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A Regional Model for the Portuguese Economy Based on a Regional Accounting Matrix

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Abstract

This paper presents a model for the Portuguese economy based on a so-called "Regional Accounting Matrix" (RAM). The RAM is an accounting table made up of information provided by the standard Portuguese National and Regional Accounts, with a framework similar to the well-known social accounting matrixes. The RAM includes regionalised information concerning the generation and the use of the households' income, but other parts of the table, namely those referring to the structure of inputs in the production process, are at national level. Starting with the RAM, the paper then develops an input-output-type model closed with respect to the households' consumption. This model is based on a "hypothesis of non-existence of regional preference in supplying regions". By this hypothesis we mean that every increase in demand, even when regionally located, is matched by a national supply (and also by international imports), and not preferentially by an increase in output of the very concerned region. The model proposed for the Portuguese economy, for the year of 1995, includes the computation of several multipliers – the most striking of them describe the inter-regional income distribution process. In fact, an increase in income, at the beginning in benefit of the households living in one region, may cross the region borders and propagate into other regions, increasing then the households' income in the latter regions. The model also provides other outstanding multipliers, as those describing the effect on the regional households' income of changes in demand of 49 groups of commodities, and those computing the change in output of these 49 commodities induced by exogenous shocks in the regional income.

A Regional Model for the Portuguese Economy Based on a Regional Accounting Matrix[1]

1. Introduction

This paper presents a model for the Portuguese economy based on a so-called "Regional Accounting Matrix" (RAM). The proposed RAM, which regards the year of 1995, is an accounting table made up of information provided by the standard Portuguese National and Regional Accounts, with a framework similar to the well-known social accounting matrixes[2]. Starting with the RAM, the paper then develops an input-output-type model that allows the inferring of some interesting results at regional level: the most striking of them aim to describe the inter-regional households' income distribution process.

The peculiarity of the RAM[3] – and of the input-output model – that we are going to unfold, is that they mingle information at national level with other that is known at regional level. Indeed, the regionalised data are confined to the households' generation and use of income. As to other components of final demand, as the GFCF or the international exports, only national totals are considered. The inter-industry relationships are at national level as well. In fact, concerning the production process, we know the place where the Gross Value Added is generated – and distributed to the households – but the technological information, i. e., the structure of inputs of the intermediate consumption is not available by regions. This unusual mix of regional and national information led to a model that works under a crucial assumption that we designate as the "hypothesis of non-existence of regional preference in supplying regions". By that hypothesis we mean:

- that every increase in demand, even when regionally located, is matched by a national supply (and also by international imports), and not preferentially by an increase in output of the very concerned region; this national supply is provided by regions, and by the rest of the world, in fixed proportions that do not depend on the region where the demand emerged.

Though this is an infrequent assumption on regional input-output models, where otherwise it is much more common to assume that imports are exogenous and thus increases in demand command regional adjustments in supply, we do believe that it is a reasonable idea for Portugal. In fact, Portugal is a small country with reduced distances, both in time and cost, benefitting of well-integrated markets, with diffuse borders between regions over which economic effects easily spread.

The remaining of this paper is organised as follows: in section 2 we explain the framework that we adopted in building up the Portuguese RAM; section 3 is devoted to the input-output-type model that is associated to the RAM – the meaning of the main multipliers that are produced by that model is explained there; section 4 concludes presenting the most outstanding results that we obtained for Portugal. The paper encloses an annex where the procedure used for inverting the input-output matrix is summarised.

2. The framework of the Regional Accounting Matrix

Matrix **A** below depicts the structure of a RAM as we performed it for the Portuguese economy, where the $\mathbf{A_{ij}}$ are sub-matrices and the $\mathbf{q_i}$, $\mathbf{q'_j}$, $\mathbf{s_3}$ and $\mathbf{t'_3}$ are vectors calculated by adding up the rows and the columns of **A** (in this paper, capital letters designate matrices and small letters vectors).

$$\textbf{(1)} \qquad \mathbf{A} = \begin{array}{c} \\ \\ \\ \Sigma \end{array} \left[\begin{array}{ccc|c} & & & \Sigma \\ \mathbf{A_{11}} & \mathbf{A_{12}} & \mathbf{A_{13}} & \mathbf{q_1} \\ \mathbf{A_{21}} & \mathbf{A_{22}} & \mathbf{A_{23}} & \mathbf{q_2} \\ \mathbf{A_{31}} & \mathbf{A_{32}} & \mathbf{0} & \mathbf{s_3} \\ \hline \mathbf{q'_1} & \mathbf{q'_2} & \mathbf{t'_3} & \end{array} \right]$$

- $\mathbf{A_{11}}$ is an inter-industry table with 49 rows and 49 columns, of the input-output type, compiled at national level, where each entry represents the sales of one commodity to be used in the production process of one industry

$\mathbf{A_{12}}$, $\mathbf{A_{21}}$, $\mathbf{A_{22}}$, $\mathbf{A_{32}}$ and $\mathbf{A_{23}}$ altogether form the households' accounts that are disaggregated by the 7 Portuguese regions according to the location of the residence of those households.

