STOCHASTIC USER EQUILIBRIUM ASSIGNMENT WITH TRAFFIC-RESPONSIVE SIGNAL CONTROL

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ABSTRACT

This paper considers the Stochastic User Equilibrium (SUE) assignment problem for a signal-controlled network in which intersection control is flow-responsive. The problem is addressed within a Combined Traffic Assignment and Control (CTAC) modeling framework, in which the calculation of user equilibrium link flows is integrated with the calculation of consistent signal settings. It is assumed that network equilibrium is dispersed due to user misperceptions of travel times, and that the intersection control system is designed to allow the persistent adjustment of signal settings in response to traffic flow variations. Thus, the model simulates real-world situations in which network users have limited information and signal control is traffic-actuated. The SUE-based CTAC model is solved algorithmically by means of the so-called Iterative Optimization and Assignment (IOA) procedure, a widely used heuristic which relies on the alternate execution of a control step (signal setting calculation for fixed link flows) and an assignment step (network equilibration under fixed signal settings). The main objective of the study is to define a methodological framework for the evaluation of the performance of various traffic-responsive signal control strategies in interaction with different levels of user information, as represented by the spread parameter of the perceived travel time distribution assumed in the SUE assignment submodel. The results are of practical relevance in a policy context, as they provide a basis for assessing the potential integration of Advanced Traveler Information Systems (ATIS) and signal control systems. Several computational experiments are carried out on a small, contrived network and using realistic intersection delay functions, in order to test the behavior of the model under a wide range of conditions; in particular, convergence pattern and network performance measures at equilibrium are analyzed under alternative information/control scenarios and for various demand levels. The issue of uniqueness of the model solution is addressed as well.

1 INTRODUCTION

It is well known in the field of traffic engineering that the most efficient use of the existing capacities of signal-controlled intersections can be accomplished by updating signal timing (and possibly also phase sequencing) so as to respond to the temporal variations of traffic flows. Operationally, such an adjustment of the signal settings may be performed in different ways, ranging from the simple periodic, manual updating of fixed-time control plans on the basis of merely historical information, to the full *actuation* of the signal, in which control decisions are linked on-line to the current (or even projected) flow conditions through the instantaneous detection of traffic volumes. A signal control system exhibiting either of the above capabilities will be referred to here as *traffic-responsive,* or *flow-responsive*, signal control, regardless of the operational mode of signal adjustment and hence of the "degree of reactiveness" of the control parameters to the variations of traffic flows.

The most distinctive feature of traffic-responsive signal control in a network-wide perspective is that link capacities can no longer be assumed fixed and known exogenously, but become endogenous, as they clearly depend on the amounts of green time assigned to the intersecting streets, and such amounts, in turn, depend on the intensity of the traffic flows using those streets. From a behavioral point of view, this mechanism translates into a significant interaction between the decisions of two actors, namely the signal setter (in practice the traffic control agency's engineer) on the one hand, and the network users on the other hand: the former is responsible for signal control decisions (implemented either manually or automatically), while the latter make travel decisions, in particular route choices. Thus, the need arises almost naturally to modify network models, and especially traffic assignment models, so as to explicitly take into account this important interaction.

Attempts at integrating equilibrium traffic assignment and intersection control into a single modeling framework under the assumption of flow-responsive signal settings have resulted in a class of *Combined Traffic Assignment and Control* (CTAC) models, also referred to as *Equilibrium Traffic Signal Setting* models by some authors. Such an integration aims at enhancing the predictive power and policy relevance of urban traffic assignment models, by focusing on the mutual interaction between user route choices and signal control decisions in the calculation of network traffic equilibria.

Therefore, CTAC models may play a key role in the assessment of the short- and long-term redistributional effects induced by the implementation of various signal control strategies. Such a modeling capability appears to be especially valuable in view of the widespread adoption of actuated signal control in many urban areas. In addition, the prospective operation of Advanced Traveler Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS) raises interesting questions as to the potential benefits that may derive from the integration of route guidance (or, more generally, real-time information to network users) and traffic-responsive signal control (see, for example, Van Vuren and Van Vliet 1992; Hu and Mahmassani 1997).

This paper considers the CTAC problem under stochastic route choice assumptions, and is organized as follows. In Section 2 we provide the essential background on the combined traffic assignment and control problem by presenting an overview of the core issues arising in the context of CTAC modeling, together with a brief review of the relevant literature. The specific purpose and the practical relevance of the present study are discussed in Section 3, while Section 4 describes the details of the modeling framework and solution algorithm adopted in the ensuing computational analyses. The experimental design and evaluation of results of such analyses are the subject of Section 5. Finally, conclusive remarks are offered in Section 6.

2 BACKGROUND ON COMBINED TRAFFIC ASSIGNMENT AND CONTROL

We consider a road network in which intersections are controlled by traffic-responsive signals as defined in the introductory remarks of Section 1. Signal control plans are determined according to some *signal control policy*, that is a criterion for calculating, at each intersection, a set of signal settings (e.g. cycle length and green time splits) for any given specification of a set of relevant traffic flows. In practice, the concept of signal control policy is rather broad, as it encompasses simple empirical rules as well as rigorous optimization algorithms. We assume that the travel demand is fixed and known and that, for given values of the signal settings, drivers' route choice behavior complies with the user equilibrium principle of Wardrop (1952).

