An analysis of airport-choice behaviour using the Mixed Multinomial Logit model

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ABSTRACT

In this paper, we describe part of an ongoing study of airport choice for passengers departing from the San Francisco Bay area. The aim of the present paper is to test for the prevalence of taste heterogeneity across travellers, using the Mixed Multinomial Logit (MMNL) model. Our results indicate the presence of significant levels of heterogeneity in tastes, especially with respect to the sensitivity to access time, characterised by significant (deterministic) variation between groups of travellers (business/leisure, residents/visitors) as well as random variation within groups of travellers. Our analysis reinforces earlier findings showing that business travellers are far less sensitive to fare increases than leisure travellers, and are willing to pay a higher price for decreases in access time (and generally also increases in frequency) than is the case for leisure travellers. Finally, the results show that the random variation between business travellers in terms of sensitivity to access time is more pronounced than that between leisure travellers, as is the case for visitors when compared to residents.

1. INTRODUCTION

During the last decade of the twentieth century, the demand for air travel grew at an average rate of 5% per annum (passenger-kilometres flown, c.f. IATA, 2002), and despite the impacts of the global economic downturn and the events of September 11th 2001, annual growth levels of 5.1% (passenger-kilometres flown) are forecast for the next 20 years (Boeing, 2003). While the growth in traffic has been accompanied by a comparable increase in the available seat-kilometres (IATA, 2002), there has been a lack of increases in airport runway and terminal capacity. As a consequence, pressure exists to expand capacity at many of world's busiest airports (c.f. DFT, 2003 and RAPC, 2000). These capacity-expansion decisions are complicated, not least because of the fact that many of the airports with capacity problems are part of a network of airports serving a multi-airport region. Thus, the case for capacity expansion depends not just on the total level of air traffic growth but also on its distribution across alternative airports. In such multi-

airport regions, the analysis of the possible impacts of capacity or policy changes on air travellers' choices of airports has become an important component of long-term transport strategies. Indeed, the shifting of demand between airports not only has an effect on the commercial viability of the single airports, but can have significant effects on the support structure of the airports (auxiliary businesses), as well as on seemingly less related businesses (e.g. local hotels). One of the most important consequences of increases in demand is the need for improvements in surface access facilities; this can cause major problems in metropolitan areas that often already suffer from severe ground-transport congestion. The effects of decreases in demand are equally significant, as the survival of many local businesses and related jobs (e.g. taxi companies) depends to a large degree on the success of the airport that was often responsible for their creation in the first place. As an example, a drop in demand for local transport services (induced by a drop in air traffic) can lead to significant job losses or even the partial writing-off of purpose-built business structure (e.g. new bus terminal). Finally, the growth in traffic has also led to high levels of congestion in some flight corridors, and especially so in the airspace surrounding large multi-airport regions, meaning that the effects of capacity changes on airspacecongestion similarly require careful consideration.

To a large degree, these factors all depend on the projected levels of passenger demand at the different airports, such that a key issue in multi-airport regions is the modelling of travellers' choice of airport. Although this area has attracted increased activity in recent years (e.g. Veldhuis et al., 1999, Pels et al., 2001, 2003, Basar & Bhat, 2002), the majority of the existing body of the work relies on basic modelling techniques, and the development of a systematic understanding of airport choice is still at a relatively early stage. In particular, compared to other dimensions of travel choice, relatively little is known about the variation in tastes across different market segments or within individual market segments, as well as about the extent of correlation between the various available alternatives along the three dimensions of airport choice, airline choice and access-mode choice.

In this paper, we present part of an ongoing study aimed at determining the optimal modelling structure for the analysis of airport choice in the San Francisco Bay (SF-Bay) area. This work builds on, and expands on, existing work by Hess & Polak (2004a, 2004b). In the present paper, we investigate specifically the prevalence of taste heterogeneity across air-travellers. To do this, we consider only the choice of airport, independently of related choice

dimensions such as those of main mode, access mode and airline. Future work will extend the current approach to include these choice dimensions. Unlike many previous studies, our analysis uses highly disaggregate data, by looking at the daily frequencies on the different routes, and by using detailed origin-destination matrices for the ground-level access dimension. In common with most previous studies, our analysis looks only at departing passengers, mainly because of the lack of detailed survey data for arriving passengers. However, by including resident as well as visiting passengers in the analysis, the models indirectly also look at the choice of arrival airport, given that the data collected from visitors corresponds to the return leg stage, for which the departure airport is in fact the arrival airport from the outbound flight (excluding the possibility of an open-jaw ticket). Connecting passengers are similarly excluded from the analysis, a decision that is motivated by the fact that such passengers rarely have a choice between different connecting airports in the same area. Even when this is the case, this choice may be influenced by a wide range of factors unrelated to the characteristics of the airports in the region of the interchange (e.g. allegiance to a specific airline, scheduled meeting at a specific connection airport). Finally, in addition to connecting passengers, travellers on indirect flights (intermediary landing without change of aircraft) are similarly excluded from the analysis.

The remainder of this paper is organised as follows. In section 2, we give a brief review of the existing literature in the field of airport choice modelling. This is followed by a presentation of the data in section 3, and a description of the modelling methodology in section 4. Section 5 discusses the various models fitted during the analysis, and section 6 reports the results. Finally, section 7 presents a validation of the modelling results, while section 8 briefly discusses avenues for further research in the context of the present application.

2. LITERATURE REVIEW

One of the first major studies of airport choice was conducted by Skinner (1976), who uses a Multinomial Logit (MNL) model for airport choice in the Baltimore-Washington DC area, and identifies flight frequency and ground accessibility as the main determining factors, with travellers being more sensitive to the latter. In a more recent study using an MNL model in this area, Windle & Dresner (1995) repeat the earlier results, and also identify a significant impact of past experience; the more often a traveller has used a certain airport in the past, the more likely he/she is to choose the same airport again.

