WHERE IS BETTER TO LIVE: in a European or American City?

First Draft

Mihai TIVADAR

MEDEE Laboratory

University of Lille 1

Hubert JAYET

MEDEE Laboratory

University of Lille 1

ABSTRACT

This paper examines the endogenous dynamics of the social structure of a city where the spatial repartition of amenities is endogenously modified by the spatial repartition of social groups. We start from the well known fact that, in most European cities, central locations are occupied by rich households; while in American cities, they are occupied by poor households. In a standard urban model without amenities, for rich households to locate downtown, their unit transport cost must be very high compared to the poor. Bruckner and alii (1999) show that, when there are historical amenities mainly located in the city center, we no longer need a high differential between transport costs: if demand for amenities by the rich is strong enough, this advantage could attract the rich households in the city centre. This explanation fits well with the fact that the most European cities have a long history, with the consequence that they accumulated many amenities in their city centre.

However, the paper by Brueckner and alii is purely static and does not explicitly consider the historical dimension of the process generating amenities. Our model explicitly takes account of time: at every period, the equilibrium spatial structure of the city is determined by the transport costs and by the spatial repartition of amenities; but, between periods, the spatial repartition of amenities changes, rich households generating local amenities in the locations they occupy, and then the spatial structure of the city changes.

We show that the city may have several long term equilibria. We explicitly analyse two of them: an "American equilibrium" with the poor living in the centre and the rich in the periphery, and a "European equibrium" with the rich living in the centre and the poor in the periphery. We show that the conditions for the existence of a European equilibrium are more restrictive than the conditions for the existence of an American equilibrium. We compare the two equilibria from an efficiency point of view. The results show that an American structure is more efficient.

1. INTRODUCTION

Our paper examines the formation of the social structures of the cities where the space distribution of the amenities is modified in an endogenous way by the space distribution of the social classes. We start from the known fact that the majority of the European cities are characterized by a central localization of the rich households and a localization of the poor households towards the periphery. In contrast, the majority of the American cities know an opposite scheme of localization: the poor live in the central areas and the rich persons in suburbs.

In the standard urban models, the rich households are attracted by the central localizations of the city, when their transportation costs are much higher compared to the poor households. Another explanation was proposed by Brueckner & alii (1999), based on the theory of the local amenities. The European cities are characterized by a longer history. Many of their centers have a strong advantage in terms of amenities on the peripheries (monuments, parks, boulevards, fine architectures, etc), which are the consequence of this history. If the

demand of the rich households for the amenities is strong enough, such an advantage can be sufficient to attract the rich households towards the central localizations, which corresponds well to their social structure.

The model of Brueckner & alii (1999) is purely static and it does not take into account the historical dimension of the process which generates the amenities. Since the historical development plays an essential role in the formation of the urban structures, we have created a dynamic model: at each period, the equilibrium space structure is determined by the spatial distribution of the amenities. Between periods, this distribution changes, because the rich households generate amenities in the areas which they occupy, but also in their vicinity.

Our model belongs to the models without durability of the capital. This type of models was developed initially by Alonso (1964), Mills (1967) and Muth (1969) within a static framework. In our model, the basic assumption is that at each period, in the neighbourhoods where are localised the rich households, but also in their vicinity, the level of amenities increases (modern amenities), this increase being added at the level of amenities inherited from the previous period (the modern amenities become historical amenities). In same time, the amenities decrease in the rich neighbourhoods, near the poor areas, because their vicinity constitutes a desamenity for the rich households.

We show that the endogenous generation of amenities has like consequence the existence of several long term equilibria. We carry an analysis on two types of equilibria: an American equilibrium, with the poor living the center and the rich person periphery and a European equilibrium, with the rich households occupying the center and the poor periphery. The conditions of existence of a European equilibrium are more restrictive and included in the conditions necessary for an American equilibrium. We compare the two equilibria from an efficiency point of view. The results show that an American structure is more efficient

The first part presents the theoretical model, with these assumptions. Then we are analyzing the possibility of existence of multiple equilibria with an efficiency comparison. The last section is devoted to the conclusions.

2 THEORETICAL MODEL

2.1 Assumptions

We created a simple model, purely residential, where the connection between periods is given by the transformation of the modern amenities into historical amenities.

We place our model in the monocentric urban models tradition (Alonso, 1964) where the CBD (Central Business District) is represented by a point in space and the only spatial variable is the distance to the center.

There are two social classes, the rich and the poor households, which are differentiated by their income respectively y_1 and y_2 and by their preferences for the amenities. The utility of the households depends on the consumption of the composite good (z) whose price is standardized with the unit, on the living space (s) and on the local amenities level ($a(x)$). We are using a Cobb-Douglas utility function $U_i\left(z, s, a^t(x)\right) = z^\alpha s^\beta \left(a^t(x)\right)^{\gamma_i}$, where $\alpha + \beta = 1$. The rich households have stronger preferences for the amenities than the poor ($\gamma_1 > \gamma_2$) and by simplification purposes, we can pose $\gamma_1 = \gamma$ and $\gamma_2 = 0$. This assumption is explained by the fact that we the amenities are considered as a superior good. The transportation costs are linear to the distance and identical for the two social categories: $C_i^t(x) = c^t x$. We choose identical transportation costs in order to avoid the effects of the differentiated costs on the structure of the city and to highlight the role played by the amenities.

Our model is in an open-city framework (there are no migration costs): the utility level of each category is exogenous and equal at the national level (u_i^t) and the population of the city is endogenous.

2.2. Model equations

Our model is structured as a succession of static equilibiums. At each period we are analyzing the equilibrium localization of each social category and the effects on the amenities. These effects will be taken into account during the next periods and will have an influence in the new decisions of localization.

At each period, the households maximize their utility under budgetary constraint:

$$\max_{z,s,x} U_i(z, s, a^t(x)) = z^\alpha s^\beta a^t(x)^{\gamma_i} \qquad \text{b.c.} \qquad y_i^t - C_i^t(x) = z + R^t(x)s$$

where $C_i^t(x)$ is the commuting cost to CBD and $R^t(x)$ is the market urban rent at the period t.

At equilibrium, each household will reach a utility level equal to the national level u_i^t. We define the bid-function as the maximum price per space unit which the household can pay to reside at distance x by reaching the level of utility u_i^t:

$$\psi_i^t(x, u_i^t) = \max\left\{\frac{y_i^t - C_i^t(x) - z}{s} \,\middle|\, U(z, s, a^t(x)) = u_i^t\right\} \qquad (1)$$

By the resolution of this maximization, we are obtaining the bid-function and the bid-surface function:

$$\psi_i^t(x, u_i^t) = A\left(y_i^t - c^t x\right)^{1/\beta} a^t(x)^{\gamma_i/\beta} (u_i^t)^{-1/\beta} \qquad (2)$$

$$S_i^t(x, u_i^t) = \alpha^{-\alpha/\beta} \left(y_i^t - c^t x\right)^{-\alpha/\beta} a^t(x)^{-\gamma_i/\beta} (u_i^t)^{1/\beta} \qquad (3)$$

where $A = \beta \alpha^{\alpha/\beta}$.

