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Forecasting stock market volatility conditional on macroeconomic conditions.

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Abstract

This paper presents a GARCH type volatility model with a time-varying unconditional volatility which is a function of macroeconomic information. It is an extension of the SPLINE GARCH model proposed by Engle and Rangel (2005). The advantage of the model proposed in this paper is that the macroeconomic information available (and/or forecasts) is used in the parameter estimation process.

Based on an application of this model to S&P500 share index returns, it is demonstrated that forecasts of macroeconomic variables can be easily incorporated into volatility forecasts for share index returns. It transpires that the model proposed here can lead to significantly improved volatility forecasts compared to traditional GARCH type volatility models.

Keywords

Volatility, macroeconomic data, forecast, spline, GARCH.

JEL Classification C12, C22, G00.

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1 Introduction

Understanding and forecasting financial market volatility is clearly of great importance. To these ends, literature has evolved along two themes. For the purposes of forecasting volatility, a myriad of time series approaches initially motivated by Engle (1982) and Bollerslev (1987) have been developed. One recurring empirical fact is the strong persistence in the conditional volatility of financial asset returns. Studies such as Ding and Granger (1993), Ding, Granger and Engle (1996), Dacorogna *et al.* (1993) and Baillie, Bollerslev and Mikkelsen (1996), among others, have found equity and currency returns to be long memory processes. While it is accepted that a high degree of persistence is present in the volatility of such series, there is little consensus regarding the underlying causes for this persistence, or generally for the long-term evolution of volatility. Liu (1997) suggests that a heavy-tailed regime switching process may produce series exhibiting long-memory properties. While Andersen and Bollerslev (1997) suggest that heterogeneous information flows may be the cause, Muller *et al.* (1997) and Kirman and Teyssière (2002) propose heterogeneous agent explanations. A different explanation for the apparently high persistence in conditional volatility is that there may be breaks in the volatility process. Diebold and Inoue (2001) have established that breaks in an otherwise short memory process can explain the appearance of long memory properties when such a break is not taken into account. Indeed, there is ample evidence for the existence of structural breaks in the volatility process (see for example, Andreou and Ghysels, 2002, Mikosch and Starcia, 2000 and Hamilton and Lin, 1996). Often these breaks are linked to underlying economic events or conditions.

Another line of research has broadened our understanding of volatility dynamics, by linking the evolution of volatility to changing economic conditions. The majority of this work, Officer (1973), Schwert (1989), Glosten, Jagannathan and Runkle (1993) and Hamilton and Lin (1996) study the link between US macroeconomic conditions and S&P500 share price index volatility. Officer (1973) explains the drop in stock market volatility in the 1960s with a reduced variability in industrial production. Schwert (1989) and Hamilton and Lin (1996) find that stock market volatility is increased during recessions and Glosten *et al.* (1993) find interest rates to be an important factor.

While useful for expanding our understanding of the links between the real economy and the financial markets, none of these studies provide any formal time series model for forecasting relatively high frequency volatility. Recent contributions to this end have been Engle and Rangel (2005)

and Engle, Ghysels and Sohn (2007). Engle and Rangel (2005) specify a modified GARCH model, SPLINE GARCH that allows for time-variation in the unconditional level of stock market volatility. This approach is an important extension to the standard GARCH framework which imposes that the unconditional mean of volatility, the level of volatility to which the process reverts, is constant through time. Essentially, their model extracts an unconditional volatility sequence around which a standard dynamic process of the GARCH type is imposed. The resulting time-series of unconditional volatility is filtered merely on the basis of the returns data to which the model is applied with certain smoothness restrictions. While the work of Engle and Rangel (2005) is certainly an important contribution to our understanding of the pure time series dynamics of volatility, they also examine the link between the unconditional volatility and a number of macroeconomic variables. While possible in a two-stage process, they however do not consider the issue of volatility forecasting. Engle, Ghysels and Sohn (2007) build upon this idea and directly condition unconditional volatility on macroeconomic data by utilizing the Mixed Interval Data Sampling (MIDAS) approach of Ghysels, Santa-Clara and Valkanov (2005 and 2006). This leads to a direct way of linking unconditional volatility in a high-frequency volatility model, to low-frequency macroeconomic information. The resulting unconditional volatility is fixed within certain time intervals, say months or quarters. In comparison to Engle and Rangel (2005), this approach leads naturally to a forecasting tool for volatility. While this is the case, the forecasting issue was not addressed.

The aim of this paper is to develop a forecasting procedure that directly incorporates macroeconomic information. Unconditional volatility is linked to the macroeconomic information by utilizing a spline approach in a similar manner to Engle and Rangel (2005). Here, the values (or height) of the knotpoints are a function of the macroeconomic data. A cubic spline is then fit to these knotpoints. This is an alternative approach to that of Engle, Ghysels and Sohn (2007) that makes no assumption regarding the constancy of the unconditional volatility during discrete periods of time. It does, however, impose smoothness restrictions on the resulting series of unconditional volatility. This paper addresses the issue of longer term volatility forecasting conditional on forecasts of the macroeconomic variables and highlights the benefit of this approach relative to standard GARCH models.

The paper now proceeds as follows. The next section reviews the SPLINE GARCH style models proposed by Engle and Rangel (2005) and Engle, Ghysels and Sohn (2007). Section 3 describes

the approach proposed here. In Section 4 the data to which the model is applied is described and results of the empirical analysis are presented in Section 5. The forecasting procedure is described and illustrated in Section 6 and conclusions are offered in Section 7.

2 SPLINE GARCH

In a recent paper Engle and Rangel (2005) extend a standard GARCH model to cater for a time-varying level of unconditional volatility. Consider the GARCH(1,1) model

$$\begin{aligned} r_t &= \mu + \varepsilon_t \quad \varepsilon_t \sim N(0, h_t) \\ h_t &= \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} \end{aligned}$$

which implies that the unconditional volatility is $\beta_0 / (1 - \beta_1 - \beta_2)$. This is the level of volatility to which the volatility process reverts and is assumed to be time-invariant. While numerous extensions to this basic GARCH(1,1) model have been proposed, extending the volatility model to cater for asymmetry and long-memory amongst others, extensions that cater for variation in the unconditional volatility are rare. A notable exception are regime switching models for volatility, which allow for a different level of unconditional volatility in each regime.

Engle and Rangel (2005) take a fundamentally different approach and allow the unconditional volatility to follow a flexible structural form to be determined by the data. They propose the following specification for their SPLINE GARCH model

$$\begin{aligned} r_t &= \mu + \varepsilon_t \quad \varepsilon_t \sim N(0, h_t) \\ h_t &= g_t \tau_t \\ g_t &= (1 - \beta_1 - \beta_2) + \beta_1 \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} \right) + \beta_2 g_{t-1}. \end{aligned} \tag{1}$$

The function τ_t is a deterministic spline function and will be explained in more detail later in this section. The unconditional expectation for ε_t^2 can be derived as follows:

$$\begin{aligned} E(\varepsilon_t^2) &= h_t = E(g_t) \tau_t \\ E\left(\frac{\varepsilon_t^2}{\tau_t}\right) &= \frac{h_t}{\tau_t} = E(g_t) \end{aligned}$$

where it should be noted that τ_t is a deterministic function. Given that $g_t = h_t / \tau_t$, g_t is a forecast of ε_t^2 / τ_t . Of course actual realizations of ε_t^2 / τ_t will in general not equal g_t . Adding ε_t^2 / τ_t to both

sides of the definition of g_t to obtain

$$\begin{aligned}
g_t + \frac{\varepsilon_t^2}{\tau_t} &= (1 - \beta_1 - \beta_2) + \beta_1 \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} \right) + \beta_2 g_{t-1} + \frac{\varepsilon_t^2}{\tau_t} \\
&= (1 - \beta_1 - \beta_2) + (\beta_1 + \beta_2) \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} \right) - \beta_2 \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} - g_{t-1} \right) + \frac{\varepsilon_t^2}{\tau_t} \\
\frac{\varepsilon_t^2}{\tau_t} &= (1 - \beta_1 - \beta_2) + (\beta_1 + \beta_2) \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} \right) - \beta_2 \left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}} - g_{t-1} \right) + \left(\frac{\varepsilon_t^2}{\tau_t} - g_t \right).
\end{aligned}$$

