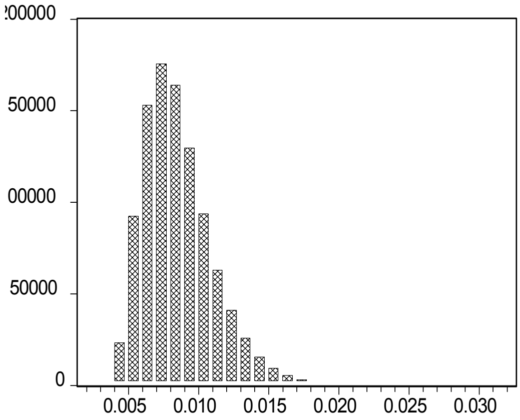
Exhibits 9: Portfolio Loss Density with 7 factors; constant recovery



Exhibits 10: Portfolio Loss Density with 7 factors; stochastic dependent recovery

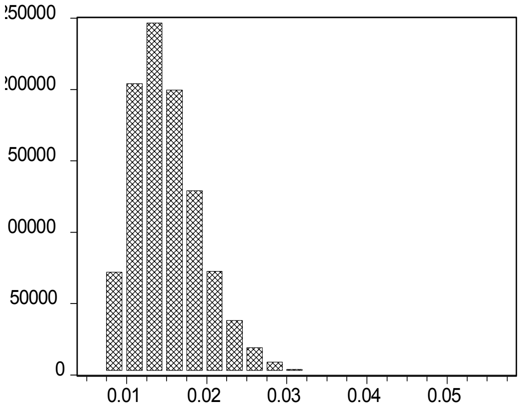


Exhibits 11: Portfolio Loss Density with 8 factors; constant recovery



Series: ULOSS8	
Sample 1 1000000	
Observations 1000000	
Mean	0.008712
Median	0.008316
Maximum	0.031719
Minimum	0.002761
Std. Dev.	0.002517
Skewness	0.982599
Kurtosis	4.460776
Jarque-Bera	249828.0
Probability	0.000000

Exhibits 12: Portfolio Loss Density with 8 factors; stochastic dependent recovery



Series: ULOSS8_SR	
Sample 1 1000000	
Observations 1000000	
Mean	0.015440
Median	0.014733
Maximum	0.056676
Minimum	0.005445
Std. Dev.	0.004457
Skewness	0.978344
Kurtosis	4.434297
Jarque-Bera	245243.3
Probability	0.000000

Appendix 1: Macro-variables dataset

Code	Data description	Transformation
EUR001M	Euribor 1 mesi	3
EUR003M	Euribor 3 mesi	3
EUR006M	Euribor 6 mesi	3
ILRSGVTG	Italy rendista govt bond	3
ITISCOKE	COKE SA SALES	3
ITISELEC	ELECTRICS SA SALES	3
ITISFOOD	FOOD SALES	3
ITISFSAT	FOREIGN SALES SA	3
ITISLEAT	LEATHER SA SALES	3
ITISMACH	MACHINERY SA SALES	3
ITISMANF	MANUFACTORING SA SALES	3
ITISMETL	METALS SA SALES	3
ITISMINE	MINERALS SA SALES	3
ITISNMET	NON METALS SA SALES	3
ITISNSAT	DOMESTIC SALES SA	3
ITISOTHR	OTHERS SA SALES	3
ITISPAPR	PAPER SA SALES	3
ITISRUBB	RUBBER SA SALES	3
ITISSCO	CONSUPTION GOODS SA SALES	3
ITISSEN	ENERGY SA SALES	3
ITISSIN	INVESTIMENT GOODS SA SALES	3
ITISSINT	INTERM GOODS SA SALES	3
ITISTEXT	TEXTILES SA SALES	3
ITISTRAN	TRANSPORT SA SALES	3
ITISTSAT	TOTAL SALES SA	3
ITISWOOD	WOOD SA SALES	3
ITORFSAL	ITALY FOREIGN INDUSTRIAL ORDER SA	3
ITORNSAL	ITALY NATIONAL INDUSTRIAL ORDER SA	3
ITORTSAL	ITALY INDUSTRIAL ORDER SA	3
ITPRENS	ITALY INDUSTRIAL PRODUCTION ENERGY SA	3
ITPRINS	ITALY INDUSTRIAL PRODUCTION INVESTIMENT GOODS SA	3
ITPRITS	ITALY INDUSTRIAL PRODUCTION INTERMED GOODS SA	3
ITPRSAN	ITALY INDUSTRIAL PRODUCTION SA	3
ITPRSCI	ITALY INDUSTRIAL PRODUCTION CHEMICALS SA	3
ITPRSDI	ITALY INDUSTRIAL PRODUCTION FOOD SA	3
ITPRSEI	ITALY INDUSTRIAL PRODUCTION ELECTRICS SA	3
ITPRSFI	ITALY INDUSTRIAL PRODUCTION MANUFACTURING SA	3
ITPRSHI	ITALY INDUSTRIAL PRODUCTION MACHINERY SA	3
ITPRSKI	ITALY INDUSTRIAL PRODUCTION COKE SA	3
ITPRSLI	ITALY INDUSTRIAL PRODUCTION LEATHER SA	3
ITPRSNI	ITALY INDUSTRIAL PRODUCTION NON METALS SA	3
ITPRSOI	ITALY INDUSTRIAL PRODUCTION OTHER SA	3
ITPRSPI	ITALY INDUSTRIAL PRODUCTION PAPER SA	3
ITPRSRI	ITALY INDUSTRIAL PRODUCTION RUBBER SA	3

