

## **Fractal Analysis of the Urbanization at the Outskirts of the City: Models, Measurement and Explanation**

**ABSTRACT:** The present research concentrates on the urbanization at the edges of the city, the outskirts of metropolitan areas, which can be considered the examples par excellence of complex, fractal-like urban structures, revealing dynamic processes of growth and transformation. The basic models for the estimation of fractal dimension are presented, with the focus on the interpretation of the calculated values. Fractal dimensions can be used as indicators of urban sprawl, of the degree of fragmentation of the urban landscape and of the transition from monocentric to polycentric structures. The exploration of the above notions is based on the application of the models on an area at the outskirts of Thessaloniki, Greece. The important changes observed between 1945 and 1990 at the urban spatial structure of the area under investigation, are discussed in the light of fractal analysis.

### **Introduction**

The issue of treating cities as complex and emergent structures has been raised during the past two decades, posing serious questions and casting further light upon the understanding of the urban phenomena. Cities are considered to be systems of organized complexity with a distinct hierarchical order. The application of fractal geometry at the analysis of the urban development patterns has been widely investigated during the last two decades, together with the cellular automata and the agent-based models (Batty 2005, Benguigui, Chamanski, Marinov 2001). Based on the recognition that urban patterns are highly complex, heterogeneous and hierarchically ordered revealing self-similarity across scales, numerous models of fractal analysis have been applied at the study of the urbanization processes.

The box-counting method and the radial analysis are two commonly applied models calculating the fractal dimension of an urbanized area (Frankhauser 1998, Benguigui et al 2000). Important attention has also been given to the analysis of urban boundaries (Batty & Longley 1994) which have been considered as fractal-like structures and are related with urban sprawl at the edges of the city (Frankhauser 2004). The fractal dimension is thus considered an extremely useful indicator of the urban spatial structure (Anas, Arnott, Small, 1998) and its transformation through time, used both at the study of the perimeter of the city, and at the study of whole urbanized surfaces.

The objective of the present study is to explore the basic relations between fractal analysis and the urbanization at the outskirts of the city. It is widely accepted that the urban structures at the edges of the metropolitan areas are characterized by dynamic processes of growth, revealing a high degree of complexity and heterogeneity. Those areas can be considered the examples par excellence of complex, fractal-like urban structures presenting a promising field of inquiry for the new geometry.

At the same time the accentuated growth of the outskirts of many metropolitan areas has led many to recognize their importance, referring to the emergence of edge cities (Garreau 1991) and polynucleated urban landscapes (Batty 2001, Soja 2000). In European cities the main tendency is the reinforcing of existing peri-urban settlement structures leading gradually to more decentralized patterns (Batty 2001). An application of fractal geometry to the study of these rapidly growing areas can provide a valuable perspective and lead to an explanation of the urban transformations.

The present research is composed of three basic parts: First the two basic models (box-counting method and radial analysis) and their relation to the analysis of urban development patterns are presented. Ways of evaluating the relationship between the fractal dimension and the degree of fragmentation, the distribution of the land uses and generally the structure of the urban area under investigation, are the main focus. The exploration of these notions is carried out by an application of the box-counting method at the outskirts of Thessaloniki, Greece. The area is located at the north-west sector of the city's outskirts, characterized by rapid growth. The transformation of the area between 1945 and 1990 is analyzed, while some further current trends are discussed. Finally the evaluation of the fractal analysis is presented and the explanation of the results is based on the relationship between the fractal dimension with various socioeconomic and population data.

## Fractal models and the measurement of urban structures

The most important characteristic of a fractal object is that it possesses a motif that repeats itself on an ever reduced scale. Fractals are by definition complex, hierarchically ordered structures revealing self-similarity across scales (Batty & Longley 1994). The fractal dimension  $D$  is a basic parameter of a fractal, revealing important aspects of its structure, as it quantifies the degree of irregularity or fragmentation. It also indicates the level of complexity or the amount of details through scales and describes the distribution of the mass around a center (Sobreira & Filho 2002).

Many ways for relaxing the strictly self-similar, deterministic fractals have been suggested, in order to relate the fractal concept with urban forms and urban systems. First it has been recognized that urban patterns reveal the property of self-affinity rather than self-similarity (Batty 1994) and thus the fractal dimension is estimated statistically. It has also been shown that sometimes the urban built-up patterns can better be described as multi-fractals or as the overlay of different fractal patterns (Frankhauser 1998). In this case the value of  $D$  can change across space and scale and also through time (Λαγapias 2005).

There are two basic ways for examining a fractal structure, either by examining changes in form by varying the scale, or by keeping the scale fixed and varying the size (Batty & Longley 1994). The basic models used to calculate the fractal dimension are referred to as the *box-counting method* and the *radial analysis*<sup>1</sup>. The approach is based on a binary logic that distinguishes between the urbanized and non-urbanized area, and describes the distribution of the built-up area on the surface that contains it.

---

<sup>1</sup> Many other methods have been devised such as the dilation, correlation and structured walk algorithms

The *box-counting method* is based on a grid analysis of the structure. The object is covered by a grid made of squares of size  $l$ , and the number  $N$  of squares in which a part of the object appears is counted. The size of the grid is varied progressively, usually through a sequence of grid lengths equal to  $l_i = L/2^i$  and the scatter diagram of  $l$  against  $N$  is presented. The relation that is expected between the two variables is expressed as  $N = (L/l)^D$  (1) where  $D$  is the fractal dimension and  $L$  the total size of the grid. The estimation of  $D$  is based on a logarithmic transformation of (1) which takes the form  $\text{Log}N = a - D \text{log}l$  (2), where  $a$  is a constant and is used as the basis for linear regression.