Taking expectations and noting that $E(\varepsilon_t^2/\tau_t - g_t) = 0$ yields

$$E\left(\frac{\varepsilon_t^2}{\tau_t}\right) = (1 - \beta_1 - \beta_2) + (\beta_1 + \beta_2) E\left(\frac{\varepsilon_{t-1}^2}{\tau_{t-1}}\right).$$

Imposing the usual stationary conditions on this process it can easily be seen that $E(\varepsilon_t^2/\tau_t) = 1$, which in turn implies that

$$E(\varepsilon_t^2) = h_t = E(g_t) \tau_t = \tau_t.$$

This convenient result was utilized by Engle and Rangel (2005) to argue that the deterministic function τ_t can be interpreted as the unconditional volatility. Engle and Rangel (2005) utilize a quadratic spline to approximate the time varying unconditional volatility. The ordered sequence $\{t_i\}_{i=1}^k$, where $t_1 > 1$ and $t_k \leq T$, represents the division of the time line into k equally spaced subsections. The spline will fit a smooth curve to a sequence of points $\{\tau_{t_i}\}_{i=1}^k$. These values for the unconditional volatility at times $\{t_i\}_{i=1}^k$ are unobserved and are based on the spline parameters. While the location (in time) of these points is determined a priori, $\{t_i\}_{i=1}^k$ the values $\{\tau_{t_i}\}_{i=1}^k$ will be determined such that the likelihood of the SPLINE GARCH model in (1) is maximized.

A quadratic spline can be specified as the sum of k truncated quadratic basis functions:

$$\tau_t = \gamma_0 \exp\left(\gamma_1 t + \gamma_2 t^2 + \sum_{i=1}^k \omega_i ((t - t_i)_+)^2\right). \quad (2)$$

Given estimates for $\boldsymbol{\gamma} = (\gamma_0 \ \gamma_1 \ \gamma_2)'$ and ω_i , $i = 1$ to k , a sequence $\{\tau_t\}_{t=1}^T$ can be calculated. This model setup requires the estimation of $k + 3$ parameters in the unconditional volatility part of the SPLINE GARCH specification. Engle and Rangel propose to determine k by means of BIC to guard against overfitting.

It was the objective of Engle and Rangel (2005) to uncover the macroeconomic causes of changes in the unconditional volatility τ_t . To that end, in addition to estimating the SPLINE GARCH model (1) they perform a second stage analysis (across a number of countries) in which they relate

the obtained sequences of unconditional volatility to observed macroeconomic data. Their findings suggest that variations across time and countries in unconditional volatility can be explained by GDP growth, inflation and variables reflecting the stage of development of a particular financial market.

Engle, Ghysels and Sohn (2007) estimate the unconditional volatility, τ_t as a direct function of lagged macroeconomic information. A generic form for the specification proposed by Engle, Ghysels and Sohn (2007) is given by

$$\tau_t = m + \sum_{k=1}^K \varphi_k (\theta' x_{t-k}). \quad (3)$$

In this case, τ_t reflects a level of unconditional volatility that is fixed within discrete periods of time such as months or quarters. Thus even when modelling volatility at a daily frequency, the unconditional volatility is fixed within these periods. x_{t-k} is a vector containing low frequency macroeconomic information and θ is a parameter vector to be estimated. Weights attached to x_{t-k} are given by φ_k which is based on the MIDAS function of Ghysels, Santa-Clara and Valkanov (2005 and 2006). In total the parameters to be estimated are a constant, m , θ and any parameters required to generate φ_k . We now turn to the spline based approach proposed here for directly incorporating macroeconomic information.

3 SPLINE GARCH using Macroeconomic Data

It is the objective of this paper to demonstrate how macroeconomic variables can be used for longer-term forecasts of volatility within the SPLINE GARCH framework, and offer an alternative to Ghysels, Santa-Clara and Valkanov (2005 and 2006). The proposed approach is discussed here with the issue of forecasting being addressed in Section 6.

In traditional volatility models of the GARCH type, macroeconomic conditions do not feature in the information set due to the low frequencies at which they are observed. Exceptions are GARCH models which include interest rates as an exogenous variable in the GARCH equation (see Glosten *et al.*, 1993). As interest rates are observable at a high frequency they can be easily incorporated into a GARCH model which is typically estimated on high frequency financial returns. Most other macroeconomic variables, however, are only observed at much lower frequencies and hence cannot be considered. As discussed in Section 2, Engle, Ghysels and Sohn (2007) provide a method for

achieving this by utilizing the MIDAS methodology. An alternative, that requires no assumption of constancy of τ_t and is closer in spirit to Engle and Rangel (2005) is now outlined.

In the spline procedure introduced by Engle and Rangel (2005) the values of τ_{t_i} are chosen freely in order to maximize the likelihood of the data. Let $f\left(t|\{\tau_{t_i}\}_{i=1}^k\right)$ be a cubic spline, fitting the sequence of knotpoints $\{\tau_{t_i}\}_{i=1}^k$ observed at a sequence of times $\{t_i\}_{i=1}^k$, evaluated at time t . Specifically we assume that the unconditional volatility in the SPLINE GARCH model can be expressed as

$$\tau_t = \exp \left[f \left(t | \{ \tau_{t_i} \}_{i=1}^k \right) \right].$$

In the context of this paper the knotpoints $\{\tau_{t_i}\}_{i=1}^k$ are constrained to be a linear function of macroeconomic data. Let x_{t_i} be a $(1 \times p)$ vector of macroeconomic data that are observable at times $\{t_i\}_{i=1}^k$, the first element of which may be a constant. In particular we assume that $\tau_{t_i} = x_{t_i} \theta$, such that the unconditional volatility can be represented as

$$\tau_t = \exp \left[f \left(t | \{ \tau_{t_i} \}_{i=1}^k \right) \right] = \exp \left[f \left(t | \{ \theta' x_{t_i} \}_{i=1}^k \right) \right]. \quad (4)$$

Given a particular $(p \times 1)$ dimensional parameter vector θ , k knotpoints ($k > p$) and τ_t can be calculated. As in the SPLINE GARCH model the number of knotpoints k has to be chosen a priori and is independent of the dimension of the vector θ . Conditional on the normality assumption in (1) the likelihood can be written as a function of τ_t and maximized by any standard nonlinear optimization routine.

As with Engle and Rangel (2005), one could express this spline as a function of a series of truncated power functions (Hastie and Tibshirani, 1990)

$$f \left(t | \{ x_{t_i} \theta \}_{i=1}^k \right) = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 + \sum_{i=1}^k \omega_i (t - t_i)_+^3.$$

The $k + 4$ parameters are functions of the sequence $\{\theta' x_{t_i}\}_{i=1}^k$.¹ This specification carries no economic meaning and merely illustrates that a spline function can be understood as a linear combination of truncated polynomial basis functions. In practice, these basis functions tend to be unstable and thus an alternative set of basis functions, so-called B-splines are commonly utilized. A computationally efficient algorithm which calculates parameter values corresponding to these basis functions on the basis of the sequence $\{\theta' x_{t_i}\}_{i=1}^k$ is described in de Boor (1978) and implemented in MATLAB.

Engle and Rangel (2005) proposed to choose γ and ω_i , $i = 1$ to k in equation (2) and hence the knotpoints such that the likelihood of the SPLINE GARCH in equations (1) and (2) is maximized. Here, θ and thus the knotpoints will be chosen such that they maximize the likelihood of the MODIFIED-SPLINE GARCH (MS GARCH) in equations (1) and (4), but they are clearly constrained to be a linear combination of the macroeconomic information². Hence, for an equal number of knotpoints one would expect the SPLINE GARCH to achieve a better fit, as it has to comply with fewer constraints.

It should also be noted that the knotpoints need not be equally spaced in time. Any sequence of gridpoints sequence of times $\{t_i\}_{i=1}^k$ could be used in the spline calculation and one may consider using a denser grid where the unconditional volatility is presumed to have more variation³.

