Multiple Paths Through a Network

Britton Harris
May 2002

Abstract

The most sophisticated iterative algorithm for balancing network congestion for a given set of desired vehicle movement from origins to destinations can generate thousands of paths of equal cost to connect a single O-D pair. Some sets of paths are combinations of minor variations on one main path, while other sets contain various degrees of difference, possibly up to complete independence. Present methods for comparing paths do not take into account the multi-dimensional nature of similarities and differences between paths, or the different character of sets of paths—especially from a geographic point of view. I develop a battery of methods of making comparisons, and apply them to illustrative sets of paths identified in the highly disaggregated Chicago network.

Introduction

This paper is largely methodological, but it is hoped that the methods developed and described here will find application in support of additional methods, of new studies of scientific approaches, and of policy investigations.

The underlying motivation for this study is my belief that the problem of assignment of trips to a network offers the last unopened frontier of micro-behavioral studies. The powerful empirical force of the gravity model, supported by the later development of maximum entropy and discrete choice models, has long since overcome the idea of deterministic choice in locational and travel decision making—but with one exception. For a variety of reasons, models of transportation behavior have almost always assumed that all travelers take the least 'costly' paths connecting origins and destinations. This assumption flies in the face of common experience, and I will later discuss the nature of methods which try to overcome this difficulty.

Meanwhile, it appears likely that if many different paths connecting an origin-destination (O-D) pair are to be made available for study, then it will be desirable to compare not only paths or routes, but groups of them. Until the present, large groups of such routes have not been available for comparison , but a new source of data has opened up a limited possibility.

The idea of network equilibrium has been an essential part of transportation modeling for years, since with limited resources congestion is inevitable, but should be balance so that all users are equally well-served. At equilibrium, the levels of congestion on all links are just those which will lead to an assignment of desired trips to the shortest routes so that the original congest ion is exactly reproduced. While this equilibrium may be unique, finding it requires multiple iterations. Previous best methods generated a sequence of shortest routes which differed somewhat in their costs from the final result. These different routes may have been available for comparative analysis, but have not to my knowledge been so employed.

Within very recent years, a new method called 'Origin-Based Assignment' has been developed by Hillel BarGera (2002) at the University of Illinois at Chicago, with the collaboration of David Boyce (2002). This method distributes flows from each of successive origins to all destinations, in repeated iterations. When part of the flow for one O-D pair is divided between two sets of links, the distribution of flows is adjusted to equalize the times on those branches. This introduces the possibility of multiple paths, in a way which can be exploited for analysis, and this possibility lays the basis for this paper.

The Data and Its Patterns

BarGera's approach generates routes for any O-D pair which are equal in cost. The method does not specify routes as such, because the operational variable is flows on links, made up in part by flows between O-D pairs. While all of the links with flows from a given O-D pair are used by that pair, not all possible routs are so used. We cannot deduce directly which routes are used and which not, and since all routes connecting an O-D pair are of equal cost, there is no economic criterion in the model for deciding this issue. BarGera has devised methods for recovering possible routes, and making an ad hoc assignment of total O-D flows to them.

Despite the lack of an economic explanation for choice, there is a behavioral explanation which parallels the operation of the assignment model itself. The model starts with an assignment to a single route for each O-D pair. Subsequent operations are carried out for each origin. The model attempts to find by-passes for each of many parts of each route, such that it is possible to balance their times by shifting flows from the route to the by-passes. Such shifting is limited by the availability of capacity on the by-passes, or of flows to be shifted from the routes. Drivers who regularly make a given O-D trip can be imagined to start by taking a known route (known from any source), and occasionally testing by-passes large and small for improvement. The results will determine whether the by-pass is accepted, rejected, or used randomly. In this way, and over time, many users will generate many working by-passes.

BarGera made an assignment of this kind for all the O-D trips in Chicago region for a given congested time period. (There are over 1.6 million O-D pairs in the region>) He later recovered all possible routes and estimated flows for them. For numbers of routes per pair, the modal number is one. Short trips and trips close to the four grid directions of the street system account for many of these, and the frequency of cases falls off rapidly as the number of alternative routes increases. Nevertheless there are many O-D pairs with large numbers of multiple routes—in one extreme case almost half a million. An explanation of how these cases arise is necessary for their analysis.

There are two fundamental ways which in which a multiplicity of routes arises, and these operate together.

First, in a grid of any size, there are many paths from an origin to a destination, unless they lie on the same line in the grid. In fact, if travel through the grid requires the use of m horizontal links and n vertical ones, then the number of possible routes is the number of combinations of m+n objects taken m (or n) at a time—or C(m+n,m). For example if the separation is 10 blocks in each direction, we have C(20,10)=184,756 possible routes.

Second, if the network of all links used by a given O-D pair can be segmented, then the total number of routes is the product of the number of routes through each segment. An additional segment is created whenever all trips for an O-D pair must pass through a single node which is not connected by other such nodes with either the O or D. Thus consider a network in which all trips for O to D must pass through A. Assume that there are r routes from O to A and s routes from A to D. Since the two sets of choices are independent, the total number of options is r x s. Thus if r=s=4, then there are 16 routes, and so on.

Third, these two rules may operate together recursively. A segment of the network may contain a subset of its routes, all of which pass through a single node. This subset will then have two pats, each of which can be analyzed separately. Such analysis however rapidly becomes very complex.

The interaction of these three rules leads to two conclusions. Almost any number of routes can arise in some situation, and it impractical to try to deuce the structure from the number of routes. Given the combinatorial nature of the problem, the number of routes can be much larger than is indicated directly by the nodes and links involved. We reserve the first of these points for later discussion, and illustrate the interaction of the second with a simple table.

