ENDOUGENOUS AMENITIES AND THE SPATIAL STRUCTURE OF CITIES

First Draft

ABSTRACT

This paper examines the endogenous dynamics of the social structure of a city where the spatial repartition of amenities is endogenously modified by the spatial repartition of social groups. We start from the well known fact that, in most European cities, central locations are occupied by rich households; while in American cities, they are occupied by poor households. In a standard urban model without amenities, for rich households to locate downtown, their unit transport cost must be very high compared to the poor. Bruckner and alii (1999) show that, when there are historical amenities mainly located in the city center, we no longer need a high differential between transport costs: if demand for amenities by the rich is strong enough, this advantage could attract the rich households in the city centre. This explanation fits well with the fact that the most European cities have a long history, with the consequence that they accumulated many amenities in their city centre.

However, the paper by Brueckner and alii is purely static and does not explicitly consider the historical dimension of the process generating amenities. Our model explicitly takes account of time: at every period, the equilibrium spatial structure of the city is determined by the transport costs and by the spatial repartition of amenities; but, between periods, the spatial repartition of amenities changes, rich households generating local amenities in the locations they occupy, and then the spatial structure of the city changes.

We show that with endogenous generation of local amenities, when the city develops, it may move from an American equilibrium to an European one. If the city starts without amenities, poor households locate in the city centre, rich households in the periphery. However, the production of new local amenities by the rich generates a lock in effect: rich go on occupying locations where they were living previously and, as the city develops, these locations become central ones.

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1. INTRODUCTION

The spatial structure of the European cities is different from that of the North-American cities. The European cities are characterized by a central localization of the rich households and a localization of the poor towards the periphery. In contrast, the majority of the American cities know an opposite scheme of localization: he poor live in the central areas and the rich person in suburbs.

A possible explanation on this difference was proposed by Brueckner & alii (1999), based on the theory of the local amenities. The European cities are characterized by a longer history. Many of their centres have a strong advantage in terms of amenities compared the peripheries (presence of the monuments, the parks, the boulevards, fine architectures, etc) which are the consequence of this history. If the demand of the rich households for the amenities is significant, such an advantage can be sufficient to attract the rich households towards the central localizations, which corresponds well to their social structure.

The explanation provided by Brueckner & alii (1999) appears insufficient because historical dimension is not explicitly taken into account. If one takes into account this one, this leads to the idea that the social structure of the European cities is the consequence of a locking phenomenon: the rich households live in the centre of the cities because they remain in localizations which were peripheral at the beginning, but that the urban development transformed into central localizations.

Thus, the historical development plays an essential role in the formation of the European urban structures. From where, the need to create a dynamic model which explains the formation of the social structure of a city and the impact played by the amenities.

According to Brueckner & alii (1999) the urban amenities are divided into three categories: natural amenities (which are generated by the topographic characteristics of the area), historical amenities (generated by the monuments, the buildings, the parks or the other urban infrastructures which hold of the past) and modern amenities (which depend on the current conditions). The natural and historical amenities are exogenous and the modern amenities are endogenous.

In our paper, we suppose that the natural amenities do not cause differentiation of urban space and we are interested only in the effects of the modern and historical amenities on the city's structure. The historical amenities are not exogenous any more, but given in the model. To endogenize the historical amenities it is necessary to build a dynamic model.

Our model belongs to the models without durability of the capital. This type of models was developed initially by Alonso (1964), Mills (1967) and Muth (1969) within a static framework.

The basic assumption is that at each period, in the districts where are localised the rich households, but also in their vicinity, the level of amenities increases (modern amenities), this increase being added at the level of amenities inherited from the previous periods (i.e. the modern amenities become historical amenities). In same time, the amenities decrease in the rich areas, near the poor areas, because their proximity constitutes a desamenity for the rich person.

The intuition behind this formalization is the historical evolution of many the European cities. Thus, initially, the rich households live rather towards periphery, but in the course of time, they remain there, and as the city increases, these areas do not represent more the periphery of the city.

In a first part we present the theoretical model, with these assumptions. Since the model is not solvable analytically, we are making a series of numerical simulations. The last part is devoted to the conclusions.

The main result of paper is the fact that the historical development of a city, translated by the formation of located amenities, plays a role determining in the space structuring of the cities.

2 THE MODEL

2.1 Assumptions

We created a simple model, purely residential, where the connection between periods is given by the transformation of the modern amenities into historical amenities.

We remain in the monocentric urban models tradition (Alonso, 1964) where the CBD (Central Business District) is represented by a point in space and the only variable of localization of the households is the distance to the centre (x).

There are two social classes, the rich and the poor households, differentiated by their income respectively y_1 and y_2 and by their preferences for the amenities. The utility of the households depends on the consumption of the composite good (z), whose price is standardized with the unit, on the living space (s) and the level of amenities ($a(x)$). We are

using a Cobb-Douglas utility function $U_i\left(z,s,a^t(x)\right)=z^{\alpha}s^{\beta}\left(a^t(x)\right)^{\gamma_i}$, with $\alpha+\beta=1$. The rich households have stronger preferences for the amenities than the poor $(\gamma_1>\gamma_2)$ and we pose $\gamma_1=\gamma$ and $\gamma_2=0$. This assumption is explained by the fact that we regard the amenities as a higher good. The transportation cost is linear with the distance and identical for the two social categories: $C_i^t(x)=c^t x$. We choose identical costs in order to avoid the effect of the differentiated transportation costs on the structure of the city and to highlight the role played by the amenities.