We let:

$\mathbf{f}^{UE} \mid \mathbf{g}$ be the vector of user-equilibrium link flows given a vector $\mathbf{g}$ of signal settings, and

$\mathbf{g}^{P} \mid \mathbf{f}$ be the vector of signal settings determined through the control policy P given a vector $\mathbf{f}$ of link flows. Then, the CTAC problem consists of finding a pair of vectors $(\mathbf{f}^*, \mathbf{g}^*)$ such that:

$$\mathbf{f}^* = \mathbf{f}^{UE} \mid \mathbf{g}^* \tag{1}$$

$$\mathbf{g}^* = \mathbf{g}^{P} \mid \mathbf{f}^* \tag{2}$$

If there exists a pair of vectors satisfying (1) and (2), then it is called a *mutually consistent flow-control equilibrium*, since under flows $\mathbf{f}^*$ and signal settings $\mathbf{g}^*$ none of the decision-makers involved (the network users and the signal setter) has an incentive to modify his or her current course of action.

The CTAC problem can be broken down into a *traffic assignment subproblem*, in which user-equilibrium link flows are computed for fixed signal settings, and a *signal control subproblem*, in which signal settings are determined under fixed link flows. Delay functions for signalized links are traffic-engineering based mathematical relationships expressing the dependence of average vehicular delays on signal settings (typically cycle length and green split) and link flows for given values of the saturation flows. The "rules" governing the two submodels are, respectively, the route choice paradigm and the control policy, while the delay functions play the key role of "interface" between the two submodels, since they act as link cost functions in the assignment model, and therefore convey the impact of signal settings upon route choice. It is, therefore, not surprising that the behavior of CTAC models may be strongly affected by the choice of a specific type of delay function.

The CTAC problem has been studied for over two decades, the work of Allsop (1974) being commonly regarded as the pioneering contribution in this area. Since an extensive literature review is outside the scope of this paper, the reader is referred to Meneguzzer (1997) for a recent survey of studies on combined traffic assignment and control. In the remainder of this section, we provide only an essential overview of methods adopted in CTAC research to date, together with a brief discussion of the key issues involved.

Broadly speaking, CTAC problems can be solved by either of two main approaches, namely the *iterative scheme approach* and the *optimization approach*. The first is essentially a "naive" imitation of the real-world interaction between user route choices and signal control decisions, since it relies on the alternate execution of a control step (signal setting calculation for fixed link flows) and an assignment step (network equilibration under fixed signal settings). For this reason the method, first suggested by Allsop (1974), is commonly known as the *Iterative Optimization and Assignment* (IOA) procedure. Though conceptually simple and appealing from a behavioral standpoint, IOA lacks an explicit underlying model formulation and does not necessarily converge to a mutually consistent flow-control equilibrium, as shown, for example, by Smith (1979).

The second approach aims at computing a mutually consistent flow-control equilibrium that solves a constrained optimization problem, in which the objective is some measure of system performance (e.g. total travel time) and the constraints are technical restrictions on the signal settings plus the requirement that link flows are in user-equilibrium. This is known to be a special instance of the more general Equilibrium Network Design (END) problem, the design variables being the signal settings. As pointed out by Fisk (1984), the game-theoretical counterpart of this approach is the so-called *Stackelberg game*, as it is reasonable to assume that the upper-level player (the signal setter in our case) is able to anticipate the reactions of the lower-level players (the network users) to his/her decisions, but not vice-versa (the drivers are usually unaware of the control strategy adopted by the signal setter).

Both the above approaches suffer from major shortcomings. The iterative scheme, even though simple and suitable for large-scale implementations, may converge to flow-control equilibria that are not optimal in terms of total travel time, or even lead to a decline in network performance as compared to the initial conditions (Dickson 1981). The reason is that signal control policies commonly adopted by traffic engineers, such as the delay minimization policy and its approximation, Webster's (1958) equisaturation policy, are not designed to take explicitly into account the rerouting effects induced by their implementation. As pointed out by Smith (1979), this results in an inefficient use of the network's physical capacity. It is possible, however, to devise less conventional control policies that overcome this limitation, such as the so-called P_0 policy (Smith 1980; 1981), which tends to divert traffic toward higher-capacity routes by assigning them green splits that result in lower delays (see Section 4). It has also been noted that the behavior of the iterative scheme may depend significantly on the type of delay function used (Smith and Van Vuren 1993).

On the other hand, owing to the nonlinearity of the user-equilibrium constraint, the optimization formulation has a non-convex character, which may result in multiple local optima. As a consequence, the solution to the CTAC problem may not be unique and, in fact, depend on the starting values of the signal settings. This problem is shared by the iterative approach, and, due to its importance, will be re-emphasized at the end of this section. Another limitation of the optimization approach is that exact solution algorithms for END problems are known to be computationally impractical for networks of realistic size. Interestingly, the effect of the latter circumstance is to provide a connection between the two approaches in real-world applications, as the IOA procedure is usually regarded as a viable heuristic for solving large-scale END problems.

Apart from considerations of computational tractability, there appears to be also a behavioral justification for the use of IOA in the context of combined traffic assignment and control. As noted by Watling (1996), the iterative scheme accounts in a realistic manner for the effect that initial conditions (e.g. current signal settings determined on the basis of local traffic engineering knowledge) may have on the evolution of network flow patterns and responsive signal control in subsequent time periods and on the eventual equilibrium, whereas the globally optimal flow-control equilibrium, whose calculation is the ultimate aim of the END approach, may correspond to a routing pattern too far from current route choice behavior, and therefore unlikely to evolve from current conditions.