The SF-bay area has been used in several case studies of airport choice, mainly thanks to the availability of very good data. An early example is that of Harvey (1987), who uses an MNL model, and finds access time and flight frequency to be significant for both leisure and business travellers, with lower values of time for leisure travellers. More recently, Pels et al. (2001) have conducted an analysis in this area using a Nested Logit (NL) model to look at the combined choice of airport and airline. The results indicate that both business and leisure travellers have a nested choice process in which airline choice is nested within the choice of airport. In a later study, Pels et al. (2003) again make use of the NL model structure, this time in the joint analysis of airport and access-mode choice, revealing high sensitivity to access time, especially for business travellers. In one of the most innovative studies of airport choice, Basar & Bhat (2002) propose the use of a two-level modelling structure in which the actual airport choice process is preceded by a choice-set generation stage, thus acknowledging the fact that some travellers only consider a subset of the available airports. The results show that this parameterised choice set consideration (PCMNL) model outperforms the MNL model, and suggest that flight frequency is the most important aspect in choice set composition, while access time is the dominating factor in the actual choice of airport.

There have also been a number of studies of airport choice in the United Kingdom. Ashford & Bencheman (1987), who use an MNL model for airport choice at five airports in England, find that access time and flight frequency are significant factors, with flight fares only having an impact for domestic passengers and for international leisure travellers. In a study that is of particular relevance to the present analysis, Ndoh et al. (1990) find that the NL model outperforms the MNL model in a study of passenger route choice in central England. Thompson & Caves (1993) use an MNL model to forecast the market share for a new airport in North England; access time, flight frequency and aircraft-size (as a proxy for comfort) are found to be significant, with access time being most important for travellers living close to the airport and frequency being more important for travellers living further afield. Finally, in an MNL analysis of the distribution of passengers between airports in the Midlands, Brooke et al. (1994) find flight frequency to be most the important factor.

In other studies from around the world, Ozoka & Ashford (1989) use an MNL model to forecast the effects of adding a third airport to a multi-airport region in Nigeria; the results show access-time to be very significant, making the choice of location and the provision of good

ground-access facilities important determinants in the planning process. On the quality-of-service side, Innes & Doucet (1990) use a binary logit model for airport-choice in Canada, and show that travellers prefer jet services to turboprop flights. Furuichi & Koppelman (1994) use an NL model for departure and destination airport choice in Japan, showing significant effects by access time, access journey cost and flight-frequency. Finally, Veldhuis et al. (1999) produce the comprehensive Integrated Airport Competition Model, showing that passenger behaviour is represented most appropriately by a sequential NL choice process that models the choice of main mode followed by the combined choice of airport and air-route, and finally the choice of accessmode at the chosen airport.

3. DATA

The SF-Bay area is served by three major airports; San Francisco International (SFO), San Jose Municipal (SJC) and Oakland International (OAK). SFO is the largest of the three airports, with, in 1995, some 15 million emplaned passengers (~55.8%), compared to around 4.2 million passengers at SJC (~15.6%), and some 7.7 million passengers at OAK (~28.6%). Forecasts by MTC (2000) predict significant increases in traffic; these will inevitably lead to problems with capacity, and different expansion schemes are already under consideration (RAPC, 2000). Data from various sources were used in this analysis; these data sources are described in this section.

3.1. Air-passenger survey data

Data on individual travellers' airport choice were obtained from the 1995 Airline Passenger Survey conducted by the Metropolitan Transport Commission (MTC) in August and October 1995, containing information on over 21,000 departing air-travellers (MTC, 1995). Passenger interviews were conducted at the three main SF-Bay area airports, as well as at the minor Sonoma County airport (STS), which was not included in the present study. The sample of passengers interviewed at the three main airports is not entirely representative of the real-world traffic at the airports; indeed, SJC is over-sampled, while OAK is under-sampled. This sampling needs to be taken into account in the modelling in order to avoid any risk of biased results. In the present analysis, we account for the sampling effects by using the Weighted Exogenous Sampling Maximum Likelihood (WESML) approach, in which each observation is assigned a weight in the likelihood function that represents the relative real-world market share of the chosen alternative compared to its market-share in the sample used in the analysis. Appropriate

weights were calculated separately for each of the sub-samples used in the various models. This procedure is described in more detail in section 5.3.

The data selection process is based on that used by Hess & Polak (2004a, 2004b). It was decided to use only destinations that could be reached by direct flight from all three airports, on every day of the week (at the time of the survey), leading to 14 destinations, and a sample of 9,924 respondents (after initial data cleaning). This contained some 3,246 travellers who indicated that they could not have flown out of a different airport. Possible reasons for this include unavailability (at the time of booking) for flights from other airports on the chosen flight date and time (especially likely for travellers with inflexible timing), misinformation of the traveller, or an *a priori* decision not to consider any of the other airports. Hess & Polak (2004a) show that the inclusion of these travellers leads to biased results, leading to the decision to exclude these observations from the analysis. In a way, this acts as an approximation to a model that incorporates choice-set generation. From the resulting sample of 6,678 travellers, a further 1,581 passengers needed to be excluded, for two main reasons. Initially, five main journey purposes were identified in the dataset; business, holiday, visiting friends and relatives, extraordinary events, and others. As the extraordinary events group contained a wide range of trips (from funerals to weddings), it was decided that it would be wrong to simply include them in a wider leisure group, as has been done previously. Indeed, the short-term planning and/or precise timing associated with some of these events make the decision-process more similar to that of a business trip. As there are however arguably also important differences between this type of trip and a business trip, and as it was not possible to fit a sensible separate model for these 299 trips, it was decided to exclude them from the analysis. An identical decision was taken for the 360 trips that had some other purpose. A further 922 observations had to be excluded due to the absence of information on household income (required to allow for a segmentation by income) or because of various other missing-information problems. The final sample thus contained 5,097 observations, with flights to 14 destinations. The data used are summarised in table 1, which illustrates the oversampling of SJC, where a larger sample is used than at SFO, despite the fact that traffic levels at SFO were over three times higher than at SJC in 1995. The specific choice of destinations had little or no effect on the distribution of flights across other dimensions, such as journey purposes and household income. Clearly, the sampling

has an effect on the market shares of the different airlines; as this study does not explicitly model the choice of airline, this is however of little importance.