The basis of the table is the following simple arrangement. We imagine a succession of grid squares with m links on a side, and we calculate the number of routes for a given m from one corner to the diagonally opposite one. A set of uniform grid squares can be linked at their corners, with the destination in one sharing the origin in the next. We develop these ideas diagrammatically in Figure 1 and numerically in Table 1, in the appendix. In the table, for m=1,5 we show the effects of multiplication for sets of grid squares whose number is selected to give a reasonable large number of routes. We also tabulate the number of links in each case, and the ratio of routes to links.

The simplest case shown here with m=1 is a succession of very short bypasses. The existence of 18 or twenty of these in a long trip is readily possible, and using any combination of them is not ruled out. At the same time, if each route were to use only one by-pass, all of the same links would appear in the network but the multiplicity of routes would be reduced. Other combinations shown are similar, but are marked by an increasing average displacement of routes from the line joining the origin and destination.

Features of the Analysis

The general thrust of the methods developed for this analysis lies in the direction of three main goals. The first is the reduction of the enormous redundancy in the descriptions of the routes in the larger cases; such files for a single O-D pair may contain tens of megabytes of data. The second is the preservation of geographic information regarding the location and structure of routes. The third is the development of measures which can be understood and used by analysts and in further computation without graphic presentation, but which do not obliterate the basis for graphics. These goals are met with a group of steps, and more are under development.

Transformed Coordinates

For each O-D pair, the coordinates of the nodes (and implicitly of the links) are translated so that the coordinate origin is at the origin node, rotated so that the Y-axis passes through the destination node, and scaled so that the O-D distance is unity. The X-Y position of each link or its entry node thus gives the (+ or -) displacement from a direct line of travel, and the distance removed from the origin to the destination (usually positive). The standardization of the scale facilitates programming and analysis, and comparison with other O-D pairs.

Abstracting the Network

Each O-D pair has routes using a collection of links which is a small fraction (less than one percent) of the total Chicago area network. We identify all of these links by their nodes of entry and exit, and count the nodes and links. We arrange the links in order of the Y-coordinate of their entry nodes. This keeps links emanating from the same node together, and mimics (though not precisely) the progression along any route away from the origin. For each link we accumulate and preserve the number of routes using it and the sum of their flows.

The separate routes which make for the bulk of the original input are concealed but not extinguished, and could be reconstructed. The individual route flows are more deeply buried, and essentially lost in the aggregated totals. For each link I provide entry node coordinates, geometric link length, and link slope (used in later calculations).

Route Profiles

The course of a single route from O to D can be traced by plotting the entry nodes of its links, and possibly connecting them. This not a standard representation, since every route uses different links. We standardize this representation by imagining a set of cutting lines, perpendicular to the airline from O to D, and defining equal intervals along it. By interpolation, we can identify where each route crosses these lines. (This is done link by link, and many links do not cross a line.)

Now suppose we have a pair of routes, which possibly do not share any link. If both routes are assigned to cutting lines in this way, their separation at each interval defines a running measure of their similarity or difference. This assignment can be generalized to a number of routes, and for each cutting line we can find the range of the displacements, their average, and their dispersion, using different weighting schemes for all but the range. As will be shown, this provides a useful profile.

The Condensed Network

The network we have used so far is complete for this O-D pair, but abstracted from the entire original network. We now consider a further abstraction.
We have noted that many (and usually most) links are used by many routes. There are, however sequences of links over which there is no change in the participation of the original routes. These sequences are initiated or terminated by the divergence or separation of routes at the terminal node of a prior link (or the origin), or by the convergence or joining of routes at the entry node of a following link, or the destination. Convergences and divergences can follow each other in virtually any order. We will consolidate the sequences of links which are defined in this way, and for which there is no change in participation, into links in a condensed network, or condensed links. In this condensed network there will be only single links of more variable length than before between directly connected convergences and divergences. The topology of the network remains the same, in that the cutting lines each encounter the same number of links, and that there is a one-to-one correspondence between the smaller link and the condensed link which contains it, along any cutting line, although the order may be perturbed.

The number of condensed links ranges from one to the number of all links in the abstracted network. If there are one or more completely distinct routes, there is the same number of condensed links. But if routes are completely distributed over a gridded network, then every original link is also a condensed link and there is no difference between the two networks. Most cases fall between these two extremes, and the distribution of condensed link lengths provides yet another characterization of the system of routes. At this stage of the investigations I have chosen to examine the mean, standard deviation, and skewness of these distributions of the summed geometric lengths of the constituent links. These are computed under different weighting schemes.

The mean length is inversely related to the number of condensed links. The standard deviation of the lengths indicates how widely dispersed they are. The cube root of the mean sum of cubed lengths around the mean indicates how symmetrically the lengths are distributed. Negative values are skewed to the greater lengths, positives are skewed to the shorter lengths, and values around zero indicate a symmetrical distribution. These values may be standardized by dividing each of the second and third mean moments by its predecessor. In this calculation, condensed links which serve all routes should be omitted. The mean and the standardized skewness move in opposite directions, and a large number of short condensed links depends on having a large number of them, which in turn requires a larger link-to-node ratio.

Sampling Routes

The idea of sampling routes runs the risk of omitting potentially important examples. At the same time, it seems unlikely that all of many thousand routes are actually used. The calculated flows, however arbitrary, seem to confirm this. Limiting the routes to be considered in one case to those having at least one part in one hundred thousand of the total O-D flow actually still collects over 99.97 percent of that flow. There is in this case only minor reduction in the condensed network's complexity. This issue deserves further attention, not because of any saving in computing time, but because of the simplification provided in exploring network structures.