A critical feature of CTAC models is that conditions that are known to be *sufficient* for the existence of a unique flow-control equilibrium do not hold under responsive signal settings. In essence, this is due to the reactive nature of the control actions, which allows link capacities to change in response to varying demand flows, thus yielding cost-flow relationships that need not be monotone, unlike those used in ordinary equilibrium traffic assignment. It is, however, important to realize that, once again, alternative control policies may perform quite differently with respect to solution uniqueness (Van Vuren and Van Vliet 1992). In practice, the possibility of multiple flow-control equilibria may be a major concern when the model is used as a tool for the evaluation of alternative schemes in the context of planning or TSM applications. This is because multiple equilibria are likely to result in different network flow patterns, and therefore it may be difficult to distinguish such differences from the effects of the options being evaluated. Even though the possible occurrence of multiple equilibria has been demonstrated on simple networks (e.g. Allsop and Charlesworth 1977; Van Vuren and Van Vliet 1992), it should be noted that the conditions being violated in the presence of responsive control are not *necessary* for solution uniqueness, so that single-equilibrium behavior cannot be ruled out *a priori*. This has been confirmed recently by extensive computational tests on a realistic network (Meneguzzer 1996).

3 OBJECTIVES AND PRACTICAL RELEVANCE OF THE STUDY

It is well known that traffic equilibria in real networks do not obey Wardrop's first principle, as not all drivers select minimum-time routes from their origin to their destination due to the prevalence of travel time misperceptions. Such misperceptions are mainly attributable to the less-than-perfect information on which the network users base their route choices. Assuming that perception errors are random variables distributed across the population of drivers, Stochastic User Equilibrium (SUE) assignment (Daganzo and Sheffi 1977) provides a more realistic model of user route choice behavior in the presence of limited information. At the aggregate level, stochastic route choice results in "dispersed" network equilibria, in which system travel time is higher than in the deterministic case, and tends to increase as driver information deteriorates.

Advanced Traveler Information Systems (ATIS) are currently regarded as effective tools for alleviating congestion and improving the performance of traffic networks by providing users with exogenous information on network conditions. Accordingly, in recent years there have been substantial efforts in the development of models aimed at predicting and evaluating the impacts of ATIS operation. Also, various levels of integration of ATIS and traffic-responsive signal control have been envisaged (see, for example, Bell 1992).

In light of these considerations, the main goal of this study is to define and test a modeling framework for the evaluation of the performance of flow-responsive signal control strategies in interaction with varying levels of driver information. This is accomplished by abandoning the assumption of deterministic route choice behavior, adopted in most implementations of CTAC models documented in the literature to date, in favor of the more realistic SUE paradigm, in which the spread parameter of the perceived travel time distribution can be taken as a proxy for the level of user information. As the primary purpose of our study is to highlight the basic effect of information upon the mechanism of interaction between signal control and route choice, the model is highly simplified, and some issues that would have to be addressed in view of a real-world implementation are not considered. In particular, there is no attempt to differentiate between informed and uninformed drivers, nor to account for ATIS market penetration levels. Information is assumed to be purely descriptive, and the possibility of prescriptive routing strategies is not taken into account.

The above ideas motivate the work presented in this paper, in which the properties of combined traffic assignment and control under SUE are investigated from a computational standpoint. More specifically, the study aims at: (a) comparing the performance of alternative signal control policies as a function of the level of driver information, and for fixed values of key exogenous (non-policy) factors; (b) providing indications for the identification of "optimal" information/control scenarios as the above exogenous factors are allowed to vary; and (c) assessing the robustness of the model's forecasts under selected information/control scenarios, by testing the consistent flow-control equilibria for uniqueness.

4 MODELING APPROACH AND SOLUTION ALGORITHM

Following the discussion presented in the previous sections, the approach taken in this study is to solve the SUE-based CTAC problem using the Iterative Optimization and Assignment procedure. The overall model is *directly separable* in the sense of Meneguzzer (1997), meaning that at signalized intersections only between-phase interactions among traffic movements are allowed. This is the same as saying that signal phasing is designed so that conflicting flows are never given way simultaneously; the main implication is that, *within the traffic assignment subproblem*, cost functions are separable in the usual sense. Route choice is represented by a Logit-based SUE model (Fisk 1980), whose solution is accomplished through the Method of Successive Averages (MSA); see, for example, Sheffi (1985).

Two signal control policies are selected for testing their interaction with route choice under varying levels of driver information. The first is the well-known equisaturation policy (Webster 1958), which yields approximately delay-minimizing signal settings by allocating green splits to phases so as to equalize the degrees of saturation of critical movements across phases. The second is a capacity-maximizing policy due to Smith (1980), known as P_0 , which tends to induce an efficient use of network capacity by diverting traffic toward routes that have higher saturation flow; this is accomplished by assigning these routes green splits that result in lower delays. Even though P_0 does not, in general, minimize total travel time, it does satisfy sufficient conditions for the existence of a mutually consistent flow-control equilibrium, as shown by Smith (1981).

Consider, for example, a simple two-phase signal controlling the intersection of two links (i = 1, 2). Assume that cycle length is fixed, and there are no lost times. Let:

s_i : saturation flow for link i;

f_i : flow on link i;

$y_i = f_i / s_i$: flow ratio for link i;

λ_i : green time split assigned to link i;

$d_i (f_i, \lambda_i)$: delay on link i as a function of flow and green split on the same link.