Table 1: Destinations used in the analysis																
		Desti	nation a	irport												
		BURBANK, CA	CHICAGO, O'HARE, IL	DALLAS, FT. WORTH, TX	DENVER, CO	LAS VEGAS, NV	LOS ANGELES, CA	ONTARIO, CA	ORANGE COUNTY, CA	PHOENIX, AZ	PORTLAND, OR	RENO, NV	SALT LAKE CITY, UT	SAN DIEGO, CA	SEATTLE, WA	TOTAL
Domontuno	SFO	54	97	45	74	65	203	35	38	129	140	1	48	261	220	1410
Departure Airport	SJC	161	71	97	89	158	370	113	246	128	105	147	66	237	157	2145
	OAK	203	6	26	12	67	370	130	168	47	96	35	42	133	207	1542
Total		418	174	168	175	290	943	278	452	304	341	183	156	631	584	5097

Special care is required in the presence of destinations that are themselves located in multi-airport regions. After careful consideration, destinations from two such multi-airport regions were included in the present analysis, namely destinations in the wider Los Angeles area, and one of the two main Chicago airports. The decision to include airports from the Los Angeles area was motivated primarily by the high representation of these destinations in the survey data. It is in this case important to consider whether passengers' choice of departure airport in the SF-Bay area may have been influenced by their choice of destination airport in the Los Angeles region, especially for passengers whose return journey started in the Los Angeles area. During the period of observation, daily flights were available between each of the three SF-Bay area airports and each of the five airports in the wider Los Angeles region. As there is relatively high frequency on all routes, passengers can be expected to make a specific choice of airport in the SF-Bay area, independently of the choice of airport in the Los Angeles area. This assumption is further supported by the fact that the differences in overall level-of-service characteristics between the three SF-Bay area airports are similar across the five destinations in the Los Angeles area, such that the choice-sets faced are similar across the five airports in the Los Angeles area. In the case of Chicago, only O'Hare (ORD) airport was included in the analysis. The inclusion of ORD was motivated by the comparatively very low frequency of direct flights to Chicago's alternative airport; Midway (MDW), almost guaranteeing that the choice of airport in the SF-bay area takes precedence. A comparison of the results produced in two small-scale separate analyses

that included, respectively excluded these destinations, revealed no major differences, suggesting that the inclusion of these airports has no significant effects on the modelling analysis.

3.2. Air-travel level-of-service data

The air-travel level-of-service dataset contains information on flights between given pairs of airports. For the present analysis, air-travel level-of-service data were obtained from BACK Aviation Solutions¹. The dataset contains daily information on the different operators serving the selected routes in August and October 1995, thus making the data more detailed than that of many previous studies that have relied on the use of weekly or even monthly data. Besides the frequencies for the different operators, the dataset also contains information on flight times and the aircraft used on the different flights. Finally, the dataset contains information on the average fares paid on a given route operated by a given airline. This clearly involves a great deal of aggregation, as no distinction is made between the fares for the different classes of travel. Furthermore, advance purchase discounts applying to a certain traveller on a given date cannot be taken into account when using this data. As in previous studies, it must thus be assumed that the fare structure is the same at the different airports, in terms of advance purchase discounts and availability of different types of tickets at a given time. This is clearly a major assumption, not least because the rate at which tickets sell out can vary across airports. Unfortunately, this assumption cannot be avoided, given the quality of the data. The availability of given fareclasses can be modelled in the presence of adequate data on the distribution of fares across classes (c.f. Battersby 2003); this is beyond the scope of the current analysis, but the inclusion of such an approach into a wider framework for air-travel related choice modelling is an important avenue for further research.

3.3. Ground-access level-of-service data

As access journeys are known to play an important role in airport choice, information on the access options at the different airports is a prerequisite for any model-fitting exercise, even if the models do not explicitly look at the choice of access-mode. For the present analysis, ground-access level-of-service information was obtained from the MTC in the form of origin-destination travel time and cost matrices for the 1099 travel area zones (TAZ) used for the SF-Bay area. Some information is clearly lost due to the aggregation into travel zones, this is however unavoidable due to the high number of possible ground-level origins, and the effects should be

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¹ Back Aviation Solutions, 6000 Lake Forrest Drive, Suite 580, Atlanta, GA 30328, www.backaviation.com

minimal, given the small differences between the origins situated in a given TAZ. These matrices contain information on travel distance, travel time and tolls for car travel (under peak and off-peak conditions, and for varying car-occupancy) and on access time, wait time, travel time, egress time and fares for public transport. Corresponding data for other modes, such as taxi, limousine and special airport bus services were calculated separately, based on current prices (September 2003) and the change in the Consumer Price Index for California from August and October 1995 to September 2003.

3.4. Data assembly and choice-set construction

As the mode-choice dimension is not analysed explicitly in the present analysis, the travel time and cost information for a given respondent corresponds to the mode actually chosen by this respondent. This is clearly a significant simplification of the actual situation (as it assumes that the same mode would have been chosen at a different airport), but does at least give some idea of the differences in access journeys to the different airports. The impacts of this assumption are also weakened by the low elasticity for access-mode changes in the SF-bay area (c.f. Hess & Polak, 2004b). As the present study also ignores the airline-choice dimension, aggregate airtravel level-of-service data were used, assigning to each passenger the industry-level information on frequencies, fares, number of operators, flight time and minimum equipment size, for flights from each of the three airports to the desired destination on the actual date of travel.

4. MODELLING METHODOLOGY

Discrete choice models have been used extensively for over twenty years in the field of transportation research. Initially, virtually all applications were based on the Multinomial Logit (MNL) model and basic Nested Logit (NL) models; more recently, the use of more flexible model forms, such as advanced Generalised Extreme Value (GEV) models and the Mixed Multinomial Logit (MMNL) model has increased dramatically. For an extensive review of existing discrete choice models, see for example Train (2003).