Conclusions and Prospects

The work carried out so far supports a few methodological and substantive conclusions. Information supporting most of these conclusions may be found in Table 2. The remaining tables at the end of the paper provide information in more detail for the interested reader, based on a single O-D pair. Table 3 provides a complete description of the network actually used by the entire collection of routes; each link is referenced to its beginning and ending nodes in the original study, the frequency of use is recorded in two ways, and the coordinates of the starting node are provided. Table 4 shows a part of the linkwise description of routes as if coded in bits; this format may be useful for comparing small numbers of routes but is not developed in the rest of the paper. Table 5 provides an example of the analysis of the profile of a route, with different ways of measuring the spread of routes from their general tendency; the range of spread between minimally and maximally displaced links at steps on the path seems economical and intuitively attractive. Table 6 shows the nature of the set of condensed links which preserve the topology of the network with minimal information.

Substantively, the cases examined all show a high proportion of 'universal links', over which all routes travel. These sets of links range upward from fifty percent of the total, and in three out of four cases split the routes into two segments. As we have shown, numerous routes can be generated out of small sets of

links, and this possibility is realized in most of these cases, where the average length of condensed links is low and
the relative frequency of very short links is high. The principal conclusion is that the dispersion of the routes is relatively small. Around more than half of the links there is no dispersion at all, and this fact is more marked when the proportion is measured by actual length, rising to over 90 percent. In the freer ten percent of our example, the range of displacement if about a half of what might be possible, and the standard deviation is about a half the range.

Thus it may be possible tentatively to conclude that the routes generated by BarGera's method are not substantially different in character. This is not a disastrous outcome, since degrees of difference can be recognized and since the routes do not in any event provide a basis for a discrete choice analysis.

Operationally, the methods developed here provide a basis for approaching two issues about groups of two or more routes between the same origin and destination, and possibly for comparing sets of routes between different O-D pairs on the same dimensions. One of these issues is the mode in which the generation of alternate paths has taken place. This is approached by analyzing the available number of links in various segments of the routes, as related to the minimum requirement, and by examining the distribution of condensed links. If the latter is skewed toward the smaller links, the potential for a multiplicity of paths increases. A second issue is the dispersion of the displacement of routes in their development, sideways away from the essential direction of travel. This dispersion is only partly related to the first issue of branching tendencies, and is not fully captured by examining just the links used. We have developed a 'profile' not only of the displacement of routes from their main direction, but of the dispersion at intervals along the path. (The sampling done here is not of routes, but of the location of cuts—since every route is crossed by any cutting line perpendicular to the main travel direction.) These measures are sufficiently realistic to capture real qualities of the routes, but sufficiently abstract to permit a wide range of comparisons.

The next stages of this study will approach two issues, one minor and one major. The minor issue will be to correct errors and shortcomings in the present model. Many of these are obvious to the reader, but others are either not apparent even to the author or have not been developed in this short exposition.

The major issue to be addressed revolves around an unsolved problem of transportation analysis. While it is relatively easy to look at two routes, and to decide that they are significantly different, it has proved technically difficult to specify in advance a clear definition of 'significant difference' and to use this as a basis for generating such routes. We have to deal in this paper with routes of the same cost and decide whether with small deviations they are significantly different. Routes which are more costly, or 'slightly more costly' may embody significant differences, but there is no suitable method for generating all routes which differ by such amounts from known least cost routes. For methodological experiments, we plan to make use of sets of equal-cost system-optimal routes generated by the same methods as employed before this work, but the significance of such comparisons for later behavioral studies is unclear. Studies based on user-reported route choices raise many other difficult questions.

One direction which seems obvious from the point of view of behavior is the selection of best or optimal routes on the basis of different defined preferences. Schneider (1959), on the basis of his work in the Chicago Area Transportation Study (1959), recognized that if neither of two routes dominated the other as to two different criteria (e.g., cost and time), then they should be considered two different modes ,and consumer choice should be allocated to them accordingly. How many such 'modes' exist in reality, and how many different sets of optimal paths they might generate, is unknown—even using an all-or-nothing approach to assignment as distinct from BarGera's origin-based approach.

This cluster of issues presents enormous practical difficulties for both research and application. In the interests of realistically understanding and predicting the behavior of system users they must be at least partially resolved, within practical limits. At present we do not even know the costs of ignoring them.

References:

BarGera, H.(2002). "Origin-Based Algorithms for Transportation Network Modeling." *Transportation Science*, in press.

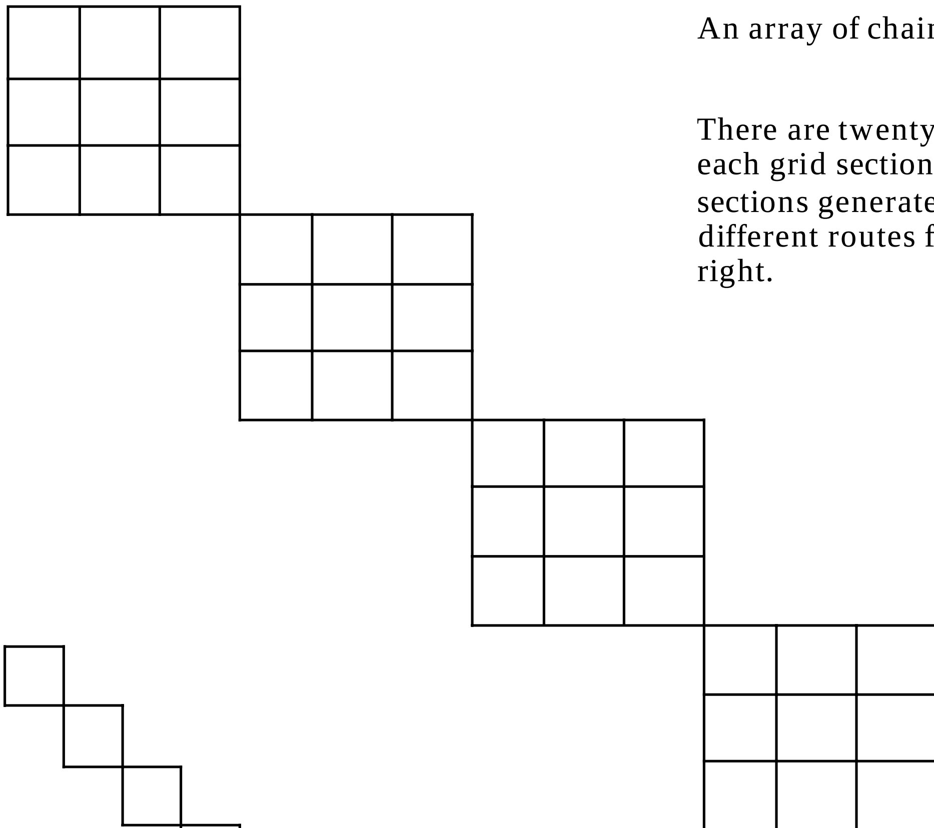
BarGera, H, and Boyce, D(2002). Origin-Based Algorithms for Combined Travel Forecasting Models." *Transportation Research, Part B*, in press.