The city's structure will be the result of competition for the space between the various uses (residential, agricultural). Each localization will be occupied by the strongest bidder. Thus, the urban rent will be the higher envelope of the bid-functions and the agricultural rent.

$$R^t(x) = \max\left\{\psi_i^t(x), RA^t\right\} \qquad (4)$$

where RA^t is the agricultural rent (the opportunity cost of land) at the period t.

The segregation points between social classes are the solution of the equality between the bid-functions:

$$x_s^t \equiv \text{sol}\left\{\psi_1^t(x) = \psi_2^t(x)\right\} \Rightarrow x_s^t = \text{sol}\left\{\frac{y_1^t - c^t x}{y_2^t - c^t x}\left(a^t(x)\right)^\gamma = \frac{u_1^t}{u_2^t}\right\} \qquad (5)$$

We are using a binary spatial variable $K(x)$ to define the social category of the household which lives at distance x of center:

$$K(x) = \begin{cases} 1, & \text{if } \psi_1^t(x) > \psi_2^t(x) \\ 2, & \text{if not} \end{cases}$$

The border of the city is determined where the bid-function of the category localised in the peripheral zone of the city is equal to the agricultural rent:

$$x_f^t \equiv \text{sol}\left\{\psi_{K(x)}^t(x) = RA^t\right\} \Rightarrow x_f^t = \text{sol}\left\{\left(y_{K(x_f)}^t - c^t x\right)\left(a^t(x_f)\right)^\gamma = \left(\frac{RA}{A}\right)^\beta u_{K(x_f)}^t\right\} \qquad (6)$$

The key of the model, which makes the connection between periods, is the amenities function $a^t(x)$. During the first period the amenities are constant $a^0(x)=a$. At each period $t+1$, in the zones occupied by the rich households, the amenities increase with a unit compared to the previous period. These amenities also increase in the vicinity of these zones. If we move away from the rich zones, the positive effect decreases. We are making the assumption that this decreasing is linear with the distance. It is considered that d represents the distance where there are no more positive externalities.

The amenities are influenced negatively by the proximity of the poor zones. Thus, the level of amenities starts to decrease not at the point of segregation but a certain front distance.

To simplify the writing of the model, we consider this distance equal to *d*, what we called the externality distance (the maximum distance where one feels the effect of vicinity between the social categories).

For example, if there is only one segregation point (the city is divided into two completely segregated zones) and if the rich households live in the peripheral zone, the amenity function at the second period is represented in Graphic 1.

The dissymmetry of the amenity function in graphic 1 is explained by the fact that in the vicinity of the segregation point $x \in [x_s - d, x_s + d]$, there is the double effect: the amenities also increase in the poor zone $x \in [x_s - d, x_s]$, because of the proximity of rich households, but there is a negative effect in the rich zone $x \in [x_s, x_s + d]$, because of the proximity of the poor households. At the outside of the city $x \in [x_f, x_f + d]$, since there is no proximity with the poor households, the only effect is the presence of the rich households.

Graphic 1 : The amenity function at the beginning of the second period
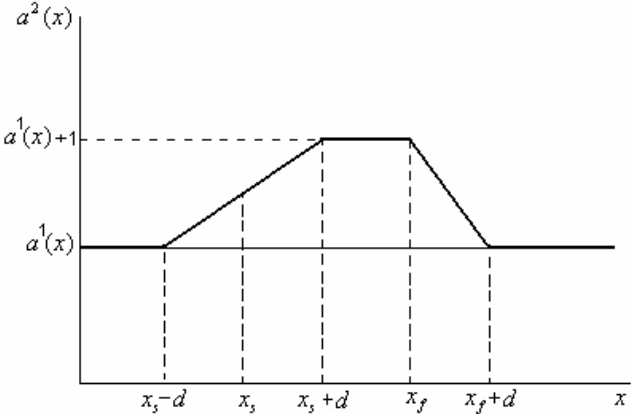
The dissymmetry of the function of amenities in graph 1 is explained by the fact that in the proximity of the segregation point $x \in [x_s - d, x_s + d]$, there is a double effect: the amenities also increase in the poor area, because of with the proximity of rich households $x \in [x_s - d, x_s]$, but there is a negative effect in the rich area, because of with the proximity of the poor households $x \in [x_s, x_s + d]$. At the outside of the city $x \in [x_f, x_f + d]$, since there is no proximity with the poor households, the only effect is the presence of the rich person

With this modelling the amenities are unlimited. To solve this problem, we supposed that they suffer a constant depreciation at a fixed rate $\delta, (0 < \delta < 1)$. Thus, the amenity function with constant depreciation, when there are *J* segregation points, is:

$$a^{t+1}(x) = \begin{cases} (1-\delta)a^t(x) + (1-K(x)), & \text{zones without proximity} \\ \max \left\{ \begin{array}{l} (1-\delta)a^t(x) + \dfrac{x+d-x_s^t(j)}{2d} \\ (1-\delta)a^t(x) + \dfrac{x_s^t(j)-x+d}{2d} \end{array} \right\}, & \text{proximity zones} \\ \left(2-K(x_f^t)\right)\left((1-\delta)a^t(x)+1+\dfrac{x_f^t-x}{d}\right), & \text{out of the city} \end{cases} \qquad (7)$$

In the zones where there are no proximity effects the amenities will be $(1-\delta)a^t(x)+1$

in the rich zones and $(1-\delta)a^t(x)$ the poor zones. Inside the proximity zones, the amenities

increase/decrease linearly, if the rich zone is outside/inside of the segregation point. Finally, if

the farthest zone from the center is occupied by the rich households, the amenities will also

increase across the city border, but the negative effect of the poor proximity disappears. This

increase decrease linearly from the border of the city: $(1-\delta)a^t(x)+1+\dfrac{x_f^t-x}{d}$.

We are noting that if a rich zone is surrounded by the poor, the amenities are

symmetrical: the two effects of proximity are identical from the both sides. This symmetry is

lost when the rich households occupy the farther zone from the center, since there's no more

the negative effect of the poor proximity.