At the first period, the amenities are constant (the city is located in a perfectly plane plain, without topographic specificities). The amenities level at the time *t* depends on their past level (historical amenities) and on the localization of the rich and poor households (modern amenities). Thus, in the rich areas and their proximity, the amenities are increasing. We can explain this assumption by a better quality of the buildings, but also of the environment (public investments, fine architecture, localization of the theatres, museums, parks etc). But since for the rich households, the proximity of the poor households constitutes a desamenity, the amenities in the rich areas decrease in the vicinity of the poor areas.

We are placed in an open city framework (there are no costs of migration): the utility level of each category is exogenous, equal at the national level (u^t_i) and the population of the city at each time is given in the model

2.2. Model equations

At the first period, there are no amenities, which corresponds to the standard urban models in the tradition of Alonso (1964). Fujita (1980) shows that with identical transportation costs of the two categories, the bid-functions are decreasing with the distance to the centre. Thereafter, there is only one point of segregation x_s and the city's border x_f is unique. This situation corresponds to the localisation scheme of the American cities: the rich households live the periphery and the poor the downtown.

Our model is structured like a succession of static equilibriums. At each period we determine the equilibrium localization of each social category and the effects on the amenities level. These effects will be taken into account during the following time and will have an influence in the new decisions of localization.

At each period, the households maximize their utility under budgetary constraint:

$$\max_{z,s,x} U_i(z,s,a^t(x))=z^{\alpha}s^{\beta}a^t(x)^{\gamma_i} \qquad \text{s. c.} \qquad y_i^t-C_i^t(x)=z+R^t(x)s$$

where $C_i^t(x)$ is the commuting cost to (CBD) and $R^t(x)$ is la market land rent at period *t*.

At equilibrium, each household will reach a utility equal to the national level u^t_i. We define the bid-function as the maximum price per unit of surface which the household can pay to reside at distance *x* to reach a level of utility $u^t_{i:}$:

$$\psi_i^t(x,u_i^t) = \max\left\{ \frac{\mathrm{y}_i^t - C_i^t(x) - z}{s} \,|\, U(z,s,a^t(x)) = u_i^t \right\} \tag{1}$$

By the resolution of maximization (1) one obtains the bid-function and the bid-surface functions:

$$\psi_i^t(x,u_i^t) = A\left(y_i^t - c^t x\right)^{1/\beta} a^t(x)^{\gamma_i/\beta} (u_i^t)^{-1/\beta} \tag{2}$$

$$S_{\mathrm{i}}^t(x,u_i^t) = \alpha^{-\alpha/\beta}\left(y_i^t - c^t x\right)^{-\alpha/\beta} a^t(x)^{-\gamma_i/\beta} (u_i^t)^{1/\beta} \tag{3}$$

where $A = \beta\alpha^{\alpha/\beta}$.

The structure of the city will be the result of competition for the land between the various uses (residential, agricultural). Each localization will be occupied by the strongest bidder. Thus, the urban rent will be the higher envelope of the bid-funtions and the agricultural rent (the opportunity cost of the land):

$$R^t(x) = \max\left\{\psi_i^t(x), RA^t\right\} \tag{4}$$

where RA^t is the agricultural rent or the opportunity cost of the land at period *t*.

The points of segregation between social classes are given by the solution of equalization of the bid-functions:

$$x_s^t \equiv \mathrm{sol}\left\{\psi_1^{\mathrm{t}}(x) = \psi_2^{\mathrm{t}}(x)\right\} \Rightarrow x_s^t = \mathrm{sol}\left\{ \frac{y_1^t - c^t x}{y_2^t - c^t x}\left(a^t(x)\right)^{\gamma} = \frac{u_1^t}{u_2^t} \right\} \tag{5}$$

We define a binary spatial variable *K*(*x*) to show the social category of the household which lives at distance *x* of centre:

$$K(x) = \begin{cases} 1, & \text{si } \ \psi_1^t(x) > \psi_2^t(x) \\ 2, & \text{sinon} \end{cases}$$

The border of the city is determined by the equalization of the bid-function of the category localised in the peripheral area of the city and the agricultural rent:

$$x_f^t \equiv \mathrm{sol}\left\{\psi_{K(x)}^{\mathrm{t}}(x) = RA^t\right\} \Rightarrow x_f^t = \mathrm{sol}\left\{ \left(y_{K(x_f)}^t - c^t x\right)\left(a^t(x_f)\right)^{\gamma} = \left(\frac{RA}{A}\right)^{\beta} u_{K(x_f)}^t \right\} \tag{6}$$

It is supposed that the land is allocated entirely to the residential use. Since our city is in a perfectly plane area, surface available for the residences to distance *x* is given by the perimeter of the circle $L(x) = 2\pi x$. The population which lives at distance *x* of CBD is determined by the ratio of the surface available for the residences and the size of each house:

$$n^t(x) = \begin{cases} \dfrac{2\pi x}{S^t_{K(x)}(x)}, & x \in \left[0, x^t_f\right] \\ 0, & x > x^t_f \end{cases} \tag{7}$$

The density of the population at distance *x* is defined as the number of households per unit of surface:

$$\rho^t(x) = \begin{cases} \dfrac{n^t(x)}{2\pi x} = \dfrac{1}{S^t_{K(x)}(x)}, & x \in \left[0, x^t_f\right] \\ 0, & x > x^t_f \end{cases} \tag{8}$$

The total population of the city is the sum of the number of the households located at each distance from centre: $N^t = \int_0^{x^t_f} n^t(x)\mathrm{d}x$. We can distinguish the population from each category ($N^t = N^t_1 + N^t_2$), where N^t_1 is the rich population and N^t_2 the poor population:

$$N^t_1 = \int_0^{x^t_f} \left(2 - K(x)\right) n^t(x)\ \mathrm{d}x = 2\pi \int_0^{x^t_f} \left(2 - K(x)\right) \frac{x}{S^t_{K(x)}(x)}\ \mathrm{d}x \tag{9}$$