Chicago Area Transportation Study (1959), *Final Report, Volume 1*, Chicago.

Schneider, M. (1959) "Gravity Models and Trip Distribution Theory," *Papers of the Regional Science Association*, 5,51-56. Appendix: Tables

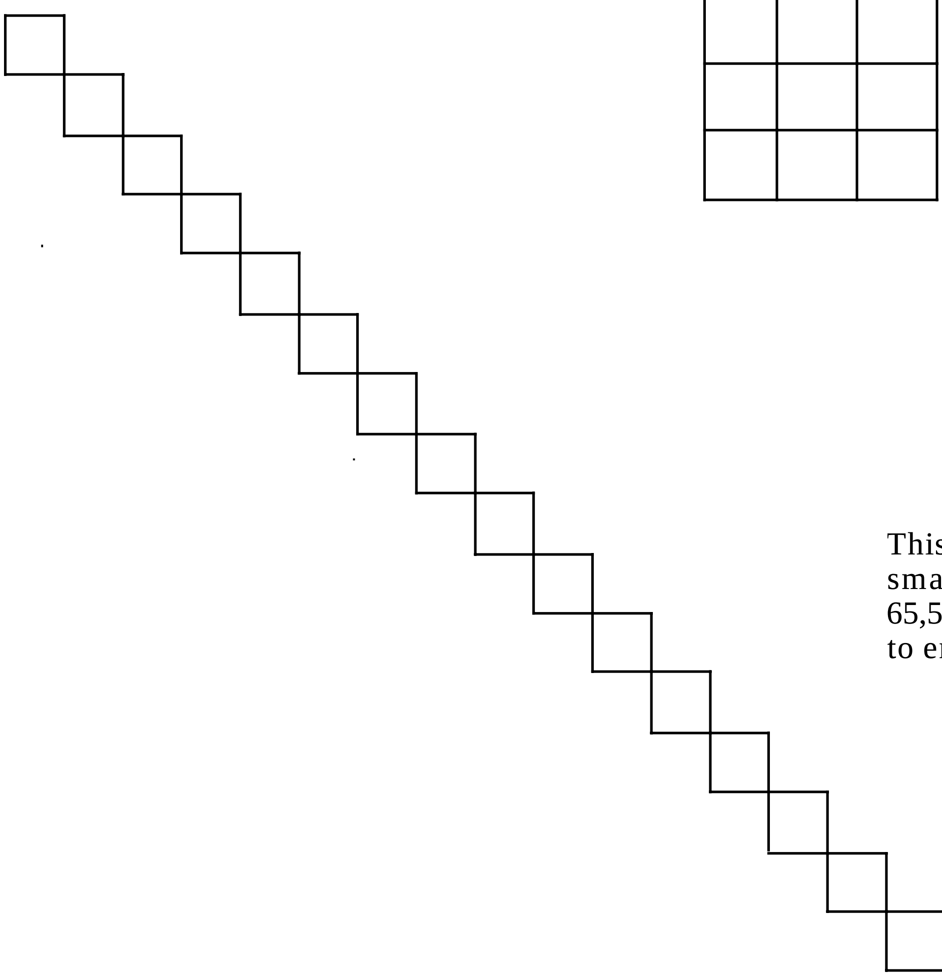
Appendix

Figure 1 — Generating Multiple Paths (also see Table 1)



An array of chained grid sections:

There are twenty paths from NW to SE in each grid section, and the chain of four sections generates 160,000 possible different routes from upper left to lower right.



This narrower chain of sixteen small bypasses generates 65,536 possible routes from end to end.

Table 1 Numbers of Paths through Square Grids, and Their Multiples

Characteristics of Gridded Squares					Features of Chains of Squares		
Links Per Side	Nodes per Square	Links per Square	Routes through Square	Ratio R/L	No. of Squares Chained	Routes Through Chains	Ratio S/L
M	N	L	R	r1	n	S	r2
1	4	4	2	.5	18	262744	3641
2	9	12	6	.5	7	279936	3333
3	16	24	20	.8	4	160000	1667
4	25	40	70	1.8	3	343000	8560
5	36	60	252	4.0	2	63504	1058
6	49	84	924	11.8	2	853776	10164
7	64	112	3432	30.6			
8	81	144	12870	89.4			
9	100	180	48630	270.2			
10	121	220	184756	839.8			
11	144	264	705432	2672.1			