3 STATIONNARY EQLUIBRIA STUDY

In this part we will study the possibility of existence of multiple equilibria. Since there

are an infinity possibilities of localization, we are interested only in two extreme cases: the

rich located in the center versus the poor located in the center. Concretely, we want to

determine under which conditions these two types of structure can be a long-run equilibrium.

To find the long-run equilibriums, we made the assumption that an infinite time passed.

3.1. Amenity function and bid-functions at stationary state

For a depreciation rate δ, the maximum level of the amenities (in the zones which was

inhabited successively by the rich households) is $1/\delta$ [1]. For example, for a fixed depreciation

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[1] To determine that, we put the condition that where the amenities are increasing more (in the rich zones) they
remain constant: $a^{t+1}(x) = a^t(x) \Leftrightarrow (1-\delta)a^*(x)+1 = a^*(x) \Leftrightarrow a^*(x) = 1/\delta$

of 10% the maximum level of the amenities is 10. In the zones inhabited successively by the poor households and which are not in the proximity of the rich zones, but also outside the city (except vicinity of a rich zone) the amenities suffered a continuous depreciation, thus they tend towards zero.

We will determine the amenity function in the proximity areas. Where the amenity function is increasing, the external area by the segregation point $x_s^*(j)$ is occupied by the rich households. The condition that the amenities are in a stationary state is $a^{t+1}(x) = a^t(x)$. By replacement and simplification one obtains:

$$a^*(x) = \frac{x + d - x_s^*(j)}{2\delta d}$$

By making the same steps in a proximity area where the amenity function is decreasing, the amenities in that zone are defined:

$$a^*(x) = \frac{x_s^*(j) + d - x}{2\delta d}$$

It remains us to determine the amenities at the stationary state outside the city, but in the proximity of the last populated area, if this area is inhabited by the rich households:

$$a^*(x) = \frac{x_f^* + d - x}{\delta d}$$

Thus, the amenity function of amenities at stationary state is:

$$a^*(x) = \begin{cases} \dfrac{(1 - K(x))}{\delta}, & \text{zones without proximity} \\[2ex] \max \left\{ \begin{array}{c} \dfrac{x + d - x_s^*(j)}{2\delta d} \\[1.5ex] \dfrac{x_s^*(j) + d - x}{2\delta d} \end{array} \right\}, & \text{proximity zones} \\[3ex] \left(1 - K(x_f^t)\right)\left(\dfrac{x_f^* + d - x}{\delta d}\right), & \text{out of the city} \end{cases} \qquad (8)$$

To simplify the notation, the steady state symbols of the variables are removed, but it is known that they are in their stationary state. For the poor households, the bid-function is easy to calculate, because they do not have preferences for the amenities:

$$\psi_2(x, u_2) = A\left(y_2 - cx\right)^{1/\beta} (u_2)^{-1/\beta}$$

It is a decreasing and convex function to the distance from the center. For the rich households, this function has a different shape, according to the form of the amenity function.

In the areas occupied successively by the poor households, and which are far away from the area rich person so that there are no amenities, the rich bid-function is null: $\psi_1^a(x) = 0$.

In the proximity areas where the amenities increase with the distance to the center, to determine the stationary rich bid-function, it is necessary to replace the steady amenity function in the bid-function of the rich households:

$$\psi_1^b(x, u_1) = A(2d\delta)^{-\gamma/\beta} (y_1 - cx)^{1/\beta} (x + d - x_s)^{\gamma/\beta} (u_1)^{-1/\beta}$$

There is a double effect which exploits the form of this function: a direct negative effect (the distance to the center) and a positive one, played by the increase of the level of the amenities. Thus, this function is increasing until $\tilde{x} = [\gamma_1 y_1 + c(x_s - d)]/c(\gamma_1 + 1)$ and decreasing after this value. One can check easily that $\tilde{x} > x_s - d$, but $\tilde{x} < x_s + d$ only if $y_1 < c(x_s + d + 2d/\gamma_1)$. Therefore the function of bidding in this interval is increasing and in certain situations (for example when the costs of transport are rather significant compared to the incomes and thus the distance effect carries on the amenities effect) it can be decreasing starting from a certain point $\tilde{x}$.

In the "rich" areas, where the amenities are on their maximum level, the biddings of the rich households take this form:

$$\psi_1^c(x, u_1) = A\delta^{-\gamma/\beta} (y_1 - cx)^{1/\beta} (u_1)^{-1/\beta}$$

One notes that this function equal the rich bid-function without amenities multiplied by a constant. Thus, the bid-function is decreasing with the distance to the center. If the most last area is occupied by the rich households, their biddings outside the city will be:

$$\psi_1^d(x, u_1) = A(d\delta)^{-\gamma/\beta} (y_1 - cx)^{1/\beta} (x_f + d - x)^{\gamma/\beta} (u_1)^{-1/\beta}$$

This function is decreasing, but more sloping than the preceding one, because the negative effect of the distance is reinforced by the reduction of the amenities. The rich bid-function in the proximity areas where the amenities are decreasing is:

$$\psi_1^e(x, u_1) = A(2\delta d)^{-\gamma/\beta} (y_1 - cx)^{1/\beta} (x_s + d - x)^{\gamma/\beta} (u_1)^{-1/\beta}$$

Now we can pass to the study of the steady state equilibria. We take the situation in which all the variables are stationary and we want to show that the two spatial configurations (American and European type) can be a long-run equilibrium. More exactly, we want to determine all the combinations u_1 and u_2 for which the two long-run equilibria are possible.

3.2. The existence of American type equilibrium

We will start with the existence of the American type equilibrium, with a central localization of the poor households and a peripheral localization of the rich households.