- $\mathbf{A_{12}}$ is the private consumption of the households, by commodities and by regions[4]

- $\mathbf{A_{21}}$ describes the generation of income in each one of 49 industries that benefits the households residing in the seven regions; that income includes the compensation of employees and the operating surplus appropriated by the own-account workers that are classified in the households' institutional sector

- $\mathbf{A_{22}}$ is a transfer matrix where we record some flows that redistribute households' income among regions (we only consider here the income of the landlords that live in a region different from the one where their estates are located, other cases being neglected)

- $\mathbf{A_{32}}$ refers to other uses of the households' income as income taxes, social contributions, other transfers to other institutional sectors, and of course saving, all these flows being computed at regional level

- $\mathbf{A_{23}}$, at last, includes other sources of households' income, namely compensations received from abroad, dividends, interests and other payments and transfers from the other institutional sectors to the households residing in different regions.

$\mathbf{A_{31}}$ and $\mathbf{A_{13}}$ are the constituent parts of the total supply and demand of the outputs that are resources or uses of other institutional sectors but households. The matrix $\mathbf{A_{31}}$ includes a row with the international imports. It also includes some rows, concerning by-products and incidental sales of some branches of activity, that convert industry outputs into commodity outputs[5]. $\mathbf{A_{13}}$ mainly joins the investment, the collective consumption and the international exports columns. Matrices $\mathbf{A_{31}}$ and $\mathbf{A_{13}}$ are not regionalised.

The column-vector $\mathbf{q_1}$ and its transposed row-vector $\mathbf{q'_1}$ measure the total demand and supply (which are of course equal) of the 49 groups of commodities. In the same way,

$\mathbf{q_2}$ and $\mathbf{q'_2}$ represent the total flows of generation and use of the households' income. On the other hand, $\mathbf{s_3}$ and $\mathbf{t'_3}$ are respectively the total resources and the total uses of the other institutional sectors (as a matter of fact $\mathbf{s_3}$ includes the households' saving and $\mathbf{t'_3}$ its application through the households' capital accounts). We do not assume $\mathbf{s_3} = \mathbf{t_3}$, we only enforce $\mathbf{i's_3} = \mathbf{i't_3}$ where $\mathbf{i'}$ is a row-vector fulfilled with 1s[6].

3. The input-output model based on the RAM

Turning the RAM structure, which we described in the previous section, into an input-output model essentially requires that we decide which part of A is exogenous and which one includes the endogenous entries. Our assumption, in this section, is that $\mathbf{A_{11}}$, $\mathbf{A_{21}}$, $\mathbf{A_{12}}$ and $\mathbf{A_{22}}$ are the endogenous ones, depending on the total outputs of the 49 commodities $\mathbf{q_1}$ (sometimes they do depend on industry outputs; please see the Annex below) and on the 7 regional total incomes of the households $\mathbf{q_2}$:

$$
\begin{aligned}
\mathbf{A_{11}} &= \mathbf{A^*_{11}} \cdot \hat{\mathbf{Q}}_1 \\
\mathbf{A_{21}} &= \mathbf{A^*_{21}} \cdot \hat{\mathbf{Q}}_1 \qquad (2) \\
\mathbf{A_{12}} &= \mathbf{A^*_{12}} \cdot \hat{\mathbf{Q}}_2 \\
\mathbf{A_{22}} &= \mathbf{A^*_{22}} \cdot \hat{\mathbf{Q}}_2
\end{aligned}
$$

$\mathbf{A^*_{11}}$, $\mathbf{A^*_{21}}$, $\mathbf{A^*_{12}}$ and $\mathbf{A^*_{22}}$ are technical coefficients matrices and $\hat{\mathbf{Q}}_1$ and $\hat{\mathbf{Q}}_2$ are diagonal matrices whose main diagonals share the entries of $\mathbf{q_1}$ and $\mathbf{q_2}$. Because $\mathbf{A_{21}}$, $\mathbf{A_{12}}$ and $\mathbf{A_{22}}$ are assumed to be endogenous, our model is stated to be closed with respect to the private consumption.