The method can be applied both at the study of whole urbanized surfaces and at the study of inner and outer urban boundaries. The value of the dimensions is estimated in the range  $1 < D < 2$  (Batty 2005). The relation between the fractal dimension of a surface (areal fractal dimension, Anas, Arnott, Small, 1998) and the corresponding boundary can be understood if we consider a perfect homogeneous circle: The value of  $D_a$  (dimension of the surface) is equal to 2 while the value of  $D_b$  (dimension of the boundary) is equal to 1. The more fragmented a pattern appears, the smaller is the value of  $D_a$  and the greater of  $D_b$  (figure 1). The fractal dimension therefore describes how the built up area is distributed on the surface, and how dense or fragmented is the spatial patterning of the city. It has been shown through various studies that the box-counting dimension of the scattered, irregular urban patterns at the edges of the city present a lower value, especially on the first stages of development. Progressively the fragmented structure is transformed through processes of coalescence, while the dendritic outline border is smoothed. The quantification of those processes provided by the box-counting fractal dimension offers important insights on the analysis and explanation of those transformations.

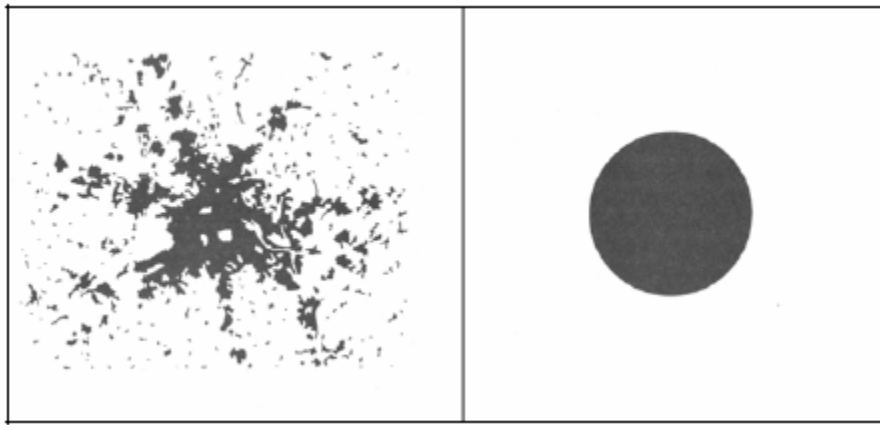


Figure 1: Comparison between a perfect Euclidean circle and the heterogeneous fractal distribution of Berlin (Frankhauser 2004)

The *radial analysis* refers to a specific point known as the counting center, and gives the law of distribution of the occupied sites around this point. The scale of the analysis is fixed and we vary the radius  $R$  of the distance from the center. At each step, the total number of occupied points  $N$  inside the circle is counted and it is expected that  $N = a R^D$  (2).

In the case of a homogeneous, uniform distribution  $D$  is calculated close to 2, while the more concentrated is the mass around the center the closest to 1 is the value of  $D$  estimated. The radial analysis offers therefore a convenient way to examine the hierarchical organization of an urban area and to determine whether the structure is monocentric or reveals a greater degree of decentralization.

The two methods do not give identical results when applied over the same area, as a cause to the fact that the urban patterns are not equivalent to the theoretical fractals and often display properties of multi-fractals. Each one however measures a different attribute of the area under investigation and therefore they should be applied in combination.

The case study presented here concentrates on the box-counting dimension of the built-up area and the corresponding boundary of the northwest sector of Thessaloniki<sup>2</sup>. Tables 1 and 2 therefore present the basic theoretical relationships between those fractal dimensions and the urban context to which they refer. In order to explore the divergence of values in space we can state that central, densely built-up zones of a city display a greater value of  $D_a$ , (Frankhauser 1998) while the peripheral a lower value, especially on the first stages of their development<sup>3</sup> (Laurence de Keersmaeker, Frankhauser, Thomas 2003), as a cause to their irregularity and fragmentation. An increase in the value of  $D_a$  in an outskirt area indicates a denser, homogeneous pattern of development with a fill-in of the existing vacant land. On the contrary a decrease in the value of  $D_a$  indicates a process of urban growth characterized by fragmented, leap-frog development. The inverse is expected for the dimension of the urban boundary.

**TABLE 1: Divergence of values in space:**

	<b>More central, densely built-up areas</b>	<b>Irregular outskirts characterized by recent sprawl</b>	<b>Densely built-up, homogeneous outskirts, next stages of development</b>
$\Delta D_a$	High values	Low values	High values
$\Delta D_b$	-	High values	Low values

**TABLE 2: Divergence of values in time:**

	<b>Increase in density at the outskirts of the city – fill in of available non-urban land between the settlements</b>	<b>Leapfrog development at the edges – extension of the urban boundary and increase of fragmentation and complexity</b>
$\Delta D_a$	+	-
$\Delta D_b$	-	+

<sup>2</sup> As the case study does not treat the city of Thessaloniki as a whole but is limited to the northwest suburban sector, the radial analysis was not applied.

<sup>3</sup> Outskirt areas which have been developed by an organized plan and with a great control in their building process, display high values just like central areas. We can state therefore that  $D$  is not equivalent to density but takes also into account the distribution and the structure of the patterns of development

