40th EUROPEAN CONGRESS. Barcelona 2000

TITLE OF THE PAPER: A STUDY OF THE INEQUALITY IN GDP IN CATALONIA

AUTHORS NAME: J. BARÓ LLINÀS, M.A. CABASÉS PIQUÉ, M.J. GÓMEZ

AFFILIATION OF THE AUTHORS: Professors Applied Economics Department

ADDRESS OF EACH AUTHOR

Dr. JOAN BARÓ LLINÀS, DRA. M.A. CABASÉS PIQUÉ, DRA. M.J. GÓMEZ ADILLÓN
Departamento de Economía Aplicada
Facultad de Derecho y Economia
Universidad de Lleida
Plaza Victor Siurana, 1 25003 LLEIDA (SPAIN)
M.Jesus.Gomez@econap.udl.es
Tf. 973 70 20 31
Fax 973 70 21 15

THEME TO WHICH PAPER RELATES: THEORY AND METHODOLOGY OF REGIONAL SCIENCE

ABSTRACT

STUDY OF THE INEQUALITY OF GDP IN CATALONIA

The concentration of indicative size variables in Economics allows an evaluation of the degree of equity in distributions. This study aims to quantify inequalities in the per capita GDP in the Catalonian regions, during the years 1990 and 1996, and by economic sectors.

First of all, we carry out a descriptive analysis is given using the concentration measurements. We calculete measures in order to point out the difference in the concentration in the two periods, which are analysed. The indicators proposed are measures derived from the concentration graph, so that we can observe the degree of greater or less concentration depending on the degree of convexity of the graph line.

Second, considering the economic nature of the variable, we adjust Kakwani´s concentration model, from which we obtain measurements of inequality. We have obtained an estimate of the model, and we have calculated the values of the functions that participate in the concentration curve at the truncation points of the median, mean, and medial. We obtaining distances of interest between the straight line of equidistribution, and the graph line which reflect the inequality in the distribution of the GDP.

In addition, we have obtained the Gini index for lower and higher truncation values, for values above and below, as well as the breakdown of the total index as a function of the truncation values, and finally we applied quadratic curves with concentration models of economic variables.

A STUDY OF THE INEQUALITY IN GDP IN CATALONIA

Introduction

Currently one of the most debated topics of public interest regards the inequality which exists between different territories. Consequently this is a question for political and social organisations, for which the impact of specific actions affecting the distribution of wealth are of prime importance.

As a result, in the field of economics there is an abundance of empirical studies to find measures for quantifying the degree of inequality that could result from the distribution of income levels or of any economic variable. These measures provide differentiation of the size in a variable contained in different sectors of the population, that is, the degree of concentration of the variable.

This study deals with per capita GDP at the level of the *comarca* (district) in the Autonomous Region of Catalonia (which comprises 41 *comarcas*), a subject chosen because it is our local environment and the territory appears to display many differences.

In the first part of the study we perform a descriptive analysis using measures of concentration based on calculations of dispersion and calculations of the Lorenz curve. In the second part, considering the economic nature of the variable, the Kakwani model is applied, from which measures of inequality are obtained.

1. Obtaining descriptive measures of inequality

On the basis of data for the variable of GDP per *comarca* corresponding to the year from 1991 to 1996 in Catalonia, calculations are produced of those descriptive measures which are most representative in the study of inequality. Table I shows GDP for each *comarca* in millions of pesetas, while Table II shows per capita GDP in millions of pesetas.

Table I. GDP per *comarca* in millions of pesetas

Comarca	1991	1992	1993	1994	1995	1996
Alt Camp	71414	90857	92871	100258	108408	92251
Alt Empordà	153773	164319	171675	189506	208124	211579
Alt Penedès	104716	109483	113278	120067	129123	135520
Alt Urgell	31638	36372	37189	39027	44109	41284
Alta Ribagorça	5933	6613	6784	7125	7605	7434
Anoia	115026	119652	123172	130619	141199	145891
Bages	211257	222210	229762	244059	265710	275167
Baix Camp	242612	301082	309450	330749	349659	305527
Baix Ebre	109785	132701	135590	144302	156183	139004
Baix Empordà	146145	154542	160299	176241	194101	195966
Baix Llobregat	757806	800782	828358	885919	970241	1005895
Baix Penedès	71604	87565	89640	96188	103325	89070
Barcelonès	3843245	4167309	4354570	4643495	5072553	5215042
Berguedà	51709	53821	55726	58455	62322	65520
Cerdanya	20961	22342	23235	25872	27855	27735
Conca de B.	28677	35376	35991	39037	42227	37244
Garraf	102823	109516	114065	121998	133094	138006
Garrigues	19980	23037	23200	24370	26940	28613
Garrotxa	75708	76855	79406	87059	93868	100352
Gironès	242646	258913	270396	296198	324813	329780
Maresme	360611	380685	395012	424529	458613	472782
Montsià	83472	99292	100670	106810	113760	102558
Noguera	43107	49231	49918	53031	58655	58171
Osona	182875	190286	196463	209329	225956	236909
Pallars Jussà	18857	21514	21943	22527	24529	23718
Pallars Sobirà	8055	9341	9514	9885	10907	10557
Pla d'Urgell	41426	46328	47059	49176	55080	54051
Pla de l'Estany	32512	32826	33871	36748	40646	43894
Priorat	11479	13363	13345	14478	15154	14695
Ribera d'Ebre	60881	78649	79984	84407	89327	79906
Ripollès	42264	42749	44165	48018	52838	56567
Segarra	29995	32002	32568	34329	37625	36963
Segrià	263292	305765	312422	333730	373464	346421
Selva	164105	171602	178360	196680	215208	221212
Solsonès	17030	19348	19761	20364	22368	21827
Tarragonès	361837	452850	467323	499448	534421	462411
Terra Alta	16198	18747	18658	19495	20985	21153
Urgell	43731	49094	50006	52162	58341	56078
Val d'Aran	14119	16437	16897	18184	20794	19146
Vallès Occ.	943618	997076	1030599	1104400	1205544	1244311
Vallès Oriental	421490	440714	454161	486619	534877	555251