Under these assumptions we can write:

$$
\underbrace{\begin{bmatrix} \mathbf{A^*_{11}} & \mathbf{A^*_{12}} \\ \mathbf{A^*_{21}} & \mathbf{A^*_{22}} \end{bmatrix}}_{\mathbf{A^*}} \cdot \underbrace{\begin{bmatrix} \mathbf{q_1} \\ \mathbf{q_2} \end{bmatrix}}_{\mathbf{q}} + \underbrace{\begin{bmatrix} \mathbf{A_{13}} \\ \mathbf{A_{23}} \end{bmatrix}}_{\mathbf{A_3}} \mathbf{i} = \underbrace{\begin{bmatrix} \mathbf{q_1} \\ \mathbf{q_2} \end{bmatrix}}_{\mathbf{q}} \qquad (3)
$$

or:

$$(3')\quad \mathbf{A}^{*}.\,\mathbf{q} + \mathbf{A}_3\,\mathbf{i} = \mathbf{q} \quad \Leftrightarrow \quad \mathbf{A}^{*}.\,\mathbf{q} + \mathbf{e} = \mathbf{q}$$

where $\mathbf{e} = \mathbf{A}_3\,\mathbf{i}$ (**i** is a vector of 1s) means the exogenous demand and income vector obtained by adding up the columns of $\mathbf{A}_3$. This vector is composed of two parts: $\mathbf{A}_{13}\,\mathbf{i}$, which is the exogenous final demand excluding private consumption; and $\mathbf{A}_{23}\,\mathbf{i}$, where is recorded the exogenous income of the households residing in the 7 regions that is not proceeding from their production activity. Solving the matricial equation (3'), we have:

$$(4)\quad (\mathbf{I} - \mathbf{A}^{*}).\,\mathbf{q} = \mathbf{e} \quad \Rightarrow \quad (5)\quad \mathbf{q} = (\mathbf{I} - \mathbf{A}^{*})^{-1}.\,\mathbf{e}$$

Matrix $(\mathbf{I} - \mathbf{A}^{*})^{-1}$ in (5), which we designate as **B** as well, computes the impacts of the increases in the entries of **e** – that is to say in exogenous final demand and in exogenous income – over the entries of **q**, i. e. over the commodities' output or the total income of the households residing in the seven regions. Matrix **B** is therefore the multipliers' matrix that we can split in four sub-matrices:

$$(6)\qquad \mathbf{B} = \left[\begin{array}{c|c} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \hline \mathbf{B}_{21} & \mathbf{B}_{22} \end{array}\right]$$

in which:

- $\mathbf{B}_{11}$ refers to the national multipliers on output, which mean the effects of increasing exogenous final demand on commodity outputs besides the place where they are produced (income induced effects are included)

- $\mathbf{B}_{21}$ is a matrix of multipliers describing the impacts on regional households' incomes of changing the final demands of the 49 groups of commodities in any place of the national territory

- $\mathbf{B}_{12}$ brings together the multipliers representing the changes in outputs at national level induced by changes in the households' regional incomes, and

- **B**$_{22}$, last but not least, is the matrix of the inter-regional households' income redistribution process, whose multipliers reckon the total impacts on the income of each region of exogenous changes on the income of the other (or the same) regions[7].

4. Some results for the Portuguese economy

The inter-regional income redistribution analysis allowed by the **B**$_{22}$ multipliers is beyond any doubt the most interesting outcome of this study. Our main idea is that regions cannot fully profit from the increases in income that their households benefit in a first wave. On the contrary, changes in income in one region do cross the region borders and propagate into other regions, because those increases in income and then in consumption in the former region imply a growth of the output everywhere, which compel further increases in other regions' income as well. Table 1 shows us those inter-regional income effects for the Portuguese economy; the upper part of the table is matrix **B**$_{22}$.

TABLE 1

INTER-REGIONAL INCOME REDISTRIBUTION MULTIPLIERS (**B**$_{22}$)

	Northern Region	Central Region	Lisbon and T. Valley	Alentejo	Algarve	Azores	Madeira
Northern Region	1.1479	0.1388	0.1456	0.1341	0.1600	0.1361	0.1317
Central Region	0.0736	1.0693	0.0720	0.0665	0.0796	0.0682	0.0650
Lisbon and Tagus Valley	0.2052	0.1904	1.2032	0.1877	0.2213	0.1839	0.1784
Alentejo	0.0226	0.0209	0.0221	1.0207	0.0237	0.0211	0.0200
Algarve	0.0212	0.0194	0.0210	0.0193	1.0238	0.0185	0.0182
Azores	0.0099	0.0091	0.0098	0.0091	0.0106	1.0095	0.0089
Madeira	0.0105	0.0097	0.0105	0.0096	0.0117	0.0094	1.0095
Population (10^3)	3524.9	1712.5	3309.6	526.4	345.1	241.0	257.0
GDP per capita	1409.1	1363.4	2007.1	1316.1	1584.6	1117.6	1241.6

The lower part of the table includes the population and the GDP per capita of the Portuguese regions in 1995. Looking at the main diagonal of the matrix, we can conclude that the intra-regional income multipliers are higher in the biggest and in the richest regions: the Northern Region and the Lisbon and Tagus Valley region. The reason why we find right there the most meaningful multipliers is that those regions guarantee the major share of the national output. Hence, indirect effects on output remain inside the regions in a large scale; leakages for other regions are reduced to a minimum. On the other hand, smallest regions as the Alentejo, Algarve or the islands (Azores and Madeira) have a more specialised production apparatus, so income effects are very often re-routed towards other regions where consumption goods are mainly produced.

