

Railways Competition in a Park-and-Ride Model*

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Abstract

Scale effect of city size and cost advantage of railway over automobiles are examined in a simple park-and-ride commuter system. The main results are:

- The unit fare charged by a monopoly railway firm is irrelevant to the city size.
- The unit fare in a symmetric equilibrium under monopolistic competition is decreasing in the city size.
- The unit fare in a symmetric zero-profit kinked equilibrium is increasing in the city size.
- The unit fare in the social optimum is decreasing in the city size.
- The operation constraints are relaxed in a larger city.

1 Introduction

From the view point of global environmental issues (e.g., reduction of CO₂, NO_x or so), it is urgent to save energy in transport sector as well. There have been several ways proposed for saving energy in urban transportation, such as congestion tax on automobile, utilization of light rail in down towns, introduction of ITS technology for improving efficiency of transport energy, and development of battery cars. One of those classic proposals is so-called "park-and-ride" system that we use automobiles from houses to railway station nearby and transfer to railways to CBD, that is, working places. By this way, the private use of automobiles should be reduced so that congestion as well as negative externality for environment in and around downtown would be improved much.

In fact, they sometimes try to facilitate parking lots with reasonable charge around railway stations in suburban areas in several countries. However, it seems that the system is employed in very limited number of stations

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and there should be much room we could introduce the system for the purpose above.

In the literature of urban economics, there is a stream on related issues of railway competition (see Kanemoto 1984 among others), yet it lacked for the environmental view point. While transportation engineers argued the importance of such system, policy implication of the system has not been derived enough from economic point of view.

In this paper, we will develop a simple model of park-and-ride system in urban setting. That is, we suppose a monocentric city where several railways start from a single CBD to suburb area spoke-wise, and households use automobiles first from each own house to nearest railway station circumferentially where they transfer to railways bound for CBD, or they may choose to ride private cars directly through to CBD. Hence, there are competitions between railways as well as between railway and automobiles in this model. Since direct commuting by automobiles works as "outside good" in monopolistic competition among railways those are differentiated in location or space, we follow the model by Salop (1979) that analyses the work of outside good in horizontally differentiated market and obtains perverse characteristics due to the introduction of kinked demand curve.

In this paper, moreover, we focus in scale effects of city size and cost advantage of railway over automobiles. Our main results are as follows: The fare per mile charged by a monopoly railway firm may not be relevant to the city size; the fare in a symmetric equilibrium under monopolistic competition is decreasing with the city size; the fare in a symmetric zero-profit kinked equilibrium is increasing with the city size; the fare of social optimum is decreasing with the city size.

This paper is organized as follows. Section 2 describes the basic model and presents the results with market competition. Social optimum is examined in Section 3 followed by Concluding Remarks in Section 4.

2 Model

2.1 Basic Structure

Spatial configuration of the city

The radius of the city considered is denoted by m . The central business district, so-called CBD, is assumed as a point for simplicity. The distance from the CBD is denoted by x . Each household lives in a unit interval circumferentially. The number of households on the concentric circle at x is $2\pi x$.

Total population or the number of households (let us assume each household consists of one person for simplicity) in the city is

$$N = 2\pi\rho \int_0^m x dx = \pi\rho m^2 \quad (1)$$

where ρ denotes population density as a constant. That is, we assume each household consumes same space of land here.

Each household commutes to an office at CBD everyday, when she chooses to use solely an automobile or the combination of automobile and railway (i.e., park-and-ride system) as a commute vehicle.

Automobile

It is assumed that roads radiate from CBD in all directions. In other words, each household could drive to her office radially from any residence in the city. This seems so simplified, yet it is for contrasting the characteristics of automobile and railway. The cost function of automobile from a residential lot at x is given as

$$C(x) = cx^\alpha \quad (2)$$

where $\alpha > 0$ stands for a distance elasticity of the cost of driving automobiles. If $\alpha > 1$, marginal cost of automobile increases as one drives for a longer distance. It is either because a higher heat makes the engine less efficient or because a longer driving makes the driver less comfortable.

One-way transportation cost facing to a household who lives at x and uses solely an automobile is cx^α . If all households in the city use solely automobiles, the total transportation costs are $2\pi c \int_0^m x^\alpha dx = 2\pi \frac{c}{\alpha+1} m^{\alpha+1}$.