Graphic 2: The bid-functions and the amenities at American long-run equilibrium
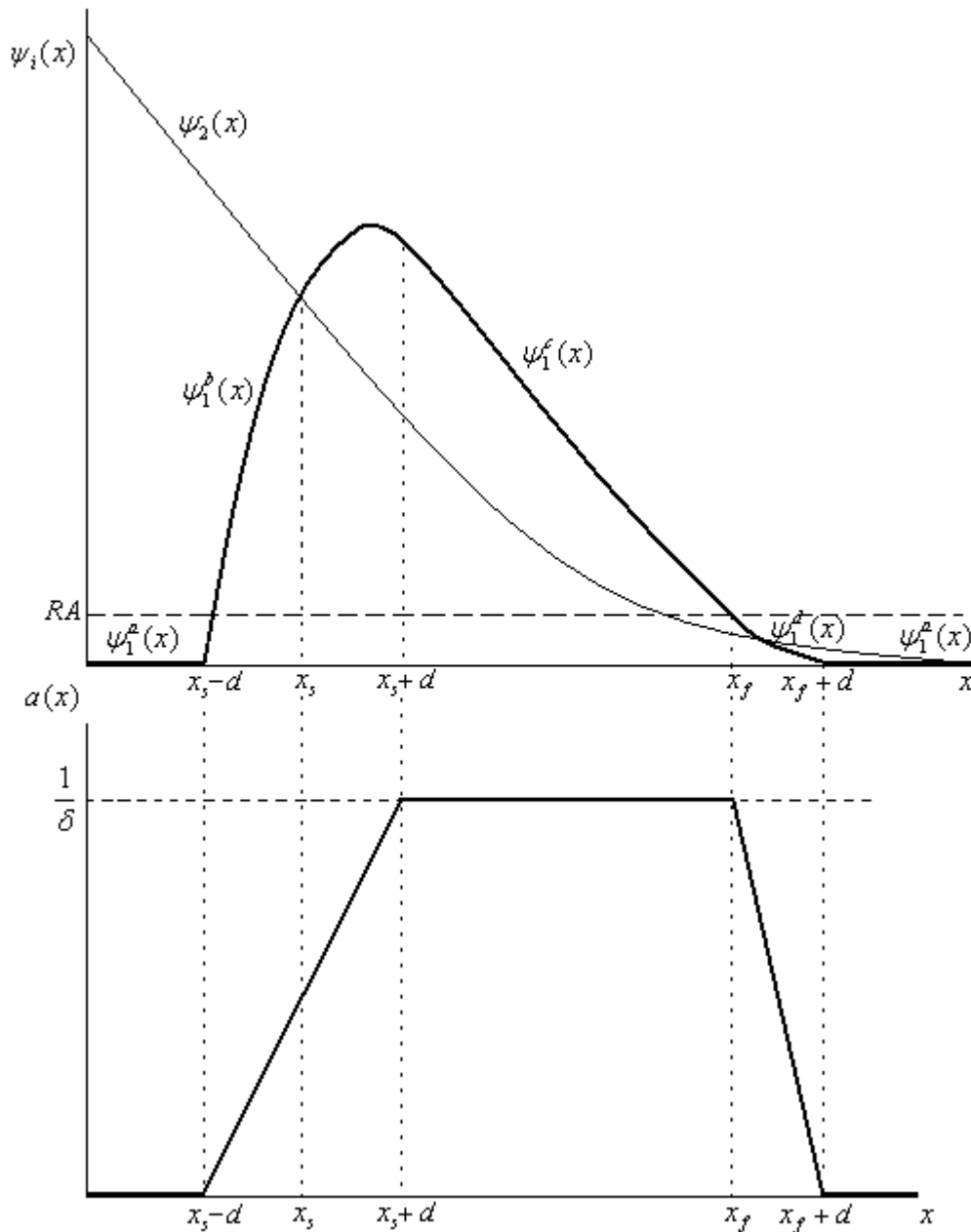
Since the poor households are insensitive with the amenities, their bid-function $\psi_2(x)$ is continuous and decreasing. For the rich households, there are four situations concerning

their bid-function. Thus, in the areas where there are no amenities (for $x \in (0, x_s - d)$ and $x \in (x_f + d, \infty)$), the biddings of the rich households are also null: $\psi_1(x) = \psi_1^a(x) = 0$. In the areas where the amenities are increasing ($x \in [x_s - d, x_s + d]$), the rich bid-function $\psi_1(x) = \psi_1^b(x)$ is initially increasing, and in certain situations it can be decreasing. In the area occupied by the rich person ($x \in [x_s + d, x_f]$), where the amenities on their stationary level are constant, the bid-function $\psi_1(x) = \psi_1^c(x)$ is decreasing. Finally, in the areas where the amenities are decreasing ($x \in [x_f, x_f + d]$), the reduction in these amenities goes reinforces the effect of the distance, and thus, the bid-function is more sloping.

First, we determine the equilibrium level of the endogenous variables and then we will check under which conditions this equilibrium exists. To determine the value of the segregation point, it is necessary to equalize the biddings in the proximity area: ($\psi_1^b(x) = \psi_2(x)$):

$$x_s^a = \frac{1}{c}\left[\frac{(2\delta)^\gamma y_2 u_1 - y_1 u_2}{(2\delta)^\gamma u_1 - u_2} \right] \tag{9}$$

For a known agricultural rent, one can easily determine the border of the city by the equalization of the rich bid-rent with the agricultural rent ($\psi_1^c(x) = RA$):

$$x_f^a = \frac{1}{c}\left[y_1 - \left(\frac{RA}{A}\right)^\beta \delta^\gamma u_1 \right] \tag{10}$$

The first conditions of existence of American equilibrium are $x_s^a > 0$ (if not the city will be inhabited only by the rich households) and $x_s^a < x_f^a$ (if not the city will be entirely poor). Since the denominator of x_s^a is positive (see Annexes), these conditions can be rewritten:

$$x_s^a > 0 \Leftrightarrow u_2 < (2\delta)^\gamma \frac{y_2}{y_1} u_1 = u_2^M(u_1) \tag{11}$$

$$x_s^a < x_f^a \Leftrightarrow u_2 > (2\delta)^\gamma u_1 - 2^\gamma \left(\frac{A}{RA}\right)^\beta (y_1 - y_2) = u_2^m(u_1) \tag{12}$$

To have rich households in the city, the condition (11) shows that the poor must have a lower utility than a maximum level $u_2^M(u_1)$ and to have the poor households in the city, the condition (12) shows that the utility of the poor must exceed the minimum level $u_2^m(u_1)$. Thus,

whatever, we have a fork in which the poor utility level must be. This condition is necessary for the existence of American type equilibrium.

After we put the conditions that the two social classes are present in our city, we will check that this equilibrium is of American type: there is a single point of segregation and the poor households live the center.

For the most central area of the city $x < x_s^a - d$, the bid-function of the rich households are null and those of the poor are positive. Thus, we are sure that in this area, there isn't a segregation point and it will be occupied by the poor.

In the proximity area ($x \in \left[x_s^a - d, x_s^a + d \right]$), there is a point of segregation, but it should be checked if that it is unique. In this area, the two equilibrium conditions are surely observed if the rich bid-function is increasing in x_s, because the poor-bid function is decreasing.