## An application of fractal analysis at the outskirts of Thessaloniki

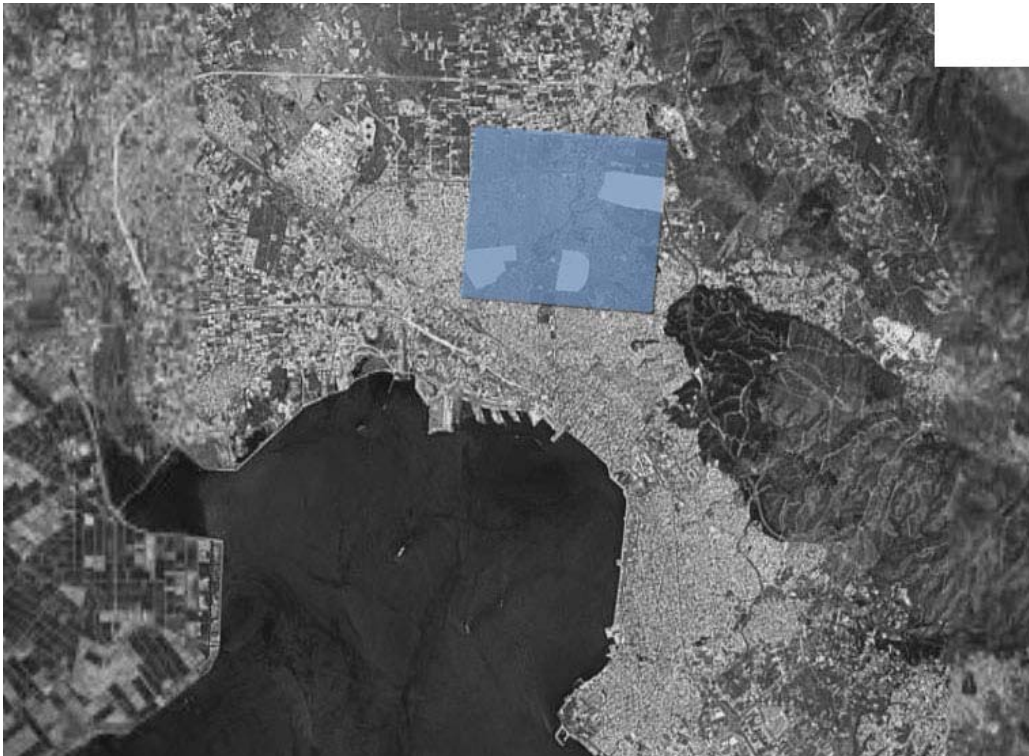


Figure 2:  
Satellite image of Thessaloniki showing the area under investigation located at the North West sector

The area chosen for the fractal analysis is located at the North West sector of the city containing the municipalities of Stauroupoli and Polichni, and also a part of Euosmos and Efkarpia (figure 2). The area is close enough to the center and connected to it by the major transportation axis of Lagada Street, while its furthest part is located at the edge of the city's administrative boundary, close to the inner ring road. A densely populated area during the last decades, it has gone through major transformations since the middle of the century when it was a low density, suburban area characterized by the growth of spontaneous settlements in areas not included in the city plan.

The analysis was carried out by comparing the patterns of development of the years 1945 and 1990. As a basis for the application of the models<sup>4</sup> a combination of aerial photos was used, which were digitized, transformed to the same scale and cropped to display exactly the same area. The two photos were then converted to binary images while the built up area of each period was extracted by setting a proper threshold<sup>5</sup> (figures 3 and 4).

<sup>4</sup> The analysis was carried out by a combination of the software programmes, Fractalyse 2.1, Photoshop 7.0, Image J and the plug-in, Fraclac 2.2.

<sup>3</sup> The complete absence of maps of the urban development of Thessaloniki, displaying the land uses has made necessary the extraction of the urban built up area directly from the photos. A certain degree of inaccuracy of the binarisation process is inevitable, though it does not seem to alter in any way the basic results of the analysis (Filho & Sobreira 2005)