Railway

A railway radiates from CBD in a direction, if exists. The marginal cost is constant and normalized to zero. A fixed cost $F > 0$ is required for laying one unit of railway.

The railway fare for passing through x is denoted by $p(x)$. The cumulative railway fare from CBD to a station at x is given by $P_x \equiv \int_0^x p(x) dx$.

If there is a station at x from CBD, households residing at x in any directions from CBD can use the park-and-ride system.

A household who lives at x from CBD and apart from the station by y drives circumferentially from her lot to the station, and switches to the railway. The park-and-ride transportation cost of the agent is $C(y) + P_x$. She will use the park-and-ride system if and only if

$$C(y) + P_x \leq C(x). \quad (3)$$

2.2 Monopoly

Suppose that n railway firms operate in the city yet each market is not overlapped. Let us call this phase as monopoly. In this subsection, we focus on this situation while each railway firm in the city lays railroads to z ($1 \leq z \leq m$).

Proposition 1 For any x within z , (i) the ratio of the railway fare to the automobile cost is $\alpha/(1 + \alpha)$, and (ii) the share of the park-and-ride system is $1/\pi(1 + \alpha)^{\frac{1}{\alpha}}$.

Proof. A marginal consumer at x , y_x , is characterized by

$$cy_x^\alpha + P_x = cx^\alpha.$$

The demand for railway at x is given by

$$d_x^M = 2y_x = 2 \frac{\mu}{x^\alpha - \frac{P_x}{c}}^{\frac{1}{\alpha}}. \quad (4)$$

Maximization problem for the monopoly is formalized as

$$\Pi^M = \max_{P_x} \int_0^z P_x d_x^M - zF, \quad (5)$$

subject to (4).

The optimality condition for P_x is

$$d_x^M + P_x \frac{\partial d_x^M}{\partial P_x} = 0,$$

which gives the optimal fare

$$P_x^M = \frac{\alpha}{1 + \alpha} cx^\alpha. \quad (6)$$

$$p(x) = \frac{\alpha^2}{1 + \alpha} cx^{\alpha-1} \quad (7)$$

Substituting (6) into (4), the demand for railway at x is given by

$$d_x^M = \frac{2}{(1 + \alpha)^{\frac{1}{\alpha}}} x. \quad (8)$$

The share of park-and-ride system of each firm is

$$d_x^M / 2\pi x = 1/\pi(1 + \alpha)^{\frac{1}{\alpha}}. \quad (9)$$

■ Equation (6) states that the fare charged by a monopoly railway firm is a fraction $\alpha/(1 + \alpha)$ of the transportation cost of automobiles, cx^α . The greater α , the greater the cost advantage of monopolist, and the higher the fare is. Equation (9) states that the share of park-and-ride system is constant for any x , and decreasing in α . Note that it is implicitly assumed here that the maximized profit of a firm is nonnegative, or we assume sufficiently small F to guarantee for it.

Lemma 2 Under monopoly, the unit fare is constant if and only if the transportation cost of automobiles is linear, i.e., $\alpha = 1$. The unit fare is given by $p = c/2$.

Proof. The lemma follows immediately from equation (7). ■

Lemma 3 If a monopoly firm lays railways to z and gains positive profits, then it has an incentive to extend the railway to $z + \Delta z$.

Proof. Substituting (6) and (8) into (5), the monopoly profit is

$$\Pi^M(z) = z \frac{1}{2\alpha(1+\alpha)^{-\frac{1+\alpha}{\alpha}}} cG(z) - F,$$

where $G(\cdot)$ stands for a measure of city size;

$$G(m) \equiv \frac{1}{m} \int_0^m x^{\alpha+1} dx.$$

The firm operates if and only if $\Pi^M(z) \geq 0$, i.e.,

$$F \leq 2\alpha(1+\alpha)^{-\frac{1+\alpha}{\alpha}} cG(z). \quad (10)$$

The net return from extending the line to the next circle is

$$\begin{aligned} \Pi^M(z + \Delta z) - \Pi^M(z) &= 2\alpha(1+\alpha)^{-\frac{1+\alpha}{\alpha}} c(z + \Delta z)^{\alpha+1} - F \\ &\geq 2\alpha(1+\alpha)^{-\frac{1+\alpha}{\alpha}} c \left[(z + \Delta z)^{\alpha+1} - G(z) \right]. \end{aligned}$$

The inequality comes from equation (10). It follows that $(z + \Delta z)^{\alpha+1} > G(z)$ because $x^{\alpha+1}$ is convex. Therefore, $\Pi^M(z + \Delta z) > \Pi^M(z)$. ■

Lemma 2 says that a monopoly railway firm will extend the line to the edge of the city, if it is profitable. The next lemma follows immediately.