If $\psi_1^b(x)$ is decreasing in x_s^a ($cd > \gamma(y_1 - cx_s)$), the equilibrium condition for two decreasing bid-functions [2] is:

$$-\left. \frac{\partial \psi_1^b(x)}{\partial x} \right|_{x=x_s^a} < -\left. \frac{\partial \psi_2(x)}{\partial x} \right|_{x=x_s^a}$$

If the bid-function of the poor is more sloping in x_s, they will occupy the central area and the rich person the peripheral area. After replacements and simplifications the equilibrium condition is:

$$-\frac{\gamma}{d} < \frac{c(y_1 - y_2)}{(y_1 - cx_s^a)(y_2 - cx_s^a)} \tag{13}$$

This condition is always observed because the term on the left is negative and the term on the right positive. In the peripheral area of the city ($x \in \left[x_s^a + d, x_f^a \right]$), to check that there is not a second point of segregation, knowing that the two functions of bidding are continuous and decreasing and $\psi_1^c(x_s^a + d) > \psi_2(x_s^a + d)$, it should be checked that $\psi_1^c(x_f^a) > \psi_2(x_f^a) \Leftrightarrow u_2 > \delta^\gamma u_1 - (y_1 - y_2)(A/RA)^\beta$. This condition is true if the condition of presence of the rich households in the city (12) is observed because: $2^\gamma > 1$: $u_2 > u_2^m(u_1) > u_2^m(u_1)/2^\gamma = \delta^\gamma u_1 - (y_1 - y_2)(A/RA)^\beta$. Therefore, so that the American equilibrium exists, the only conditions are those of presence of the two social categories.

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[2] see Fujita (1980)

3.3. The existence of European type equilibrium

A European equilibrium is characterized by a central localization of the rich households and a peripheral localization of the poor households.

Graphic 3: The bid-functions and the amenities at European long-run equilibrium
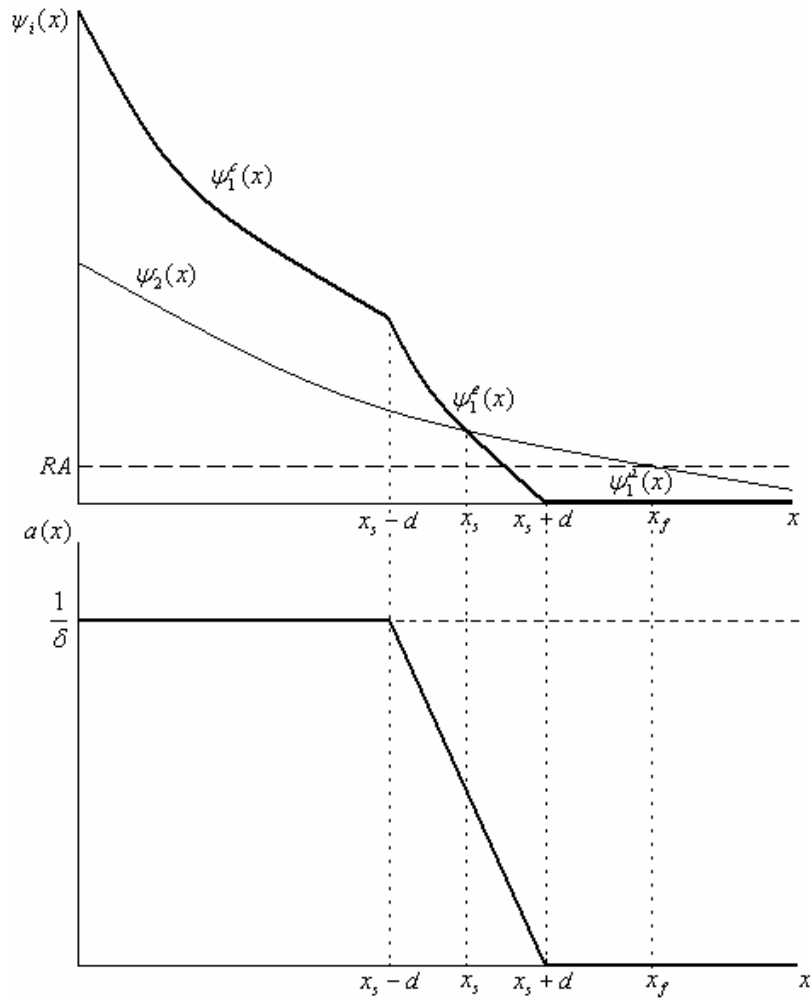
In the area where the amenities are at the maximum level (the central area), the rich bid-function is equal to ψ_1^c and in the proximity area (for $x \in [x_s - d, x_s + d]$), it is ψ_1^e. At the outside of the point $x_s + d$ the biddings of the rich are null.

We will use the same approach for a reversed scheme of localization, with the rich households occupying the central area of the city. We will determine the equilibrium values of the endogenous variables of this scheme of localization and then the conditions necessary for the existence of this equilibrium.

The segregation point between the two social classes is obtained by the equalization of their biddings in the proximity area ($\psi_1^e(x) = \psi_2(x)$):

$$x_s^e = \frac{1}{c}\left[\frac{(2\delta)^\gamma\, y_2 u_1 - y_1 u_2}{(2\delta)^\gamma\, u_1 - u_2}\right] = x_s^a \qquad (14)$$

One finds the same value of the segregation point as in the American scenario. That is explained by the fact that the biddings of the rich person in the two situations are symmetrical (of reversed slope) and at the point of segregation (in the middle of the proximity area) they are identical.

The border of the city depends this time on the utility level of the poor:

$$x_f^e = \frac{1}{c}\left[y_2 - \left(\frac{RA}{A}\right)^\beta u_2\right] \qquad (15)$$

It should be checked that the two social groups are present in the city: $0 < x_s < x_f$. Since the segregation point is identical for the two scenarios, the condition $x_s > 0$ is identical to the American scenario:

$$x_s^e > 0 \Leftrightarrow u_2 < (2\delta)^\gamma \frac{y_2}{y_1} u_1 = u_2^M(u_1) \qquad (16)$$

$$x_s^e < x_f^e \Leftrightarrow u_2 > (2\delta)^\gamma u_1 - \left(\frac{A}{RA}\right)^\beta (y_1 - y_2) = u_2^{m'}(u_1) \qquad (17)$$

It is noted that the conditions (11) and (16) are identical and the functions $u_2^m(u_1)$ and $u_2^{m'}(u_1)$ have the same slope $(2\delta)^\gamma$ and $u_2^{m'}(u_1) > u_2^m(u_1),\ \forall u_1$, which means that the condition (17) is more restrictive than the condition (12).

It should now be checked that the point of segregation it is unique and that the poor households live the periphery of the city. For any localization more distant from centre than $x > x_s^e + d$, the biddings of the rich households are null, thus, this area is occupied by the poor and it does not have any segregation point.