ITPRSSI	ITALY INDUSTRIAL PRODUCTION METALS SA	3
ITPRSTI	ITALY INDUSTRIAL PRODUCTION TEXTILES SA	3
ITPRSWI	ITALY INDUSTRIAL PRODUCTION WOOD SA	3
ITPRSXI	ITALY INDUSTRIAL PRODUCTION FURNITURE SA	3
CPALIT	ALL ITEM CPI ITALIA	4
CPCLITI	CLOTHING AND FOOTWEAR CPI ITALIA	4
CPCMITI	COMMUNICATIONSCPI ITALIA	4
CPEDITI	EDUCATION CPI ITALIA	4
CPENITI	ENERGY CPI ITALIA	4
CPEXITI	CORECPI ITALIA	4
CPFDITI	FOOD CPI ITALIA	4
CPFNITI	FURNISHING CPI ITALIA	4
CPGGITI	GOODS CPI ITALIA	4
CPHLITI	HEALTH CPI ITALIA	4
CPHRITI	RESTURANT AND HOTELS CPI ITALIA	4
CPMSITI	MISCELLANEOUS CPI ITALIA	4
CPRNITI	RECREATION CPI ITALIA	4
CPTRITI	TRANSPORT CPI ITALIA	4
CPXNITI	EXCLUDING ENERGY CPI ITALIA	4
PPENIT	PPI ENERGY	4
PPMNIT	PPI MANUFACTURING ITALIA	4
PPNGIT	PPI NON DOURABLE GOODS ITALIA	4
PPTXIT	TOTAL PRODUCER PRICE EX CONSTRUCTION ITALIA	4
ITPRSPI	ITALY INDUSTRIAL PRODUCTION PAPER SA	3
ITPRSRI	ITALY INDUSTRIAL PRODUCTION RUBBER SA	3
ITPRSSI	ITALY INDUSTRIAL PRODUCTION METALS SA	3
ITPRSTI	ITALY INDUSTRIAL PRODUCTION TEXTILES SA	3
ITPRSWI	ITALY INDUSTRIAL PRODUCTION WOOD SA	3
ITPRSXI	ITALY INDUSTRIAL PRODUCTION FURNITURE SA	3

Note: In the third column, the number are associated to a specific transformation of each raw series. Specifically, the transformations are as follows: 2 = no transformation; 3 = first difference of the log level; 4 = annualised growth rate, that is, y which is the log level of the time series, is transformed into is (y_t - y_{t-12}). As for the interest rates (the first four series) variables in the second column, these are the transformed annualised rates, *r*, into monthly gross rates, using (1/12)*log(1+*r*/100)]. We then apply the first order difference transformation. Transformation 4 is for the prices series whose raw observations are not seasonally adjusted.

Appendix 2: MIB stock price data

Code	Data description
MIBFOODH	MIB Food/Grocery
MIBINSH	MIB Insurance
MIBBANKH	MIB Banking
MIBPAPH	MIB Paper Print
MIBBUILH	MIB Building
MIBCHEMH	MIB Chemicals
MIBCOMH	MIB Transport/Tourism
MIBCUMH	MIB Distribution
MIBELECH	MIB Electrical
MIBREALH	MIB Real Estate
MIBMECH	MIB Auto
MIBMINH	MIB Metal/Mining
MIBTEXTH	MIB Textiles
MIBMISCH	MIB Industrial Miscellaneous
MIBPLNTH	MIB Plants/Machinery
MIBFNCLH	MIB Financial Services
MIBFINCH	MIB Finance/Part
MIBFINMH	MIB Financial Miscellaneous
MIBPUBLH	MIB Public Utility
MIBPRNTH	MIB Media
MIB30	MIB 30

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