Lemma 4 Railway firms operates in the city if

$$F \leq 2\alpha(1+\alpha)^{-\frac{1+\alpha}{\alpha}} cG(m) \equiv \bar{F}^M. \quad (11)$$

The maximum number of profitable monopolists is $n^M = \pi(1+\alpha)^{\frac{1}{\alpha}}$ if we do not care about integer characteristic of number of firms.

2.3 Competition among railway firms

Suppose that n railway firms operate in the city and that each market is overlapped. In short run, the number of firms is supposed to be fixed. Let us call this phase monopolistic competition.

Proposition 5 In a symmetric equilibrium under monopolistic competition among railways, (i) the ratio of the railway fare to the automobile cost is $2\alpha(\pi/n)^\alpha$. (ii) If $\alpha = 1$, then the unit fare is constant at $2\pi c/n$.

Proof. Suppose that the neighboring firms charge \bar{P}_x . A marginal consumer at x, y_x , is characterized by

$$P_x + cy_x^\alpha = \bar{P}_x + c \frac{2\pi x}{n} - y_x, \quad (11)$$

which gives demand per firm, $d_x^c = 2y_x(P_x)$.

Note that

$$\frac{\partial d_x^c}{\partial P_x} = -\frac{2}{\alpha c y_x^{\alpha-1} + \frac{2\pi x}{n} - y_x}.$$

Maximization problem for a monopolistically competitive firm is formalized as

$$\Pi^c = \max_{P_x} \int_0^m P_x d_x^c - mF. \quad (12)$$

The optimality condition is $d_x^c + P_x \frac{\partial d_x^c}{\partial P_x} = 0$. From symmetry, equilibrium condition is $P_x = \bar{P}_x \equiv P_x^c$. Thus, the optimal fare is as follows:

$$P_x^c = 2\alpha \frac{\pi}{n} \cdot cx^\alpha.$$

If $\alpha = 1$, then the unit fare is given by $p_x^c = 2\pi c/n$. ■

Profit per firm is

$$\Pi^c = m \left[4\alpha c \frac{\pi}{n} \cdot G(m) - F \right].$$

In long run, the number of railway firms is determined by the free entry condition, $\Pi^c = 0$.

In equilibrium,

$$n^c = \pi \frac{4\alpha c}{F} G(m)^{\frac{1}{1+\alpha}}, \quad (13)$$

$$P_x^c = (2\alpha)^{\frac{1}{1+\alpha}} \frac{F}{2cG(m)^{\frac{\alpha}{1+\alpha}}} cx^\alpha. \quad (14)$$

The necessary condition that each market is overlapped is

$$P_x^c + c \frac{x\pi}{n^c} \leq cx^\alpha.$$

Substituting (13) and (14), the operation constraint for monopolistic competition is

$$F \leq 4\alpha(1+2\alpha)^{-\frac{1+\alpha}{\alpha}} cG(m) \equiv \bar{F}^c. \quad (15)$$

Lemma 6 $\bar{F}^c < \bar{F}^M$ for any $\alpha > 0$.

Proof. From (11) and (15), the condition $\bar{F}^c < \bar{F}^M$ is equivalent to

$$(1 + 2\alpha)^{\frac{1+\alpha}{\alpha}} > 2(1 + \alpha)^{\frac{1+\alpha}{\alpha}},$$

or

$$1 + \frac{\alpha}{1 + \alpha} > 2^{\frac{1+\alpha}{\alpha}}.$$

Define the difference function by $f(t) = 1 + t - 2^t$, where $t \equiv \alpha/(1 + \alpha) \in (0, 1)$ because $\alpha > 0$. Observe that f is continuous and differentiable, and that $f(0) = f(1) = 0$, and $f'' < 0$. Therefore $f(t) > 0$ for any $t \in (0, 1)$. ■

2.4 Competition between railway firms and automobiles

Salop (1979) examines a zero-profit kinked equilibrium in which the comparative statics results are perverse. Let us examine this phase in this subsection.