4 Data

The SPLINE GARCH and MS GARCH models will be applied to S&P500 daily log returns. Returns for the period from the 3rd of January 1957 to the 31st of December 2004 are used to estimate the various versions of GARCH models and additional daily returns up to November 4 2005 are retained to illustrate the out-of-sample forecasting procedure implied by the MS GARCH model. A number of different macroeconomic variables were considered as explanatory variables to explain variation in the unconditional volatility. GDP growth, Δgdp , growth in industrial production, Δip , inflation, π , the 3 month T-bill rate, is , the 10 year Treasury Note rate, il , and the 10 year corporate bond rate (BAA rated companies), ic were considered⁴. Further the yield curve slope implied by short and long term interest rates, $idy(= il - is)$, as well as the difference between the Treasury and BAA corporate rate, $idc(= ic - il)$ were calculated. In the first instance all data were sampled quarterly, where the interest rates were averaged over the relevant quarter. As the index returns are sampled daily, it will be necessary to allocate the economic data to a particular day. The convention chosen here is that the economic data are allocated to the last trading day in each quarter. Proxies for the uncertainty surrounding the macroeconomic data were also constructed. For each of the macroeconomic series an AR(1) model was estimated for the quarterly data and the estimated squared residuals were then calculated. Following Engle and Rangel (2005), a four quarter moving average of these residuals is treated as a volatility proxy, these being indicated by the prefix v , e.g. $v\Delta gdp$ indicating the volatility of GDP growth. Volatility proxies were also

calculated along the lines of Engle, Ghysels and Sohn (2007) as the absolute value of single residuals.

Correlations between these variables ranged from -0.3 between Δip and idc , to 0.69 between Δgdp and Δip . Not surprisingly we also find a strong correlation of 0.67 between π and is . There are also negative correlations between Δgdp or Δip and is or idc , and a negative correlation between Δgdp or Δip and idy . All uncertainty measurers are positively correlated with each other.

5 Empirical Analysis

For comparative purposes, Section 5.1 will discuss the features of the SPLINE GARCH model. Section 5.2 outlines the empirical performance of the MS GARCH model from a general perspective and draws comparisons between the SPLINE and MS GARCH performance. Section 5.3 discusses the link between the evolution of volatility and macroeconomic conditions.

5.1 SPLINE GARCH performance

To start, two benchmark models are estimated, a standard GARCH(1,1) model and the SPLINE GARCH model. The parameter estimates, log-likelihood and associated information criteria are displayed in Table 1. While there is a sizeable literature which argues that asymmetric volatility models such as the GJR and EGARCH model are preferred, no such models are estimated here, in order to focus on the role of the time varying unconditional mean⁵.

From the presented results it is apparent that the estimated GARCH model indicates an extremely persistent volatility process, as $(\beta_1 + \beta_2) = 0.997$, a common feature of GARCH models applied to long time series of returns. It is often argued (see for instance, Beltratti and Morana 2005) that this high persistence results from the presence of structural breaks in the unconditional variance and that once such breaks are taken into account, this persistence should be reduced. As the SPLINE and MS GARCH models cater for such variation, it is expected that the variance persistence is somewhat reduced in these models.

[add Table 1 about here]

The SPLINE GARCH model is estimated with 5, 10, 12, 15 and 20 knotpoints. As indicated in Table 1, every additional knotpoint requires the estimation of an additional parameter. While the estimated knotpoints in the SPLINE GARCH model are not shown in Table 1, they are shown for the 12 knotpoint model in Figure 1. For illustrative purposes, the SPLINE GARCH model with 12 knotpoints is interpreted in some detail. The persistence implied by the GARCH process,

around the time-varying unconditional volatility, has decreased to $(\beta_1 + \beta_2) = 0.967$. We can also observe an increase in the log-likelihood of the model and a decrease in the information criteria in comparison to the GARCH(1,1) model. Note that the SPLINE GARCH model does not have a constant parameter in the volatility process, β_0 .

Restricting all 12 knotpoints to be equal will restrict the SPLINE GARCH model to be equivalent to the standard GARCH model. This restriction can be tested by means of an LR test. The test statistic is $LR = -2(52796.2 - 52888.6) = 184.8$ which is clearly significant at any conventional significance level when evaluated against the χ_{11}^2 distribution under the null hypothesis. The results suggest that allowing the unconditional volatility to vary through time delivers a statistically significant improvement in the fit of the model. Figure 1 displays the conditional and the unconditional volatility from this model. The unconditional volatility proxies the long-term swings in the conditional volatility, with smoothness imposed by the number of knotpoints.

[add Figure 1 about here]

While each of these different SPLINE GARCH models can be tested against the standard GARCH model, models with different numbers of knotpoints are not nested. Therefore, it is necessary to compare these models by means of information criteria. It is interesting to note that the persistence of the GARCH volatility process decreases as the number of knotpoints is increased.

5.2 MS GARCH performance

Prior to examining the impact of various macroeconomic factors upon volatility in detail in the next section, the overall performance of the MS GARCH model will be examined. Performance will be evaluated with respect to the following two aspects of the model. *First*, model performance will be evaluated with respect to number of spline knotpoints, equidistant grids with 47, 23 and 12 knotpoints being utilized. These choices correspond to knotpoints placed 1, 2 and 4 years apart. It may be argued that using an equidistant grid is inefficient as the unconditional variance may exhibit times when it changes more frequently than others, with the former possibly requiring a denser grid. Experiments with more knotpoints during the 1970's and 1980's did not deliver any obvious advantages and thus results are not reported here. While this is the case here, this approach is flexible in that it can accommodate irregular knot placement required. *Second*, when utilizing the macroeconomic information, should these data be aggregated over time in any way? To address this issue, the macroeconomic data is averaged over the 1, 2, 4, 6, 8, 12 and 16 quarters preceding

each knotpoint. These choices reflect increasing smoothing of the data.

A selection (a wider range is discussed below) of models which vary according to these dimensions are estimated and their Schwarz Information Criterion (SIC) are displayed in Table 2. SIC is reported for 2 models, $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$ and $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, \pi_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$. We focus on models without the volatility proxies for a number of reasons. Many models had been estimated with combinations of level and volatility variables. The volatility variables were often found to be insignificant with changes in the signs of parameters across the various sampling frequencies and aggregation levels, a pattern observed irrespective of the of the definition of the volatility variable used. This was not the case with the level variables reported here that were always of a consistent sign. Overall, models containing the volatility variables did not outperform those with only the level variables. This is a different result to that of Engle, Ghysels and Sohn (2007) in that they found level variables to be unimportant. We consider a wider range of level variables here and are using quite a different approach to estimating the unconditional level of volatility, τ_t . As reported in the following section, the coefficients on the macroeconomic variables reveal quite logical links from the real economy to equity market volatility.

[add Table 2 about here]

Analyzing the results in Table 2 offers a number of interesting insights. In all cases except when the macroeconomic data is aggregated over the proceeding 16 quarters, it is optimal to place knotpoints every 2 years. Irrespective of the sampling frequency considered, a moderate level of smoothing of the macroeconomic data is preferred. Across the two models reported here, the optimal smoothing ranges between 2 and 6 quarters. Therefore, it appears as though knotpoints spaced 2 years apart with macroeconomic data aggregated over 2 or 4 quarters seem to balance having enough resolution in terms of knotpoints and eliminating noise in the macroeconomic data. These patterns were consistent across other models not reported here.