The very same reasons also explain, if we look at the multipliers located out of the main diagonal, which represent the cross-effects between regions, why increasing income in the smallest and the poorest regions has a more important impact on other regions than the opposite effect. That means that income in Portugal mainly flows from the smallest and poorest regions as Alentejo or the islands into the biggest and richest regions as Lisbon and Tagus Valley or the Northern Region. We should stress, however, that that conclusion concerns only the kind of income effects that are embraced by an input-output-type model, namely the automatic effects on the production income, appropriated by the households, which are induced by a growing consumption.

Regarding the two main Portuguese regions - Northern Region and Lisbon and Tagus Valley, which have a similar size in terms of their population - it is interesting to note that the Lisbon region benefits more in its inter-relationship with the Northern Region than the opposite. That happens in spite of the fact that Northern Region is the main industrial region in Portugal (but its industry is mainly directed to the international exports and not as much to the domestic consumption); on the other hand, the Lisbon and Tagus Valley production structure is more based on services. In fact, the reason for Lisbon's advantage seems to be its higher GDP per capita.

Tables 2 and 3 depict respectively the impacts on regional income of exogenous shocks on final demand (table 2 shows $\mathbf{B}_{21}$ transposed) and the impacts on commodity output supplies (including import supplies) of the changes in the regional incomes ($\mathbf{B_{12}}$)[8]. It is interesting to find out that in the greater regions – Northern Region and Lisbon and Tagus Valley – the "Health and Veterinary Market Services" is the industry where a change in demand has a stronger income effect (note that that change in demand is not regionally located but can, on the contrary, occur anywhere). In Alentejo and Azores that role is assumed by "Agriculture and Hunting", while the demand for "Hotels and Restaurants" has a prominent effect on the households' income of Algarve.

TABLE 2

REGIONAL INCOME MULTIPLIERS OF CHANGES

IN COMMODITY DEMANDS (transposed $\mathbf{B}_{21}$)

	Northern Region	Central Region	Lisbon T. Valley	Alentejo	Algarve	Azores	Madeira
Agriculture and hunting	0.3088	0.1881	0.3878	0.0855	0.0445	0.0373	0.0230
Forestry	0.0812	0.0714	0.1142	0.0407	0.0130	0.0031	0.0036
Fishing	0.1197	0.0616	0.1807	0.0165	0.0478	0.0215	0.0126
Coal mining and transform.	0.0473	0.0221	0.0768	0.0061	0.0084	0.0023	0.0030
Petroleum extr. and refining	0.1689	0.0751	0.2597	0.0250	0.0215	0.0087	0.0110
Electricity, gas and water	0.1128	0.0508	0.1642	0.0191	0.0127	0.0111	0.0116
Metal ores mining and transf.	0.1048	0.0495	0.1531	0.0040	0.0095	0.0041	0.0047
Non-metal. mineral pr. ext. &tr.	0.2446	0.1439	0.3320	0.0613	0.0267	0.0091	0.0147
Porcelain and pottery	0.1594	0.2278	0.2725	0.0178	0.0166	0.0070	0.0080