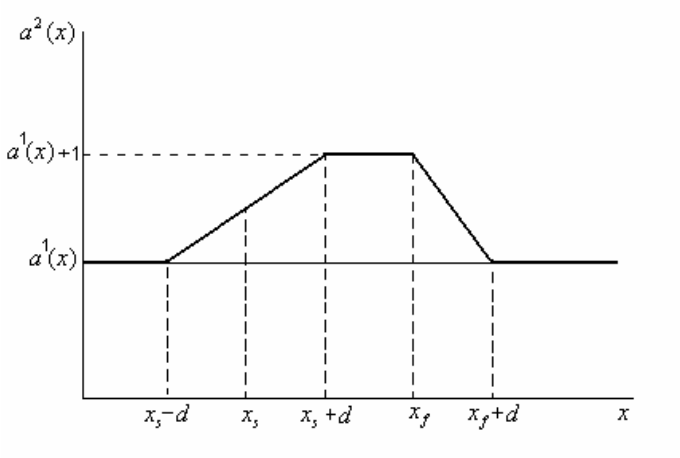
$$N^t_2 = \int_0^{x^t_f} \left(K(x) - 1\right) n^t(x)\mathrm{d}x = 2\pi \int_0^{x^t_f} \left(K(x) - 1\right) \frac{x}{S^t_{K(x)}(x)}\ \mathrm{d}x \tag{10}$$

The key of the model, which makes the connection between periods, is amenities function $a^t(x)$. At the first period the amenities are constant $a^t(x)=a$. During the period $t+1$, in the rich zones, the amenities increase with a unit compared to the previous period. These amenities also increase in the proximity of these areas. The more one moves away from the rich areas, the more the positive effect of those decreases. We are making the assumption that this reduction is linear with the distance. It is supposed that *d* represents the distance where one does not feel any more the positive externalities.

The amenities are influenced negatively by the proximity of the poor areas. Thus, the level of amenities starts to decrease not at the point of segregation but a certain front distance. To simplify the writing of the model, we consider this distance equal to *d* what is called the proximity distance (the maximum distance where one feels the effect of proximity between the social categories).

For example, if there is only one point of segregation x_s (the city is made up only by two completely segregated areas) and if the rich households live in the peripheral area, the amenities function at the second period is represented graphically:

Graphic 1 : Amenity function at the second period



The dissymmetry of the function of amenities in graph 1 is explained by the fact that in the proximity of the segregation point $x \in [x_s - d, x_s + d]$, there is a double effect: the amenities also increase in the poor area, because of with the proximity of rich households $x \in [x_s - d, x_s]$, but there is a negative effect in the rich area, because of with the proximity of the poor households $x \in [x_s, x_s + d]$. At the outside of the city $x \in [x_f, x_f + d]$, since there is no proximity with the poor households, the only effect is the presence of the rich person

With this modelling the amenities are unlimited. To solve this problem, it is supposed that they suffer a constant depreciation at a fixed rate $\delta, (0 < \delta < 1)$. Thus, the amenity function with constant depreciation, when there is J points of segregation, is:

$$a^{t+1}(x) = \begin{cases} (1-\delta)a^t(x) + (1-K(x)), & \text{zones without proximity} \\ \max\left\{ \begin{array}{l} (1-\delta)a^t(x) + \dfrac{x + d - x_s^t(j)}{2d} \\ (1-\delta)a^t(x) + \dfrac{x_s^t(j) - x + d}{2d} \end{array} \right\}, & \text{proximity zones} \\ \left(2 - K(x_f^t)\right)\left((1-\delta)a^t(x) + 1 + \dfrac{x_f^t - x}{d} \right), & \text{out of the city} \end{cases} \quad (11)$$

In the zones where there are no proximity effects the amenities will be $(1-\delta)a^t(x)+1$ in the rich zones and $(1-\delta)a^t(x)$ the poor zones. Inside the proximity zones, the amenities

increase/decrease linearly, if the rich zone is outside/inside of the segregation point. Finally, if the farthest zone from the centre is occupied by the rich households, the amenities will also increase across the city border, but the negative effect of the poor proximity disappears. This increase decrease linearly from the border of the city: $(1-\delta)a^t(x)+1+\frac{x_f^t-x}{d}$.

We are noting that if a rich zone is surrounded by the poor, the amenities are symmetrical: the two effects of proximity are identical from the both sides. This symmetry is lost when the rich households occupy the farther zone from the centre, since there's no more the negative effect of the poor proximity.

With this formalization, the amenities will be limited, and their maximum level in a stationary state will be reached in the areas which were lived successively by the rich households. Thus, after an infinity of periods, this maximum level is $1/\delta$ [1]

3. NUMERICAL SIMULATIONS

In our simulations, we are analyzing two different scenarios: European and American urban structure. The European scenario shows that for the same set of parameters, the city can evolve from an American social structure to a mixed structure (rich person-poor-rich person) to arrive at a European localization scheme. The second simulation shows that city can be "blocked" in a social structure of American type.

What differentiate the two series of simulations are the values of the certain key parameters. According to Tivadar and Jayet (2006) the long term European equilibrium is more restrictive than the American equilibrium and the factors which support the existence of the European type exploit two dimensions. First, there are the factors which increase the role played by the amenities in the space structuring: strong preference of the rich households for these amenities and a weak depreciation of the amenities what leads to high stationary level. The other factors have a direct impact on the biddings of the two categories, by increasing those of the rich households compared to the poor: strong difference between the incomes, a level of utility raised for the poor and weak for the rich households.