[add Table 3 about here]

A number of particular MS GARCH models will now be discussed in more detail, with all results reported in Table 3. All models are based on knotpoints every 2 years with 2 quarter aggregation of the macroeconomic data. The first model (Model A) is based on the full set of macroeconomic variables, $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, \pi_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$. Parameter estimates and relevant results are shown in the first column of Table 3. Overall, it appears as though macroeconomic conditions are

useful in explaining long-term movements in the unconditional volatility, a plausible attractor for the conditional variance. The persistence implied by this model is $(\beta_1 + \beta_2) = 0.966$, which is approximately equal to the persistence exhibited by the SPLINE GARCH with 12 knots discussed earlier. The improvement relative to a standard GARCH model is significant as indicated by an LR test statistic of $-2(52796.2 - 52869.20) = 146.0$ which is clearly significant when compared to its critical values from a χ^2 distribution with 10 degrees of freedom, the 1% critical value being 23.21. In this instance inflation, π , is found to be insignificant, and its exclusion has no impact on the model as indicated by the results in for Model *C*. Figure 2 shows the conditional and unconditional variance along with the estimated spline knotpoints for this model excluding inflation. The unconditional volatility in this case is somewhat more variable than that in the SPLINE GARCH model shown in Figure 1. Growth in GDP and industrial production have opposite signs of similar magnitude, but given they are strongly correlated this is of little surprise. However as the results for Model *B* and *D* indicate, when Δip is removed the performance of the model deteriorates.

[add Figure 2 about here]

We are now in a position to compare the results from the MS GARCH model to those of the SPLINE GARCH model. The best performing SPLINE GARCH model according to SIC is that with 20 knots (sampling every 2 years) with the best MS GARCH model being based on $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$ (Model *C* in Table 3). As the models are not nested, it is impossible to use an LR test statistic to decide between these models. Information criteria may shed some light on which of the two specifications would be preferred. Based on the *SIC* the SPLINE GARCH displays a marginally better fit. It should be taken into consideration that the knotpoints in the SPLINE can be chosen freely, whereas the knotpoints in the MS GARCH have to meet additional restrictions in terms of their relationship to the macroeconomic variables. It is therefore not surprising to find the SPLINE GARCH exhibiting a marginally better fit. In the light of these results one may question the additional value of MS GARCH as compared to the SPLINE GARCH. Two reasons are put forward to highlight the additional value provided by tying the knotpoints to macroeconomic variables. *First*, the spline function used to fit the unconditional volatility is essentially a nonparametric approximation to an unknown function and it is well known that the use of such an approximation carries the danger of overfitting. In this light the additional constraints imposed are desirable as a guard against overfitting. *Further*, the MS GARCH model allows for a

natural forecasting procedure for volatility based on forecasts of the macroeconomic variables. The advantage, again, stems from the link between the knotpoints and macroeconomic variables. The forecasting issue will be directly addressed below.

5.3 Role of Macroeconomic information in MS GARCH

The link between each of the macroeconomic variables and the level of the unconditional volatility, in the context of the MS GARCH models in Table 3 will now be considered, with a number of robust conclusions being drawn. There is very clear evidence that information contained in the term structure of interest rates has significant explanatory power for the unconditional variance of the S&P500 returns. Unconditional volatility is positively related to the level of the short term rate is , negatively related to slope of the yield curve for government interest rate instruments, idy and positively related to the spread between commercial and government rates, idc .

It is well known that the yield curve slope is a useful leading indicator for the business cycle with inverse yield curve slope is indicative of forthcoming recessionary periods (see Hamilton and Kim, 2002 and references therein). Therefore the negative coefficient on idy is logical in that unconditional share market volatility increases with the prospect of such recessionary periods. Unconditional volatility is positively related to the premium on commercial interest rates, idc . An increased risk premium is associated with increased default risk for the interest rate instruments considered, here those issued by BAA rated commercial operations, and it is again plausible that a riskier commercial environment is reflected in increased share market volatility. The positive coefficient on short term rates, is , would seem to reflect the fact volatility has been high during periods of high interest rates that have been associated with high inflation.

When both Δgdp and Δip are included they have opposite signs that have a marginally positive sum, and when Δip is removed the performance of the model falls significantly. This may be due to a number of reasons. The fact that Δgdp and Δip are measuring similar effects and are highly correlated makes the interpretation of this result not straightforward. The sum of these coefficients being positive (but small) suggests that larger current economic growth is associated with increased share market volatility. Prima facie, these results are inconsistent with the findings of Schwert (1989) and Hamilton and Lin (1996), who seem to suggest that the share market volatility is increased during recessions. This result can be justified given the importance of the information contained in the yield curve. As argued earlier, the slope of the yield curve can be interpreted

as a leading indicator of the business cycle. Given that share prices are believed to reflect the present value of expected future dividends, it is sensible that expectations about the future state of the business cycle may bear greater importance for share prices than its current state. In the light of this, one may argue that the positive relation of current economic activity indicated by the estimation results does not contradict previous findings, as the latter may not have sufficiently differentiated between the current and the forecast state of the business cycle. Estimation results relating to the inflation coefficient shows that π is not important, as it is insignificant when included and has no detrimental impact on the model when removed. Again, it is possible to argue that a forecast of future inflation is already incorporated in the term structure of interest rates. In this light, the positive coefficients on *is* and *idy* can be interpreted as capturing the increased uncertainty in the share market during times with higher expected inflation.

6 Volatility forecasts with the MS GARCH model

It is not the purpose of this paper to comprehensively demonstrate the superiority of the MS-GARCH model in comparison to a range of alternative volatility models of the S&P500 share index, but rather to illustrate how the MS-GARCH model can be used to forecast stock market volatility. Emphasis will be put on the conditioning role of the macroeconomic variables believed to drive the level of unconditional stock market volatility. Initially an out-of-sample forecast, utilizing genuine forecasts for macroeconomic variables, is presented. In order to demonstrate that the favorable properties of this out-of-sample forecast is not limited to the particular forecast period chosen, we consider further in-sample forecasts. The general principle behind how the MS GARCH approach may be used to generate forecasted will now be described.

Assume that the following data are available: $\{r_t\}_{t=1}^T$ log index returns. Further, the following observations for the macroeconomic variables are available, $\{x_{t_i}\}_{i=1}^k$ where $\{t_i\}_{i=1}^k = (1 \leq t_1, t_2, \dots, t_k \leq T)$. The task is to produce volatility forecasts conditional on the information available at time T and on forecasts for the relevant macroeconomic variables $x_{t_{k+1}}^f$ where $t_{k+1} > T$. Using information up to time T , the parameters of the MS GARCH model can be estimated and used to obtain estimates of the in-sample knotpoints $\{\hat{\tau}_{t_i}\}_{i=1}^k = \left\{ \hat{\theta}' x_{t_i} \right\}_{i=1}^k$. We continue by obtaining a forecast of a future knotpoint at time t_{k+1} , $\hat{\tau}_{t_{k+1}} = x_{t_{k+1}}^f \hat{\theta}$. Using the extended series of knotpoints $\{\hat{\tau}_{t_i}\}_{i=1}^{k+1}$ the spline function can be used to obtain estimates of the unconditional

volatility $\{\tau_t\}_{t=1}^{T^*}$ where $T < t_{k+1} \leq T^*$ ⁶. This sequence can then be used to obtain volatility forecasts for h_t using the recursive relation in the SPLINE GARCH specification (1).

We start with the out-of-sample forecast where the end of the estimation period is December 31 2004. This forecast is based on the model with $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$. The estimation utilizes return data up to December 31 2004, placing the last in-sample knotpoint at this date. The macroeconomic forecasts to be considered in the volatility forecast are forecasts for 2005, and will be located at December 31 2005. Forecasts for real GDP growth, inflation as well as short and long-term interest rates were retrieved from the OECD Economic Outlook (OECD, 2005)⁷. It should be noted that this report was only published in March 2005 and some of this information might, therefore, not have been available at the end of 2004. Nevertheless, this section is mainly aimed at demonstrating the forecasting principle of the MS GARCH model.

[add Figure 3 about here]

Figure 3 plots the in-sample unconditional volatility and the revised estimate / forecast of the unconditional volatility based on the extended sequence of knotpoints which now also includes the knotpoint forecast for t_{k+1} , $\hat{\tau}_{t_{k+1}}$ (December 2005). The inclusion of this knotpoint also results in a slight revision of the unconditional variance some periods before T . The reason for this is that the cubic spline enforces continuity of the second derivative. In this particular case the procedure predicts an almost unchanged level of unconditional volatility during the year of 2005.