Table 2 Characteristics of Illustrative Cases and Their Complete Networks

Features by Class:	Origin and Destination Nodes				
	2-700	4-1429	5-624	9-292	9-292*
Size information:					
Actual O-D distance	77231	274874	112339	136891	136891
Number of routes	4	12	185	17028	112
Number of nodes	4	130	107	137	110
Number of links	43	138	123	170	128
Average # links/route	37.5	108.2	67.9	70.3	66.2
# of universal links	30	95	43	35	44
U-links / average	.80	.88	.63	.50	.66
Profile information:					
Segment 1					
Max # route crossings	2	4	4	5	5
Max mean displacement	.011	.040	.161	.010	.014
Max standard dev.	.018	.007	.038	.025	.026
Max range	. 036	.030	.083	.062	.062
Segment 2					
Max # route crossings	2		2	5	3
Max mean displacement	.196		.295	.081	.088
Max standard deviation	.025		.002	.014	.009
Max range	.052		.004	.038	.019
Branching information:					
A-links: total less u-links	13	43	80	135	84
Excess of links over nodes	2	5	16	33	18
Excess over route average	5.5	29.8	55.1	99.7	61.8
C-links: condensed links	7	23	44	88	50
B-links: c-links less cu-links	4	21	41	85	47
Ratio b-links to a-links	0.31	0.49	0.51	0.63	0.56
Mean length	0.11	0.02	0.03	0.02	0.03
Sigma/mean	0.25	0.71	0.97	0.60	0.66
Skewness/mean	0.11	0.69	1.63	0.82	0.83
Skewness/sigma	0.38	0.96	1.43	1.15	1.25

* Analysis as in preceding column, sampled to recover 99.98% of total flows.

Table 3 Basic data defining links used in routes for O-D pair 5-624

Link	Node 1	Node 2	Routes on Link	Flows on Link	Node 1 Y	Node 1 X	Link Length	Inverse Slope
1	5	10297	185.	.040000	.000000	.000000	.002235	-4.692183
2	10297	12724	40.	.022681	.000466	-.002186	.022254	-.628812
3	10297	12722	145.	.017319	.000466	-.002186	.023298	.256496
4	12724	2370	40.	.022681	.019305	-.014032	.016558	.071535
5	12722	10296	40.	.001975	.023033	.003603	.019584	-.628812
6	12722	2594	105.	.015344	.023033	.003603	.011572	1.590301
7	2594	10295	105.	.015344	.029193	.013399	.019441	.897011
8	2370	12721	40.	.022681	.035821	-.012850	.008207	.310251
9	10296	12721	40.	.001975	.039611	-.006822	.005415	-.888606
10	12721	2595	80.	.024656	.043659	-.010419	.005630	-.244277
11	10295	7860	105.	.015344	.043665	.026381	.023968	.925017
12	2595	2591	80.	.024656	.049128	-.011755	.011943	.607359
13	2591	2599	80.	.024656	.059336	-.005555	.007657	.410133
14	7860	7857	105.	.015344	.061260	.042656	.024051	1.466866
15	2599	7850	80.	.024656	.066420	-.002649	.018351	.121575
16	7857	7855	60.	.014267	.074808	.062529	.011572	-.628812
17	7857	10293	45.	.001077	.074808	.062529	.028499	.347807
18	7855	2378	45.	.003136	.084604	.056369	.020493	.433930
19	7855	7846	15.	.011131	.084604	.056369	.032058	-.668262
20	7850	10411	80.	.024656	.084637	-.000434	.034762	.134200
21	10293	2406	45.	.001077	.101725	.071891	.005700	.347807
22	2378	7775	15.	.001189	.103404	.064526	.021364	-.628812
23	2378	7433	30.	.001947	.103404	.064526	.006419	.448323
24	2406	9155	15.	.000062	.107109	.073763	.029173	.163757
25	2406	7433	30.	.001015	.107109	.073763	.006952	-3.071797
26	7433	9155	30.	.001092	.109261	.067152	.028946	.425164
27	7433	6737	30.	.001870	.109261	.067152	.017803	-.628812
28	7846	7775	15.	.011131	.111259	.038557	.017825	1.426845
29	10411	7837	80.	.024656	.119091	.004189	.034013	.189255
30	7775	6737	30.	.012320	.121489	.053154	.005341	1.590301
31	6737	9155	60.	.014190	.124332	.057675	.023802	1.798488
32	9155	7711	70.	.001756	.135899	.078478	.023036	1.024841
33	9155	7773	35.	.013588	.135899	.078478	.028028	-.706919
34	7711	7710	35.	.001665	.151987	.094965	.021401	-.272344
35	7711	10399	35.	.000091	.151987	.094965	.023144	-.628812
36	7837	5880	80.	.024656	.152510	.010514	.017825	1.426845
37	7773	10399	35.	.013588	.158785	.062299	.024034	1.590301
38	5880	6735	80.	.024656	.162741	.025111	.005341	1.590301
39	6735	7768	80.	.024656	.165584	.029633	.023161	1.462392
40	10399	7710	70.	.013679	.171579	.082645	.006779	6.339835
41	7710	7707	125.	.021658	.172635	.089342	.018427	1.628453
42	7768	7705	80.	.024656	.178658	.048752	.023161	1.462392
43	7707	7664	145.	.030593	.182278	.105045	.041153	.876103
44	7709	7710	20.	.006314	.182626	.085163	.010829	-.418306
45	5214	7707	20.	.008935	.184373	.102991	.002933	-.980194
46	7706	5214	20.	.008935	.191521	.094291	.011260	-1.217295
47	7705	7703	40.	.000561	.191731	.067870	.007121	1.590301
48	7705	7704	40.	.024095	.191731	.067870	.007962	10.204082
49	7704	7709	20.	.006314	.192508	.075794	.013617	-.948039
50	7704	7702	60.	.018342	.192508	.075794	.016937	1.793007

Table 3 (continued)