Then, according to the equisaturation control policy the green splits are:

$$\lambda_i = y_i / (y_1 + y_2) \qquad i = 1, 2 \tag{3}$$

whereas under P_0 we compute λ_i (i = 1, 2) such that:

$$s_1\, d_1 (f_1, \lambda_1) = s_2\, d_2 (f_2, \lambda_2) \tag{4}$$

Even though the statements of both policies can be generalized for intersections of more than two links and signal plans with more than two phases, the simple case presented here is sufficient to understand intuitively why the two policies exhibit a quite different behavior when applied in interaction with route choice: equisaturation tends to favor more congested links/routes in the allocation of green times, and this, in turn, attracts even more traffic onto those links/routes; P_0 , on the other hand, encourages the use of links/routes having higher saturation flow, thus exploiting more fully the network's available capacity.

A function due to Akçelik (1988) is employed to model delay for signalized links, which acts effectively as the interface between the traffic assignment submodel and the signal control submodel; see the discussion in Section 2. Like other so-called *sheared* delay formulae, the Akçelik function is suitable for use within an equilibrium assignment framework, since it covers oversaturated conditions through a time-dependent overflow delay term, thus overcoming a major limitation of the classical steady-state formulae (for a discussion of this issue in the context of CTAC models, see Meneguzzer 1997). The Akçelik formula takes one of the following expressions, depending on the value of the degree of saturation x:

for $x \le 0.5$:

$$d = \frac{0.5C(1-\lambda)^2}{1-\lambda x} \tag{5}$$

for $x > 0.5$:

$$d = \frac{0.5C(1-\lambda)^2}{1-\lambda \cdot \min(x,1)} + 900T\left[x - 1 + \sqrt{(x-1)^2 + \frac{8(x-0.5)}{KT}}\right] \tag{6}$$

where:

d : average delay incurred by vehicles on subject link (sec.);

C : length of signal cycle facing subject link (sec.);

λ : green time split facing subject link;

T : duration of demand flow period on subject link (hrs);

K : capacity of subject link (vehicles/hr);

x : degree of saturation (flow-to-capacity ratio) of subject link.

From an algorithmic standpoint, the model solution is accomplished by embedding the MSA, employed for the solution of the Logit-based SUE subproblem, into the IOA procedure, thus resulting in the following steps:

STEP 0: INITIALIZATION
$\mathbf{g} = \mathbf{g}^0$ (initial signal settings)
k = 0 (iteration counter)

STEP 1: TRAFFIC ASSIGNMENT SUBPROBLEM
k = k+1
Compute link flows $\mathbf{f}^k$ solving SUE by MSA under signal settings $\mathbf{g}^{k-1}$

STEP 2: SIGNAL CONTROL SUBPROBLEM
Compute signal settings $\mathbf{g}^k$ via chosen control policy under link flows $\mathbf{f}^k$

STEP 3: STOPPING RULE
For k > 1:
IF $\delta(k-1,k) \le \varepsilon$ STOP and set $\mathbf{f}^* = \mathbf{f}^k$, $\mathbf{g}^* = \mathbf{g}^k$
ELSE go to STEP 1

where $\delta(k-1,k)$ is a measure of distance between the solutions of two successive iterations (based on the values of link flows and/or signal settings), ε is a prespecified tolerance, and ($\mathbf{f}^*$, $\mathbf{g}^*$) represents the mutually consistent flow-control equilibrium which solves the CTAC problem. Note that the algorithm has a nested structure: assuming we run k = 1, ..., K iterations of IOA (also called outer iterations), and solve each SUE subproblem by means of N_k iterations of MSA (also called inner iterations), each consisting of a stochastic network loading, the overall computational effort will amount to $L = \Sigma_k N_k$ loadings. Also note that, in our application, each SUE subproblem is solved to convergence, and there is no attempt to explore computational tradeoffs between outer and inner iterations; for a systematic analysis of this issue in a CTAC context, see Meneguzzer (1996).

5 NUMERICAL EXPERIMENTS

5.1 Experimental design

The model is applied to a small artificial test network in order to conduct a number of computational experiments, whose main purposes are:

1) to evaluate the behavior of combined traffic assignment and control under alternative information/control scenarios in terms of: (a) existence of a mutually consistent flow-control equilibrium; (b) network performance at equilibrium; (c) speed of convergence of the IOA procedure; (d) equilibrium values of the signal settings. The information/control scenarios are defined by combining the two previously described signal control policies (equisaturation and Smith's P_0) with various values of the coefficient of variation of the perceived travel time distribution, which are intended to represent situations ranging from very accurate to very poor information;

2) to explore the effect of some key exogenous factors in interaction with selected information/control scenarios. These factors, which are varied parametrically in the computational experiments, include the level of congestion in the network, the duration of the demand flow period assumed in the time-dependent term of the delay formula, and the length of the signal cycles;

3) to provide limited empirical evidence as to the uniqueness of the solution to the SUE-based CTAC problem under selected information/control scenarios. This is accomplished by repeatedly solving the model, *ceteris paribus*, starting from different initial solutions as determined by different values of the green splits assigned to signalized links.

The network employed in the numerical tests consists of a single origin-destination pair, seven nodes and ten links (see Figure 1). There are three signals, each controlling two intersecting links and operating on a two-phase plan with fixed cycle length; for simplicity, change intervals and lost times are not considered, so that the sum of the green times for each pair of intersecting links equals the respective cycle length, and thus there is effectively a single control variable to be determined at each junction. To make the example more realistic from a traffic engineering standpoint, a minimum green split of 0.1 is assumed in the signal setting calculations. Note that the network topology has been purposely designed so that each route traverses at least one signalized intersection. Travel time for each signalized link consists of a fixed free-flow component plus a flow-dependent delay term modeled according to the Akçelik formula (see Section 4), while the remaining links are assumed to be uncongested.