In the proximity area $x \in (x_s^e - d, x_s^e + d)$, the two bid-functions are decreasing, therefore the necessary and sufficient condition for equilibrium is that $\psi_2(x)$ is less sloping than $\psi_1^e(x)$ in x_s^e. This condition is:

$$y_1 > y_2 + \frac{cd\left[(2\delta)^\gamma u_1 - u_2\right]^2}{\gamma (2\delta)^\gamma u_1 u_2} \qquad (18)$$

Once again, European equilibrium is more restrictive than that American, because except the conditions of presence of the two social categories, European equilibrium requires an additional condition for the unicity of the segregation point and the "correct" localization of the households.

According to the condition (18), the factors which support the existence of the European equilibrium are: a significant difference between the incomes of the two social categories, strong preferences of the rich households for the amenities, a short distance of proximity and weak transportation costs. Knowing that $(2\delta)^\gamma u_1 > u_2$ (see Appendix A), a low level of utility of the rich households and strong level of the poor households also support the existence of this equilibrium. In our simulations made in the third part of paper, this condition is always observed.

It remains to be checked that in the central area $x \in \left[0, x_s^e - d\right]$, there is not another point of segregation. The bid-functions of the two social categories are continuous and decreasing in this area. Knowing that at the limit of this zone $\psi_1^c\left(x_s^e - d\right) > \psi_2\left(x_s^e - d\right)$ (because of the condition of unicity of the segregation point in the area of proximity), it is enough to check if $\psi_1^c(0) > \psi_2(0)$:

$$y_1 > \delta^\gamma \frac{u_1}{u_2} y_2 \qquad (19)$$

We cannot determine which conditions (18) and (19) is more restrictive, but the condition (19) presents the same elements supporting the European equilibrium. First, there are the factors which increase the role played by the amenities in the space structuring: strong preference of the rich households for these amenities and a weak depreciation of the amenities what leads to high stationary level. The other factors have a direct impact on the biddings of the two categories, by increasing those of the rich compared to the poor households: strong difference between the incomes, a strong level of utility for the poor and weak one for the rich households.

3.4. The possibility of multiple equilibria

We are seeking the couples $\left(u_1, u_2\right)$, for which the two schemes of localization can be equilibrium. For that, we will present another approach, more general, to find the conditions of presence of the two categories in the city, which can be applied in the situation when the point of segregation cannot be found analytically (for example when the parameters of the

functions of utility are no more identical). Since $x_s^a = x_s^e$, we will use only x_s to simplify the notation.

In the "American" scenario, the rich households occupy the peripheral area of the city and thus we can fix u_1 and then determine all the values of u_2 for which this space configuration is an equilibrium.

We know that there is a segregation point where the bid-functions are equal. Thus, we can express u_2 like a function of u_1 and x_s :

$$u_2 = f(x_s, u_1) = (2\delta)^{\gamma} \left(\frac{y_2 - cx_s}{y_1 - cx_s} \right) u_1 \qquad (20)$$

This function is decreasing on x_s. The behaviour of $f(x_s, u_1)$ is explained by the fact that a widening of the poor area results in an increase in their biddings, which makes a reduction in their level of utility.

We can determine the upper and lower value u_2 as a function of u_1. For that, it is necessary to replace x_s with the minimal and maximal value in (14), which will give us the maximal and minimal value of u_2 :

$$u_2^M(u_1) = f(0, u_1) = (2\delta)^{\gamma} \frac{y_2}{y_1} u_1 \qquad (21)$$

$$u_2^m(u_1) = f(x_f^a, u_1) = (2\delta)^{\gamma} u_1 - 2^{\gamma} \left(\frac{A}{RA} \right)^{\beta} (y_1 - y_2) \qquad (22)$$

It is not surprising to find the same values of the conditions (11) and (12). The two functions are increasing and linear in u_1, $u_2^m(u_1)$ being more sloping than $u_2^M(u_1)$ [3]. The values of these functions in origins are $u_2^m(0) = 2^{\gamma}(A/RA)^{\beta}(y_2 - y_1) < 0$ and $u_2^M(0) = 0$. If $\delta < 1/2$, the functions have a slope lower than the unit. The point of intersection of the two lines is: $u_1^{ii} = \delta^{\gamma}(A/RA)^{\beta} y_1$.

We must restrict the couples (u_1, u_2) only to the values with economic significance. The level of utility cannot be negative, considering our specification of the utility function (Cobb-Douglas). Thus, for all the area $(0, u^i)$ [4] $u_2^m(u_1) = 0$. With the assumption $\delta < 1/2$ (what means that the rich have a higher utility level than the poor households), we can define the ensemble

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[3] $\partial u_2^M / \partial u_1 = (2\delta)^{\gamma_1} (y_2 / y_1) < \partial u_2^m / \partial u_1 = (2\delta)^{\gamma_1}$

[4] $u_1^i = \mathrm{sol}\left(u_2^m(u_1) = 0 \right) = (y_1 - y_2)(A/RA)^{\beta} \delta^{-\gamma}$

of couples (u_1, u_2) for which the equilibrium localization poor-rich is possible: for $u_1 \in (0, u_1^{ii})$, $u_2 \in \left(\max \left(0, u_2^m(u_1) \right), u_2^M(u_1) \right)$.

As in the American case, we will proceed the same approach to determine the conditions necessary for the presence of the two types of households in an "European" city, knowing that the additional conditions (18) and (19) are necessary for the existence of this equilibrium. This time we will fix u_2 because the poor households live in the periphery. By equalizing the bid-functions at the segregation point one obtains u_1 function of u_2:

$$u_1 = g(x_s, u_2) = (2\delta)^{-\gamma} \left(\frac{y_1 - cx_s}{y_2 - cx_s} \right) u_2 \qquad (23)$$

Even if with the first sight this property appears surprising, it is explained by the fact that the widening of the rich area has two impacts. The first is a negative income-effect: to occupy a larger area, the rich households must raise their biddings, which decreases their net incomes and their utility level. The second is the direct effect played by the amenities: in the widened area, the level of amenity is higher (on its maximum level stationary) what results in an increase of the utility level. The function $g(x_s, u_2)$ is increasing with x_s if the amenities effect gains over the income effect.