[1] To determine that, we put the condition that the amenities where they increase more (in the rich areas) remain constant:: $a^{t+1}(x)=a^t(x)\Leftrightarrow(1-\delta)a^*(x)+1=a^*(x)\Leftrightarrow a^*(x)=1/\delta$

In our simulations we will especially exploit the preferences of the rich households for amenities. Very strong preferences for amenities determine a European social structure and weak preferences an American urban structure.

3.1. The reference scenario: European urban structure

The "European" equilibrium represents a social urban structure characterized by a central localization of the rich households and a peripheral localization of the poor households.

3.1.1. The parameters

In our simulation, the exogenous variables are defined as $\mu_t = \mu_{t-1} + \tau_\mu t^{-\varepsilon}$, where μ is the variable in question (income, utility level and the agricultural rent), τ_μ represents the increment rate of the variables μ, t is the period of simulation, and ε $(0 \le \varepsilon \le 1)$ is a parameter which influences the speed of variation in time: a raised value of ε means a slow evolution of the exogenous variables. For $\varepsilon = 0$, there is a linear evolution. This formalization was chosen so that the variables know a concave evolution in time. The transportation costs are constant and identical for the two categories, in order to avoid an influence of those in the localization of the households in time.

Table 1: Parameters value in the European scenario

Parameter	**Rich Households**	**Poor Households**
$\alpha / \beta / \gamma$	0,6 / 0,4 / 0,45	0,6 / 0,4 / 0
Income	100	90
Income increment rate	5	4,95
Utility level	10	7
Utility increment rate	0,5	0,3
Agricultural rent	3	
Increment rate of the agricultural rent	0,1	
Depreciation rate of amenities	0,10	
Proximity distance	5	
Amenity level at first period	1.5	
ε	1/8	

The income is expressed in K/period. The utility levels were determined to obtain a satisfactory solution. The proximity distance corresponds to 50 meters.

Whatever their initial level, the amenities will be limited at $1/\delta$, which will be stationary state. For a depreciation rate of 10% the limit of the amenities is 10. At the beginning of each period the amenities decrease by 10%. We are finding the equilibrium before passing to the following period and we apply the procedure of increase of the amenities level, function of the households' localization in the city.

We choose $\gamma_2 = 0$ (the poor do not have preferences for the amenities) because we regard these amenities as a higher good. Moreover, if the rich person and the poor have the same preferences for the amenities ($\gamma_1 = \gamma_2$), in this case the rich households will always occupy the peripheral area, because the amenities do not play any more any role.

In this scenario the rich households' preferences for amenities are strong (thus they will be attracted by the areas occupied before by the rich households). Whatever the initial level of the amenities, they have the same level at steady state. We choose an initial level higher than the unit so that the two categories are present in the city. We choose a rapid evolution the exogenous variables (a low value of ε). With this set of parameters the amenities will play a significant role in the structuring of urban space.

3.1.2. The results

At the beginning, since the amenities are constant, the city is divided in two areas according to an American structure: the central part is occupied by the poor households and periphery by the rich households (see Appendix 1.1). All the functions have the characteristics specific to the standard urban models: decreasing bid-functions and urban rent with the distance to centre. The surface of the residences is increasing and the density of the population is decreasing, with a point of discontinuity for these two functions at the segregation point. After this period the amenities increase in the area occupied by the rich person and in his proximity, but with a negative effect towards the centre, because of the poor vicinity. We note a strong increase in the amenities level in the peripheral area of the city which was inhabited successively by the rich households.

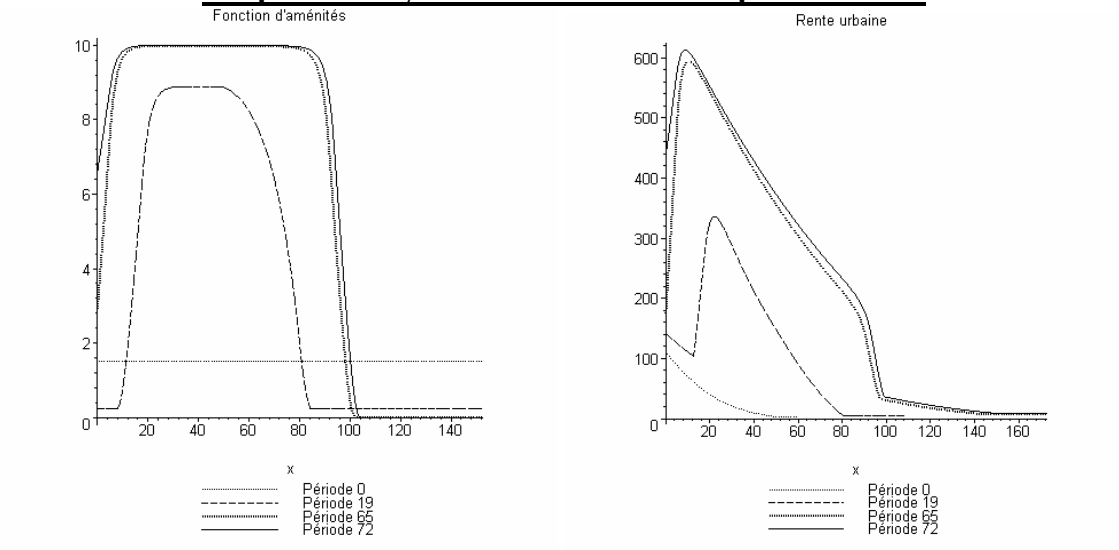
With time, the size of the city increases. The new rich households are attracted by the areas where, owing to the fact that their predecessors lived there or were localised in the vicinity, the amenities are higher. The poor are remaining in the centre. But, wedged by the ring occupied by the rich households, they pile up until a part of them have interest to leave in periphery The city is now made up of three areas: poor – rich – poor (Appendix 1.2). The

functions do not have any more the usual forms. The rich bin-function presents a strong increase in the rich area, according to their preferences for the amenities. They are ready to yield a part of their living space, which determines a concentration of the population.