In the implementation of the solution algorithm, the following stopping criterion is adopted for both the IOA and MSA procedures:

$$\max_{l} \frac{\left| f_l^{\,k} - f_l^{\,k+1} \right|}{f_l^{\,k}} \le \varepsilon \qquad \forall l : f_l^{\,k} \ne 0 \tag{7}$$

where the maximum is taken over all links. Different values of the tolerance ε are used for IOA (0.001) and for MSA (0.01), in order to ensure a certain degree of "streamlining" of the algorithm, which was found to enhance computational efficiency by previous studies (e.g. Meneguzzer 1996). Also, the maximum number of iterations was set at 100 for both procedures, a bound which turned out to be active only for IOA and in very few instances (see Sections 5.2 and 5.3 below).

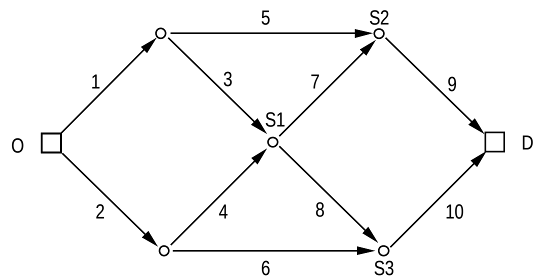

Figure 1. Test network (signalized intersections are S1, S2, S3)

5.2 Behavior of CTAC under alternative information/control scenarios

The combined effect of driver information and signal control policy upon the equilibrium performance and convergence behavior of the model was investigated in the first part of the numerical tests, whose results are summarized in Table 1. Five levels of driver information, as measured by the value of CV, were considered in interaction with the two assumed control policies, yielding a total of ten model runs. Note that the lowest value of CV

(0.01) is intended to represent a situation of very accurate driver information, and results in a nearly deterministic model. In this round of experiments, the three exogenous factors were held fixed at "base-case" values, corresponding to a demand level equal to the "actual" origin-destination trip rate, an oversaturation period of 0.5 hrs and a signal cycle length of 90 sec. (common to all intersections).

First, we note that convergence to a mutually consistent flow-control equilibrium was attained in most cases, as only the two runs corresponding to CV = 0.01 failed to meet the stopping test within the prespecified maximum number of iterations (100), and tended to exhibit a typical flip-flopping pattern of the convergence measure (7). If we consider the cases of CV = 0.01 to be an approximation to a deterministic model, these results appear to be consistent only in part with the findings of previous research: based on the studies carried out by Smith (1979, 1981) in a deterministic route choice context, one would expect the equisaturation and P_0 control policies to behave quite differently in terms of convergence to a mutually consistent flow-control equilibrium.

Table 1. Equilibrium values of mean travel time, IOA iterations and signal settings for various levels of information and two control policies

		CV = 0.01	**CV = 0.05**	**CV = 0.15**	**CV = 0.25**	**CV = 0.35**
ES	MTT	30.355	28.398	27.206	27.150	27.173
	NIT	100	70	25	16	12
	S(3)	0.100	0.449	0.464	0.466	0.467
	S(5)	0.845	0.540	0.512	0.506	0.499
	S(6)	0.900	0.867	0.664	0.619	0.596
P_0	MTT	27.067	26.988	27.027	27.070	27.105
	NIT	100	41	18	12	10
	S(3)	0.635	0.515	0.490	0.485	0.482
	S(5)	0.255	0.426	0.456	0.458	0.458
	S(6)	0.764	0.605	0.554	0.538	0.529

CV: Coefficient of Variation of perceived travel times **ES:** Equisaturation
MTT: Mean Travel Time (min.) NIT: Number of IOA iterations
S(i): Green split for link i, i = 3,5,6

Second, we observe that, for both control policies, the number of IOA iterations needed for convergence decreases monotonically as CV increases, suggesting that the spread of traffic over alternative routes, induced by driver misperceptions, tends to accelerate the redistributional effect of responsive signal control. Also, convergence appears to be slower under equisaturation, especially for low and medium values of the perceived travel time variance. A possible explanation is that this control policy encourages the use of more congested routes, and hence causes some links to operate at high volume-to-capacity ratios, where the delay function is considerably steep, and therefore sensitive to small flow variations: at the network level, this may result in a longer "time" (i.e. number of iterations) needed to reach equilibrium.

The results of the ten "base" runs are less clear-cut in terms of mean travel time. A somewhat counterintuitive result is obtained under equisaturation, where system performance is seen to improve as driver information becomes less accurate: this is in contrast with the expected network inefficiency normally ascribed to the perception errors inherent in the stochastic nature of route choice behavior. On the other hand, the equilibrium mean travel times obtained under P_0 appear to be rather insensitive to the values of CV. Also, we observe that, for the assumed demand level, the network performance of P_0 is consistently superior to that of equisaturation, even though the difference tends to vanish as information deteriorates. The latter tendency is evident from the experiments described in the subsequent sections as well, and suggests that the impact of the supply action (the control strategy in our case) on network performance becomes less important as driver perception errors dominate route choice behavior.