Anuario Caixa de Catalunya 1998.

Table II: per capita GDP in millions of pesetas

Comarca	1991	1992	1993	1994	1995	1996
Alt Camp	2.088	2.656	2.715	2.931	3.169	2.697
Alt Empordà	1.672	1.787	1.867	2.061	2.263	2.301
Alt Penedès	1.464	1.531	1.584	1.679	1.805	1.895
Alt Urgell	1.665	1.914	1.957	2.053	2.321	2.172
Alta Ribagorça	1.682	1.874	1.923	2.020	2.156	2.107
Anoia	1.358	1.413	1.454	1.542	1.667	1.722
Bages	1.386	1.458	1.508	1.602	1.744	1.806
Baix Camp	1.783	2.213	2.274	2.431	2.570	2.245
Baix Ebre	1.682	2.033	2.078	2.211	2.393	2.130
Baix Empordà	1.572	1.662	1.724	1.896	2.088	2.108
Baix Llobregat	1.209	1.278	1.322	1.413	1.548	1.605
Baix Penedès	1.672	2.045	2.094	2.247	2.413	2.080
Barcelonès	1.734	1.880	1.964	2.095	2.288	2.353
Berguedà	1.333	1.388	1.437	1.507	1.607	1.689
Cerdanya	1.668	1.778	1.849	2.059	2.217	2.207
Conca de B.	1.581	1.950	1.984	2.152	2.327	2.053
Garraf	1.229	1.309	1.363	1.458	1.591	1.649
Garrigues	1.033	1.190	1.199	1.259	1.392	1.479
Garrotxa	1.632	1.657	1.712	1.877	2.024	2.164
Gironès	1.904	2.031	2.121	2.324	2.548	2.587
Maresme	1.178	1.244	1.291	1.387	1.499	1.545
Montsià	1.531	1.821	1.846	1.959	2.086	1.881
Noguera	1.245	1.422	1.441	1.531	1.694	1.680
Osona	1.522	1.583	1.635	1.742	1.880	1.971
Pallars Jussà	1.469	1.676	1.710	1.755	1.911	1.848
Pallars Sobirà	1.434	1.663	1.694	1.760	1.942	1.880
Pla d'Urgell	1.431	1.600	1.625	1.698	1.902	1.867
Pla de l'Estany	1.448	1.462	1.509	1.637	1.810	1.955
Priorat	1.229	1.430	1.428	1.550	1.622	1.573
Ribera d'Ebre	2.676	3.457	3.516	3.710	3.927	3.513
Ripollès	1.579	1.597	1.650	1.794	1.974	2.113
Segarra	1.743	1.860	1.893	1.995	2.187	2.148
Segrià	1.612	1.872	1.913	2.044	2.287	2.121
Selva	1.616	1.690	1.757	1.937	2.119	2.179
Solsonès	1.551	1.762	1.799	1.854	2.037	1.988
Tarragonès	2.227	2.788	2.877	3.075	3.290	2.847
Terra Alta	1.269	1.469	1.462	1.527	1.644	1.657
Urgell	1.458	1.637	1.668	1.739	1.946	1.870
Val d'Aran	2.134	2.485	2.554	2.749	3.143	2.894
Vallès Occ.	1.413	1.493	1.544	1.654	1.806	1.864
Vallès Oriental	1.539	1.609	1.659	1.777	1.953	2.028

The following descriptive measures are used:

1. Coefficient of variation (V): a measure of relative dispersion obtained from the quotient between the standard deviation and the arithmetic mean.

   $$V = \frac{DesvSt(X)}{\overline{X}}.$$

2. Logarithmic variation (Varlog): corresponding to the following transformation.

   Var log = Var (logX)

3. Pietra index (P): measures the maximum distance between accumulations of *comarcas* and the mass of variables obtained from the arithmetic mean. It corresponds to the greatest distance between the actual distribution curve and the curve of equal distribution in a Lorenz diagram.