Glass and glass prod. manuf.	0.1675	0.1925	0.2741	0.0184	0.0162	0.0075	0.0092
Other construction materials	0.1590	0.1621	0.2966	0.0231	0.0283	0.0116	0.0101
Chemical prod. manufacture	0.1188	0.0575	0.2275	0.0217	0.0138	0.0062	0.0069
Metallic products manufact.	0.1810	0.0905	0.2310	0.0151	0.0158	0.0064	0.0072
Non-electrical machinery	0.1039	0.0461	0.1374	0.0099	0.0092	0.0035	0.0041
Electrical prod. and machinery	0.1186	0.0457	0.1436	0.0123	0.0085	0.0037	0.0042
Vehicles and transport equip.	0.1019	0.0601	0.1817	0.0132	0.0109	0.0045	0.0055
Manufact. of meat products	0.2108	0.1202	0.3053	0.0460	0.0280	0.0197	0.0146
Manufact. of dairy products	0.2257	0.1347	0.3017	0.0466	0.0275	0.0261	0.0151
Manufact. of fish products	0.1308	0.0672	0.1830	0.0146	0.0280	0.0134	0.0088
Manufacture of fat products	0.1625	0.0830	0.2682	0.0315	0.0202	0.0126	0.0103
Man. of grain and legum. pr.	0.2184	0.1148	0.2808	0.0389	0.0258	0.0162	0.0140
Man. of other food products	0.1635	0.0953	0.2675	0.0360	0.0225	0.0150	0.0110
Beverages	0.1378	0.0650	0.1845	0.0187	0.0164	0.0088	0.0103
Tobacco	0.0347	0.0174	0.0639	0.0058	0.0046	0.0047	0.0033
Textiles and textile products	0.2950	0.0894	0.1923	0.0199	0.0156	0.0077	0.0085
Leather and leather products	0.2454	0.0527	0.1714	0.0146	0.0132	0.0059	0.0066
Wood and cork products	0.2471	0.1017	0.2324	0.0281	0.0193	0.0073	0.0093
Pulp, paper, publ. and printing	0.1612	0.0946	0.2702	0.0214	0.0172	0.0074	0.0092
Rubber and plastic products	0.1561	0.0739	0.1826	0.0145	0.0128	0.0055	0.0066
Other manufactured products	0.1724	0.0586	0.1951	0.0144	0.0137	0.0057	0.0069
Construction	0.2250	0.1139	0.3028	0.0246	0.0267	0.0129	0.0159
Recycling and repair services	0.2251	0.1096	0.3899	0.0306	0.0305	0.0117	0.0128
Wholesale and retail trade	0.2559	0.1163	0.4007	0.0333	0.0333	0.0134	0.0168
Hotels and restaurants	0.1836	0.0980	0.3069	0.0336	0.0606	0.0146	0.0271
Land transport and pipelines	0.2458	0.1347	0.4039	0.0380	0.0379	0.0146	0.0174
Water and air transp. serv.	0.1337	0.0514	0.3010	0.0174	0.0252	0.0192	0.0615
Supporting aux. transp. ser.	0.1624	0.0643	0.3422	0.0276	0.0332	0.0134	0.0248
Communication services	0.1498	0.0741	0.2349	0.0244	0.0217	0.0094	0.0114
Banking and financial serv.	0.1729	0.0696	0.3428	0.0218	0.0206	0.0113	0.0105
Insurances	0.2729	0.1410	0.7394	0.0383	0.0428	0.0200	0.0176
Housing services	0.4968	0.2609	0.6138	0.0638	0.0787	0.0295	0.0365
Business services	0.1211	0.0526	0.3145	0.0169	0.0213	0.0073	0.0085
Educat. and res. market serv.	0.1908	0.0862	0.2788	0.0246	0.0198	0.0116	0.0176
Health and vet. market servic.	0.5000	0.2471	0.7748	0.0501	0.0502	0.0220	0.0273
Other market services	0.2163	0.0964	0.4043	0.0341	0.0328	0.0153	0.0175
Public adm. non-market serv.	0.2921	0.1584	0.5301	0.0609	0.0429	0.0314	0.0360
Edu. and res. non-mark. serv.	0.4265	0.2201	0.5077	0.0713	0.0510	0.0294	0.0273
Health and v. non-market ser.	0.3426	0.1768	0.5039	0.0388	0.0343	0.0299	0.0325
Other non-market services	0.3698	0.1762	0.4770	0.0562	0.0419	0.0258	0.0305