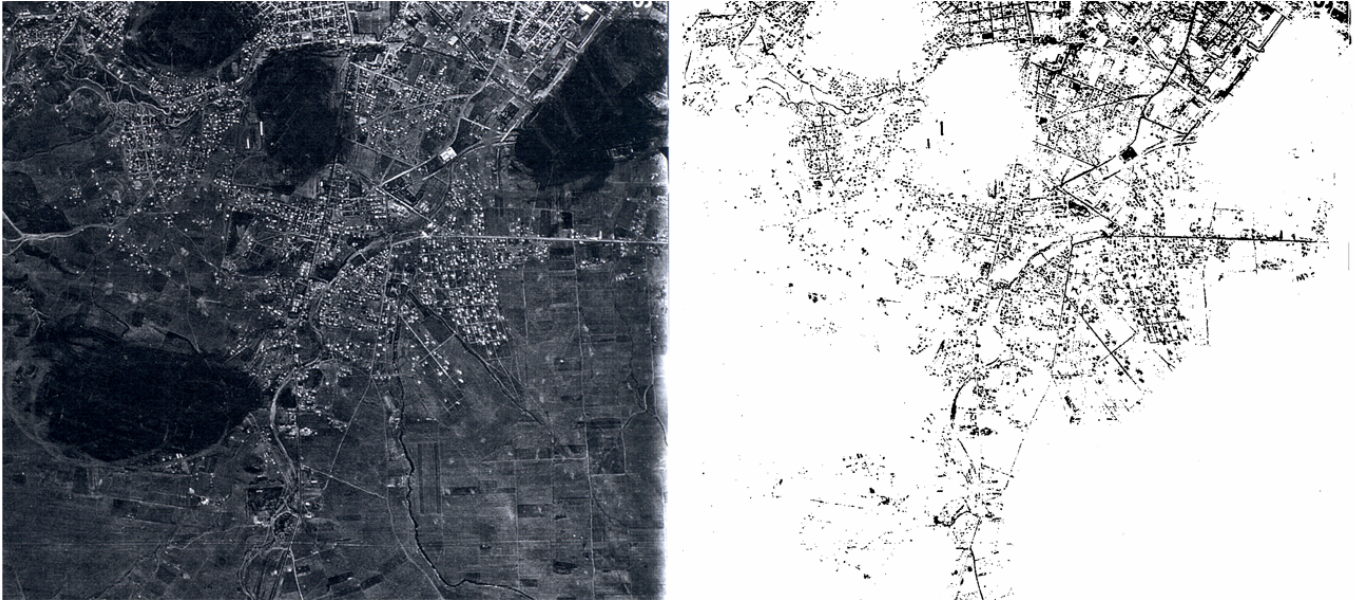


Figure 3: Aerial photo and the extracted built-up area in 1945

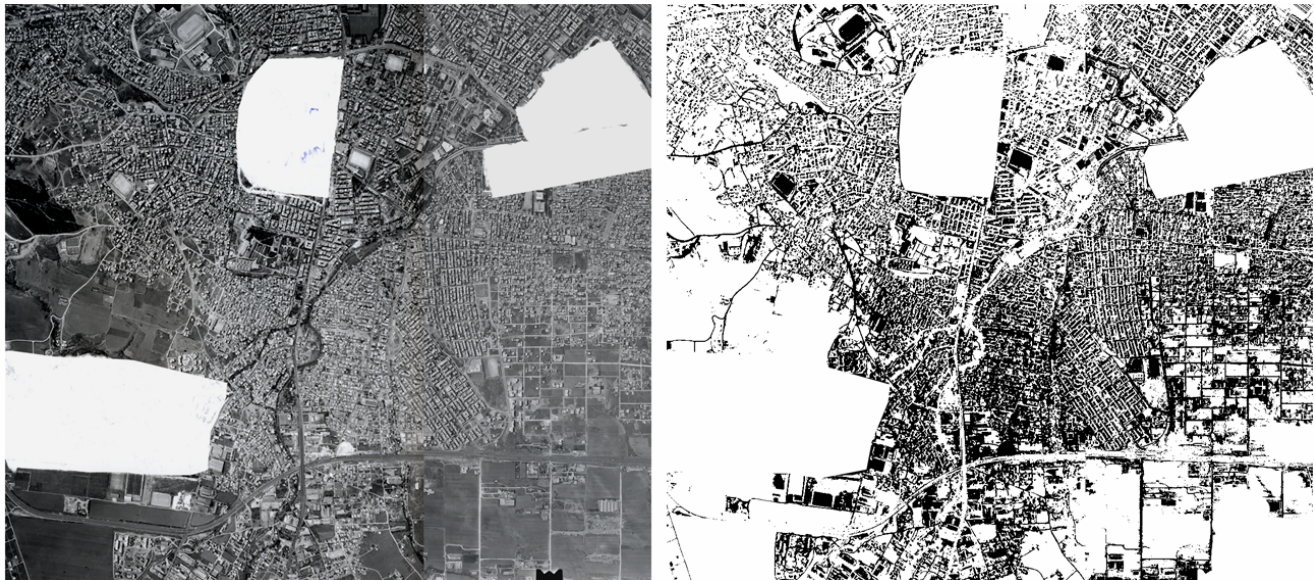


Figure 4: Aerial photo and the extracted built-up area in 1990

The box-counting method was first applied at the map of 1945 using a sequence of grid sizes 1-2-4-8-16-32-64-128-256-512. The corresponding scatter diagrams and the logarithmic transformation are presented at figure 5. A liner regression was performed<sup>6</sup> and the fractal dimension was estimated equal to  $D=1.462$  with a constant  $a=11.624$ . Therefore the law of distribution of the developed urban area across scales is given by the function:

$$\text{Log}N = 11.624 - 1.462\text{log}l \quad (3)$$

<sup>6</sup> The values of  $N$  for grid size over 64 were excluded from the estimation of  $D$ . This is a common technique (Batty 2005) for at the grid sizes over a certain limit we obtain a rather poor approximation of the object

It must be noted that the reliability of the calculated D value is examined by the correlation coefficient  $r$ , which in the present case is 0,997 giving an extremely good fit of the line. We can therefore state that the urban pattern under investigation clearly displays a fractal structure, with a rather small value.

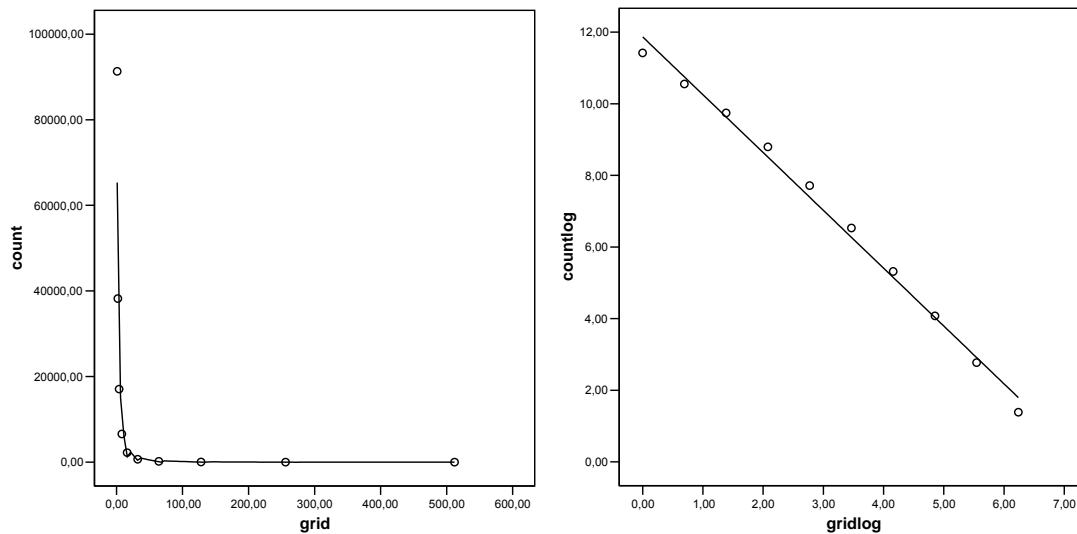


Figure 5: Scatter diagrams of  $I$  against  $N$  in 1945 and the corresponding plot of their logarithms

In order to give a visual impression of the model, a series of images displaying the form that the structure takes at the different scales of examination is presented (figure 6). What can easily be concluded is that the form of developed area displays great complexity and the amount of details increases at the finer scales of examination, exactly like the basic fractal patterns.