   P = F (m) - q (m).

4. Gini index (g): equal to double the area enclosed by the actual distribution curve and the curve of equal distribution in a Lorenz diagram.

   $$g = 1 - \sum_{\forall x} f(x)(q(x)+q(x-1)).$$

5. Theil index (T): represents a general group of indicators as a measure of the entropy of distributions which will in turn be used to evaluate inequality.

   $$T(1) = \frac{1}{m}\sum_{\forall x} x\,Ln\left(\frac{x}{m}\right) f(x).$$

6. Atkinson index (A): the most common index, which gives a coefficient of deviation from inequality equal to 0.5.

   $$A(0.5) = 1 - \frac{1}{m}\left(\sum_{\forall x}\sqrt{x}\; f(x)\right)^2$$

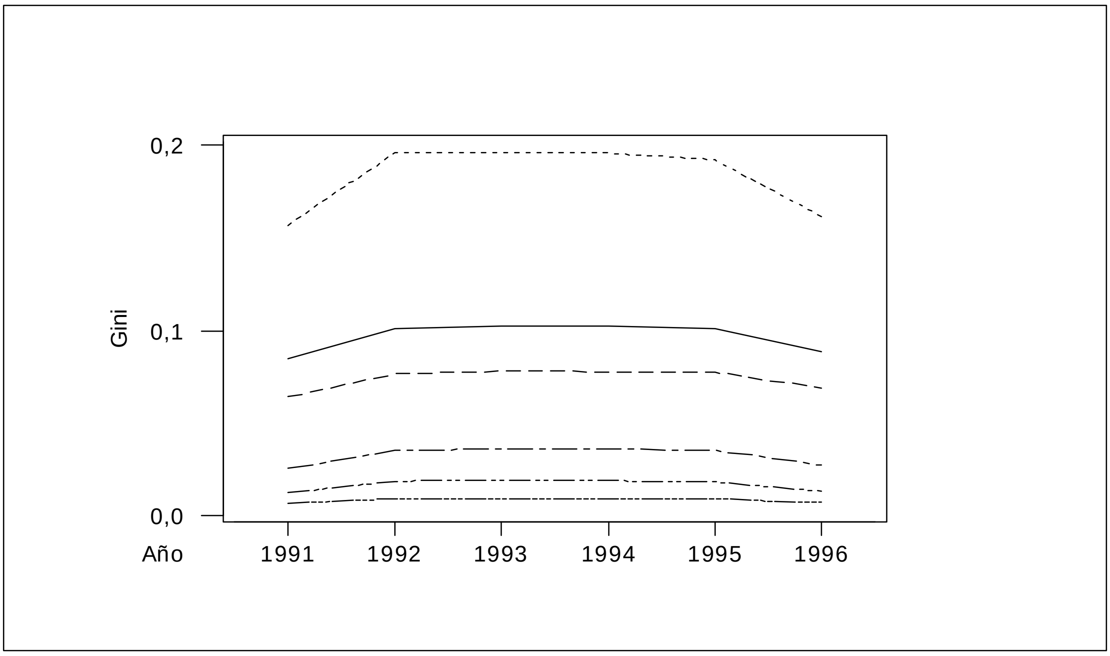
Table III and Figure 1 shows the results of concentration measures applied to GDP by *comarca*, identifying *x* as per capita GDP and *n(x)* as the population which in this case has been taken as the average population in the years 1991 and 1996 to represent the years from 1992 to 1995. given the availability of official statistical information.

Table III. Descriptive concentration measures of per capita GDP (1990-1996).

MEASURES	1991	1992	1993	1994	1995	1996
Media	1.5790	1.7188	1.7823	1.9070	2.0792	2.0895
Coef. variación	0.1456	0.1963	0.1964	0.1962	0.1924	0.1828
Varlog	0.0203	0.0352	0.0357	0.0358	0.0352	0.0350
Pietra	0.0537	0.0764	0.0779	0.0776	0.0773	0.0811
Gini	0.0738	0.1011	0.1022	0.1021	0.1007	0.1018
Theil (1)	0.0103	0.0184	0.0185	0.0185	0.0179	0.0168
Atkinson (0.5)	0.0051	0.0090	0.0091	0.0091	0.0088	0.0084