[add Figure 4 about here]

The value of the MS GARCH model lies in its capacity to generate forecasts. While GARCH models, as well as asymmetric GARCH models, such as the GJR or EGARCH model will produce volatility forecasts that eventually converge to the time-invariant unconditional volatility, the MS GARCH model will generate a volatility forecast that will also converge towards the unconditional volatility, but the latter will depend on the current economic environment. To better evaluate the conceptual difference between the volatility forecast generated by the MS GARCH model and a standard GARCH model the different forecasts for 2005, made on December 31 2004 are illustrated graphically. Figure 4 depicts the conditional volatility estimated by the GARCH and MS GARCH model throughout the years 2003 and 2004. It is apparent that the differences between these two are only minimal during the in-sample estimation period. The unconditional volatility of the GARCH model is 0.013 and hence the estimated value for the conditional volatility as of December 31, 2004,

is well below its unconditional value. Consequently the GARCH model predicts a reversion of the conditional volatility towards this unconditional value, as indicated by the solid grey line in the last third of the graph. The situation is significantly different for the MS GARCH model. To start out with, as of December 31, 2004, the MS GARCH model estimates the conditional volatility to be below its unconditional value. Therefore it will predict an upward convergence of the conditional volatility towards the unconditional value.

[add Figure 5 about here]

In order to show that the two forecasts are qualitatively different, Figure 5 includes a proxy of the actual realized volatility during the forecast period. The thin black line in this Figure is the 21-day moving average of squared daily returns. After an initial increase in the first half of 2005, for most of 2005, volatility has reverted to a level below (and thus on average around) the unconditional volatility estimate from the MS GARCH model. Only early in November did the volatility move above the unconditional volatility predicted by the MS GARCH. Quite clearly, however, there is very little evidence of reversion towards the unconditional volatility as predicted by the standard GARCH model.

To show that the favorable result of the out-of-sample forecast above is not due to the particular choice of the forecast period, a number of in-sample forecasts have been generated. These are based on the premise that we treat the subsequent realizations of the macroeconomic data as forecasts. Conditional volatility forecasts are then generated in the same manner as the preceding discussion. Forecasts are generated with the estimation sample being restricted to end on 31 December 1994, 1998, 2002. For each of the three estimation samples the last in-sample knotpoint was placed on the last day of the estimation period. The forecast periods cover the periods up to 31 December 1995, 1999, 2003 and 2004 (the latter two both starting from the end of 2002). All these forecasts are based on the same model, using $x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$. Plots of forecasts of MS GARCH unconditional and conditional volatility, GARCH volatility forecasts along with observed volatility are shown in Figures 6 through 9 for each of the four forecasting periods. In each case, the conditional MS GARCH forecast broadly tracks the observed volatility more closely than the GARCH forecast, with the differences being quite marked in a number of instances. The forecasts shown in Figure 7 are the most similar even though the MS GARCH does track the general fall in volatility during 1999 somewhat better than the GARCH model. In general the forecasts from the

MS-GARCH models fare significantly better towards the end of the forecast period. This highlights that the incorporation of macroeconomic information has the most potential for improvement in longer range forecasts. This is, of course, entirely reasonable, as macroeconomic circumstances do not tend to change at very short notice and also considering the well documented success of standard volatility models for short-range forecasts.

[add Figure 6 about here]

[add Figure 7 about here]

[add Figure 8 about here]

[add Figure 9 about here]

These forecast exercises clearly demonstrate the value of tying the unconditional volatility forecast to macroeconomic information. In all examples used here the volatility forecasts based on the MS-GARCH models show some qualitative improvement when compared to a standard volatility model that treats the level of unconditional volatility to be constant. Especially when one is interested in long range volatility forecasts it appears to be important to allow for the explanatory power of macroeconomic information. Neither standard volatility models nor the SPLINE GARCH model are capable of achieving this. At this stage the MS GARCH model presented here or the MIDAS based GARCH model by Engle, Ghysels and Sohn (2007) are the only volatility models that cater for these considerations.

7 Conclusion

This paper extends the recent advances in volatility modeling proposed by Engle and Rangel (2005), where a slowly moving unconditional volatility is incorporated into a GARCH type model. The extension offered in this paper is to make this unconditional volatility directly dependant on macroeconomic information in a MS (Modified Spline) GARCH model. The MS GARCH model is applied to the S&P500 share index data and shows a number of macroeconomic variables prove to have significant explanatory power for explaining variation in unconditional volatility. Empirical evidence suggests that information in the yield curve is particularly useful for capturing the behavior of unconditional volatility.

It is further demonstrated that the MS GARCH specification is a natural forecasting tool. It is straightforward to incorporate forecasts for macroeconomic variables into the volatility forecasts

for the share index returns. Using the particular example at hand, it can be seen that the forecasts generated from this model differ significantly from those of standard GARCH models. Moreover they appear to significantly better capture the movements of the volatility process over the forecast period than the GARCH forecasts.

On a more general level, this paper illustrates how low frequency information data can be put to use in a model for high frequency variables. The work presented here used this information to allow for a time varying level of unconditional volatility, but it is straightforward to let the unconditional mean of a high frequency process be determined by low frequency information.

Notes

¹The extra 4 parameters can be determined thanks to additional smoothness constraints imposed on the spline function.

²It should be noted that macroeconomic data used here are only one example of lower frequency data that can be incorporated into a high frequency volatility model. When building a volatility model for an individual stock one may want to consider firm-specific accounting information that is released infrequently.

³Eventually one may even consider to estimate the gridpoint locations as parameters.

⁴While interest rates are also available at higher frequencies we take the view that they essentially reflect macroeconomic factors and hence they are dealt with in the same manner as the remainder of the macroeconomic variables that are only observable on a quarterly basis.

⁵The extension of GJR GARCH models to include a spline function capturing time variation in the unconditional volatility is straightforward.

⁶As the spline is essentially an interpolation algorithm it is advisable to set $t_{k+1} = T^*$. It is well known that the behaviour of the spline outside the last knotpoint may be unreasonable. This is, in fact, the reason why the standard *SPLINE GARCH* is not suitable for out-of-sample forecasts.

⁷For the period in which the OECD data and our in-sample data overlap the in-sample yield curve slope is on average 20 basis points larger than that reported in the OECD Economic Outlook. Hence the OECD forecast for the yield curve slope has been adjusted accordingly. The OECD does not produce an explicit forecast for corporate interest rates. The forecast for $idc_{t_{k+1}}$ was generated such that its recent change reflects that in $idy_{t_{k+1}}$.

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	GARCH		SPLINE GARCH			
		5 knots	10 knots	12 knots	15 knots	20 knots
μ	$5.0E - 4$ [8.16]	$5.0E - 4$ [8.16]	$5.0E - 4$ [8.61]	$5.1E - 4$ [8.39]	$5.1E - 4$ [8.11]	$5.1E - 4$ [7.94]
β_0	$5.0E - 7$ [8.16]					
β_1	0.082 [575]	0.091 [15.81]	0.091 [17.33]	0.093 [16.48]	0.094 [15.49]	0.094 [15.25]
β_2	0.915 [3.6E7]	0.893 [131]	0.884 [120]	0.874 [109]	0.862 [96.80]	0.860 [88.09]
<i>LLik</i>	52796.2	52839.7	52862.7	52888.6	52906.6	52912.3
<i>SIC</i>	-8.7022	-8.7087	-8.7117	-8.7157	-8.7180	-8.7183
p	4	8	13	15	18	23

Table 1: Parameter estimates for GARCH(1,1) and SPLINE GARCH model. p is the number of parameters to be estimated. t-statistics in brackets.

	$x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$						
	Aggregation						
<i>Sampling</i>	1	2	4	6	8	12	16
4	-8.7069	-8.7074	-8.7085	-8.7096	-8.7074	-8.7075	-8.7092
2	-8.7116	-8.7135	-8.7115	-8.7129	-8.7100	-8.7084	-8.7073
1	-8.7082	-8.7088	-8.7088	-8.7078	-8.7076	-8.7073	-8.7078
	$x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, \pi_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$						
	Aggregation						
<i>Sampling</i>	1	2	4	6	8	12	16
4	-8.7081	-8.7082	-8.7092	-8.7094	-8.7073	-8.7077	-8.7103
2	-8.7116	-8.7133	-8.7115	-8.7129	-8.7099	-8.7082	-8.7071
1	-8.7081	-8.7088	-8.7088	-8.7078	-8.7075	-8.7072	-8.7078

Table 2: Schwarz Information Criterion for models which differ in their sets of explanatory variables, number of knotpoints (sampling interval in years) and their aggregation level (in quarters).