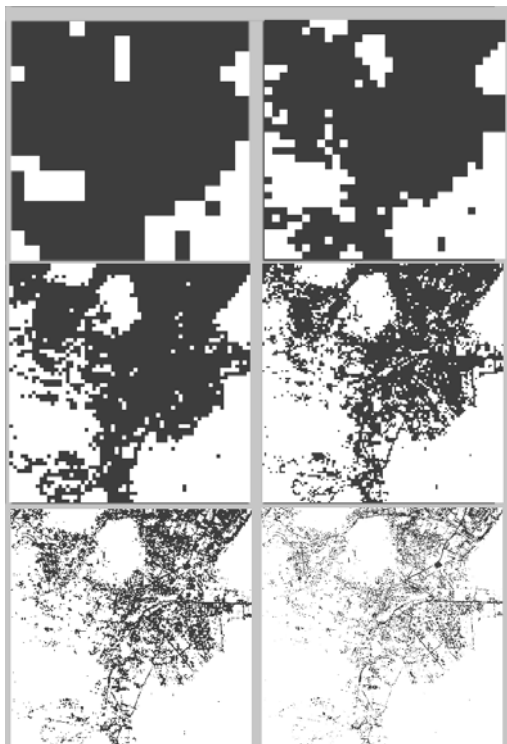


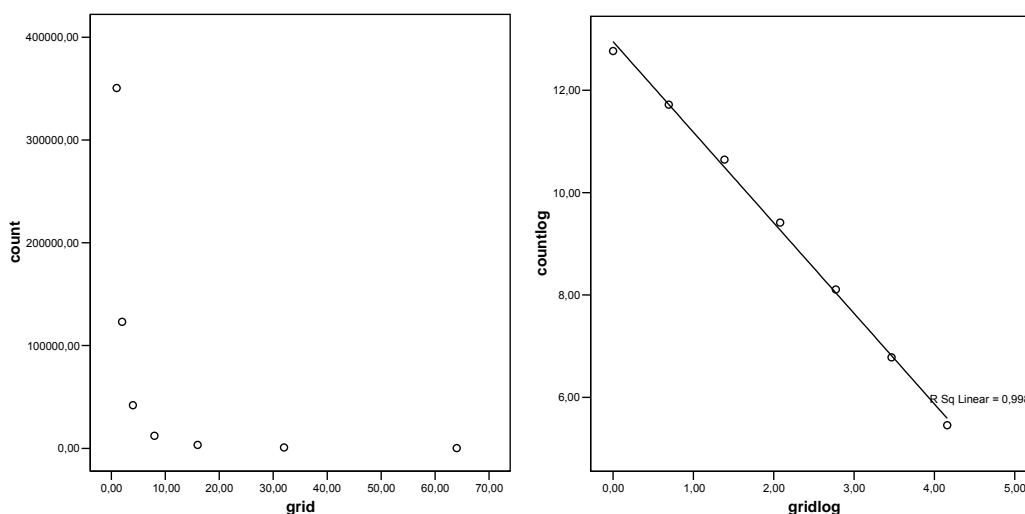
Figure 6:  
The form of the  
urban area at  
different scales.

A further examination of the structure was carried out through the extraction of the urban boundary and the calculation of its fractal dimension. The map was dilated incrementally and the outline was extracted, as presented in figure 7. The inner boundaries created by the significant empty areas between the settlements were at first included in the boundary and then removed before the estimation. The fractal dimension was estimated and  $D_b = 1.413$  with a correlation coefficient equal to 0.989.



*Figure 7:  
The urban boundary in 1945*

The same calculations were applied to the map of 1990 giving a fractal dimension equal to 1.77 and the constant factor  $a=12.951$ . The law of the distribution across scales is  $\text{Log}N=12.951 - 1.77\text{log}l$ . The greater value of  $D_a$  implies an important change of the form of the urban area, which will be discussed later. The same technique was applied for the extraction of the boundary of 1990 and  $D_a$  gave a lower value equal to 1.247 with a correlation coefficient  $R = 0.981$ . Therefore, the change of the dimension of the urbanized surface is followed by a change of the dimension of the boundary.



*Figure 8: Scatter diagrams of  $l$  against  $N$  in 1990 and the corresponding plot of their logarithms*



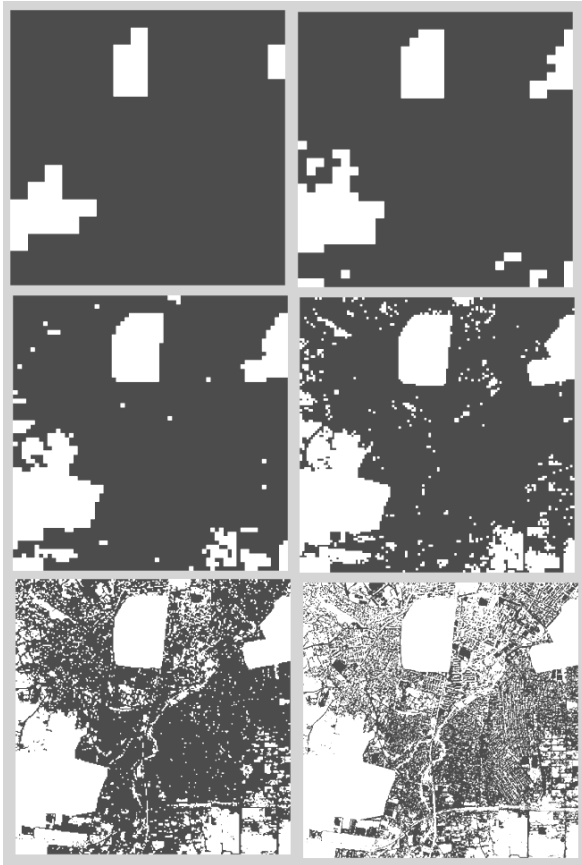
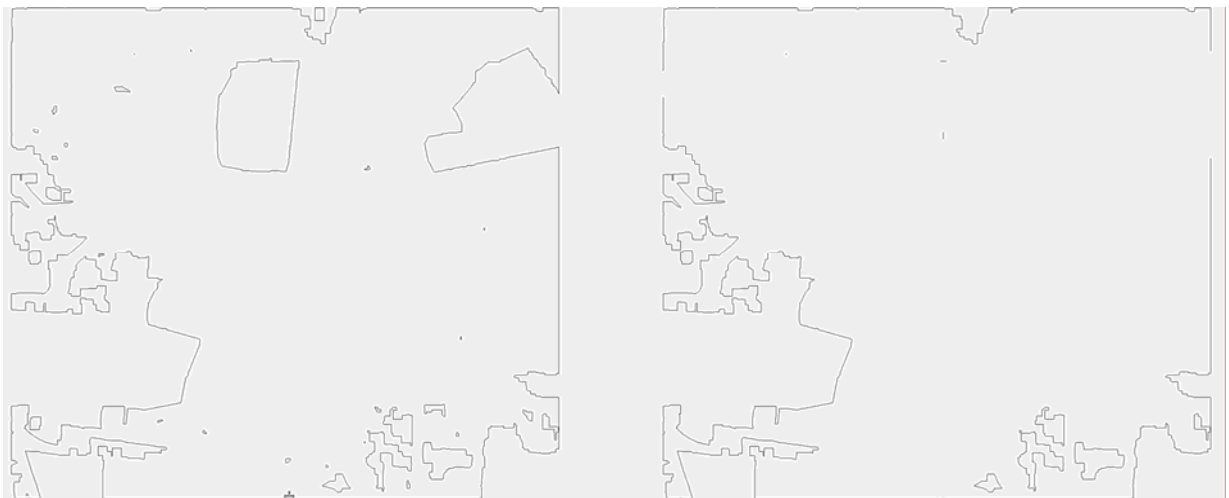