More and more new poor households are delocalized towards the periphery, while the extension of the area occupied by the rich households gradually leads them to absorb the central area. So, we find a European city with two areas, the rich living the centre now whereas the poor are in periphery. There is a strong comparative advantage in terms of amenities of the centre against the periphery (Appendix 1.3). This process of segregation is reinforced in time (Appendix 1.4) with a stronger differentiation between the two areas. The bid-functions and the amenities take a form very close to that at steady state (see Tivadar and Jayet, 2006).

The changes of urban social structure slow down, arriving at very weak variations. This evolution is also explained by the formalization of the exogenous variables, which vary in a concave way and in time their increase slowed down. Practically, into terms of structure, the city does not change any more (Appendices 1.3 and 1.4) and we note a stronger differentiation between the poor area rich person and areas, in terms of rent, density of the population and amenities. We can consider this structure close of that at long-term equilibrium, because the amenities are very close to their stationary state and the exogenous variables vary very little and less and less.

Graphic 2: City's evolution in the European scenario



In graphic 2, we represent on the same graph the evolution of the principal characteristics of the city. This representation is very useful because it enables us to see the "locking" effect of the rich households. Their initial localization is peripheral, but by the

evolution of the city, their area does not represent any more the external one of the city, but the centre. The urban rent strongly increases in the rich area.

3.2 American scenario

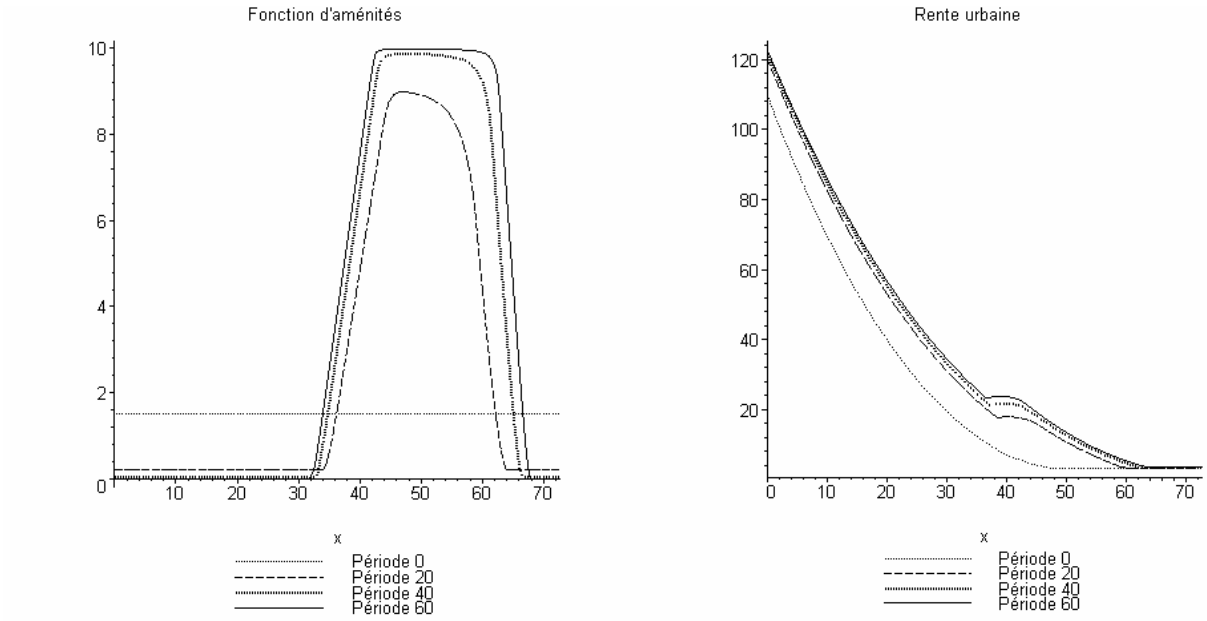
An American urban structure, represent a spatial structure characterized by a complete segregation between the rich and the poor households, with a central localization of the poor households and a peripheral localization of the rich households.

To arrive at this equilibrium, we start from the reference simulation and we strongly decreases the role played by the amenities in the structuring of space (by decreasing the preference of the rich person for the amenities) and we slowed down the evolution of the exogenous variables:

Table 2: Parameters changes for the American scenario

Parameter	Rich Households	Poor Households
γ	0,15	0
ε	1	

Graphic 3: City's evolution in the American scenario



For this play of the parameters we find the same tendency in the localizations as for the reference simulation, but the amenities cannot change the localization of the two social classes (Appendices 2.1-2.4). Thus the city is completely segregated in two areas: the poor centre and rich periphery. The amenities are concentrated in the peripheral area of the city, where the rich households are. They know a deceleration in their evolution arriving close to

their stationary state. The rent increases in time, more accentuated in the rich area. It is noted that at the last period of simulation (Appendix 2.4) the form of the bid-functions and the function of amenities is very close to the steady state form (see Tivadar and Jayet, 2006).

4. CONCLUSIONS

The main result of paper is the fact that the historical development of a city, translated by the formation of located amenities, plays a determining role in the social structuring of the urban space.