We continue our reasoning by determining the extreme values of u_1 :

$$u_1^m(u_2) = g(0, u_2) = (2\delta)^{-\gamma} \frac{y_1}{y_2} u_2 \qquad (24)$$

$$u_1^M(u_2) = g(x_f^e, u_2) = (2\delta)^{-\gamma} \left[u_2 + \left(\frac{A}{RA} \right)^{\beta} (y_1 - y_2) \right] \qquad (25)$$

These two functions are increasing and linear with u_2, $u_1^m(0) = 0$ and $u_1^M(0) = (A/RA)^{\beta} (2\delta)^{-\gamma} (y_1 - y_2) > 0$. To be able to compare the conditions of the two scenarios, it is necessary to express the inversed functions of (24) and (25). Since the segregation point is identical in both scenarios, $u_1^m(u_2)$ is the inversed function of $u_2^M(u_1)$, while $u_1^M(u_2)$ is not the inversed of $u_2^m(u_1)$. We will find the same expression as in the relation (17):

$$u_2^{m'}(u_1) = (2\delta)^{\gamma} u_1 - \left(\frac{A}{RA} \right)^{\beta} (y_1 - y_2) \qquad (26)$$

With the assumption $\delta < 1/2$ (a higher level of utility of the rich households) we can define the values of (u_1, u_2) that allow the European equilibrium (with the additional conditions (18) and (19)): for $u_1 \in \left(0, u_1^{iii}\right)^5$, $u_2 \in \left(\max\left(0, u_2^{m'}(u_1)\right), u_2^{M}(u_1)\right)$.

The values of (u_1, u_2) which allow the simultaneous existence of the two types of equilibrium (if the conditions (18) and (19) are observed) are represented by dark grey surface in the graph 4. The clear grey surface represents the couples (u_1, u_2) which allow only the existence of the American equilibrium:

<u>Graphic 4: The utility levels for which two equilibrium are possible</u>
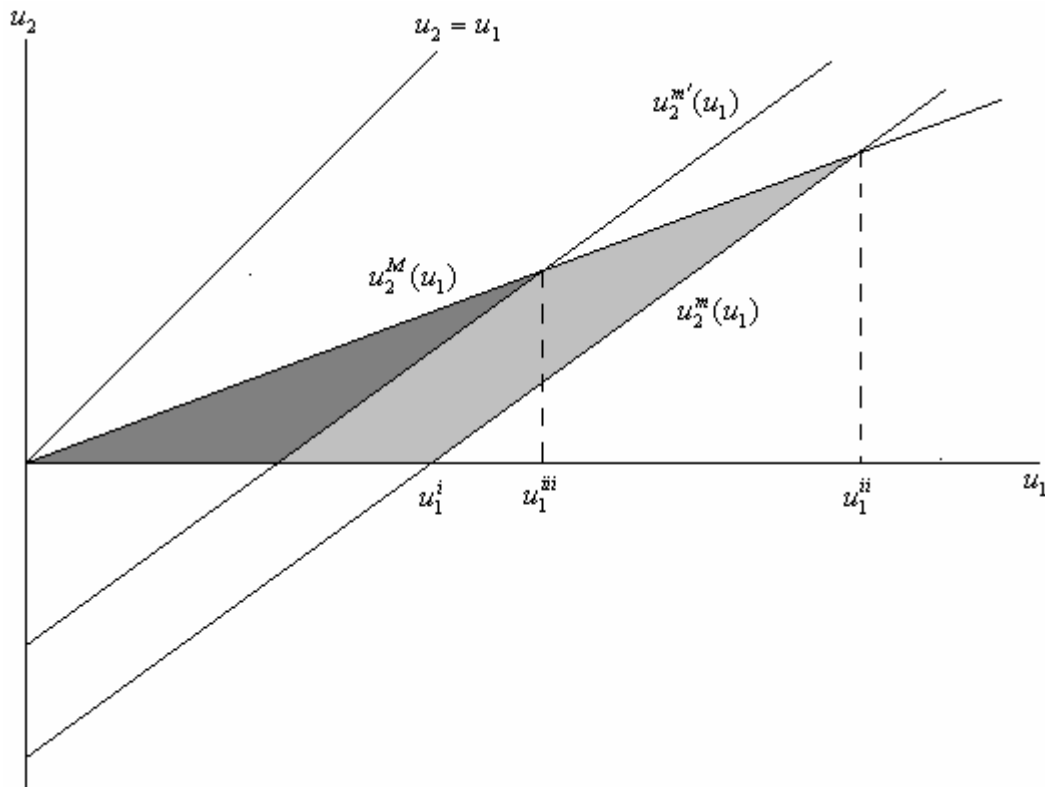
The couples (u_1, u_2) which simultaneously allow both equilibriums (when the conditions (18) and (19) are checked) are the same ones as those for European equilibrium: for $u_1 \in \left(0, u_1^{iii}\right)$, $u_2 \in \left(\max\left(0, u_2^{m'}(u_1)\right), u_2^{M}(u_1)\right)$. Our analysis shows that under certain conditions, the two equilibriums are possible, but the conditions necessary for the balance

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5 $u_1^{iii} = (2\delta)^{-\gamma}\left(A/RA\right)^{\beta} y_1$ is the point of intersection between $u_2^{M}(u_1)$ et $u_2^{m'}(u_1)$

European one are more restrictive. More, European equilibrium is "included" in the American one: if European equilibrium is possible, that American it is too

4. EFFICIENCY COMPARISONS

Since our analysis carries within an opened city framework, the utility levels of the two categories of households are exogenous and identical for the two types of social structures. Thus, to carry out an efficiency comparison between an American and a European, it is necessary to compare the surplus of the economy, which in our case is represented only by the differential rent.

In the American case, the differential rent is:

$$RD^a = \int_0^{x_s} \left(\psi_2(x) - RA \right) dx + \int_{x_s}^{x_s+d} \left(\psi_1^b(x) - RA \right) dx + \int_{x_s+d}^{x_f^a} \left(\psi_1^c(x) - RA \right) dx$$

and in a european spacial configuration of the city, the differential rent becomes:

$$RD^e = \int_0^{x_s-d} \left(\psi_1^c(x) - RA \right) dx + \int_{x_s-d}^{x_s} \left(\psi_1^e(x) - RA \right) dx + \int_{x_s}^{x_f^e} \left(\psi_2(x) - RA \right) dx$$

These two expressions cannot be calculated analytically, thus we will carry out simulations.

In this series of simulation, we fix the stationary parameters relating to the preferences, the incomes, the transportation costs and the agricultural rent and thereafter we determine all the combinations (u_1, u_2) that are respecting the conditions of existence of the two types of equilibrium. For all the values (u_1, u_2) which respect these conditions, we calculate the differential rent in the two situations and then we compare them.

Table 1 : The values of the stationary parameters

Parameter	Rich Households	Poor Households
$\alpha / \beta / \gamma$	0,6 / 0,4 / 0,25	0,6 / 0,4 / 0
Income	100	90
Transportation cost	1,5	
Agricultural rent	3	
Depreciation rate of amenities	0,10	
Proximity distance	5	

We choose $\gamma_2 = 0$ (the poor do not have preferences for the amenities) because we regard these amenities as a higher good. Moreover, if the rich and the poor have the same

preferences for the amenities ($\gamma_1 = \gamma_2$), in this case the rich will always occupy the peripheral area of the city, because the amenities do not play any more any role. The income is expressed in K/period and the proximity distance corresponds to 50 meters.