TABLE 3

COMMODITY OUTPUT MULTIPLIERS OF CHANGES

IN REGIONAL INCOMES (**B$_{12}$**)

	Northern Region	Central Region	Lisbon T. Valley	Alentejo	Algarve	Azores	Madeira
Agriculture and hunting	0.1465	0.1297	0.1405	0.1337	0.1371	0.1444	0.1280
Forestry	0.0118	0.0102	0.0091	0.0071	0.0085	0.0081	0.0069
Fishing	0.0220	0.0192	0.0284	0.0226	0.0360	0.0250	0.0182
Coal mining and transform.	0.0033	0.0030	0.0036	0.0034	0.0035	0.0035	0.0036
Petroleum extr. and refining	0.0975	0.0905	0.0992	0.0971	0.1025	0.0750	0.0883
Electricity, gas and water	0.0917	0.0845	0.1004	0.0949	0.0980	0.0976	0.1020
Metal ores mining and transf.	0.0096	0.0092	0.0094	0.0086	0.0096	0.0086	0.0074
Non-metal. mineral pr.ext. &tr.	0.0019	0.0019	0.0019	0.0017	0.0021	0.0019	0.0017
Porcelain and pottery	0.0021	0.0022	0.0024	0.0016	0.0016	0.0020	0.0028
Glass and glass prod. manuf.	0.0038	0.0031	0.0035	0.0032	0.0037	0.0034	0.0031
Other construction materials	0.0023	0.0023	0.0022	0.0020	0.0028	0.0023	0.0021
Chemical prod. manufacture	0.0849	0.0762	0.0890	0.0862	0.0920	0.0906	0.0724
Metallic products manufact.	0.0145	0.0138	0.0142	0.0134	0.0153	0.0146	0.0127
Non-electrical machinery	0.0104	0.0099	0.0098	0.0087	0.0097	0.0092	0.0078
Electrical prod. and machinery	0.0291	0.0277	0.0306	0.0259	0.0284	0.0295	0.0233
Vehicles and transport equip.	0.0789	0.0779	0.0736	0.0681	0.0703	0.0562	0.0459
Manufact. of meat products	0.0803	0.0736	0.0813	0.0723	0.0768	0.0756	0.0658
Manufact. of dairy products	0.0307	0.0289	0.0345	0.0334	0.0325	0.0419	0.0306
Manufact. of fish products	0.0228	0.0193	0.0215	0.0175	0.0159	0.0163	0.0101
Manufacture of fat products	0.0141	0.0140	0.0134	0.0153	0.0147	0.0108	0.0101
Man. of grain and legum. pr.	0.0474	0.0426	0.0440	0.0486	0.0456	0.0528	0.0497
Man. of other food products	0.0451	0.0389	0.0467	0.0439	0.0448	0.0542	0.0404
Beverages	0.0209	0.0164	0.0204	0.0187	0.0231	0.0173	0.0152
Tobacco	0.0175	0.0113	0.0180	0.0186	0.0186	0.0222	0.0172
Textiles and textile products	0.0916	0.0812	0.0918	0.0848	0.0896	0.0809	0.0853
Leather and leather products	0.0242	0.0216	0.0236	0.0246	0.0245	0.0228	0.0293
Wood and cork products	0.0276	0.0272	0.0324	0.0215	0.0282	0.0286	0.0215
Pulp, paper, publ. and printing	0.0331	0.0317	0.0360	0.0305	0.0353	0.0315	0.0330
Rubber and plastic products	0.0181	0.0204	0.0194	0.0184	0.0205	0.0196	0.0159
Other manufactured products	0.0173	0.0159	0.0168	0.0146	0.0170	0.0170	0.0142
Construction	0.0174	0.0178	0.0163	0.0154	0.0217	0.0175	0.0160
Recycling and repair services	0.0862	0.0781	0.0789	0.0732	0.0793	0.0604	0.0634
Wholesale and retail trade	0.1812	0.1656	0.1824	0.1707	0.1830	0.1651	0.1543
Hotels and restaurants	0.1209	0.0924	0.1231	0.1070	0.1333	0.0717	0.0875
Land transport and pipelines	0.0228	0.0194	0.0271	0.0173	0.0187	0.0189	0.0289
Water and air transp. serv.	0.0066	0.0069	0.0081	0.0058	0.0068	0.0092	0.0109
Supporting aux. transp. ser.	0.0150	0.0131	0.0173	0.0132	0.0157	0.0145	0.0140
Communication services	0.0319	0.0314	0.0381	0.0359	0.0376	0.0346	0.0355
Banking and financial serv.	0.0046	0.0044	0.0044	0.0042	0.0051	0.0042	0.0040
Insurances	0.0087	0.0074	0.0085	0.0070	0.0083	0.0069	0.0066
Housing services	0.0789	0.0874	0.0683	0.0708	0.1081	0.0823	0.0752
Business services	0.0606	0.0556	0.0634	0.0545	0.0634	0.0538	0.0522
Educat. and res. market serv.	0.0200	0.0172	0.0254	0.0158	0.0185	0.0134	0.0165
Health and vet. market servic.	0.0320	0.0295	0.0319	0.0226	0.0396	0.0267	0.0251
Other market services	0.0339	0.0263	0.0318	0.0282	0.0354	0.0248	0.0261
Public adm. non-market serv.	0.0011	0.0013	0.0014	0.0014	0.0014	0.0009	0.0012
Edu. and res. non-mark. serv.	0.0006	0.0005	0.0008	0.0004	0.0005	0.0004	0.0005
Health and v. non-market ser.	0.0006	0.0005	0.0006	0.0004	0.0007	0.0004	0.0004
Other non-market services	0.0155	0.0159	0.0149	0.0185	0.0190	0.0121	0.0164

ANNEX

The matrix inversion procedure

Let **q** be the vector of commodity outputs – or commodity supplies since imports are included – and **g** be the vector of industry outputs (in fact, those outputs only fill 49 of the 56 entries of those vectors; the last 7 entries, which comprise the total incomes of the households residing in the 7 Portuguese regions, are the same in both vectors). In the main text of this paper we assumed that the technical coefficients matrix **A*** is computed by dividing the entries of **A** by the outputs of **q**. However, these technical relationships shall be settled instead at industry level, because the same commodity can be produced by a different technology in a different industry. That means that the basic input-output equation we consider in this paper is really:

$$\textbf{(7)} \quad \mathbf{q} = \mathbf{A^*}\,\mathbf{g} + \mathbf{e}$$

instead of (3') in the main text, where **e** is the final demand vector. However, the output vectors **q** and **g** can be related as follows:

$$\textbf{(8)} \quad \mathbf{q} = \mathbf{g} - \mathbf{tr_1} + \mathbf{tr_2} + \mathbf{im} + \mathbf{tm} + \mathbf{t}$$