Figure 8: The form of the urban area in 1990 at different scales.

Figure 9: The urban boundary in 1945



In a discussion of the results of the models it must first be stated that the urban patterns examined display a clear relationship with the fractal structures. The application of the fractal models at the case of Thessaloniki gave thus results that conform well to existing theory. The scatter diagrams show a strong linear association between  $\text{Log}N$  and  $\text{Log}l$  and therefore we can consider both the structure of the built-up area and of the boundary, fractal. The change of  $D_a$  and  $D_b$  follows an inverse logic and the increase of  $D_a$  through time is consistent with the results of various applications all over the world. (Benguigui, Chamanski, Marinov 2000, Shen 2002). Finally the range of the values of  $D$  lies between  $1,4 < D_a < 1,8$  και  $1,1 < D_b < 1,5$ , as expected from various case studies all over the world.

## Towards explanation: Fractal analysis and the transformation of the area under investigation

The focus of the final part is in the interpretation of the results and the explanation of what has happened in the area under investigation. The sector of the city analyzed here is part of the west suburban area. A rapid urbanization process took place during the middle of the century mainly due to the proximity of the industrial zone, the harbor, and the main transportation axes of the city. The largest part of the area has been built illegally through a process of land colonization by rural migrants settling at the outskirts of the city (Tsoulouvis 1985). There was always a belated expansion of the city boundary, and large areas where urban growth had already taken place, were gradually incorporated in the city plan (Triantafillidis 1968).

Taking into account the results of fractal analysis, we can conclude that the distribution of the built-up area was highly heterogeneous and fragmented especially in the year 1945. During the period 1945 -1990 significant changes led to a remarkable increase of density on every scale leading into an increase in the value of  $D_a$  from 1.462 to 1.77. What can further be stated is that the fragmentation and heterogeneity of the urban development were reduced, while the scattered settlements of 1945 coalesced into larger clusters. The form of the urban boundary confirms the above conclusion as its fractal dimension reduced from 1.413 to 1.247. The urban boundary has been extended to areas that in 1945 were empty especially in the northern part of the area around the intersection of Lagada Street and the Ring Road. We can mark two basic processes of urban growth:

- The further sprawl of the built-up area at the areas that surround the development of 1945.
- The filling of many empty spaces inside the urban fabric, leading to a nearly saturation of the more central parts of the area, and a considerable increase in density.

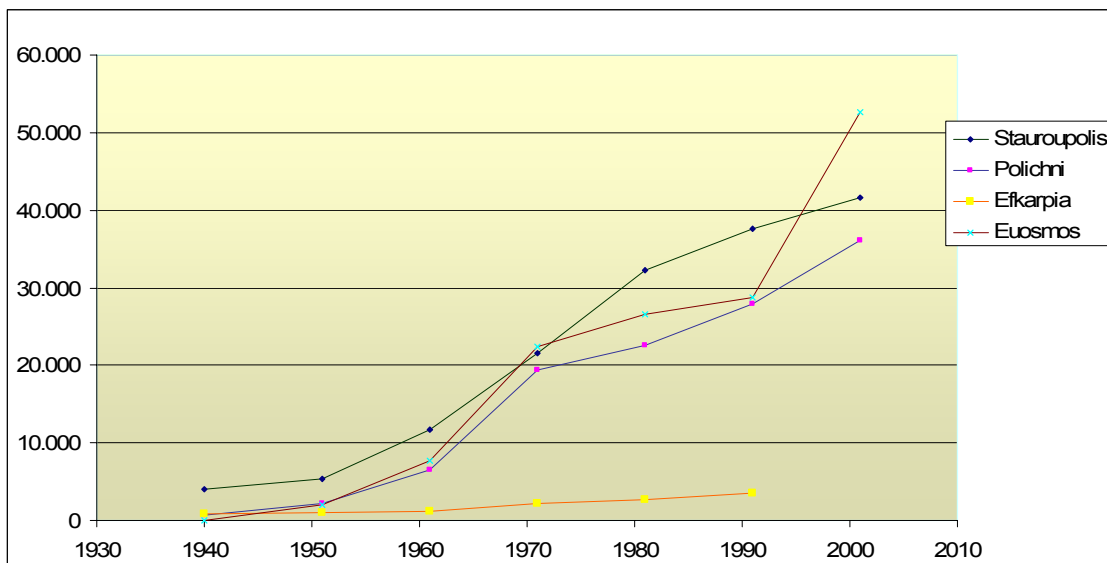


Figure 10:  
Population in thousands for the municipalities of the area from 1940 to 2001

The examination of the population data between 1940 and 2000, accords with the above conclusion. As we can see in figure 10 the population of the municipalities of the area has risen rapidly from 5.500 to 97.000 in 1990 and still continues to grow according to the census data of ESYE (National Statistical Service of Greece) for the year 2001. Important has also been the rise in the number of buildings which is presented in figure 11. The area was transformed from a spontaneous low-density outskirts into a more central, denser sector of the city. The fractal analysis provides important insights on how this urbanization process took place in a spatial sense and quantifies the distribution of the new development.