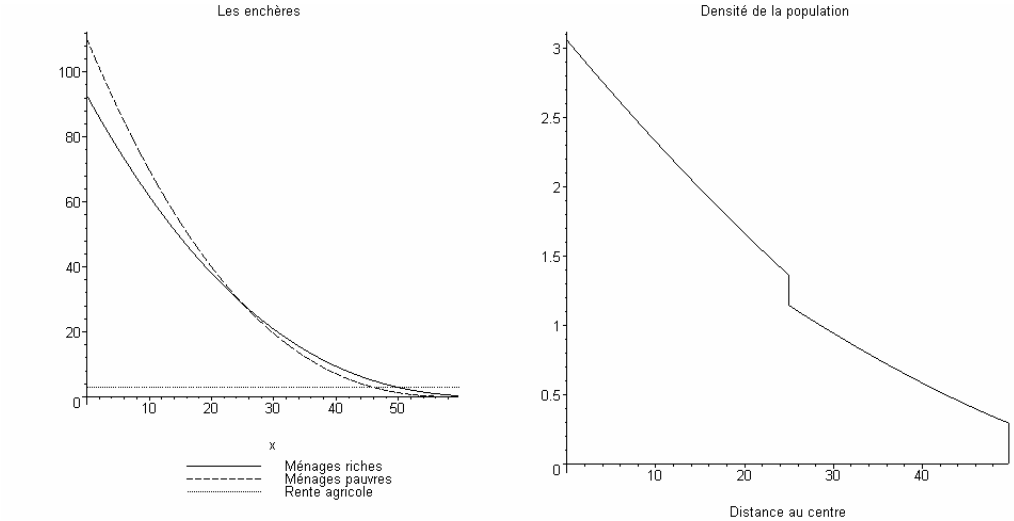
In our simulations, we note that the amenities alone can transform the structure of a city and that this process occurs in time. A possible explanation for the difference between the spatial structure of the European cities and the North-American cities is the fact that the last are more recent and they did not arrive yet in their stationary state. This situation corresponds to the reference scenario. At the beginning the city has weak amenities and the rich households locate towards the periphery, while the poor households occupy the central area of the city. This type of social structure corresponds to the North-American cities, whose their history is recent, and thus, the amenities are very weak. On the contrary, the European cities are much more “old”. Thus, initially the rich households were localised in the periphery, but because the city experienced a long development, the rich person remained there, which constitutes today the central area of the city. Since the localization of the rich households determines an increase in the amenities level, the European centres have a strong comparative advantage in terms of amenities compared to the suburbs.

Another possible explanation of contrasts between the American and European cities can be simply a difference in preferences, incomes and/or utility levels of the households. In the case with weak preferences of the rich households for amenities, in the long run, the rich households are located towards the periphery, while the poor households occupy the central area of the city. This type of social structure corresponds to the North-American cities. The fact that the preferences of the rich households for the amenities are weak makes the rich to continue to occupy the periphery of the city, and so the amenities will be concentrated there.

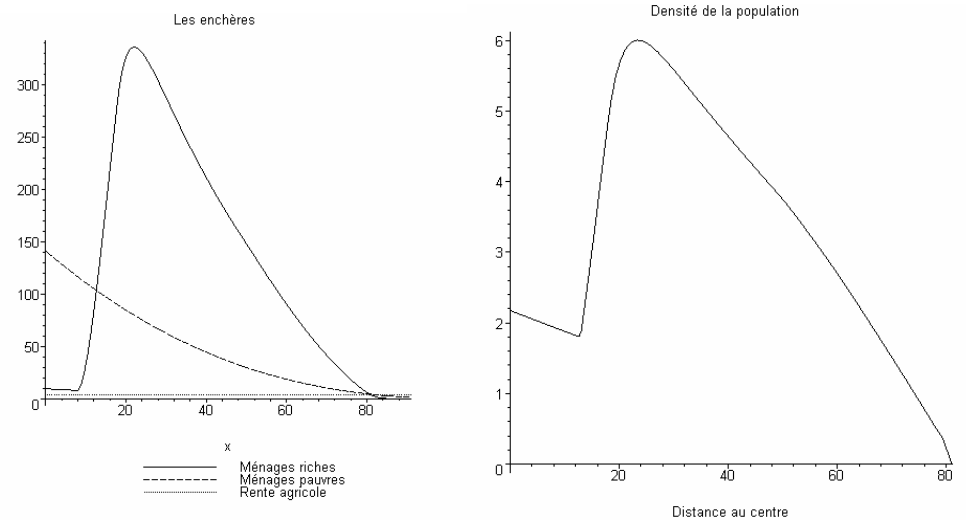
Appendix: Graphic results of the simulations