in which $\mathbf{tr_1}$ and $\mathbf{tr_2}$ are by-products and incidental sales of commodities that are transferred from the industry where they actually are produced (and then subtracted by $\mathbf{tr_1}$) to the commodity group where they belong by their very nature (and then added by $\mathbf{tr_2}$). **im** is the vector of imports (including taxes on imports), **tm** are the trade margins and **t** means the VAT. Note that in the Portuguese National Accounts, and therefore in our RAM as well, $\mathbf{tr_1}$ and $\mathbf{tr_2}$ only record residual flows: secondary production is already included in the industry where it belongs by its nature. On the other hand **t** only bears deductible VAT that falls upon the private consumption. Although the outputs are evaluated at purchasers' prices, we have excluded non-deductible VAT from the sales to intermediate consumption and to GFCF, and then from the value of the total output of each commodity (however, it is assumed to be a cost and then included in the total value of the output of the industry that buys the commodities).

We assumed that:

$$\mathbf{tr_1} = \hat{\mathbf{Z}}_1\mathbf{g} \quad \text{and} \quad \mathbf{tr_2} = \mathbf{z_2\, i'\, tr_1} = \mathbf{z_2\, i'}\, \hat{\mathbf{Z}}_1\mathbf{g}$$

where $\hat{\mathbf{Z}}_1$ is a diagonal matrix and $\mathbf{z_2}$ a vector of coefficients; $\mathbf{i'}$ is a row-vector fulfilled with 1s. By-products and incidental sales are supposed to depend on the output of the industries where they are generated ($\mathbf{tr_1}$ depends on $\mathbf{g}$), while their structure by commodities is deemed to be invariant, i. e. $\mathbf{tr_2}$ depends on the total entries of $\mathbf{tr_1}$.

Imports and trade margins are supposed to be fixed shares of the commodity outputs (VAT excluded):

$$\mathbf{im} = \hat{\mathbf{Z}}_3(\mathbf{q} - \mathbf{t}) \quad \text{and} \quad \mathbf{tm} = \hat{\mathbf{Z}}_4(\mathbf{q} - \mathbf{t})$$

VAT is assumed to depend on households' consumption, which is a function of their total income

$$\mathbf{t} = \hat{\mathbf{Z}}_5\, \mathbf{c} = \hat{\mathbf{Z}}_5\, \mathbf{A^*}\, \hat{\mathbf{J}}\, \mathbf{q}$$

where $\mathbf{c}$ is the vector of the households' consumption, $\mathbf{A^*}$ is the technical coefficients matrix and $\hat{\mathbf{J}}$ is a square matrix with 1s in the last 7 entries of the main diagonal and 0s elsewhere ($\hat{\mathbf{J}}\,\mathbf{q}$ is a vector of 0s, except in its last 7 entries, which are the households' incomes).

Doing $$\mathbf{q} - \mathbf{t} = (\mathbf{I} - \hat{\mathbf{Z}}_5\, \mathbf{A^*}\, \hat{\mathbf{J}})\, \mathbf{q} = \mathbf{W}\, \mathbf{q}$$

we can write $$\mathbf{im} = \hat{\mathbf{Z}}_3\, \mathbf{W}\, \mathbf{q} \quad \text{and} \quad \mathbf{tm} = \hat{\mathbf{Z}}_4\, \mathbf{W}\, \mathbf{q}$$

Those equations can now be inserted into (8), getting:

$$\mathbf{q} - \mathbf{t} = \mathbf{W}\, \mathbf{q} = \mathbf{g} - \hat{\mathbf{Z}}_1\, \mathbf{g} + \mathbf{z_2\, i'}\, \hat{\mathbf{Z}}_1\, \mathbf{g} + \hat{\mathbf{Z}}_3\, \mathbf{W}\, \mathbf{q} + \hat{\mathbf{Z}}_4\, \mathbf{W}\, \mathbf{q}$$

$$(\mathbf{I} - \hat{\mathbf{Z}}_3 - \hat{\mathbf{Z}}_4)\, \mathbf{W}\, \mathbf{q} \;=\; (\mathbf{I} - \hat{\mathbf{Z}}_1 + \mathbf{z}_2\, \mathbf{i'}\, \hat{\mathbf{Z}}_1)\, \mathbf{g}$$

$$\textbf{(8')} \quad \mathbf{g} = (\mathbf{I} - \hat{\mathbf{Z}}_1 + \mathbf{z}_2\, \mathbf{i'}\, \hat{\mathbf{Z}}_1)^{-1} (\mathbf{I} - \hat{\mathbf{Z}}_3 - \hat{\mathbf{Z}}_4)\, \mathbf{W}\, \mathbf{q} = \mathbf{H}\, \mathbf{W}\, \mathbf{q}$$

On the other hand, rewriting equation (7) we have:

$$\textbf{(7')} \quad \mathbf{q} = \mathbf{A^*}\, \mathbf{g} + \mathbf{Z}_4\, (\mathbf{q} - \mathbf{t}) + \mathbf{e} = \mathbf{A^*}\, \mathbf{g} + \mathbf{Z}_4\, \mathbf{W}\, \mathbf{q} + \mathbf{e}$$

The reason why we added up $\mathbf{Z}_4$ $(\mathbf{q} - \mathbf{t})$ above is that the technical coefficients matrix **A*** has its 33rd row, concerning the retail and wholesale trade, entirely fulfilled with 0s; instead, the trade output matches the trade margins (the difference between the matrices $\hat{\mathbf{Z}}_4$ and $\mathbf{Z}_4$ is that the former is a diagonal matrix while the latter has the same coefficients on its 33rd row, and 0s elsewhere).