MS GARCH				
	Model <i>A</i>	Model <i>B</i>	Model <i>C</i>	Model <i>D</i>
μ	0.0005	0.0005	0.0005	0.0005
β_1	0.0915	0.0898	0.915	0.0897
β_2	0.8745	0.8840	0.8745	0.8842
γ_0	-11.0596	-10.9766	-11.0594	-10.9722
Δgdp	0.6745	0.0199	0.6747	0.0122
Δip	-0.5784		-0.5788	
π	-0.0018	0.0430		
<i>is</i>	0.2309	0.1689	0.2317	0.1908
<i>idy</i>	-0.2941	-0.2264	-0.2944	-0.2312
<i>idc</i>	0.4427	0.5176	0.4431	0.5263
<i>LLik</i>	52869.20	52851.67	52869.19	52851.50
<i>SIC</i>	-8.7133	-8.7106	-8.7135	-8.7107
<i>agg</i>	2	2	2	2
<i>p</i>	10	9	9	8
<i>k</i>	23	23	23	23

Table 3: Parameter estimates for MS GARCH models. p is the number of parameters to be estimated. k is the number of knotpoints used in the spline and agg indicates the aggregation level of the macroeconomic variables used (in quarters). Coefficient estimates in bold face are significant at a 1 per cent significance level.

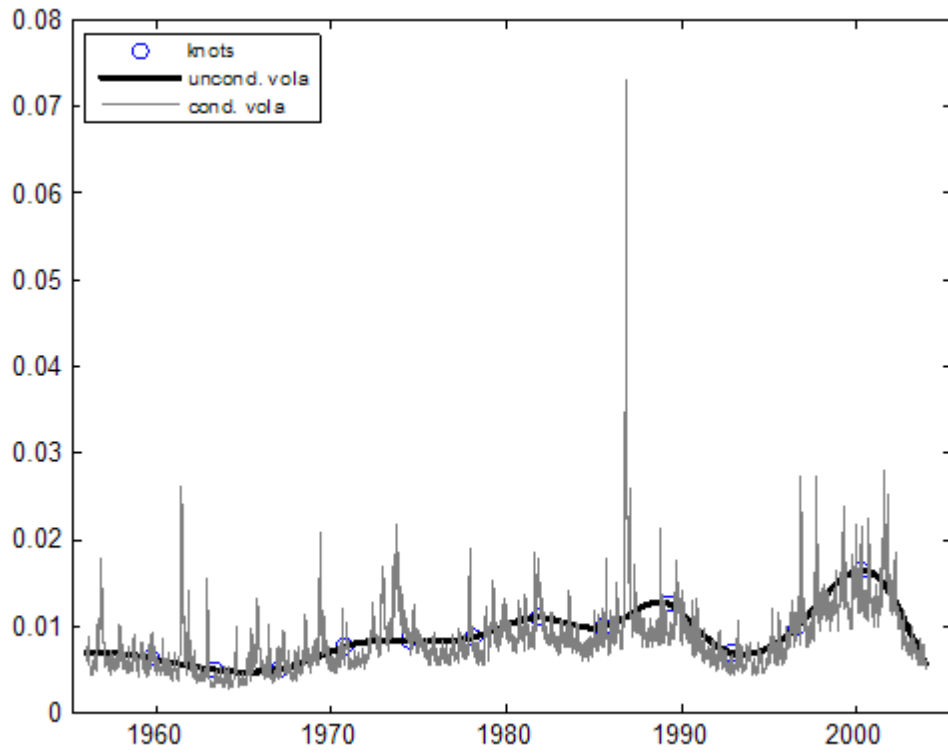


Figure 1: Conditional and unconditional variance from the SPLINE GARCH model with 12 knot-points.

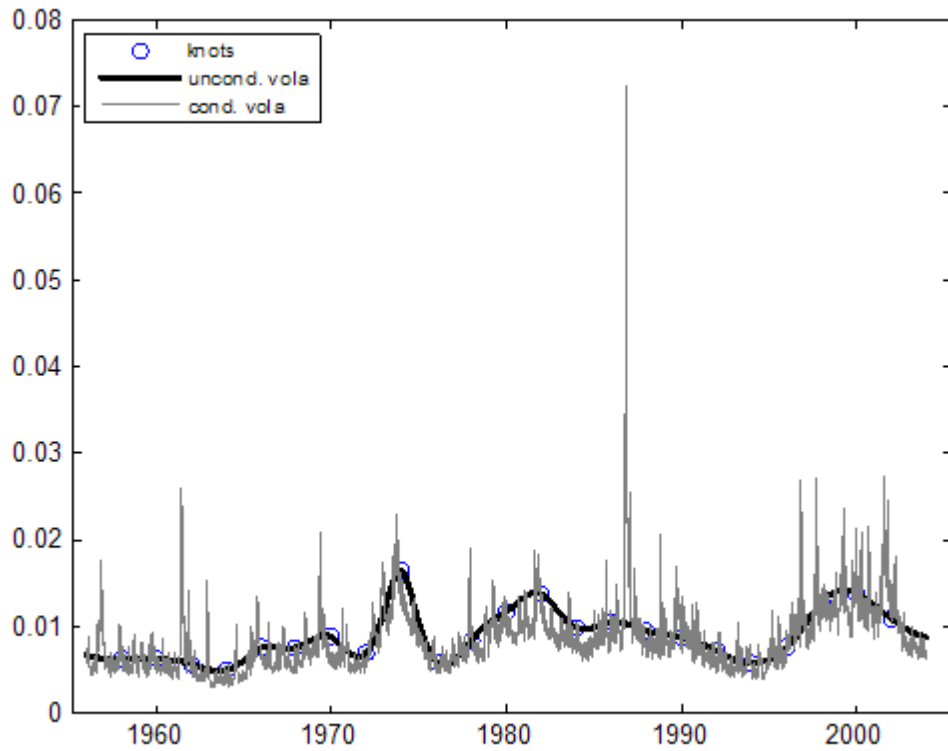


Figure 2: MS Garch model with 5 level variables ($x_{t_i} = (1, \Delta gdp_{t_i}, \Delta ip_{t_i}, is_{t_i}, idy_{t_i}, idc_{t_i})$). Sample = 2. Aggregation = 2. This models corresponds to the results reported in the third column of Table 3.

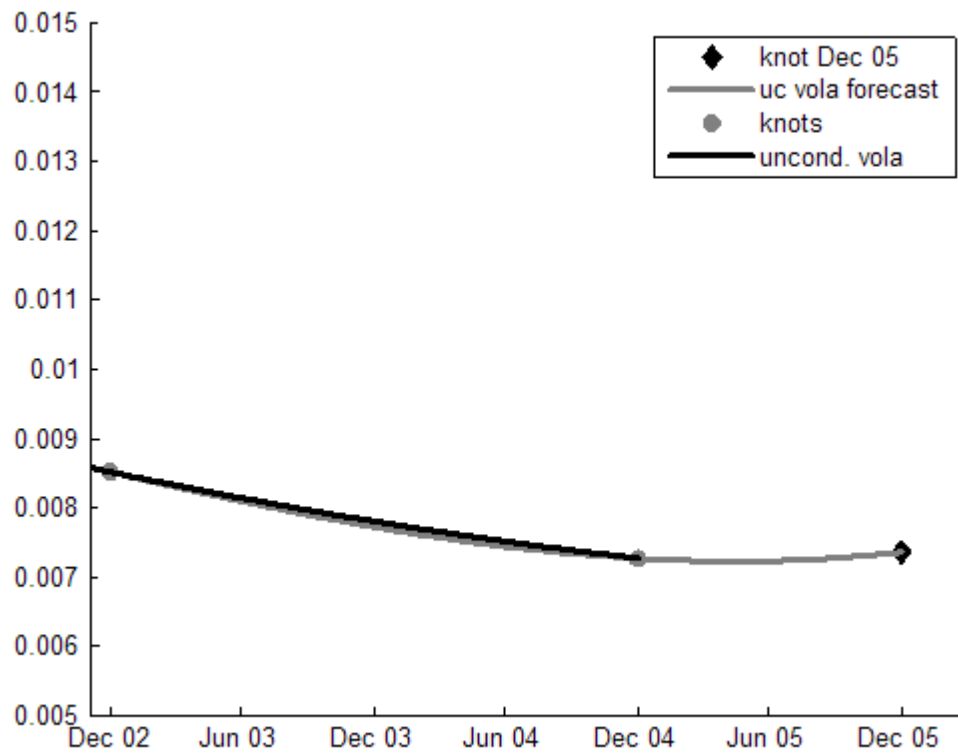


Figure 3: Plot of in-sample unconditional MS GARCH volatility (black line), knotpoint forecast for the year-end 2005 and the unconditional volatility forecast (grey line).