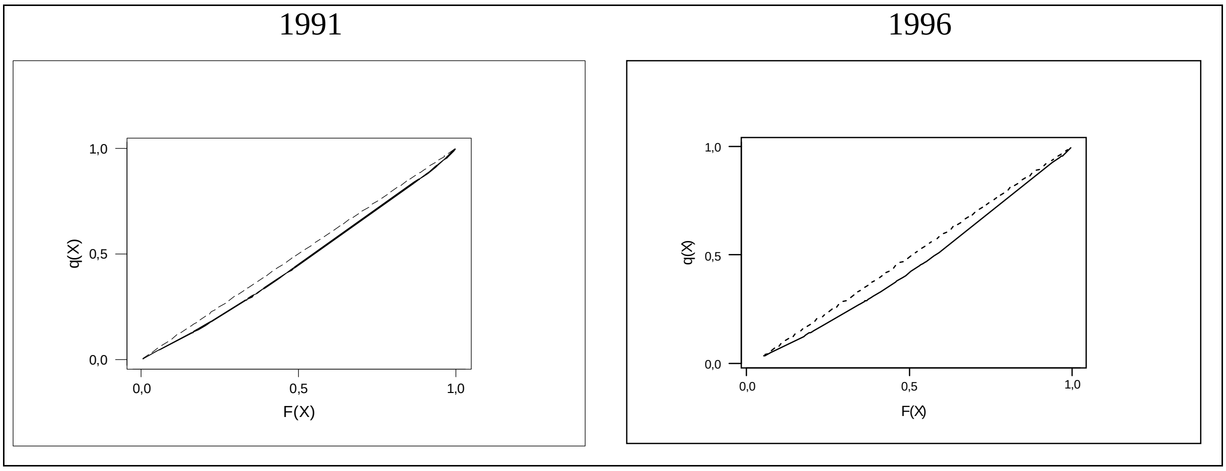
Observing the descriptive indicators collectively, it is noticeable that the year 1991 shows less inequality, as the values are less than in the other years. The year 1993 shows greater inequality in per capita GDP, because greater values are obtained. Figure 1 shows the development of the Gini index to demonstrate the growth in inequality from 1991 onwards, and its decrease from 1995 onwards, while 1991 and 1996 are the years of greatest concentration.

Figure 1. Development of the Gini index in the period 1991 to 1996.



If the study is limited to concentration for the years 1991 and 1996. from the representation of the Lorenz curve shown in Figure 2. a slight increase in inequality of the year 1996 in comparison with 1991 can be observed, given that the concentration curve is further from the straight line of equal distribution.

Figure 2. The Lorenz curve for the years 1991 and 1996.



2. Kakwani model of concentration

Based on an empirical study of the data, and given the economic nature of the variable of per capita GDP by *comarca* in Catalonia, we have performed the following theoretical functions of concentration. All the models provide adjustments in quality due to their degree of adherence in each year under consideration. Finally we have made a calculation using the Kakwani model, and we have obtained measures derived from the concentration curve itself:

Kakwani Model (1980): $q(x) = F(x) - AF(x)^{\alpha}(1 - F(x))^{\beta}$ [1]

Subsequently we have considered significant distances between the straight line of equal distribution and the curve, which reflect the inequality of the variable in the accumulation of its various values, considering that the area which separates the curve from the straight line is a good measure of the concentration of the variable.

The equation [1], having A, α and β as parameters greater than zero, and which determine measures of concentration, possess the usual properties of a concentration curve:

- a range between 0 and 1: $F \in (0.1) \rightarrow q \in (0.1)$
- increasing monotony: $dq / dF \geq 0$
- convexity: $d^2q / dF^2 \geq 0$

In addition, these equations are operative for the calculation of the different indicators of concentration. The model is calculated using the squared minimums after conversion method.

In the context of this study we can identify, in terms of the aleatory variable, F(x) as the fraction of accumulated population by *comarca*, as well as q(x) as the fraction of accumulated GDP by *comarca*, in both cases the distribution is ordered on a per capita basis.

2.1 Concentration measures used

On the basis of the calculations made, and using the Kakwani model, four concentration measures will be calculated which derive from the curve itself and which allow for a quantitative calculation of the inequality and concentration of per capita GDP by *comarca* in Catalonia for the years 1991 and 1996.

2.1.1.- g Index

Gini's coefficient of concentration, which is well known, and corresponds to double the area enclosed between the two lines of distribution, that is, double the average of all the distances between population and ERDF accumulation, as shown by Figure 3.

$$g = 2E(F-q) = 1 - 2E(q) \qquad [2]$$

The expression of the index in the models considered takes on the following form:

$$g = 2A\, B(\alpha+1.\beta+1) \quad \text{with } B(\alpha+1.\beta+1) \text{ Beta Euler}$$

2.1.2. - P Index

The P for Pietra coefficient is associated to the greater distance existing between the population and ERDF accumulations, a distance which is observed in the ERDF volume expected:

$$P = F(\mu) - q(\mu) \qquad [3]$$

It is known that the concentration curve has a unitary gradient at the point (F(μ), q(μ)), the moment in which the distances between the two lines is at its maximum (Figure 4).