The MMNL model (c.f. McFadden & Train, 2000) offers significant advantages over the MNL model by allowing for random taste variation across decision-makers, thus acknowledging the differences across agents in their sensitivities to factors such as fare and frequency. The random-coefficients formulation of the MMNL model uses integration of the MNL choice probabilities over the assumed distribution of the random taste coefficients, such that the probability of individual n choosing alternative i is given by:

$$P_{ni} = \int_{\beta} \frac{e^{\beta' X_{ni}}}{\sum\limits_{j=1}^{I} e^{\beta' X_{nj}}} f(\beta | \theta) d\beta, \qquad \dots (1)$$

where X_{ni} is a vector of explanatory variables for alternative i as faced by decision-maker n, and β is a vector of taste coefficients that measure the impact of these explanatory variables on the utility of an alternative. In the MMNL model, the vector β varies across decision-makers and reflects the idiosyncratic aspects of a given decision-maker's preferences; these terms are distributed in the population with density $f(\beta|\theta)$, where θ is a vector of parameters to be estimated that represents, for example, the mean and variance of preferences in the population.

The MMNL model not only allows for random taste variation, but also in principle avoids the unrealistic MNL substitution patterns resulting from the *Independence from Irrelevant Alternatives (IIA)* assumption, which dictates that the dependency between any two alternatives is the same across alternatives, and makes the MNL model an inappropriate choice in many scenarios. The MNL model has been used repeatedly in airport choice modelling, and several authors (e.g. Ashford & Bencheman, 1987 and Thompson & Caves, 1993) have justified the use of the MNL model by claiming that airports are independent entities, such that *IIA* would be a valid assumption. This is however far from clear, as in some cases, it seems that there is a possibility of varying cross-elasticities across pairs of airports in a multi-airport region, given the similarities, respectively dissimilarities between some of the airports (e.g. business airport versus no-frills airlines base).

The biggest drawback of the MMNL model is the fact that the integrals representing the choice probabilities do not have a closed-form expression and need to be approximated through simulation. Standard Monte-Carlo approaches can be very computationally expensive, and various alternative approaches have been proposed to reduce the computational burden. For an introduction to these quasi-Monte Carlo approaches, see for example Bhat (1999), and Hess et al. (2003, 2004). In the present application, a relatively straightforward type of quasi-Monte Carlo approach, the Halton sequence (Halton, 1960), was used, leading to important savings in simulation cost over the use of pseudo-random draws. A second issue with the MMNL model is the choice of distribution to be used for the random taste coefficients, especially in the case where an *a priori* assumption exists about the sign of a given coefficient. For a discussion of this issue, see for example Hensher & Greene (2001) or Hess & Polak (2004c).

5. MODELLING APPROACH

5.1. Explanatory variables

A large number of different model specifications were fitted to the data during the analysis. In section 6, we describe the results obtained with the final estimated models. The results produced by the more basic MNL and MMNL used in the early stages of the research are not described in detail in the present paper; more in depth descriptions and results for these models are available on request (see also Hess & Polak, 2004a). In each model, the influence of a number of attributes was explored. These attributes included fare, frequency, access journey time, access journey cost, flight time, the number of operators serving a route, the minimum size of equipment used, and the on-time performance at the different airports. Only fare, frequency and access journey time were found to have a consistently significant effect. The lack of effects by other variables could be due to the use of aggregate data (airport-specific), and different results might be expected with the use of airline-specific data; this is the topic of ongoing research.

At this point, it seems worthwhile pointing out that the frequency coefficient is of special interest. Indeed, it should be noted that, due to data limitations, it is not presently feasible to model the interaction between the available departure times and individual travellers' preferred departure times. Under the considerable assumption of a relatively even spread of flights across the day, the frequency coefficient can be seen as giving an estimate of the effect of changes in the difference between a desired departure time and the next best available departure time. In this context, higher frequency means more reliability and a lower risk of not arriving at the destination *on time*. It should also be noted that the frequency coefficients capture a *visibility effect*, in that, ceteris paribus, options with a higher frequency of service have a higher chance of being selected, due to higher representation in the choice set.

Given that it was not possible to identify a significant effect for access-cost in any of the models, it was similarly impossible to give a proper estimate of the value of access-time savings. An indication of the monetary value of access-time changes can be given by looking at the ratio between the access-time coefficient and the air-fare coefficient. The estimate of this ratio should however be expected to be higher than the actual value of access-time, given the different meaning of the fare and access-cost coefficients, and the significant differences in scale between the associated attributes. As such, a lower marginal utility would be associated with a change in air-fare by one dollar than in a change in access-cost by the same value.

5.2. Distributional assumptions

Due to limitations in model specification, but also in the quality of the data available, researchers are never able to capture all information that affects the choice of a given decision-maker. As such, the utility of a given alternative is not fully observed, and an error term, or unobserved part of utility, remains. By adding alternative specific constants (ASC) to the utility of alternatives, researchers in effect add the mean of this randomly distributed error term into the observed utility function, such that the remaining error term has a mean of zero. These ASCs thus capture the mean effect of all unobserved variables attributes, including general attitude towards an alternative, while the remaining error term captures the variation in this effect. For identification reasons, one of the ASCs needs to be normalised (typically to zero), and whereas in fixed coefficients model, the choice of normalisation is of no importance, in the MMNL model, special care is required in the normalisation of the ASCs, the means of which can themselves be distributed randomly across the population (Hensher & Greene, 2001). To minimise the error caused by not accommodating the variation in the ASC set to zero, the ASC with the least amount of variation across decision-makers should be chosen for normalisation. As significant variation could only ever be found for the ASC associated with SFO, the normalisation setting the ASC of OAK to zero could safely continue to be used.

Another important issue already referred to in section 4 is the choice of distribution for randomly distributed coefficients. A Normal distribution can safely be used for ASCs, thus allowing for positive as well as negative impacts of unmeasured variables across decision-makers. However, in the case of coefficients with an a priori sign assumption, the use of the Normal distribution should be avoided, as it leads to a positive probability of wrongly signed coefficients (c.f. Hess and Polak, 2004c). It is thus preferable to use a distribution that produces strictly positive or negative coefficients. The classic choice of such a bounded distribution is the Lognormal distribution, which produces strictly positive coefficients, such that, in the case of an undesirable attribute (e.g. cost), the sign of the attribute needs to be reversed. The use of the Lognormal distribution can occasionally lead to problems with convergence and very high standard deviations (c.f. Train, 2003); however, in the present analysis, no such problems were observed, such that the Lognormal distribution could safely be used. Besides being more intuitively appealing (strictly signed coefficients), the use of the Lognormal distribution in this case also led to better model fit. As an example, a simple MMNL model with two fixed taste

coefficients (fare and frequency) and one randomly distributed coefficient (access time) produced a log-likelihood of -3461.74 when using the Normal distribution, whereas the corresponding model based on the Lognormal distribution produced a log-likelihood of -3447.79, with the same number of parameters.