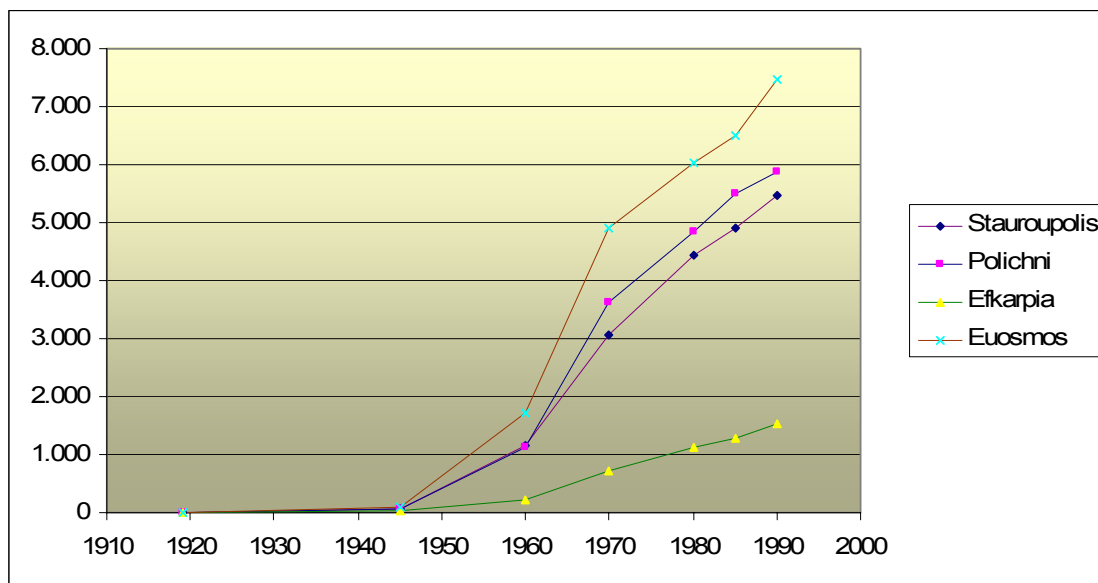


Figure 11: Number of buildings for the municipalities of the area from 1919 to 1990

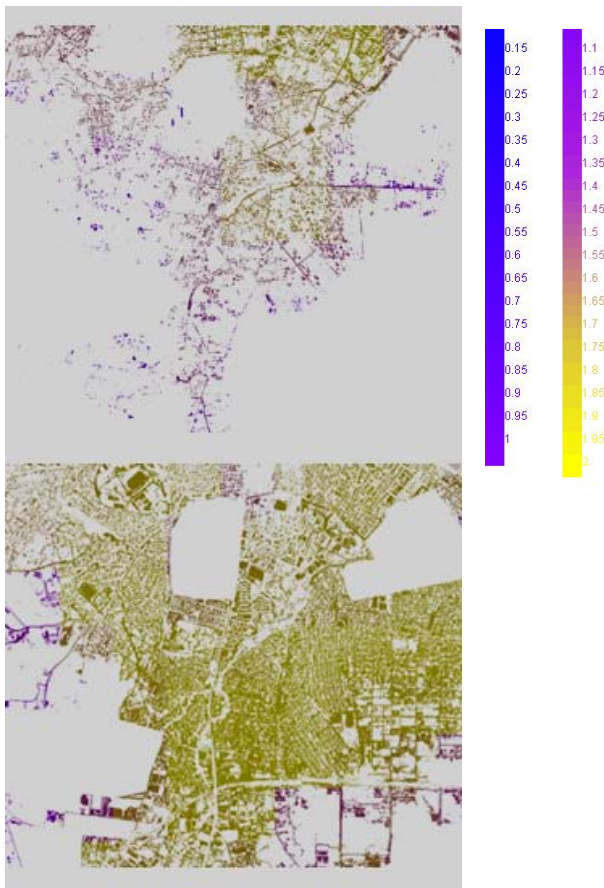
Another issue, which has not been analyzed by the models presented here, was the significant change in the heights of the buildings during the period of 45 years. The small, one or two floors height houses, where replaced by buildings up to 4 and 5 stories height, especially in the more central parts of Stauroupolis and Polichni. This change explains the enormous shift in the residential density of the whole area.

The rates of population growth decreased during the last decades. However, the growth of the built-up area continued leading to a considerable rise in density and the almost complete fill-in of every available empty land. In the same time the urban sprawl led to a further expansion of the developed area beyond the inner ring road and the growth of settlements in the peripheral zone of Thessaloniki. Therefore the conclusion that the urbanized area became denser and more homogeneous holds only for the area that was analyzed. If we consider a greater area towards the northwest we recognize an increase in the fragmentation and the leap-frog discontinuous urban sprawl (figure 12).



*Figure 12:*  
Recent photo of the  
area under  
investigation and the  
zones that surround  
it.

There is another way we can extend our analysis and relate the fractal dimension with other, more general data concerning urban growth. As we have already noted, the fractal dimension varies in space, and different parts of an urbanized area are characterized by different dimensions. We can therefore classify the different parts of the area by grouping them according to their box-counting dimension. It is obvious that the area under investigation contains sub-areas of different type and it is important to make a distinction between the more homogeneous, central parts and the fragmented development near the urban boundary. The box-counting method was applied again calculating different dimensions for the sub-areas, both at the map of 1945 and 1990. The results are displayed at figure 13 where the range of values corresponds to a colour range.