51	7703	7704	40.	.000561	.195522	.073899	.003561	-.628812
52	7702	7706	20.	.008935	.200757	.090586	.009952	-.401132
53	7702	7503	40.	.009407	.200757	.090586	.023161	1.462392
54	7664	7662	165.	.030593	.213232	.132163	.010149	3.289933
55	7503	7667	40.	.009407	.213831	.109705	.001780	1.590301
56	7667	6741	40.	.009407	.214779	.111212	.011606	1.348424
57	5475	7664	20.	0.000000	.215578	.127533	.005190	-1.973320
58	7662	7611	165.	.030593	.216184	.141874	.053358	.577124
59	10393	7661	20.	.009407	.220100	.124690	.008902	1.590301
60	10393	5475	20.	0.000000	.220100	.124690	.005341	-.628812
61	6741	10393	40.	.009407	.221692	.120534	.004451	-2.609453
62	7661	7620	20.	.009407	.224838	.132226	.024182	2.066576
63	7620	7491	20.	.009407	.235371	.153994	.006231	-.628812
64	7491	7492	20.	.009407	.240646	.150677	.023366	.841874
65	7492	7611	20.	.009407	.258521	.165725	.004794	.727514
66	7611	7577	185.	.040000	.262397	.168545	.047937	.498781
67	7577	7540	185.	.040000	.305294	.189941	.052603	.671348
68	7540	4360	185.	.040000	.348968	.219261	.029844	.853592
69	4360	7440	185.	.040000	.371666	.238637	.014922	.853592
70	7440	4548	185.	.040000	.383016	.248324	.071767	1.222319
71	4548	4547	185.	.040000	.428459	.303871	.003561	-.628812
72	4547	4544	185.	.040000	.431474	.301976	.005341	-.628812
73	4544	4543	37.	.039472	.435995	.299132	.007121	1.590301
74	4544	10365	148.	.000528	.435995	.299132	.008902	-.628812
75	4543	7511	37.	.039472	.439786	.305161	.006231	-.628812
76	10365	4542	74.	.000028	.443531	.294394	.003561	1.590301
77	10365	2495	74.	.000500	.443531	.294394	.004451	-.628812
78	7511	4540	37.	.039472	.445061	.301844	.002670	-.628812
79	4542	4533	74.	.000028	.445426	.297408	.004451	-.628812
80	2495	4533	74.	.000500	.447298	.292025	.003561	1.590301
81	4540	4530	37.	.039472	.447321	.300423	.004451	-.628812
82	4533	4524	148.	.000528	.449194	.295039	.004451	-.628812
83	4530	4521	37.	.039472	.451089	.298053	.004451	-.628812
84	4524	2491	74.	.000366	.452962	.292670	.003561	-.628812
85	4524	4521	74.	.000162	.452962	.292670	.003561	1.590301
86	4521	2781	111.	.039634	.454857	.295684	.003561	-.628812
87	2491	4519	74.	.000366	.455976	.290774	.003561	-.628812
88	2781	4517	111.	.039634	.457871	.293789	.003561	-.628812
89	4519	4513	74.	.000366	.458990	.288879	.007121	-.628812
90	4517	4512	111.	.039634	.460886	.291893	.007121	-.628812
91	4513	4512	74.	.000366	.465019	.285088	.003561	1.590301
92	4512	4499	185.	.040000	.466914	.288102	.001990	-.098000
93	4499	4483	185.	.040000	.468895	.287908	.014243	-.628812
94	4483	4587	185.	.040000	.480952	.280327	.009792	-.628812
95	4587	7883	185.	.040000	.489241	.275114	.015133	-.628812
96	7883	8050	185.	.040000	.502052	.267059	.023144	-.628812
97	8050	7870	185.	.040000	.521644	.254739	.008011	-.628812
98	7870	8201	185.	.040000	.528426	.250474	.018715	-.697311
99	8201	7443	185.	.040000	.543777	.239770	.007121	-.628812
100	7443	8051	185.	.040000	.549806	.235979	.018693	-.628812
101	8051	7119	185.	.040000	.565631	.226028	.017803	-.628812
102	7119	7666	185.	.040000	.580702	.216551	.008946	-.777715
103	7666	7896	185.	.040000	.587764	.211059	.026838	-.777715

Table 3 (continued)

104	7896	7893	185.	.040000	.608949	.194583	.009952	-.915273
105	7893	7889	185.	.040000	.616291	.187864	.023161	-.683811
106	7889	7887	185.	.040000	.635409	.174790	.032948	-.667177
107	7887	8412	185.	.040000	.662817	.156504	.041562	-.901288
108	8412	7884	185.	.040000	.693690	.128678	.010719	-.751526
109	7884	8286	185.	.040000	.702259	.122239	.021364	-.628812
110	8286	8281	185.	.040000	.720345	.110866	.039208	-.694105
111	8281	8275	185.	.040000	.752554	.088510	.057310	-.486011
112	8275	2206	185.	.040000	.804099	.063458	.041847	-.658904
113	2206	12518	185.	.040000	.839043	.040434	.043944	-.813931
114	12518	12514	185.	.040000	.873124	.012694	.010719	-.518318
115	12514	12505	185.	.040000	.882641	.007761	.036540	-.699034
116	12505	12501	185.	.040000	.912589	-.013174	.024925	-.628812
117	12501	2208	185.	.040000	.933689	-.026441	.047636	-.671351
118	8567	8963	185.	.040000	.969580	-.047959	.019604	1.440703
119	2208	8567	185.	.040000	.973239	-.052993	.006224	-1.376079
120	8963	12498	185.	.040000	.980758	-.031854	.012462	1.590301
121	12498	8958	185.	.040000	.987392	-.021304	.005341	1.590301
122	8958	10849	185.	.040000	.990235	-.016783	.017825	1.426845
123	10849	624	185.	.040000	1.000466	-.002186	.002235	-4.692183
124	624	0	0.	.000000	1.000000	0.000000	.000000	.000000

Table 4 Bit Representation of a Portion of 39 Routes used by O-D pair 5-624: Contains the 39 routes identified in first 70 links.