Finally, equilibrium green splits for links number 3, 5 and 6 are also shown in Table 1. Each of these links represents one of the two approaches to each of the three signalized junctions included in the network, so that the green splits assigned to the other intersecting links can be derived straightforwardly from those appearing in the table. The values of the splits obtained for CV = 0.01 suggest, in agreement with previous studies, that the two control policies perform quite differently under quasi-deterministic route choice: equisaturation tends to generate "extreme" signal settings (maximum green to one approach and minimum to the other), whereas P_0 produces more even green time allocations. Moreover, it is clear from the results that the prevalence of perception errors tends to move the signal settings toward a 50/50 split, and that the two policies behave in a similar fashion as information deteriorates.

5.3 Effect of exogenous factors

The second part of the numerical experiments was devoted to testing the effects of key exogenous factors upon model behavior under selected information/control scenarios, defined by four combinations of CV and control policy (equisaturation and P_0 with CV = 0.05 and 0.25). The analysis relies on the same descriptors previously considered in the base case.

Table 2 presents the results obtained with three different demand levels, corresponding to the base-case trip rate (1.0 D) plus two alternative situations of low (0.5 D) and high (1.5 D) network congestion. It can be observed that, in the scenarios with better information (CV = 0.05), P_0 tends to outperform equisaturation only at medium and high congestion levels, a result which is consistent with the findings of previous studies conducted in a deterministic route choice setting (e.g. Smith *et al*. 1987). On the other hand, as the variance of driver perceptions increases (CV = 0.25), the only noticeable difference in mean travel time occurs in the high-demand case (1.5 D), and with an opposite sign as compared to CV = 0.05. As expected, convergence of IOA is consistently faster at low demand levels, whereas equilibrium signal settings show generally limited sensitivity to network congestion.

The length of the period of oversaturation caused by demand flows temporarily exceeding the capacity of intersection approaches is another element that may have a significant impact on model behavior. Its effect is represented in the time-dependent term of the delay function (6) by T, a parameter which essentially controls the slope of the function in the region of volume-to-capacity ratios greater than unity. The values of T assumed in the tests

Table 2. Equilibrium values of mean travel time, IOA iterations and signal settings for various demand levels under selected information/control scenarios

		ES 0.05	**P_0 0.05**	**ES 0.25**	**P_0 0.25**
0.5 D	MTT	23.446	23.607	23.719	23.721
	NIT	8	6	4	3
	S(3)	0.445	0.511	0.466	0.506
	S(5)	0.523	0.446	0.463	0.435
	S(6)	0.891	0.486	0.588	0.439
1.0 D	MTT	28.398	26.988	27.150	27.070
	NIT	70	41	16	12
	S(3)	0.449	0.515	0.466	0.485
	S(5)	0.540	0.426	0.506	0.458
	S(6)	0.867	0.605	0.619	0.538
1.5 D	MTT	39.042	38.637	39.116	39.528
	NIT	67	53	16	9
	S(3)	0.456	0.622	0.467	0.513
	S(5)	0.646	0.602	0.613	0.488
	S(6)	0.843	0.435	0.699	0.484

MTT: Mean Travel Time (min.)   NIT: Number of IOA iterations
S(i): Green split for link i, i = 3,5,6

are 0.25 hrs, 0.5 hrs (the base case), and 1.0 hr, and the corresponding results are shown in Table 3. It can be seen that P_0 yields lower mean travel times in all cases; note, however, that the two control policies exhibit a markedly different performance only for T = 1.0 hr and under conditions of accurate driver information. Overall, mean travel time appears to be considerably less sensitive to the length of the oversaturation period than to the demand level for all information/control scenarios. This is not an unreasonable result if one considers that, for volume-to-capacity ratios greater than 0.5, T affects only the second term of the delay formula (6), while x, which directly reflects the congestion level, appears in both terms of the same equation. Further, we note that the speed of convergence of IOA decreases as the oversaturation period becomes longer, consistent with the previous observation on T controlling the slope of the delay function, and hence the sensitivity of link performance to flow variations. Finally, the results shown in Table 3 indicate a remarkable stability of green splits with respect to variations of T under all information/control conditions.

The last parameter investigated in this part of the numerical tests is the length of the signal cycle, assumed to be the same for all intersections. According to the results presented in Table 4, it has, among the exogenous factors examined, the lowest impact on model behavior

Table 3. Equilibrium values of mean travel time, IOA iterations and signal settings for various oversaturation periods under selected information/control scenarios

		ES 0.05	**P_0 0.05**	**ES 0.25**	**P_0 0.25**
0.25h	MTT	26.135	25.548	25.671	25.633
	NIT	54	30	11	9
	S(3)	0.447	0.501	0.466	0.483
	S(5)	0.541	0.407	0.497	0.450
	S(6)	0.900	0.643	0.615	0.536
0.5 h	MTT	28.398	26.988	27.150	27.070
	NIT	70	41	16	12
	S(3)	0.449	0.515	0.466	0.485
	S(5)	0.540	0.426	0.506	0.458
	S(6)	0.867	0.605	0.619	0.538
1.0 h	MTT	32.451	29.628	30.043	29.860
	NIT	100	38	22	15
	S(3)	0.453	0.534	0.467	0.490
	S(5)	0.534	0.478	0.515	0.473
	S(6)	0.828	0.546	0.619	0.529

MTT: Mean Travel Time (min.) NIT: Number of IOA iterations

S(i): Green split for link i, i = 3,5,6

in terms of all descriptors considered in the analysis. In particular, we observe that mean travel time always increases, and at an almost constant rate, with cycle length, reflecting the positive, albeit moderate, effect of the latter on signalized link delay. Also, it should be noted that P_0 consistently outperforms equisaturation in terms of both system travel time and speed of convergence of IOA.