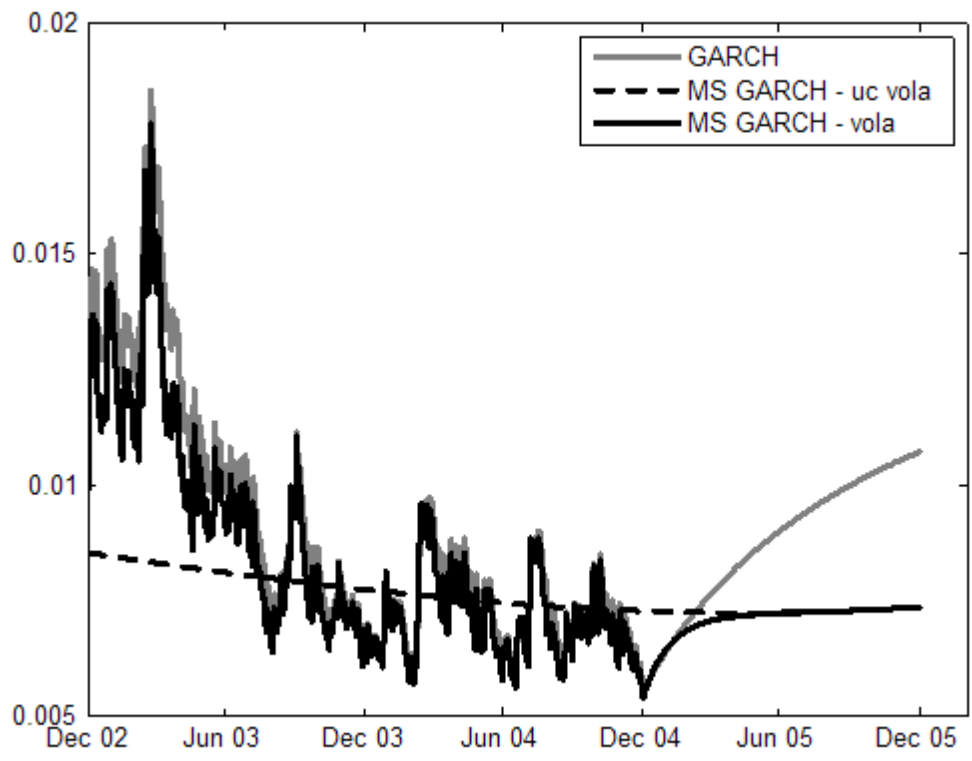


Figure 4: Plot of unconditional volatility forecast (heavy line), conditional volatility (solid line) and GARCH volatility forecast (dashed line). These are forecasts for the year 2005.

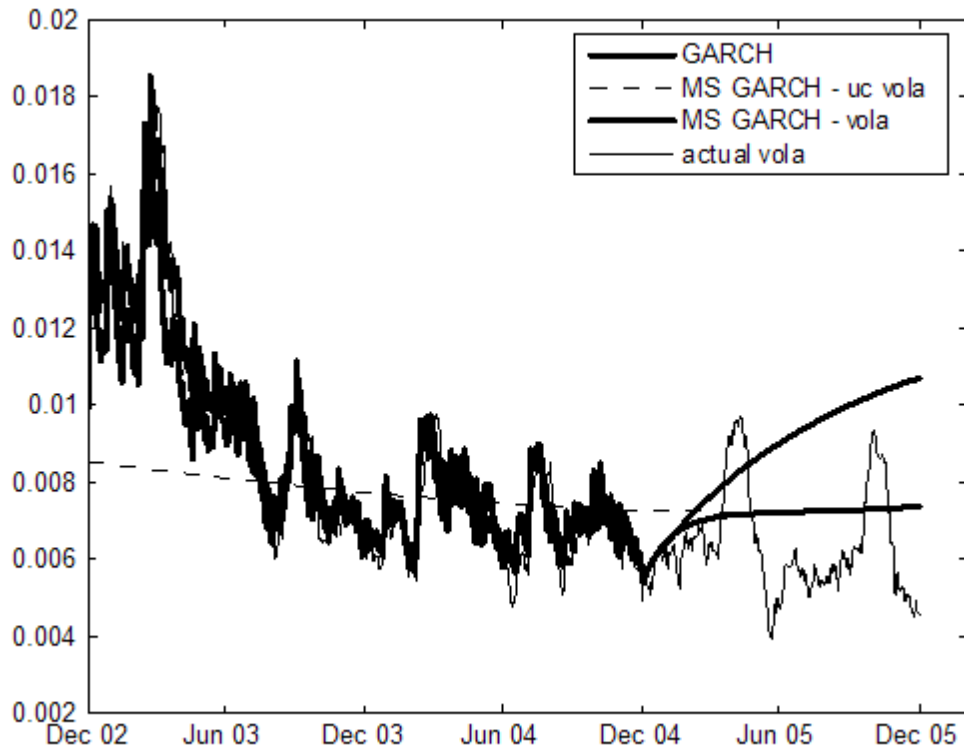


Figure 5: Plot of unconditional MS GARCH volatility forecast (dashed line), conditional volatility (heavy black line), GARCH volatility forecast (heavy grey line) and 22 day moving average volatility (light line). These are forecasts are for the year 2005.

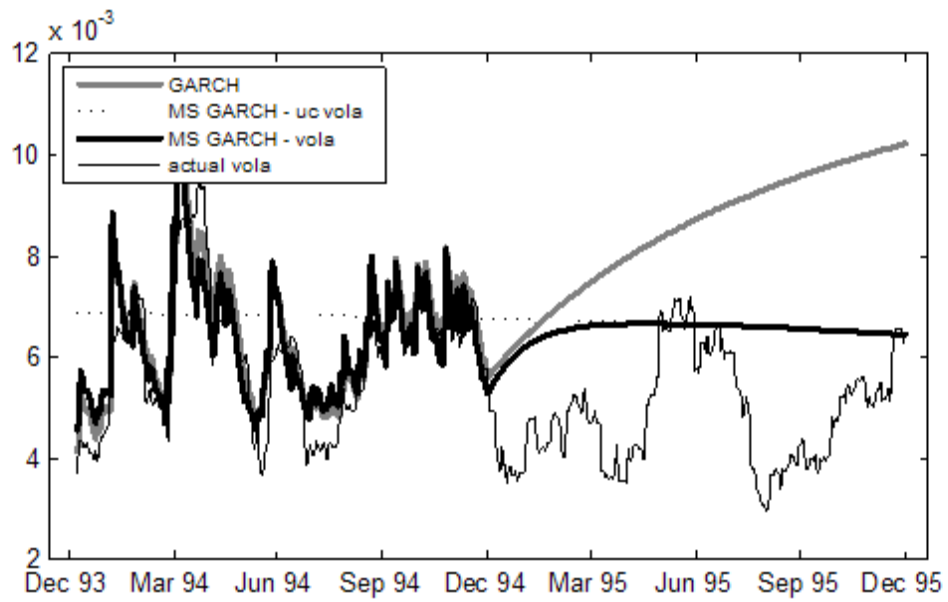


Figure 6: MS GARCH forecast of unconditional volatility (dotted line), conditional volatility (solid black line), GARCH volatility (grey line) and 22 day moving average of volatility. These forecasts are in-sample forecasts for the year 1995.