At this stage it seems worthwhile to mention that, in the remainder of this paper, the tables showing the results for models based on the use of the Lognormal distribution give the estimated parameters of the underlying Normal distribution c and s (of which the Lognormal distribution is a transformation), along with the corresponding values of the actual mean and standard deviation of the Lognormal distribution, μ and σ , where the t-test values are associated with c and s. Given that both mean and standard deviation are functions of c and s, it is important for both t-values to be significant (c.f. Hess & Polak, 2004c). Finally, for ease of interpretation, the sign of μ was reversed in the tables, reflecting the change of the sign of the associated attribute.

5.3. Deterministic taste heterogeneity

While the MMNL model gives researchers the power to explain variations in tastes with the use of statistical distributions, attempts should always be made to explain as much of this variation as possible using deterministic approaches that segment the population with the help of socioeconomic and demographic variables. This generally comes in the form of separate models for different population groups, or separate coefficients for different groups within the same model, but can also come in the form of a continuous relationship between attributes of the alternatives and attributes of the decision-makers (c.f. Train, 2003).

Three dimensions of segmentation were used in the present analysis; purpose, residency status and income. Ideally, we would have wished to explore segmentation by ticket type – e.g., business *vs* economy class, but since information on ticket type was not available, it was only possible to explore segmentation by travel purpose, in this case business *vs* leisure travellers, where the leisure group contains holiday travellers, and travellers *visiting friends or relatives*. Major differences between these two groups can still be expected, notably because of the differences in terms of flexibility and who bears responsibility for the fare paid. It is similarly important to realise that there are potentially important differences between residents and visitors in their evaluation of the different travel options. Although the selection of flight destinations (corresponding to the origins of visitors) should guarantee that for both groups of travellers, there

is an informed choice of airport from the different options available in the SF-Bay area, differences between residents and visitors can be expected, for example due to the fact that visitors generally have a less detailed knowledge of the area. Finally, it should be expected that income has a significant effect on travel behaviour, most notably in the sensitivity to fare. These differences can be accommodated quite easily by estimating separate taste coefficients in different income groups. For this, respondents were grouped into three roughly equally sized groups (less than \$21,000, between \$21,000 and \$44,000 and above \$44,000 per annum). An alternative approach would have been to explicitly model the continuous relationship between income and the sensitivity to factors such as fare and access time; this is however beyond the scope of the present analysis.

The double segmentation by residency status and by purpose leads to four separate residency/purpose groups. An important question arises with regards to which approach should be used to account for the differences between these four groups. The first approach is to estimate a common model for the four groups, and to use separate coefficients for the four groups for at least some of the attributes. The second approach is to estimate a separate model for each group, thus acknowledging potential differences across groups in all of the coefficients, including the ASCs. A separate analysis (detailed results available on request) showed that, although the approach using separate coefficients within the same model does have a slight computational advantage over the approach using separate models, the use of separate models is preferable, as the gains in combined model fit by far exceed the critical values of the appropriate statistical tests. This shows that the differences across residency/purpose groups cannot be completely explained by the use of separate taste coefficients, but that there also exist substantive differences in the effects of unmeasured variables (captured by the ASCs). On the other hand, no further gains could be made by estimating separate models for the three different income groups, such that (where necessary) separate coefficients for the three income groups would be used inside the four different models estimated.

As mentioned in section 3.1., the WESML approach was used to reflect the sampling effects caused by the selection of respondents on the basis of screening criteria (c.f. MTC, 1995). For this, aircraft occupancy data was used to calculate the total traffic on the different routes used in the analysis, across carriers. From this, relative weights were assigned to each origin-destination pair. A similar process was used to calculate corresponding weights for the sample

data used in the present analysis, where separate weights were calculated for residents and visitors in the two purpose groups, leading to four different sets of weights for the four different models fitted. For added precision, the season (August or October) was also taken into account in the calculation of the weights for the individual observations. The individual pairs of weights were then used to calculate multiplicative weights that could be used in the analysis, where the weight for a given origin-destination pair was given by dividing the actual population weight by the sample weight for this pair. This process was repeated for each observation used in the analysis, and it is easy to see that the resulting sum of weights over the total number of observations in a given group is equal to the number of observations used in this group. In the estimation process, each term in the log-likelihood function was then multiplied by the appropriate weight for the associated chosen alternative.

5.4. Non-linear utility functions

Another important point that needs to be considered in model specification is the way in which explanatory variables enter the utility function. Generally, a linear specification is used in discrete choice models, such that increases in the value of an attribute lead to linear changes in the utility of the associated alternative. However, this approach is not optimal in the case of attributes for which decreasing marginal returns in utility would be expected. In the case of airport choice modelling, the most prominent example of such an attribute is flight frequency. While a linear specification of flight frequency leads to constant marginal returns for increases in frequency, independent of the original frequency level, it is quite reasonable to expect that, beyond a given frequency level, any additional increases in frequency will not lead to important gains in utility. If this is indeed the case, then the use of a linear specification can be expected to lead to an underestimated effect at low initial frequencies and an overestimated marginal return of frequency increases at already high initial frequencies. A non-linear specification of frequency can be accommodated in the models by replacing the absolute frequency levels by a formula that gives a decreasing marginal return. With such an approach, an increase in frequency at a lower base frequency is valued more highly than an increase at higher base frequency, and the increase in utility resulting from an increase in frequency by K flights at a base frequency of f is less than K times the increase resulting from the first additional flight. Two non-linear specifications were compared in a separate analysis conducted for the present research. The first approach uses the natural logarithm, as used previously by Veldhuis et al (1999) and Pels et al. (2003), while the