5.4 Tests of uniqueness of the flow-control equilibria

Solution uniqueness is generally regarded as a highly desirable property of network equilibrium models; as stressed in Section 2, this consideration applies, in particular, to CTAC models as well. In this study, a limited analysis of uniqueness of the mutually consistent flow-control equilibria obtained under selected information/control scenarios is conducted using a computational approach. A discussion of the rationale for tackling the issue of uniqueness from a computational (as opposed to analytical) standpoint can be found in Meneguzzer (1997).

For each of the four information/control scenarios selected for the experiments of Section 5.3, the model was run five times, corresponding to five different values of the green splits *initially* assigned to signalized links, while the exogenous factors were kept at their "base-case" values. The descriptors of interest in this analysis are average and deviation measures (over five runs) of equilibrium mean travel time and flow on each link of the network, and their resulting values are presented in Table 5. As pointed out by Meneguzzer (1997), considering *only* the values of mean travel time is not appropriate when investigating uniqueness, since different equilibria could conceivably result in a similar performance at the aggregate network level.

The values shown in Table 5 seem to indicate that the solution algorithm always converges to a unique equilibrium, with the exception of scenario *s1*: in fact, under the remaining scenarios the coefficient of variation of both mean travel time and link flows is of the same order of magnitude as the tolerance assumed for testing convergence of IOA, suggesting that, within the approximation of the computational procedure, the same equilibrium is approached starting from different initial solutions. Also, we note that even under *s1* the solution would *appear* to be unique if we were to look only at the aggregate network performance measure, thus confirming our previous remark. A further examination of the deviation measures of link flows *across scenarios* reveals that solution uniqueness may depend critically on the signal control policy under conditions of accurate information, while the same

Table 4. Equilibrium values of mean travel time, IOA iterations and signal settings for various cycle lengths under selected information/control scenarios

		ES 0.05	**P_0 0.05**	**ES 0.25**	**P_0 0.25**
60 s	MTT	27.899	26.821	26.952	26.882
	NIT	91	60	15	12
	S(3)	0.457	0.514	0.467	0.484
	S(5)	0.504	0.414	0.503	0.457
	S(6)	0.842	0.613	0.613	0.539
90 s	MTT	28.398	26.988	27.150	27.070
	NIT	70	41	16	12
	S(3)	0.449	0.515	0.466	0.485
	S(5)	0.540	0.426	0.506	0.458
	S(6)	0.867	0.605	0.619	0.538
120 s	MTT	28.902	27.167	27.349	27.259
	NIT	87	40	16	12
	S(3)	0.438	0.517	0.466	0.486
	S(5)	0.567	0.431	0.509	0.459
	S(6)	0.900	0.600	0.624	0.536

MTT: Mean Travel Time (min.) NIT: Number of IOA iterations

S(*i*): Green split for link *i*, *i* = 3,5,6

Table 5. Average and deviation measures of equilibrium mean travel time and link flows obtained from different initial solutions and for selected information/control scenarios

		MTT	**f(1)**	**f(2)**	**f(3)**	**f(4)**	**f(5)**	**f(6)**	**f(7)**	**f(8)**	**f(9)**	**f(10)**
s1	A	28.456	2434.6	3065.4	1141.6	1234.6	1293.0	1830.7	1808.5	567.7	3101.5	2398.5
	C	.00222	.00818	.00649	.01641	.01684	.02655	.01016	.02643	.05067	.00479	.00620
	R	.00499	.02571	.02042	.03451	.03556	.07796	.02698	.08145	.11695	.01499	.01939
s2	A	26.991	2765.1	2734.9	1830.0	1549.9	935.0	1185.1	1952.8	1427.1	2887.8	2612.2
	C	.00016	.00501	.00507	.00534	.00520	.00469	.00498	.00402	.00658	.00120	.00133
	R	.00037	.01136	.01148	.01142	.01136	.01209	.01215	.01024	.01619	.00298	.00329
s3	A	27.152	2702.2	2797.8	1623.9	1642.2	1078.2	1155.6	1758.5	1507.7	2836.7	2663.3
	C	.00005	.00087	.00084	.00089	.00096	.00095	.00089	.00101	.00123	.00029	.00031
	R	.00015	.00207	.00200	.00228	.00225	.00260	.00260	.00279	.00352	.00081	.00086
s4	A	27.072	2768.8	2731.2	1778.4	1702.9	990.3	1028.3	1811.0	1670.4	2801.3	2698.7
	C	.00006	.00116	.00118	.00115	.00124	.00120	.00107	.00103	.00108	.00024	.00025
	R	.00015	.00285	.00289	.00287	.00299	.00293	.00272	.00254	.00275	.00061	.00063

MTT: Mean Travel Time (min.) **f(*i*):** flow on link *i*, $i = 1, ..., 10$ (veh./hr)
s1: ES and CV = 0.05 **s2:** P_0 and CV = 0.05 **s3:** ES and CV = 0.25 **s4:** P_0 and CV = 0.25
A: Average C: Coefficient of variation R: Range-to-average ratio

equilibrium is attained, regardless of the control policy adopted, when driver perception errors dominate route choice behavior.

6 CONCLUSIONS

This paper has presented a modeling framework suitable for investigating key properties of combined traffic assignment and control under the assumption of stochastic route choice. The primary aim of such a framework is the simulation of real-world signal-controlled networks in which drivers have limited information and intersection control is flow-responsive. Even though the SUE-based CTAC model developed in this study is applied to a small, contrived network, the results of the computational experiments are of practical relevance in a policy context, as they provide a methodological basis for assessing the potential integration of Advanced Traveler Information Systems and signal control systems.