$\max [F(x)-q(x)] = F(\mu) - q(\mu) \rightarrow dF(\mu) / dq(\mu) = 1$

$$F(\mu) = \frac{\alpha}{\alpha+\beta} \text{ and, } \quad q(\mu) = \frac{\alpha}{\alpha+\beta} - A \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}$$

The coefficient P is normally used as the lowest level in the Gini index, at the same time that it responds to the double of the area of the greatest triangle that can be inscribed within the figure, i.e., that it coincides with half of the average relative difference and which in any case fulfils that P = $\underline{DMR}/2 \leq g$

2.1.3.- d* Index

Picks up the lack of phasing between the accumulated population and ERDF both in the mean (Ml) and in the median (Me). On the one hand, F(Ml)-0.5 measures the inequality between two groups with equal ERDF and on the other, 0.5-q(Me) measures the inequality of the two groups with equal population, both correspond with the distances which separate the line of equal distribution and the concentration curve from the geometrical centre of the Figure (Figure 5), its simple aggregation leads us to the index:

$$d^* = F(Ml) - q\ (Me) \qquad [4]$$

Its usage is not widely extended and it is rarely used in theoretical studies on concentration.

$q(Me) = 0.5 - A\ 0.5^{\alpha+\beta}$

2.1.4.- d' Index

In the same way that we have considered central distances in the curve, it makes sense to measure the inequality gap that exists in the first quartile (0.25-q (Q1)) and in the third quartile (0.75-q (Q3)) (Figure 6.) The sum of these two distances gives sense to the d' coefficient:

$$d' = 1 - [q(Q1) + q(Q3)] \quad [5]$$

This is not a very widely used indicator either, its advantage lies in the fact that it admits the generalisation of any of the other quartiles, and in addition its calculation is immediate in any concentration function. In the Kakwani model:

$d' = A\, 0.25^{\alpha+\beta}(3^{\beta}+3^{\alpha})$

Figure 3: g Index

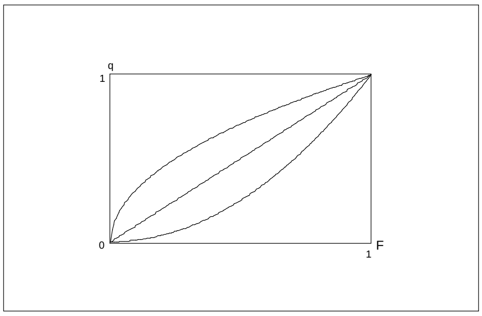


Figure 4: P Index

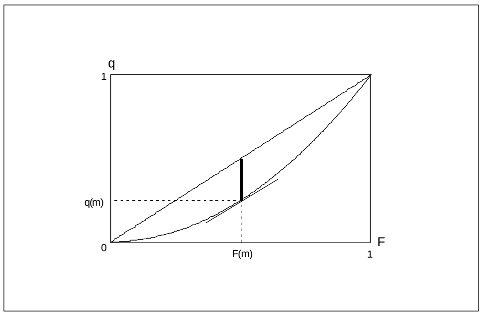


Figure 5: d* Index

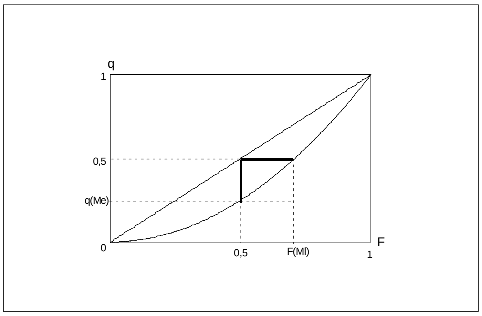
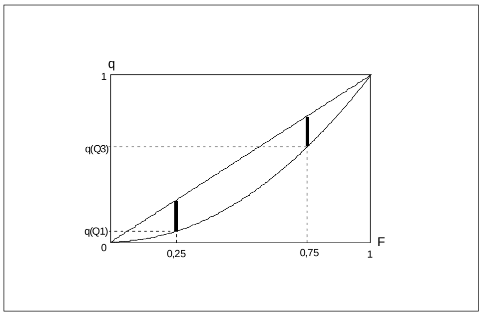


Figure 6: d’ Index



2.2 Calculation of the Kakwani concentration model

For the years included in the study we have established the order on the basis of *comarcas* according to the level of per capita GDP which has allowed us to calculate the variables F(x) and q(x), employing the aggregate of the *comarca* as the interval of the reference variable.

The model has functioned satisfactorily for each year and the results can be seen in Tables IV and V below.

Table IV: Results of the calculation of the Kakwani model for 1991.

La ecuación de regresión es:

Ln (F-q) = - 1.71 + 0.882 Ln (F) + 0.747 Ln (1-F)

Predictor	Coef	DesvEst	razón-t	p
Constante	-1.71190	0.03201	-53.48	0.000
Ln(F)	0.88214	0.01424	61.94	0.000
Ln(1-F)	0.74736	0.01373	54.45	0.000

s = 0.08925 R-cda = 99.1% R-cda(ajda) = 99.1%

Análisis de varianza

Fuente	GL	SC	CM	F	p
Regresión	2	34.069	17,034	2138.34	0.000
Error	37	0.295	0.008		
Total	39	34.363			

Table V: Results of the calculation of the Kakwani model for 1996.