second approach uses a method proposed by Brooke et al. (1994), replacing the frequency f by (f-0.5)/f, such that, beyond a frequency of around 8-9 flights, any increases in frequency lead only to minor increases in utility. A simple MMNL model using five coefficients (including two ASCs) was estimated using these two approaches. The base-model had a log-likelihood of -3447.79, while the use of a non-linear specification of frequency leads to a log-likelihood of -3389.38 with the specification proposed by Brooke et al. (1994) and -3371.92 with the natural logarithmic transform. The fact that either approach leads to a very significant gain in model fit (with no additional parameters) shows that the use of a linear specification is not appropriate, the better model fit when using the natural logarithm transform suggests that this specification is more appropriate in the context of the present dataset (where a frequency of 8 flights per day is a more common occurrence than in the case study of East Midlands airport conducted by Brooke et al., 1994). Given the important gains in model fit resulting from the use of non-linear specification, it was decided to use this approach for the remaining models developed in the analysis, with preference being given to the logarithmic transform. The effects of this specification are described in more detail in the next section, with the help of the estimates produced by the final models. Finally, attempts were also made to use a non-linear specification for the remaining coefficients of fare and access-time, where decreasing marginal returns can similarly be expected. However, the use of such a specification did not lead to any significant gains in model fit for either of these coefficients.

6. MODELLING RESULTS

In this section, we follow the recommendations made in sections 5.3. and 5.4., by fitting separate models for resident and visiting business travellers, along with separate models for resident and visiting leisure travellers, and by using the natural logarithm specification for the marginal utility of flight frequency. The final sample of 5,097 individuals was divided into 1,268 resident business travellers, 1,500 resident leisure travellers, 1,269 visiting business travellers, and 1,060 visiting leisure travellers. For each of these four groups, a random sub-sample of roughly 10% was removed and retained for later validation of the models on data not used in the estimation.

The results of the estimation process are summarised in table 2. In each one of the four models, there was sufficient variation in the sensitivity to access-time to use a random coefficient that follows a lognormal distribution. In addition, significant variation to enable the use of a normally distributed ASC for SFO was identified in each model except the model for business

trips by visitors. It was not possible to identify significant random taste heterogeneity in order to use random coefficients for fare and frequency; the lack of additional variation can be partly explained by the use of aggregate data (airport specific), and ongoing work has revealed the existence of additional levels of heterogeneity when looking at the additional choice dimensions of airline and access-mode.

In each case, the use of the MMNL specification led to statistically significant gains in model fit over the corresponding MNL structure, with the most significant gain being obtained for the model for visiting business travellers, despite the fact that this model has only one randomly distributed coefficient. Attempts were also made to use segmentation by income, however the differences in sensitivities between income classes were not generally significant. The only exceptions are the model for resident business travellers, where a significant effect of fare could only be identified for the low-income group, and the model for visiting leisure travellers, where a separate frequency coefficient was used for the high-income group, with a common coefficient for the low and medium-income groups. It was not possible to identify a significant effect of fare for the model for business trips by visitors; this comes in addition to the inability to estimate such an effect for the medium and high-income groups for resident business travellers. The failure to estimate a significant effect of fare could reflect the comparatively low sensitivity to fare for business travellers, but could also be partly due to the use of highly aggregate fare information. Other researchers have encountered similar problems with estimating a significant fare coefficient (e.g. Pels et al., 2003). Finally, it should be noted that for visiting leisure travellers, the ASC for SJC is positive, while the ASC for SFO is negative. This can be explained by noting that the ASCs are greatly affected by the use of the WESML approach, in an effort to retrieve the true market shares. Unlike in the overall sample, SFO and especially OAK are not as heavily under-sampled in this subsample, and as a consequence, SJC is less oversampled, leading to a lower need for sample weight correction.

Trade-offs between the access-time coefficient and the fare coefficient cannot be calculated for visiting business travellers, given the lack of fare coefficient for this model; the same applies for the trade-off between the frequency and fare coefficients. Trade-offs between frequency and access-time coefficients on the other hand can be calculated for all four models. In the case of trade-offs involving randomly distributed coefficients, it is of interest not just to calculate the mean values of access-time, but to incorporate the full distribution of this

coefficient in the calculation. This not only gives an account of the variation in these trade-offs across the population, but also avoids a major risk of biased estimates (c.f. Hensher & Greene, 2001, Hess & Polak, 2004c).

Table 2: Results from Mixed Logit models using segmentation by purpose and division into residents and visitors

	Resident b	ousiness	Resident leisure		Visitor business		Visitor leisure	
<u>Parameter</u>	<u>Estimate</u>	<u>t-test</u>	<u>Estimate</u>	<u>t-test</u>	<u>Estimate</u>	<u>t-test</u>	<u>Estimate</u>	t-test
Fare (common)			-0.0475	-3.8			-0.0477	-3.7
Fare (income < \$21,000)	-0.0430	-2.55						
Frequency (common)	1.9469	5.6	1.8333	5.7	1.8881	7.7		
Frequency (income < \$44,000)							1.9701	5.2
Frequency (income > \$44,000)							3.0328	5.2
Access time c	-1.8571	-15.5	-1.8916	-17.1	-1.9706	-20.6	-1.9669	-13.0
Access time s	0.6742	4.3	0.5102	3.6	0.9373	5.4	0.6934	5.5
Access time μ	-0.1960	N/A	-0.1718	N/A	-0.2163	N/A	-0.1779	N/A
Access time σ	0.1487	N/A	0.0937	N/A	0.2566	N/A	0.1398	N/A
ASC SFO mean	1.1563	4.2	0.9289	3.9	0.3632	2.5	0.5028	1.9
ASC SFO std.dev	2.0260	3.6	1.3650	2.7			1.6019	2.2
ASC SJC	-0.1045	-0.5	-0.1515	-0.8	-0.7767	-3.7	0.7784	2.8
Observations	1,140		1,347		1,142		952	
LL	-604.03		-659.67		-573.67		-514.62	
LL (MNL)	-615.53		-666.22		-592.05		-519.92	