The main findings of the numerical tests carried out in this study suggest the following conclusions. First, convergence to a point of equilibrium of the CTAC problem can be expected to occur whenever driver perception errors are not negligible in the description of route choice behavior. Also, the results show a clear positive relationship of the speed of convergence of IOA to the magnitude of the perception errors, suggesting that the prevalence of stochastic routing patterns tends to accelerate the redistributional effect of responsive signal control. A comparative evaluation of the performance of the two traffic-responsive control policies considered in this study indicates a consistent superiority of P_0 over equisaturation, even though the difference tends to vanish as perception errors dominate route choice. This suggests that the impact of the control strategy on equilibrium signal settings and network performance may be negligible under conditions of poor driver information.

The second interesting conclusion is that, among the three exogenous factors considered in interaction with alternative information/control scenarios, the demand level has the most significant effect on network performance, followed by the duration of the period of oversaturation of the intersection approaches, and by the signal cycle length. This result hints at the potential benefits of integrating signal control and information provision strategies with normative Travel Demand Management actions, such as, for example, road pricing.

Last, our tests of uniqueness of the model solution provide limited computational evidence that the possibility of multiple, starting-point dependent, flow-control equilibria may materialize only under the equisaturation policy and in the presence of accurate driver information. This can be viewed as another element in favor of the adoption of the alternative policy P_0 , except for situations in which a more realistic model might be obtained by explicitly allowing for the effect of initial signal settings upon the long-term equilibrium of signal control actions and route choice.

Finally, further efforts should be directed toward implementing the modeling framework presented in this paper on larger and more realistic networks.

REFERENCES

Akçelik R. (1988). The Highway Capacity Manual delay formula for signalized intersections. *ITE Journal*, vol. 58, no. 3, pp. 23-27.

Allsop R.E. (1974). Some possibilities for using traffic control to influence trip distribution and route choice. *Proceedings of the 6th International Symposium on Transportation and Traffic Theory*, D.J. Buckley, ed., Elsevier, New York, pp. 345-373.

Allsop R.E. and Charlesworth J.A. (1977). Traffic in a signal controlled road network: an example of different signal timings inducing different routeings. *Traffic Engineering and Control*, vol. 18, pp. 262-264.

Bell M.G.H. (1992). Future directions in traffic signal control. *Transportation Research -A*, vol. 26A, pp. 303-313.

Daganzo C. and Sheffi Y. (1977). On stochastic models of traffic assignment. *Transportation Science*, vol. 11, pp. 253-274.

Dickson T.J. (1981). A note on traffic assignment and signal timings in a signal-controlled road network. *Transportation Research -B*, vol. 15B, pp. 267-271.

Fisk C.S. (1980). Some developments in equilibrium traffic assignment. *Transportation Research -B*, vol. 14B, pp. 243-255.

Fisk C.S. (1984). Game theory and transportation system modelling. *Transportation Research -B*, vol. 18B, pp. 301-313.

Hu T-Y and Mahmassani H.S. (1997). Day-to-day evolution of network flows under real-time information and reactive signal control. *Transportation Research -C*, vol. 5C, pp. 51-69.

Meneguzzer C. (1996). Computational experiments with a combined traffic assignment and control model with asymmetric cost functions. *Proceedings of the 4th International*

Conference on Applications of Advanced Technologies in Transportation Engineering, Y.J. Stephanedes and F. Filippi, eds, ASCE, New York, pp. 609-614.

Meneguzzer C. (1997). Review of models combining traffic assignment and signal control. *Journal of Transportation Engineering*, vol. 123, pp. 148-155.

Sheffi Y. (1985). *Urban Transportation Networks: equilibrium analysis with mathematical programming methods*. Prentice-Hall, Englewood Cliffs, N.J.

Smith M.J. (1979). Traffic control and route-choice; a simple example. *Transportation Research -B*, vol. 13B, pp. 289-294.

Smith M.J. (1980). A local traffic control policy which automatically maximises the overall travel capacity of an urban road network. *Traffic Engineering and Control*, vol. 21, pp. 298-302.

Smith M.J. (1981). Properties of a traffic control policy which ensure the existence of a traffic equilibrium consistent with the policy. *Transportation Research -B*, vol. 15B, pp. 453-462.

Smith M.J., Van Vuren T., Heydecker B.G. and Van Vliet D. (1987). The interactions between signal control policies and route choice. *Proceedings of the 10th International Symposium on Transportation and Traffic Theory*, N.H. Gartner and N.H.M. Wilson, eds, Elsevier, New York, pp. 319-338.

Smith M.J. and Van Vuren T. (1993). Traffic equilibrium with responsive traffic control. *Transportation Science*, vol. 27, pp. 118-132.

Van Vuren T. and Van Vliet D. (1992). *Route Choice and Signal Control: The potential for integrated route guidance*. Avebury, Aldershot, U.K.

Wardrop J.G. (1952). Some theoretical aspects of road traffic research. *Proceedings of the Institution of Civil Engineers*, Part II, 1, pp. 325-378.

Watling D. (1996). Modelling responsive signal control and route choice. *Proceedings of the 7th World Conference on Transport Research*, D. Hensher, J. King and T. H. Oum, eds, vol. 2, Pergamon, Oxford, pp. 123-137.

Webster F.V. (1958). Traffic signal settings. *Road Research Technical Paper No. 39*, HMSO, London.