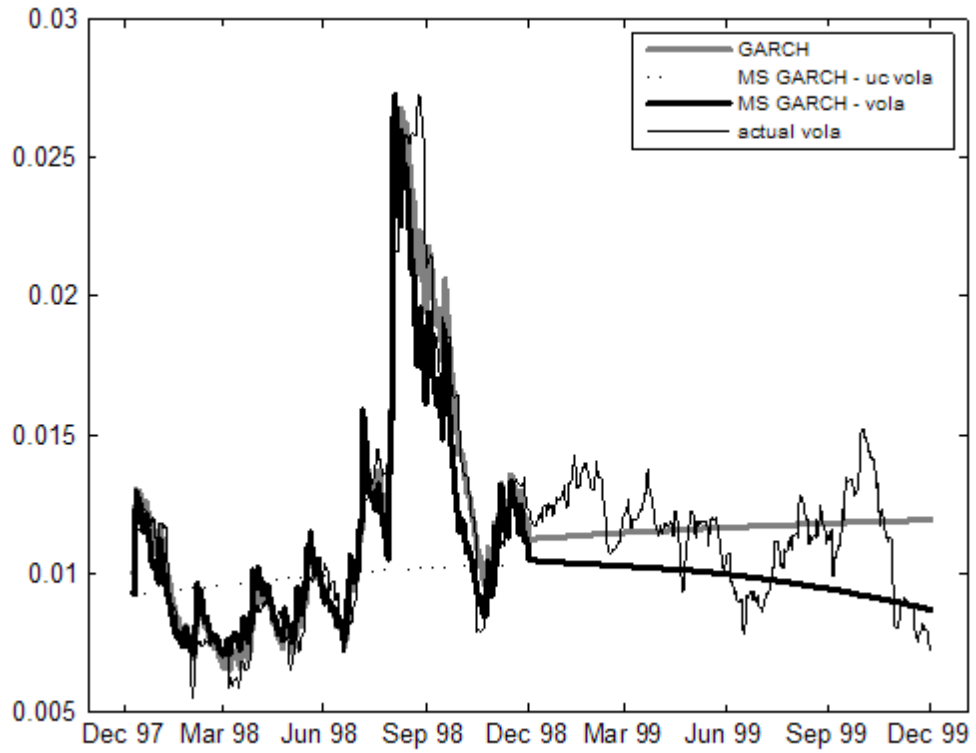


Figure 7: MS GARCH forecast of unconditional volatility (dotted line), conditional volatility (solid black line), GARCH volatility (grey line) and 22 day moving average of volatility. These forecasts are in-sample forecasts for the year 1999.

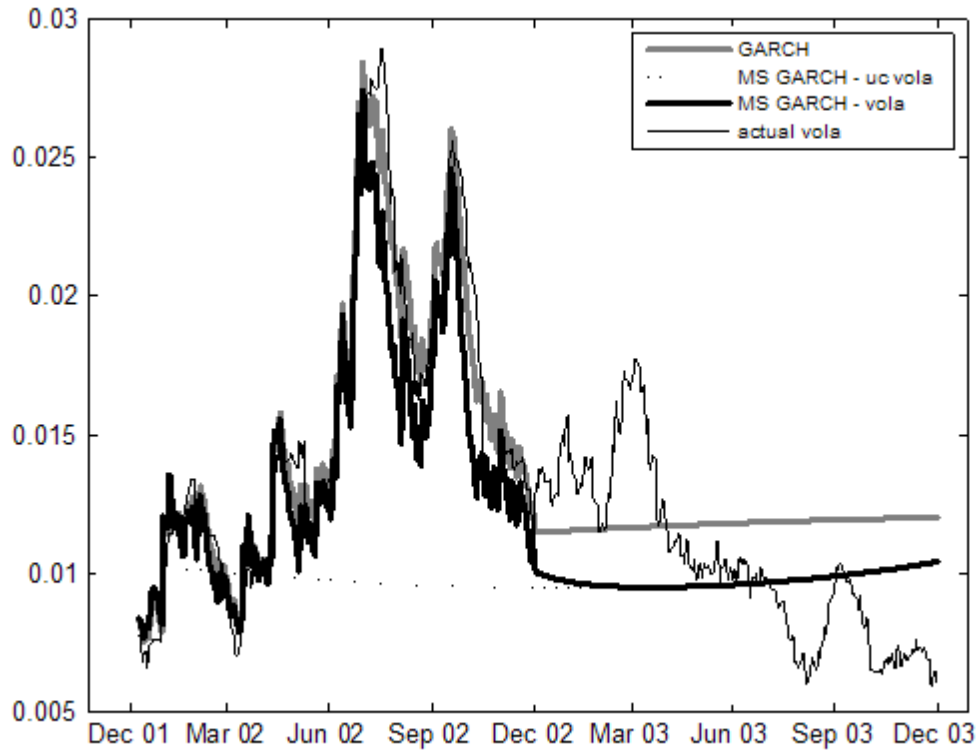


Figure 8: MS GARCH forecast of unconditional volatility (dotted line), conditional volatility (solid black line), GARCH volatility (grey line) and 22 day moving average of volatility. These forecasts are in-sample forecasts for the year 2003.

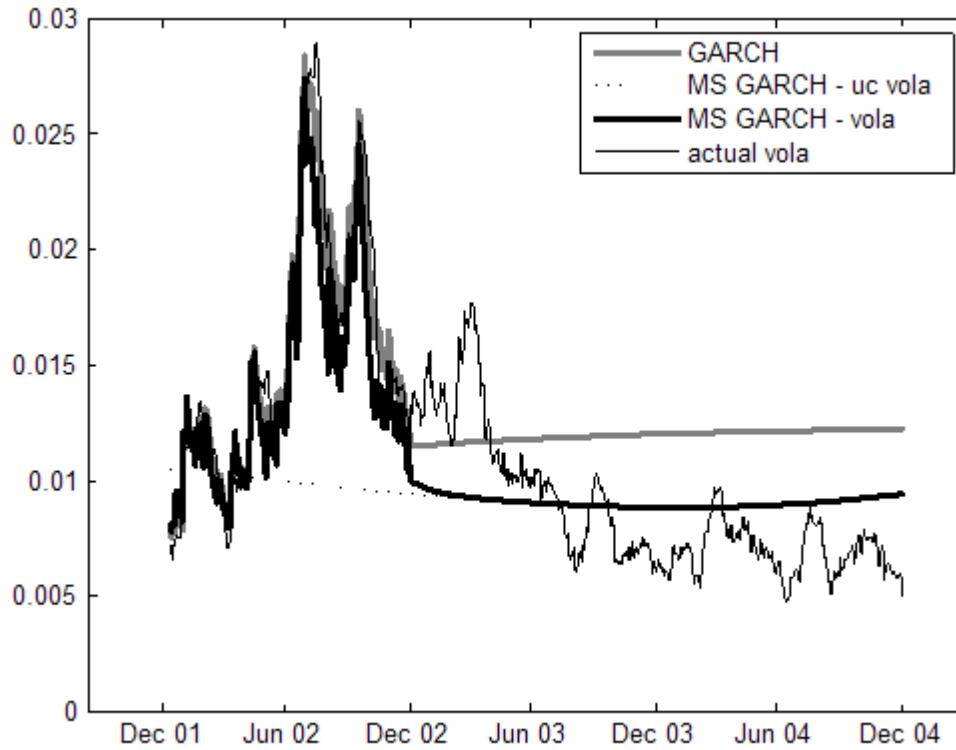


Figure 9: MS GARCH forecast of unconditional volatility (dotted line), conditional volatility (solid black line), GARCH volatility (grey line) and 22 day moving average of volatility. These forecasts are in-sample forecasts for the years 2003 and 2004.