For the trade-off between access-time and flight fare, the random variable is only used in the nominator of the quotient, such that the distributional characteristics of the trade-off, which likewise follows a lognormal distribution, can be obtained by dividing the mean and standard deviation of the access-time coefficient by the constant cost coefficient. For trade-offs where the random variable forms the denominator of the quotient, the distribution was calculated empirically. For this, a high number of draws (1,000,000) from the distribution of the access-time coefficients were produced for the different models, using the respective parameters of the lognormal distribution. The distribution of a given trade-off can then be approximated as the distribution of the values resulting from dividing the concerned fixed taste coefficient by the different draws. In the present application, an example of such a trade-off is given by the willingness to accept access-time increases in return for increases in flight frequency. Due to the

use of the logarithmic transform for frequency, this trade-off does not directly give the willingness to accept access-time increases for increases in frequency, but rather, the resulting ratio needs to be multiplied by the difference between the logarithm of the new frequency and the logarithm of the old frequency to obtain a *real* measure for the trade-off. Again, the resulting ratio roughly follows a Lognormal distribution.

The use of the Lognormal distribution leads to a long tail for the access-time coefficient, which is carried over into the empirical calculation of trade-offs involving this coefficient. A possible approach to dealing with this issue is to remove the upper few percentile points of the produced sample of draws, and to base any calculations of trade-offs on this censored sample (c.f. Hensher & Greene, 2001). Aside from the difficulty of deciding how many percentile points to remove, this approach arguably introduces a form of bias into the calculation. In our analysis, we found that the use of 1,000,000 draws without censoring led to virtually the same results as an approach using the 98 lower percentiles of 100,000 draws. This suggests that, with this high a number of draws, the impact of the tail of the distribution on the calculation of the mean and standard deviation are negligible, and the use of a censoring approach can be avoided.

A final trade-off looks at the implied willingness to pay for frequency increases, where this calculation involves no randomly distributed coefficients, and where the resulting trade-off is not available for visiting business travellers. Again, the resulting trade-off needs to be multiplied by the difference between the logarithm of the new frequency and the logarithm of the old frequency to obtain a measure of the willingness to pay.

The resulting values are shown in table 3. For the trade-offs involving frequency, the need to multiply the resulting value by the difference in the logarithms of the new and old frequencies is indicated through multiplication by K. To give a meaning to the calculated trade-offs for frequency changes, the table also gives corresponding values for an increase by one flight at a base frequency of five flights per day, where K is equal to 0.1823. The corresponding value of K at a base frequency of 10 flights is 0.0953, showing the decreasing marginal returns with this specification.

The results indicate a higher willingness to pay for access-time decreases for resident business travellers than for resident leisure travellers (no measure available for visiting business travellers), especially when taking into account that the fare coefficient estimated for resident business travellers is for the low-income groups only. The results further indicate that, while the

mean willingness to pay is very similar for resident and visiting leisure travellers, the withingroup variation is more important for visiting leisure travellers. The implied willingness to pay for access-time decreases can be expected to be even higher for visiting business travellers, given that it was not possible to identify a significant fare effect for this group of travellers. Although, as mentioned before, the calculated trade-offs should not be seen as an estimate of the value of access-time, given the use of the flight fare rather than the access cost coefficients, the estimated values are still very high. This is a direct result of the low air-fare coefficient, which is at least partly due to the relatively poor quality of the fare information used. However, the size of the ratio is clearly also a result of the high access-time coefficient, which could possibly indicate that travellers interpret increases in access time as increases in the risk of missing a flight. This explanation is supported by the high values of access-time reported in studies where an accesscost coefficient could be identified. For example, Pels et al (2003) use the same data as the present study and report values of \$2.90/min for business travellers in August and \$1.97/min for business travellers in October. Lower values were reported in older studies; for example, Harvey (1986) gives a value of \$0.69/min, while Furuichi and Koppelman (1994) give a value of \$1.21/min. This could also signal increasing values of time over time.

Table 3: Trade-offs [standard deviations in brackets where applicable]								
	Resident business	Resident leisure	Visitor business	Visitor leisure				
Trade-off between access-time and flight fare coefficient (\$/min)	4.56 [3.46] ^b	3.62 [1.97]	N/A	3.73 [2.93]				
Trade-off between frequency increases and access- time increases (min/flight) ^a	15.64 K [11.84]	13.85 K [7.55]	20.99 K [24.62]	17.91 K [14.00] 27.57 K [21.56]				
Willingness to pay for frequency increases (\$) ^a	45.27 K ^b	38.60 K	N/A	41.30 K 63.58 K				
Mean willingness to accept access-time increases for one additional flight at a base frequency of 5 flights (min)	2.85	2.53	3.83	3.27 ^c 5.03 ^d				
Willingness to pay for one additional flight at a base frequency of 5 flights (\$)	8.25	7.04	N/A	7.53 ^c 11.59 ^d				

^a K = ln(f+1) - ln(f); ^b low-income travellers only;

^c low-income and medium-income travellers only, ^d high-income travellers only

In terms of the willingness to accept access-time increases in return for frequency increases, the results indicate a higher mean willingness for visiting business travellers than for

resident business travellers (20.99K vs 15.64K), despite the fact that the simple ratio between the coefficient mean values would suggest the opposite (8.73K vs 9.93K). This is caused by the much larger standard deviation in the coefficient for visitors than for residents, and illustrates the importance of incorporating the full distribution of the coefficients in the calculation of tradeoffs. This comparison shows that the simple ratio of means approach thus not only underestimates the trade-offs, but also wrongly predicts a higher willingness to accept access-time increases for residents than for visitors, potentially leading to wrong policy implications. A similar problem occurs when using the MNL.

The models similarly show a higher relative desire for frequency increases for visiting leisure travellers than for resident leisure travellers, with increasing willingness to accept access-time increases for visiting travellers in higher income classes. The results also suggest that in both of these income groups for visiting leisure travellers, the desire for frequency increases is larger than the common trade-off for resident business travellers. The observations for the willingness to pay for frequency increases are very similar, with the exception that only the willingness to pay of high-income visiting leisure travellers' is above the common willingness of resident business travellers. In terms of the actual real-world values of one additional daily flight with a base frequency of five flights, the implied trade-offs between frequency and access-time increases seem a bit low, which this could again signal the perception of access-time increases as increases in the risk of missing a flight. The monetary values of one additional flight seem realistic, though possibly also at the low end of the *real* values.