Each line is a route, & each column is a link, which if 1 appears in this route.
No two rows are identical and each column is represented in at least one row.

1100100101	0110100001	0000000000	1001001001	0001000110	1011000111	1101111111
1010010011	0110100001	0000000000	1001001001	0001000110	1011000111	1101111111
1100100101	0110100001	0000000000	1001001001	0001001000	1011000111	1101111111
1010010011	0110100001	0000000000	1001001001	0001001000	1011000111	1101111111
1100100101	0110100001	0000000000	1001001001	0001000110	1011111110	0010011111
1010010011	0110100001	0000000000	1001001001	0001000110	1011111110	0010011111
1100100101	0110100001	0000000000	1001001001	0001001000	1011111110	0010011111
1010010011	0110100001	0000000000	1001001001	0001001000	1011111110	0010011111
1100100101	0110100001	0000000000	1001001001	0011010111	1100010000	0010011111
1010010011	0110100001	0000000000	1001001001	0011010111	1100010000	0010011111
1100100101	0110100001	0000000000	1001001001	0011011001	1100010000	0010011111
1010010011	0110100001	0000000000	1001001001	0011011001	1100010000	0010011111
1100100101	0110100001	0000000000	1001001001	1101100110	0100010000	0010011111
1010010011	0110100001	0000000000	1001001001	1101100110	0100010000	0010011111
1100100101	0110100001	0000000000	1001001001	1101101000	0100010000	0010011111
1010010011	0110100001	0000000000	1001001001	1101101000	0100010000	0010011111
1011001000	1001001000	1000001000	0100010000	1000000000	0100010000	0010011111
1011001000	1001010100	0100010001	0100010000	1000000000	0100010000	0010011111
1011001000	1001001000	1010010001	0100010000	1000000000	0100010000	0010011111
1011001000	1001010100	0001000011	0100010000	1000000000	0100010000	0010011111
1011001000	1001010010	0000100011	0100010000	1000000000	0100010000	0010011111
1011001000	1001010100	0100000100	0100010000	1000000000	0100010000	0010011111
1011001000	1001001000	1010000100	0100010000	1000000000	0100010000	0010011111
1011001000	1001001000	1000001000	0100100010	1000000000	0100010000	0010011111
1011001000	1001010100	0100010001	0100100010	1000000000	0100010000	0010011111
1011001000	1001001000	1010010001	0100100010	1000000000	0100010000	0010011111
1011001000	1001010100	0001000011	0100100010	1000000000	0100010000	0010011111
1011001000	1001010010	0000100011	0100100010	1000000000	0100010000	0010011111
1011001000	1001010100	0100000100	0100100010	1000000000	0100010000	0010011111
1011001000	1001001000	1010000100	0100100010	1000000000	0100010000	0010011111
1011001000	1001001000	1000001000	0010000110	1000000000	0100010000	0010011111
1011001000	1001010100	0100010001	0010000110	1000000000	0100010000	0010011111
1011001000	1001001000	1010010001	0010000110	1000000000	0100010000	0010011111
1011001000	1001010100	0001000011	0010000110	1000000000	0100010000	0010011111
1011001000	1001010010	0000100011	0010000110	1000000000	0100010000	0010011111
1011001000	1001010100	0100000100	0010000110	1000000000	0100010000	0010011111
1011001000	1001001000	1010000100	0010000110	1000000000	0100010000	0010011111
1100100101	0110100001	0000000000	1001001001	0001000110	1011000111	1101111111
1010010011	0110100001	0000000000	1001001001	0001000110	1011000111	1101111111
1100100101	0110100001	0000000000	1001001001	0001001000	1011000111	1101111111
1010010011	0110100001	0000000000	1001001001	0001001000	1011000111	1101111111

Table 5 Profile for routes of O-D pair 5-624
Analysis of cross-sections at 40 intervals from Origin to Destination

Numbers of crossings at 40 cutting lines:

3	2	3	4	4	3	3	3	2	2	1	1	1	1	1	1	1	2	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Distribution of link displacements (X-value for first node):