La ecuación de regresión es

Ln (F-q) = - 1.25 + 0.959 Ln (F) + 0.892 Ln (1-F)

Predictor	Coef	DesvEst	razón-t	p
Constante	-1.24809	0.03288	-37,96	0.000
Ln(F)	0.95916	0.02171	44.19	0.000
Ln(1-F)	0.89247	0.01326	67,32	0.000

s = 0.07306 R-cda = 99.2% R-cda(ajda) = 99.1%

Análisis de varianza

Fuente	GL	SC	CM	F	p
Regresión	2	24.193	12.096	2266.04	0.000
Error	37	0.198	0.005		
Total	39	24.390			

2.3 Annual values and concentration measures

The calculation of the parameters of the Kakwani concentration model allows us to calculate the measures of inequality that are derived directly from the distance between the curve and the straight line of equal distribution. The greater the distance, the greater the values of the indices calculated. Table VI shows the values of the significant points (quartiles, medial and mean) for 1991 and 1996. Table VII shows the values of the concentration measures.

Table VI: Values of the concentration curve at significant points

1991			1996		
Point	F (x)	q (x)	Point	F (x)	q (x)
Q1	0.250	0.207	Q1	0.250	0.191
Me	0.500	0.441	Me	0.500	0.420
Q3	0.750	0.700	Q3	0.750	0.686
Ml	0.558	0.500	Ml	0.578	0.500
μ	0.541	0.482	μ	0.518	0.438

Table VII: Concentration measures

	g	P	d*	d’
1991	0.082	0.059	0.117	0.093
1996	0.108	0.080	0.158	0.123

Among these results it is worth noting the small deviations that exist between the Pietra and Gini index when calculated from actual data and when calculated from the Kakwani method. On the other hand, a slight increase in inequality in 1996 relative to 1991 is detectable from the four indicators.

2.4 Breakdown of the Gini index

It is possible to break down the Gini index according to values which correspond to the division of the population. If we generalise the stratification into *k* groups arranged and defined according to $\omega_{1.}\ \omega_{2.}\ \omega_{3}.... \omega_k$ = max. ξ, we arrive with ease at the Gini calculation on the basis of:

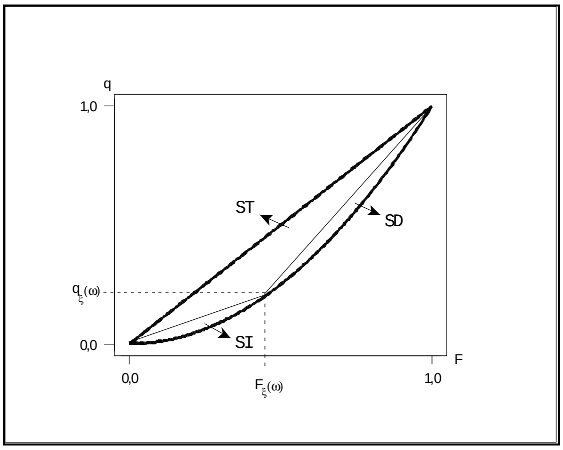
$$g = \sum_{i=1}^{k} [q_\xi(\omega_i)-q_\xi(\omega_{i-1})]\,[F_\xi(\omega_i)-F_\xi(\omega_{i-1})]\,g_i + - \sum_{i=1}^{k} [q_\xi(\omega_i)+q_\xi(\omega_{i-1})]\,[F_\xi(\omega_i)-F_\xi(\omega_{i-1})] \quad [6]$$

In this study we have firstly supposed a division into two groups, that is, a truncation at point ω, thus obtaining the following expression for the Gini index:

$$g = F_\xi(\omega)\, q_\xi(\omega)\, g_I + [1-F_\xi(\omega)]\,[1-q_\xi(\omega)]\, g_D + F_\xi(\omega)-q_\xi(\omega)$$

The value g_I indicates the inequality existent in the first group: $g_I = g\,(\,\xi \,/\, \xi \le \omega)$, whilst g_D is the index of the second group: $g_D = g\,(\,\xi \,/\, \xi > \omega)$. The independent term formed by the difference between: $F_\xi(\omega)-q_\xi(\omega)$ shall indicate the difference existent between the two groups, which contrasts with g_I and g_D that indicate the inequality within each of the groups. Figure 7 shows the new areas of concentration when considering truncation at a point of ω.

Figure 7. Breakdown of the Gini index.



$$ST = \frac{F_\xi(\omega) - q_\xi(\omega)}{2}$$

$$SI = \frac{F_\xi(\omega) q_\xi(\omega)}{2}\; g_I$$

$$SD = \frac{\left[1 - F_\xi(\omega)\right]\left[1 + q_\xi(\omega)\right]}{2}\; g_D$$

$$g = 2\,(\,ST + SI + SD\,)$$

It is of interest to show the division at the median (Me), medial (Ml) and mean (μ), corresponding to the central values of the distribution. The following shows the expressions of the breakdown of the index at these points:

- ω = median

  $g = 0.5q_\xi(Me)\, g_I + 0.5[1-q_\xi(Me)]\, g_D + 0.5-q_\xi(Me)$

The first part of the development is half of a mean calculated from truncated indices and the second part measures the imbalance shown in the mass variable possessed by half of the population, which is the inequality existent among groups of equal population.