*Figure 13:*  
Estimation of fractal dimensions  
at different parts of the area.  
The results are displayed by an  
appropriate colour range from  
blue (lower values) to yellow  
(greater values).

As the maps in figure 13 illustrate, the differentiation of the values of  $D$  is more important in the year 1945 where the parts closer to the center of the city (upper part of the map) are more dense and homogeneous, while the areas with proximity to the external boundary (lower part) are extremely fragmented and heterogeneous. On the contrary, in 1990 the different sub-areas are more similar and the homogeneity of the whole built-up area is expressed by a fractal dimension which varies less across space. The different character of those areas can also be explored through census data concerning population density, building heights, ratio of incorporated - unincorporated areas and permitted plot ratios. As a cause to the non-availability of spatially disaggregated material referring to the past of Thessaloniki (Tsoulouvis 1985), we will refer to data of the year 1979, the date when a systematic research was carried out by a working group of TEE<sup>7</sup> (Τσουλουβής 1981).

According to the data, a distinction between the different parts of the area under investigation can still be observed in 1979. First of all, the more distant areas at the edge of the boundary remain completely unincorporated, even though they present considerable residential development. It is important though that some more central sectors of Stauroupoli and Polichni are partly outside the city plan. There is a considerable higher density (number of residences per acre) in the central parts of the municipalities with a maximum of 178.26, while near the boundary the value varies between 16 and 50 people per acre. Data are also provided for the heights of buildings which vary from 1.7 to 2.4 floors in the central parts and from 1.1 to 1.7 near the boundary. It must be noted that the heights of the buildings have been further increased during the last two decades reaching the 4 floors in certain areas.

Finally the saturation degree of the area is calculated by dividing the constructed by the permitted plot ratios. These data are provided only for the incorporated parts of the area and range from 0.4 to 0.85. We can therefore understand that the urbanization process is still on the way, and will continue until complete saturation is reached. All the areas between the ring road and Lagada Street and around Efkarpia were gradually incorporated in the city plan, and the high plot ratios that were established launched a wave of new development.

Fractal analysis therefore accords well with various more general data concerning the urban spatial structure, and provides insights on how the form of the urbanization is related to its function and transformation. The case study presented here is however preliminary and highly suggestive, and its main objective was to present some ways of applying the fractal models to the analysis of urban areas at the outskirts of cities. Many extensions are possible: The need for the presentation of more time periods and the comparative analysis of many outskirt areas is obvious, while the application of radial analysis and the focusing on the variation of fractal dimension across space and scale and through time constitutes a major path of further investigation.

---

<sup>7</sup> Technical Chambers of Greece



## ACKNOWLEDGEMENTS

I wish to thank my PH.D. Supervisor, Tsoulouvis Eleftherios professor of the Aristotle University of Thessaloniki, for the interest he has shown in my work. The discussion and the ideas presented here belong to the author and I apparently accept the responsibility for any deficiencies.

## REFERENCES

- Alex Anas, Richard Arnott, Kenneth A.Small (1998) Urban Spatial Structure, *Journal of Economic Literature*, pp.1426-1464
- Batty M., Longley P. (1994) *Fractal cities, a Geometry of Form and Function*, Academic Press
- Batty M. (2001) 'Polynucleated urban Landscapes', *Urban Studies*, Vol.38 No.4 / 635-655
- Batty M. (2005) *Cities and complexity: Understanding cities with Cellular Automata, Agent-Based models, and Fractals*
- Benguigui B., Chamanski D., Marinov M. (2000) 'When and where is a city fractal?' *Environment and planning B: Planning and design*, Vol.27 / 507-519
- Benguigui B., Chamanski D., Marinov M. (2001) 'The dynamics of urban morphology: The case of Petah Tikvah', *Environment and planning B: Planning and design* vol.28, / 447-460
- Frankhauser P. (2004) 'Comparing the morphology of urban patterns in Europe – a fractal approach' *European Cities, Insights on outskirts*, Report Cost action 10 Urban Civil Engineering, Vol.2 / 79-105
- Frankhauser P. (1998) 'The Fractal Approach, a new tool for the spatial analysis of urban agglomerations', *Institut National d' Etudes Démographiques*
- Garreau, J. (1991) *Edge city: Life on the New Frontier*, Doubleday, New York
- Λαγαρίας Απ. (2005) 'Οι πόλεις είναι fractals; Εφαρμογές της fractal γεωμετρίας στη μελέτη του αστικού χώρου', 1<sup>ο</sup> Πανελλήνιο συνέδριο 'Πολοδομία, Χωροταξία & Περιφερειακή Ανάπτυξη, σύγχρονες τάσεις – νέοι ερευνητές', Βόλος, Μάιος 2005
- Laurence de Keersmaecker M., Frankhauser P., Thomas I. (2003) 'Using Fractal Dimensions for Characterizing Intra-Urban Diversity : The Example of Brussels', *Geographical analysis*, Vol.35, No.4 / 310-328
- Shen G. (2002) ' Fractal dimension and fractal growth of urbanized areas' *Int. j. geographical information science*, 2002 vol.16, no.5 / 419-437
- Sobreira Fabiano, Filho Mauro (2002) Assessing Texture Pattern in slum across scales, An unsupervised approach, CASA working paper 55, [http://www.casa.ucl.ac.uk/working\\_papers](http://www.casa.ucl.ac.uk/working_papers)
- Soja E.(2000) *Postmetropolis, Critical studies of cities and regions*, Blackwell Publishing Ltd, USA
- Tsoulouvis L. (1985) *Perceptions of urban development and planning policies in Thessaloniki (Greece)*, PHD Thesis, LSE, London
- Τσουλουβής Λ. κ.α. (1981) *Μελέτη Οικιστικής Ανάπτυξης Θεσσαλονίκης*, Τεχνικό Επιμελητήριο Ελλάδος – Τμήμα Κεντρικής Μακεδονίας
- Triantafillidis I.D. (1968) Master Plan of Thessaloniki, General Report 66, Thessaloniki