1. Reference scenario: European social structure

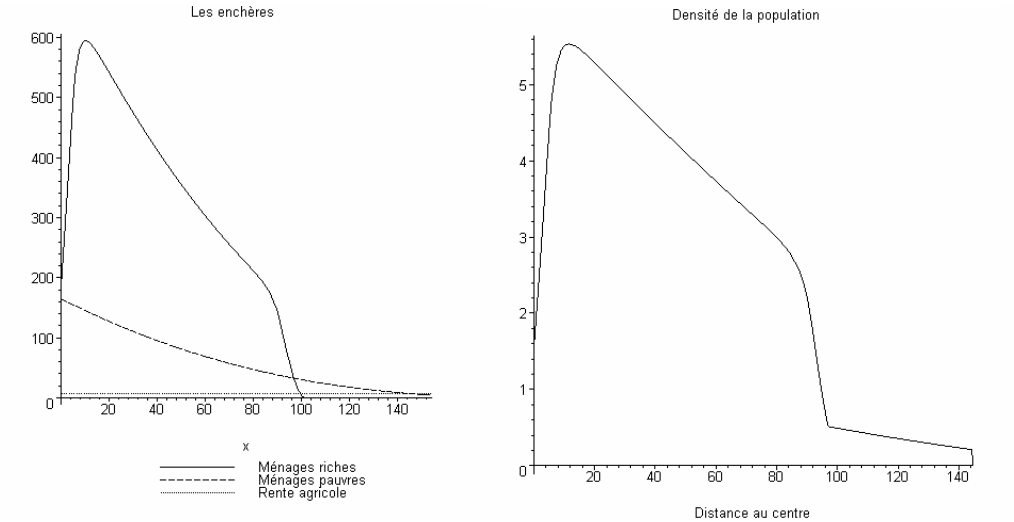
1.1. First period



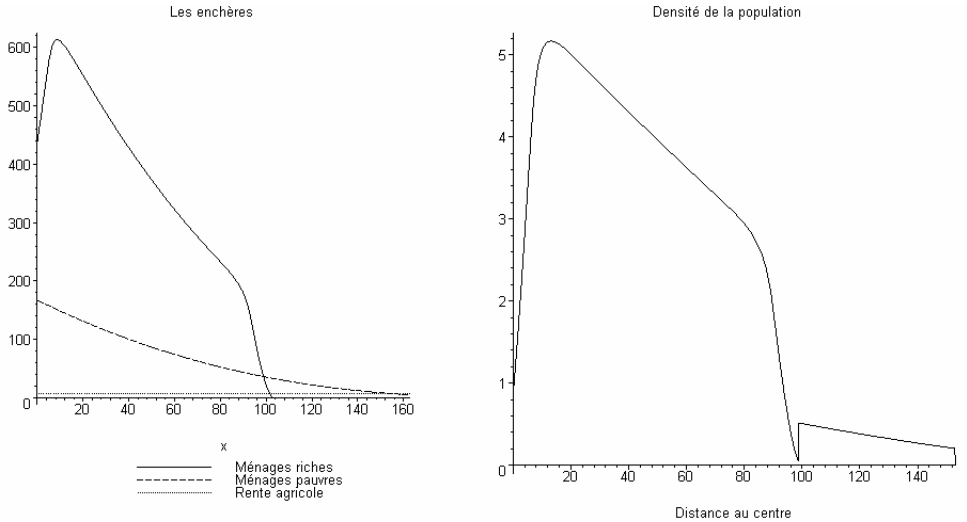
1.2. The city at 19-th period



1.3. The city at 65-th period

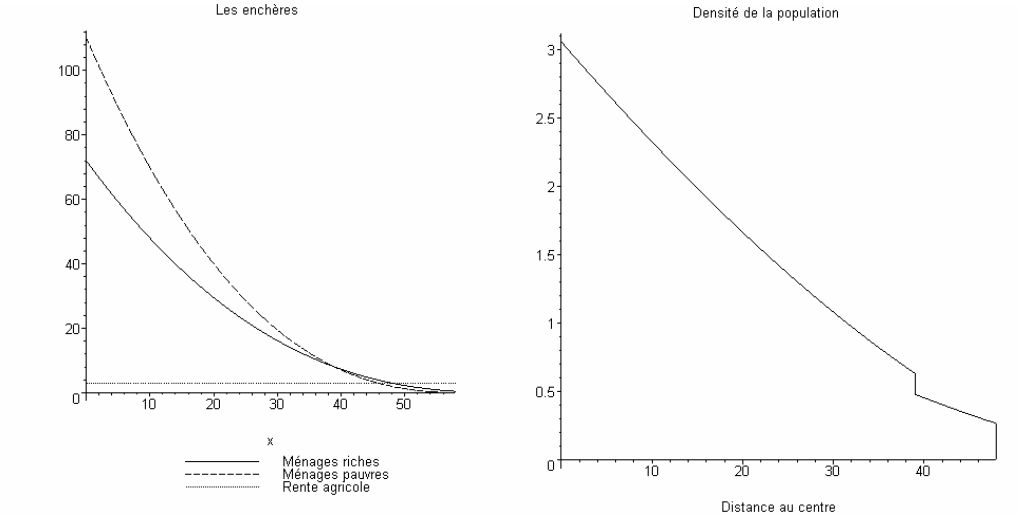


1.4. The city at 72-th period

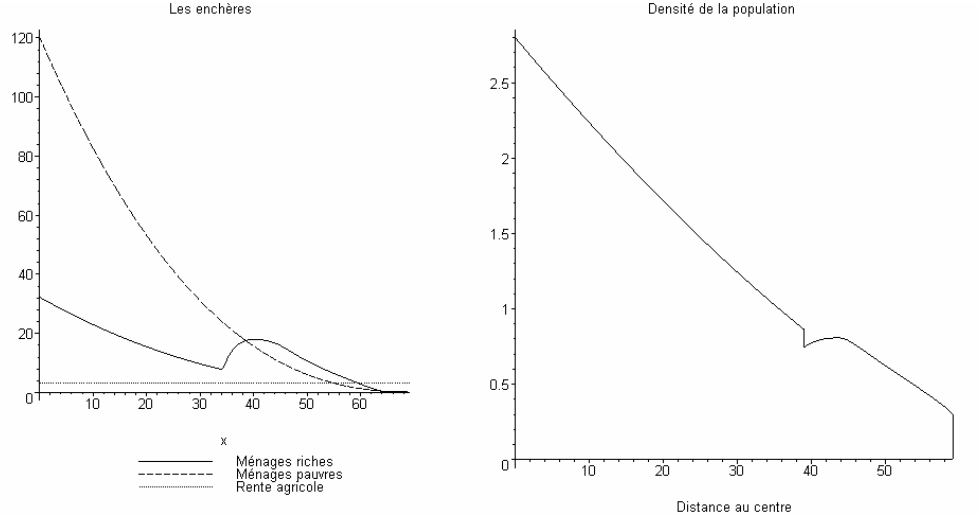


2. Second scenario: American social structure

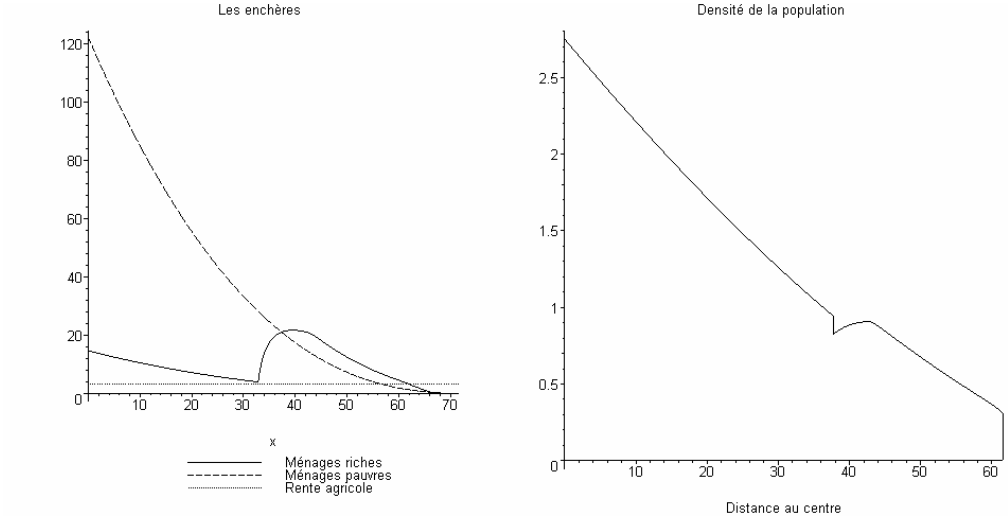
2.1. First period



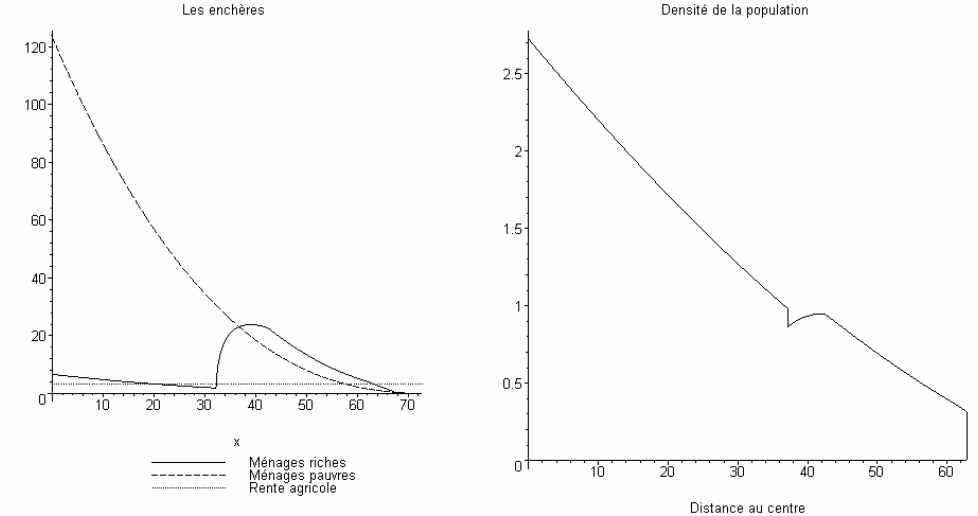
2.2. The city at 20-th period



2.3. The city at 40-th period



2.4. The city at 60-th period



BIBLIOGRAPHY

Alonso, William (1964): "Location and Land Use", *Cambridge, Harvard University Press*

Amin, K and Capozza, C.R. (1993): "Sequential Development", *Journal of Urban Economics, Volume 34, Pages 142-158*

Anas, Alex (1978): "Dynamics of Urban Residential Growth", *Journal of Urban Economics, Volume 5, Pages 66-87*

Brueckner K. Jan (1980): "A Vintage Model of Urban Growth", *Journal of Urban Economics, Volume 8, Pages 389-402*

Brueckner K. Jan (1981): "A Dynamic Model of Housing Production", *Journal of Urban Economics, Volume 10, Pages 1-14*

Brueckner K. Jan, Thisse Jacques-François and Zenou Yves (1999) : "Why is central Paris rich and downtown Detroit poor?: An amenity-based theory"**,** *European Economic Review, Volume 43, Pages 91-107*