- ω = medial

  $g = 0.5F_\xi(Ml)\, g_I + 0.5[1-F_\xi(Ml)]\, g_D + F_\xi(Ml)-0.5$

The expression of half of an average calculated from the truncated indices and the proportion of population existent between the medial and the median.

- ω = mean

  $g = F_\xi(\mu)q_\xi(\mu)g_I + [1-F_\xi(\mu)]\,[1-q_\xi(\mu)]\,g_D + F_\xi(\mu)-q_\xi(\mu)$

The initial combination of coefficients of each group increases with the maximum distance between accumulations corresponding to the Pietra index.

Tables VIII and IX show the calculations of the breakdown of the Gini index for the variable of per capita GDP in the two years examined, considering the truncation at the points analysed previously.

Table VIII. Breakdown of the Gini index for 1991.

Me	$g = 0.2208\ g_I + 0.2791\ g_D + 0.0583$
Ml	$g = 0.2793\ g_I + 0.2207\ g_D + 0.0586$
μ	$g = 0.2613\ g_I + 0.2372\ g_D + 0.0586$

Table IX. Breakdown of the Gini index for 1996.

Me	$g = 0.2100\ g_I + 0.2900\ g_D + 0.0800$
Ml	$g = 0.2890\ g_I + 0.2110\ g_D + 0.0780$
μ	$g = 0.2268\ g_I + 0.2708\ g_D + 0.0800$

It is worth noting, firstly, how the independent term increases in the year 1996 compared to 1991 for the three truncations considered, that is, equality increases between the two groups formed, by the division created.

When the truncation occurs in the median for 1991 the overall index is more sensitive to variations within the second group than in the first, whilst if it occurs in the medial and in the mean the opposite is the case. For 1996 there is a greater marginal effect in the first group only when the truncation is located in the medial.

If we suppose truncation in the quartiles, that is when ω1= Q1. ω2 = Me, ω3 = Q3 and ω4 = max. X, the calculation leads to the following expression:

$$g = 0.25\, q_\xi(Q1)\, g_1 + 0.25(q_\xi\,(Q2)-q_\xi\,(Q1))\, g_2 + 0.25(q_\xi\,(Q3)-q(Q2))\, g_3 + 0.25(1-q_\xi\,(Q3))\, g_4 + 0.75 - 0.5\,(q_\xi\,(Q1)+ q_\xi\,(Q2)+ q_\xi\,(Q3)) \qquad [7]$$

The results obtained for this study are shown in Table X.

Table X. Breakdown of the Gini index in quartiles.

1991	$g = 0.\,0517\, g_1 + 0.0585\, g_2 + 0.0647\, g_3 + 0.0750\, g_4 + 0.0760$
1996	$g = 0.\,0477 g_1 + 0.0572\, g_2 + 0.0665\, g_3 + 0.0785\, g_4 + 0.1015$

With regard to the division into quartiles the overall index is more sensitive in the last group and for the two years examined.

2.5 Gini index in a truncated distribution

The breakdown of the index provides the calculation of the concentration for each of the groups into which the population is divided. It is necessary, therefore, to calculate the area of concentration when the variable is truncated, that is when the extent of the variable is established between any two levels ω_1 and ω_2. The functions of the new truncated variable are affected and they may be expressed in terms of the original functions:

$$F_{\xi T}(x) = \begin{cases} 0 & if \quad x < \omega_1 \\ \dfrac{F_\xi(x) - F_\xi(\omega_1)}{F_\xi(\omega_2) - F_\xi(\omega_1)} & if \quad \omega_1 < x \le \omega_2 \\ 1 & if \quad x \ge \omega_2 \end{cases}$$

$$q_{\xi T}(x) = \begin{cases} 0 & if \quad x < \omega_1 \\ \dfrac{q_\xi(x) - q_\xi(\omega_1)}{q_\xi(\omega_2) - q_\xi(\omega_1)} & if \quad \omega_1 < x \le \omega_2 \\ 1 & if \quad x \ge \omega_2 \end{cases}$$

On the basis of the truncation of the variable at two new extremes, $\omega_1 < \xi_T \leq \omega_{2.}$ the truncated Gini index g_T (ξ / $\omega_1 < \xi \leq \omega_2$) will be:

$$g_T = \frac{\left[F(\omega_2) - F(\omega_1)\right]\left[q(\omega_2) + q(\omega_1)\right] - 2\int_{F(\omega_1)}^{F(\omega_2)} q(x)\,dF(x)}{\left[F(\omega_2) - F(\omega_1)\right]\left[q(\omega_2) - q(\omega_1)\right]} \qquad [8]$$