Thus, from the substitution of (8') into (7') we find:

$$\mathbf{q} = \mathbf{A^*}\, \mathbf{H}\, \mathbf{W}\, \mathbf{q} + \mathbf{Z}_4\, \mathbf{W}\, \mathbf{q} + \mathbf{e}$$

$$\mathbf{q} = (\mathbf{I} - \mathbf{A^*}\, \mathbf{H}\, \mathbf{W} + \mathbf{Z}_4\, \mathbf{W})^{-1}\, \mathbf{e}$$

and

$$\mathbf{g} = \mathbf{H}\, \mathbf{W}\, (\mathbf{I} - \mathbf{A^*}\, \mathbf{H}\, \mathbf{W} + \mathbf{Z}_4\, \mathbf{W})^{-1}\, \mathbf{e}$$

The inverse matrix **B** that is discussed in the main text of this paper is in fact:

$$(\mathbf{I} - \mathbf{A^*}\, \mathbf{H}\, \mathbf{W} + \mathbf{Z}_4\, \mathbf{W})^{-1} \quad \text{or} \quad \mathbf{H}\, \mathbf{W}\, (\mathbf{I} - \mathbf{A^*}\, \mathbf{H}\, \mathbf{W} + \mathbf{Z}_4\, \mathbf{W})^{-1}$$

depending on whether we are computing impacts on commodity or industry outputs. The impacts on households' income (i. e. the multipliers $\mathbf{B}_{21}$ and $\mathbf{B}_{22}$) are the same in both matrices.

FOOTNOTES

[1] The theoretical structure of the Regional Accounting Matrix (RAM) that we develop in this paper was presented once in a meeting of the APDR (the Portuguese Regional Developing Association) that took place in Ponta Delgada, Azores, in July 2000 (Ramos, 2000). However, the application to the Portuguese economy, which is now the most prominent part of this presentation, was not yet available at that time.

[2] On that framework see, for instance, Lager (1998) and Pyatt and Round (1998). Our model, and social accounting matrices as well, can also be seen as proceeding from the Miyazawa's models developed in the sixties and seventies, some of them reproduced and/or shortened in Miyazawa (1976). However, the regional model proposed by Miyazawa (1976), in his chapter 2, assumes a more extensive set of regional information than ours, thus avoiding an assumption such as our "hypothesis of non-existence of regional preference in supplying regions".

[3] We did not attach to this paper the RAM that we produced for the Portuguese economy because of its considerable size. However, the RAM (and the inverted matrices that we derived from that original table) can be provided under request through the e-mail address: pnramos@sonata.fe.uc.pt

[4] As a matter of fact, in building up the RAM, in a first step we split the private consumption by regions, and then we estimate the consumption by commodities within each region. The outcome of this process is a structure of the private consumption, by commodities, that is different from the one that is taken in the National Accounts. The difference between the two estimates is a vector of "errors" that was transferred to $\mathbf{A_{13}}$. This vector also includes the consumption of non-residents on the national territory (minus the consumption of residents made abroad) and the consumption of the households that are deemed to be resident in the Extra-regio. The Extra-regio is the part of the economic territory which cannot be attached directly to a single region, for instance embassies or consulates (Eurostat, 1996, 13.06).

[5] Note that the proposed RAM is still based on National and Regional Accounts that follow the 1970 European System of Accounts. That explains some peculiarities in the way we dealt with by-products. For more details in the process of turning industries into commodities, please see the Annex to this paper.

[6] It could also be assumed $\mathbf{s_3} = \mathbf{t_3}$. However, this would require that $\mathbf{A_{31}}$, $\mathbf{A_{32}}$, $\mathbf{A_{13}}$ and $\mathbf{A_{23}}$ were organised by institutional sectors and not by kind of transaction. Nevertheless, this alternative and more intricate shape of the RAM did not seem, in our opinion, to be worthwhile.

[7] We can also find an inter-regional income redistribution matrix in Miyazawa (1976), chapter 2. However, as we explained in footnote 2, the underlying model is different, incorporating more information at regional level (for instance $\mathbf{A_{11}}$ and then $\mathbf{B_{11}}$ are regionalised).

[8] We have also compiled the impacts on the industry (domestic) outputs. These results can be demanded through the same E-mail mentioned in footnote 3.

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