Brueckner K. Jan (2000): "Urban Growth Models with Durable Housing: An Overview", *Economics of Cities – Theoretical Perspectives, Chapitre 7, Pages 263-289*

Capozza, C.R. and Helsley, R.W. (1989): "The Fundamentals of Land Prices and Urban Growth", *Journal of Urban Economics, Volume 26, Pages 295-306*

Capozza, C.R. and Helsley, R.W. (1990): "The Stochastic City", *Journal of Urban Economics, Volume 28, Pages 187-203*

Fujita, Masahisa (1989): "Economic Theory: Land Use and City Size", *Cambridge University Press*

Mills, E.S.(1967): "An Aggregative Model of Resource Allocation in a Metropolitan Area", *American Economic Review, Volume 57, Pages 197-210*

Miyao, Takahiro (1979): "Dynamic Stability of An Open City with many households classes", *Journal of Urban Economics, Volume 6, Pages 292-298*

Miyao, Takahiro (1987): "Dynamic Urban Models", *Handbook of Regional Science and Urban Economics , Volume 2, Chapitre 22, Pages 877-925*

Muth, R.F. (1969): "Cities and Housing", *Chicago, University of Chicago Press*

O'Flaherty, B (1993): "Abandoned Buildings: A Stochastic Analysis", *Journal of Urban Economics, Volume 34, Pages 43-74*

Tivadar, Mihai and JAYER, Hubert (2006): "Where is better to live: in a European or American City", Work in progress