Interval	Unweighted Mean	Unweighted s.d.	Weight: No. routes Mean	Weight: No. routes s.d.	Weight: Flow Mean	Weight: Flow s.d.	Range
1	-.001509	.008750	.001386	.008071	-.005027	.009881	.020355
2	.010508	.021733	.013445	.021533	.005448	.021136	.043465
3	.041133	.030221	.034772	.031753	.022954	.031134	.064202
4	.045512	.026918	.037117	.031604	.020688	.024758	.069664
5	.053680	.028736	.039583	.030508	.026292	.026716	.071386
6	.057159	.034779	.052465	.038034	.033540	.030177	.082890
7	.074982	.022418	.075059	.023552	.069285	.024345	.049790
8	.100229	.014399	.109574	.014811	.106054	.015383	.031343
9	.139761	.007201	.145405	.004472	.143575	.006108	.014402
10	.159971	.001419	.161083	.000882	.160723	.001204	.002839
11	.174831	.000000	.174831	.000000	.174831	.000000	.000000
12	.187301	.000000	.187301	.000000	.187301	.000000	.000000
13	.203171	.000000	.203171	.000000	.203171	.000000	.000000
14	.220142	.000000	.220142	.000000	.220142	.000000	.000000
15	.241482	.000000	.241482	.000000	.241482	.000000	.000000
16	.269085	.000000	.269085	.000000	.269085	.000000	.000000
17	.299643	.000000	.299643	.000000	.299643	.000000	.000000
18	.296635	.002103	.295373	.001682	.298683	.000480	.004206
19	.284069	.000000	.284069	.000000	.284069	.000000	.000000
20	.268349	.000000	.268349	.000000	.268349	.000000	.000000
21	.252629	.000000	.252629	.000000	.252629	.000000	.000000
22	.235857	.000000	.235857	.000000	.235857	.000000	.000000
23	.220137	.000000	.220137	.000000	.220137	.000000	.000000
24	.201543	.000000	.201543	.000000	.201543	.000000	.000000
25	.181908	.000000	.181908	.000000	.181908	.000000	.000000
26	.165055	.000000	.165055	.000000	.165055	.000000	.000000
27	.145524	.000000	.145524	.000000	.145524	.000000	.000000
28	.123937	.000000	.123937	.000000	.123937	.000000	.000000
29	.107635	.000000	.107635	.000000	.107635	.000000	.000000
30	.090282	.000000	.090282	.000000	.090282	.000000	.000000
31	.077601	.000000	.077601	.000000	.077601	.000000	.000000
32	.065450	.000000	.065450	.000000	.065450	.000000	.000000
33	.049687	.000000	.049687	.000000	.049687	.000000	.000000
34	.031515	.000000	.031515	.000000	.031515	.000000	.000000
35	.011721	.000000	.011721	.000000	.011721	.000000	.000000
36	-.004374	.000000	-.004374	.000000	-.004374	.000000	.000000
37	-.020978	.000000	-.020978	.000000	-.020978	.000000	.000000
38	-.037392	.000000	-.037392	.000000	-.037392	.000000	.000000
39	-.040150	.000000	-.040150	.000000	-.040150	.000000	.000000
40	-.002850	.000000	-.002850	.000000	-.002850	.000000	.000000

Table 6 Condensed Network Analysis for O-D pair 5-624

Characteristics of condensed links:

Serial No.		Original		Link Nodes		Numbers of		Length of
Start	End	Links		Entry	Exit	Links	Routes	Cond. Link
1	2	1	2	5	10297	1	185	.0022347343
2	34	2	10	10297	12721	3	40	.0470189530
3	4	3	5	10297	12722	1	145	.0232977912
4	34	5	10	12722	12721	2	40	.0249982317
5	6	6	16	12722	7857	4	105	.0790328106
6	8	16	18	7857	7855	1	60	.0115721170
7	12	17	24	7857	2406	2	45	.0341989397
8	10	18	22	7855	2378	1	45	.0204930876
9	35	19	30	7855	7775	2	15	.0498837204
10	35	22	30	2378	7775	1	15	.0213639082
11	14	23	26	2378	7433	1	30	.0064190555
12	16	24	32	2406	9155	1	15	.0291734980
13	14	25	26	2406	7433	1	30	.0069523941
14	16	26	32	7433	9155	1	30	.0289455389
15	36	27	31	7433	6737	1	30	.0178032569
16	18	32	34	9155	7711	1	70	.0230358528
17	37	33	40	9155	10399	2	35	.0520619991
18	38	34	41	7711	7710	1	35	.0214009662
19	37	35	40	7711	10399	1	35	.0231442339
20	22	47	49	7705	7704	2	40	.0106819541
21	22	48	49	7705	7704	1	40	.0079618585
22	38	49	41	7704	7710	2	20	.0244461715
23	24	50	52	7704	7702	1	60	.0169365032
24	39	52	43	7702	7707	3	20	.0241453637
25	26	53	59	7702	10393	4	40	.0409987895
26	41	59	66	10393	7611	5	20	.0674744590
27	40	60	54	10393	7664	2	20	.0105314738
28	43	73	86	4544	4521	5	37	.0249245596
29	30	74	76	4544	10365	1	148	.0089016284
30	42	76	82	10365	4533	2	74	.0080114656
31	42	77	82	10365	4533	2	74	.0080114656
32	44	84	92	4524	4512	4	74	.0178032569
33	43	85	86	4524	4521	1	74	.0035606514
34	20	10	47	12721	7705	10	80	.1818455305
35	36	30	31	7775	6737	1	30	.0053409771
36	16	31	32	6737	9155	1	60	.0238018891
37	38	40	41	10399	7710	1	70	.0067792782
38	39	41	43	7710	7707	1	125	.0184274115
39	40	43	54	7707	7664	1	145	.0411526663
40	41	54	66	7664	7611	2	165	.0635072371
41	28	66	73	7611	4544	7	185	.2259736561
42	32	82	84	4533	4524	1	148	.0044508142
43	44	86	92	4521	4512	3	111	.0142426055
44	0	92	124	4512	624	32	185	.6759559327

Table 6 (continued)

Assignment of original links to condensed links:

1	2	3	2	4	5	5	2	4	34
5	34	34	5	34	6	7	8	9	34
7	10	11	12	13	14	15	9	34	35
36	16	17	18	19	34	17	34	34	37
38	34	39	22	24	24	20	21	22	23
20	24	25	40	25	25	27	40	26	27
25	26	26	26	26	41	41	41	41	41
41	41	28	29	28	30	31	28	30	31
28	42	28	32	33	43	32	43	32	43
32	44	44	44	44	44	44	44	44	44
44	44	44	44	44	44	44	44	44	44
44	44	44	44	44	44	44	44	44	44
44	44	44							

Analysis of distribution of condensed link lengths (universal links omitted):

	Mean	Standard Deviation	Root Mean 3rd Moment
Unweighted	.0281642528	.0301405954	.0451572248
Weighted by # Routes	.0298821753	.0340938503	.0488274069
Weighted by Flows	.0604198747	.0560138071	.0630014750

	S.D./Mean	3rd Moment/S.D.
Unweighted	1.0701720223	1.4982194035
Weighted by # Routes	1.1409427191	1.4321470416
Weighted by flows	.9270758561	1.1247490259