7. MODEL VALIDATION AND PREDICTION PERFORMANCE

The first part of the model validation process was concerned with applying the four calibrated models described in section 6 to the respective estimation samples, and calculating the average choice probabilities assigned by the models to the actual chosen alternatives, leading to the average probability of correct prediction over alternatives. This approach produces correct prediction probabilities of 64.29% for resident business travellers, 67.95% for resident leisure travellers, 66.46% for visiting business travellers and 65.85% for visiting leisure travellers. These values are lower than those reported recently by Basar and Bhat (2003), who obtained a correct prediction rate of 74.9%. However, when taking into account the use of a simplistic utility function (only three terms), the use of airport- rather than airline-specific level-of-service information, and the fact that choice-set formation was excluded from the analysis, the

performance of the models is actually very good, and reflects the relative explanatory power of the three variables used in the models (fare, frequency and access-time). The lower correct prediction is also partly due to the relatively small samples used for the models in section 6, which is a result of the division into four models. Indeed, the use of corresponding models using only segmentation by purpose led to correct prediction rates of just over 70%, with a similar rate for models using segmentation by residency status only. Nevertheless, the models using a division into residents and visitors as well as business and leisure travellers should be preferred, given their higher explanatory power in terms of highlighting the differences between the individual groups of travellers. Furthermore, it should be noted that the aim of this paper was not necessarily to produce the best performing model in terms of producing forecasts, but to test for the presence of taste heterogeneity, and to exploit the power of the MMNL model in the representation of the random component of any such heterogeneity.

Table 4: Prediction performance on validation sample									
	Resident business	Resident leisure	Visitor business	Visitor leisure					
Observations	128	153	127	108					
Share SFO	56.41%	52.35%	56.20%	52.67%					
Share SJC	15.91%	16.02%	15.28%	15.18%					
Share OAK	27.68%	31.63%	28.52%	32.15%					
Correct prediction	67.61%	66.09%	67.03%	68.25%					

The most telling test of model performance is however the ability of the final calibrated models to correctly predict the market shares and choices in data that were not used in the actual model calibration process. For this purpose, the four models were applied to the four validation samples retained for this use. The results of this validation process are shown in table 4, giving the weighted predicted market shares, along with the average probability of correct prediction. The results show that for all but the model for resident leisure travellers, the correct prediction performance on the validation sample is actually higher than that obtained with the estimation sample, suggesting that the models have not been *over-fitted* on the estimation data, and are capable of offering good performance on *unknown* data. In terms of reproducing the weighted market shares for the three airports, the performance is again very good, although the two models for leisure travellers tend to slightly overestimate the market share for OAK and underestimate

the market share for SFO (as a reminder, the overall real-world market-shares of the three airports were 55.8%, 15.6% and 28.6% for SFO, SJC and OAK respectively).

8. FURTHER RESEARCH

A number of directions for future research can be identified. An obvious extension is to the joint modelling of airport choice, ground-level access-mode choice and airline choice, which will require the use of a nesting structure to capture the (potentially complex) substitution patterns amongst alternatives. This is the topic of a parallel research project, with initial results reported by Hess & Polak (2004b). This elementary-choice-level analysis repeats findings from the aggregate-level analysis presented in this paper with regards to significant effects of flight frequency and in-vehicle access-time for all travellers, while showing that other factors such as fare and aircraft size, have a significant effect only in some of the population subgroups. The analysis repeats findings by Windle & Dresner (1995) that important gains can be made through accounting for past experience at the different airports. The study further indicates that correlation between alternatives exist along the three dimensions of choice, with correlation being at its highest along the access-mode choice dimension; ceteris paribus, travellers are more willing to accept a change of airport or airline than a change of access-mode. Finally, Hess & Polak (2004b) report correct prediction rates of up to 85%. These results, in conjunction with those produced in the present paper, would indicate that accommodating random taste heterogeneity in a study of the joint choices of airport, airline and access-mode could lead to further gains in model performance. The requirement to accommodate these substitution patterns as well as random taste variation suggests the need for a Mixed GEV structure, in which GEV choice probabilities are integrated over the assumed distribution of the random taste coefficients (c.f. Train, 2003). Two other aspects that should be considered are choice-set generation (as proposed by Basar & Bhat, 2002) and the availability of different fare classes (c.f. Battersby, 2003). These additional modelling components could relatively readily be incorporated into a wider framework of an MGEV model, and could lead to important further gains in model accuracy. Finally, it would be desirable to extend the existing models to provide an explicit characterisation of the effects of the use of aggregate fare and service frequency data, analogous to the classical treatment of treatment of aggregate or erroneous data in linear regression analysis (Maddala, 1977).

9. CONCLUSIONS AND FURTHER RESEARCH

In this paper, we have described part of an ongoing study of airport choice in multi-airport regions. In line with the results of previous research, the present analysis has shown that there exist significant influences on airport choice due to access time, fare and frequency of service. Moreover, the results also indicate that there are significant differences across travellers in their sensitivity to these factors and that while differences in sensitivity to fare and frequency can be adequately accommodated by deterministic market segmentation, the sensitivity to access time additionally varies randomly within these market segments. Finally, results show that the use of a non-linear specification of flight frequency can lead to improved model fit, reflecting the decreasing marginal return of increases in flight frequency beyond a certain level.

It should be noted that all the models assign a higher-than-expected coefficient to access time, along with rather low fare coefficients, leading to a high implied willingness to accept fare increases in return for access-time decreases. High implied values of access-time have been observed in previous research (e.g. Pels et al., 2003), and we believe that such results reflect in large part the use of highly aggregated fare data, with the resulting tendency towards the attenuation of the fare coefficient. An additional explanation may be that that travellers associate a higher risk of missing their flight with higher access time, so that the coefficient associated with access time reflects not just the sensitivity to the actual access time but indirectly also the willingness to pay for reductions in the risk of missing a flight.

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