If instead of considering a bilateral truncation, a unilateral truncation is considered, the following expressions are obtained:

- g (ξ / $\xi < \omega$)

$$g_I = \frac{\left[F(\omega)\cdot q(\omega)\right] - 2\int_{0}^{F(\omega)} q(x)\,dF(x)}{\left[F(\omega)\cdot q(\omega)\right]}$$

- g (ξ / $\xi \geq \omega$)

$$g_D = \frac{\left[(1-F(\omega))\cdot(1+q(\omega))\right] - 2\int_{F(\omega)}^{1} q(x)\,dF(x)}{\left[F(\omega)\cdot q(\omega)\right]}$$

The expressions of the Gini index for the Kakwani model are as follows:

g (Kakwani) = 2A B[α+1. β+1]

$$g_{I\,(\text{Kakwani})} = 1 - \frac{F_\xi(\omega)}{q_\xi(\omega)} + \frac{2A}{q_\xi(\omega)F_\xi(\omega)}\ \mathrm{B}[F_\xi(\omega);\alpha+1.\ \beta+1]$$

$$g_{D\,(\text{Kakwani})} = \frac{2A}{\left(1-q_\xi(\omega)\right)\left(1-F_\xi(\omega)\right)}\left(\mathrm{B}[\alpha+1.\ \beta+1] - \mathrm{B}[F_\xi(\omega);\alpha+1.\ \beta+1]\right) -$$

$$\frac{F_\xi(\omega) - q_\xi(\omega)}{1-q_\xi(\omega)}$$

with B[$F_\xi(\omega)$; α+1. β+1] incomplete Euler Beta function

Tables XI and XII below show the value of the Gini indices truncated for a selection of significant points.

Table XI. Gini indices truncated at central points. 1991.

ω	g_I	g_D
Me	0.04133	0.06499
Ml	0.04675	0.04086
μ	0.04408	0.04582

Table XII. Gini indices truncated at central points. 1996.

ω	g_I	g_D
Me	0.05982	0.07542
Ml	0.06737	0.03649
μ	0.06166	0.06353

From these results it is interesting to note that if the division is located at the medial a reduction is caused in the inequality of the second group and an increase in the inequality of the first group, of the year 1996 with respect to 1991.

REFERENCES

- ANUARIO ECONÓMICO COMARCAL,1994. Institut d'Estadística de Catalunya. Generalitat de Catalunya, pp.30-82.

- ANUARIO ESTADÍSTICO DE CATALUNYA,1997. Institut d'Estadística de Catalunya. Generalitat de Catalunya, pp.147-155.

- ATKINSON, A.B. ,1970. On the Measurement of Inequality. Journal o Economics Theory, nº2

- BAJO, O Y SALAS, R ,1998. Indices de concentración para la economía española: Análisis a partir de las fuentes tributarias. Rev: Economía Industrial, nº320.

- BARÓ, J.,1988. Función e índice de concentración de algunas distribuciones truncadas. Comunicación II reunión ASEPELT-España. Valladolid.

- BARÓ, J. ,1991. Aplicaciones del modelo de Kakwani al análisis de la distribución de la renta. Document de Treball nº 9101. Institut d'Estudis Laborals. Universtitat de Barcelona.

- BARÓ, J. y MORATAL,V. ,1984.Concentración y distribuciones truncadas. Qüestió , Vol.8 núm.3 pp.127-132

- BARÓ, J. y TORRELLES, E.,1989. Sobre las medidas de concentración. Document de Treball 8904. Institut d'Estudis Laborals. Universitat de Barcelona.

- BARÓ, J.,1989. Descomposició del índice de Gini. Comunicación en III Reunión ASEPELT-España. Sevilla

- BARÓ, J. ; CABASÉS M.A.; GOMEZ M.J. Analysis of the concentration of the structural founds. The Spanish case. 39th European Congress of the Regional Science Association, Dublin, Irland.

- GASTWIRTH, J.L.,1972. Estimation of the Lorenz Curve and Gini Index. Review of Economics and Statistics, Vol.LIV nú.3 pp. 306-316

- GIORGI,G. y PALLINI,A.,1986. Di talune soglie inferior e superiore del rapporto di concentrazione". Metron, Vol XLIV, nº1-4

- GUPTA,M.R.,1984. Notes and comments. Functional Form for Estimation the Lorenz Curve.Econometrica , 52. pp. 1313-1314.

- KAKWANI,N.C. y PODDER, N. ,1973.On the Estimation of Lorenz Curves from Grouped observations. International Economic Review,14 pp.278-292.

- KAKWANI,N.C., 1980 a. On a class of Poverty Measures.Econometrica nº48.

- KAKWANI,N.C.,1980 b. Income inequality and poverty. Oxford University Press.

- THEIL, H.,1967. Economics and Inormation Theory. Rand McNally and Company. Chicago
- ZAGIER,D, 1983. Inequalities for the Gini coefficient of Composite Populations; J. Mathematical economics, n. 12.