According to conditions' of presences of the two categories in the city, the maximum value of u_1 for the American scenario is $u_1^{ii} = 58,46$ and for the European scenario is $u_1^{iii} = 49,16$. Since we are seeking the couples (u_1, u_2) which allow the existence of both equilibriums, the maximum value selected is u_1^{iii}.

In our simulations, we make evolve the two levels of utility from 0 to u_1^{iii}, with a step of 0,1. After this series of simulations, we note that except some exceptions, for all the values of (u_1, u_2), the differential rent of an American structure is higher than the differential rent in the European configuration. These exceptions appear when the utility levels are very high and there is a very significant difference between these levels of utility (the utility of the rich households is almost double that the poor one). In these situations, the differential rents in the two situations are almost identical:[6].

To represent graphically the triplets $(u_1, u_2, \Delta RD)$ we use a step of 0,5 and one utility levels higher than 5 (by preoccupation of visibility). In graph 5 we represented in 3D, the couples (u_1, u_2) which allow the existence of two equilibriums and the difference between the American and European differential rent:

Graphic 5: Difference of the differential rents between the two urban structures

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[6] $[u_1, u_2, \Delta RD] =$ [22.1, 11.6, -0.032], [22.3, 11.7, -0.36], [22.4, 11.7, -0.28], [22.4, 11.8, -0.34], [22.5, 11.8, -0.68], [22.6, 11.9, -0.70], [22.7, 11.9, -0.98], [22.7, 12.0, -0.36], [22.8, 12.0, -1.05], [22.9, 12.1, -0.76], [23.0, 12.1, -1.37], [23.0, 12.2, -0.12], [23.1, 12.2, -1.14], [23.2, 12.3, -0.57], [23.3, 12.3, -1.50], [23.4, 12.4, -0.99], [23.5, 12.5, -0.16], [23.6, 12.5, -1.39], [23.7, 12.6, -0.63], [23.9, 12.7, -1.08], [24.0, 12.8, -0.08], [24.2, 12.9, -0.58]
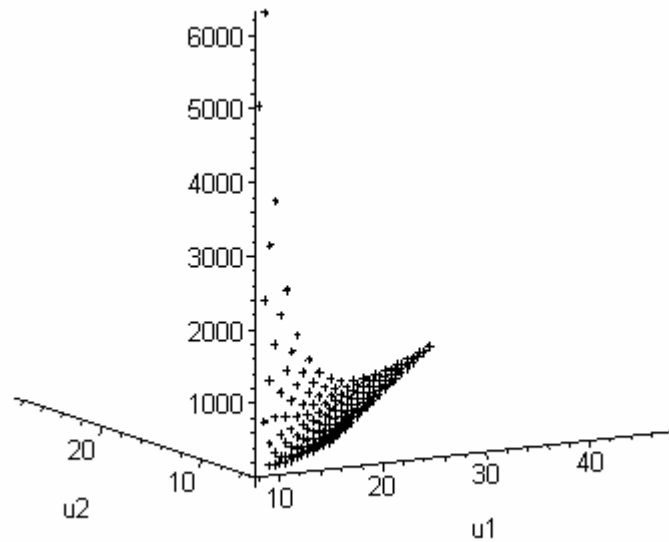
In graph 5, one can see that around the couple $(u_1 = 20,\ u_2 = 10)$ the difference between the differential rents is weaker (see negative). Also, on this graph the constraints on the two levels of utility are visible. With an increase in the utility levels, the difference between the two differential rents is lower.

We carried out a comparison within a closed city framework (there is no migration between cities, and by consequence, the population is exogenous and the utility levels are endogenous). We made a comparison between two cities of the same size, with different social structures (American type, respectively European). For the size, first we considered surface of the city, and second the population. For the two cases, the utility levels reached by the two social categories are higher in an American type structure compared to the European spatial structure, which confirms the superiority of the American configuration, from a Pareto point of view.

5. CONCLUSIONS

The study of stationary equilibriums shows that the conditions of existence of a European equilibrium are more restrictive and included in those for the American type. These conclusions are the result of the fact that a European equilibrium requires a temporal process of population "switch".

First, there are the conditions of presence of the two categories in the city, which are more restrictive for the European scenario. For this scenario, there are some supplementary restrictions, necessary for an European pattern of the localisation: unique segregation point and rich households occupying the centre of the city

Thus, for parameters which satisfy the conditions (11) (12) (17) (18) and (19), the two types of equilibriums are possible and we cannot predict which will carry. If these conditions are not observed, the only possible equilibrium is of American type. According to these conditions the factors which support European equilibrium play on two levels: on the role played by the amenities (preference of the rich households, their depreciation) and on the biddings of the two categories (the ratio between the incomes and the utility levels of the two social categories).

Since the levels of utilities are exogenous and identical for the two types of urban structures, the comparison between the surpluses released by both type of urban structures relates only to the differential rent. This efficiency comparison shows us that, except some extreme cases, an American structure is better than a European structure. This fact is true only when the parameters of the model allow the existence of the two types of equilibrium.

Appendix: Relations between the utility levels of the two social categories

To check that the denominator of x_s (in both scenarios) is positive ($(2\delta)^\gamma u_1 - u_2 > 0$), we will use the indirect utility functions:

$$V_i(x) = \alpha^\alpha \beta^\beta (y_i - cx) a(x)^{\gamma_i} \psi_i(x)^{-\beta}$$

At equilibrium, the indirect utility function must be equal in all the localizations to the exogenous utility level of each category:

$$u_1 = \alpha^\alpha \beta^\beta (y_1 - cx) a(x)^\gamma \psi_1(x)^{-\beta}$$

$$u_2 = \alpha^\alpha \beta^\beta (y_2 - cx) \psi_2(x)^{-\beta}$$

The condition to check $(2\delta)^\gamma u_1 > u_2$ becomes :

$$(2\delta)^\gamma (y_1 - cx) a(x)^\gamma \psi_1(x)^{-\beta} > (y_2 - cx) \psi_2(x)^{-\beta}$$

The point where the two bid-functions are equal (with the urban revenue) is the point of segregation. By replacing x with x_s and after simplification, we obtain $y_1 > y_2$, which is always true.

It is also noted that if $\delta < 1/2 \Rightarrow (2\delta)^\gamma < 1$ and thus since $(2\delta)^\gamma u_1 > u_2$, u_1 must be higher than u_2. This assumption appears probable because a depreciation rate of $1/2$ is very high.

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