Proposition 7 In a zero-profit kinked equilibrium, (i) the ratio of the railway fare to the automobile cost is $1 - (\pi/n)^\alpha$. (ii) If $\alpha = 1$, then the unit fare is constant at $c(1 - \pi/n)$.

Proof. There exists a critical fare, \hat{P}_x , such that

$$\begin{aligned} P_x + c \frac{d_x^M}{2} &= cx^\alpha \quad \text{if } \hat{P}_x \leq P_x \leq cx^\alpha, \\ P_x + c \frac{d_x^c}{2} &= \bar{P}_x + c \frac{2\pi x}{n} - \frac{d_x^c}{2} \quad \text{if } P_x \leq \hat{P}_x. \end{aligned}$$

The critical fare is derived by setting $d_x^M = d_x^c = 2\pi x/n$.

$$\hat{P}_x = \frac{cx^\alpha}{1 - \frac{\pi}{n}}, \quad (16)$$

$$\hat{d}_x = \frac{2\pi x}{n}. \quad (17)$$

If $\alpha = 1$, then the unit fare is

$$\hat{p}_x = c\left(1 - \frac{\pi}{n}\right). \quad (18)$$

■

Profit per firm:

$$\begin{aligned} \Pi^k &= \int_m^Z \hat{P}_x \hat{d}_x - mF \\ &= m \frac{\pi}{n} \frac{1}{1 - \frac{\pi}{n}} \left[2cG(m) - F \right]. \end{aligned}$$

The zero profit condition requires that

$$\frac{\pi}{n} \left(1 - \frac{\pi}{n} \right)^{\frac{1}{\alpha}} = \frac{F}{2cG(m)}. \quad (19)$$

Denote the LHS of (19) by a function $f(t) = t(1 - t^\alpha)$, where $t \equiv \pi/n \in (0, 1)$. The maximum is given by $f((1 + \alpha)^{-\frac{1}{\alpha}}) = \alpha(1 + \alpha)^{-\frac{1+\alpha}{\alpha}}$. The necessary condition for the existence of the kinked equilibrium is that $\alpha(1 + \alpha)^{-\frac{1+\alpha}{\alpha}} \geq F/2cG(m)$, i.e.,

$$F \leq 2\alpha(1 + \alpha)^{-\frac{1+\alpha}{\alpha}} cG(m) = \bar{F}_M. \quad (20)$$

If equation (20) is satisfied, the number of railway firms is given by equation (19), and the fare and market share are given by (16) and (17).¹

A comparative statics shows that the number of railway firms is decreasing in F , and increasing in m . Thus, the fare in the symmetric zero-profit kinked equilibrium is increasing in the city size. This seems perverse, yet intuitive explanation is as follows.

- As the city size becomes greater, more railway firms enter the market.
- The market share per firm shrinks.
- The competitor of railway firms is not the neighboring firms but automobiles.
- As the market shrinks, a marginal consumer comes closer to a railway firm.
- Each railway firm can charge a higher fare.

2.5 Social optimum

A planner chooses the number of railways, n , and fares, F_x , $x \in (0, m)$, to minimize the total transportation costs under the balanced budget and the participation constraints.

Is it optimal to allow some households to use only automobiles? In our model the answer is no because of the assumed cost structure. The following lemma will be useful.

Lemma 8 At the optimum, all households use the park-and-ride system.

¹Equation (19) has two solutions. But the relevant solution is the larger one. To see this suppose that the fixed cost F goes to zero. By equation (19), π/n must be zero or one. But the economically relevant solution is zero, which corresponds to a sufficiently large n .

Proof. Assume that each market is segmented. As a monopoly case, a marginal consumer is given by $\bar{y} = (x^\alpha - P_x/c)^{\frac{1}{\alpha}}$.

The sum of transportation cost for park-and-ride users is

$$C_{PR} = 2n \int_0^m \int_0^{\bar{x}} (P_x + cy^\alpha) dy dx \quad (21)$$

The sum of transportation cost for automobile users is

$$C_A = \int_0^m 2n \left(\frac{\pi x}{n} - \bar{y} \right) cx^\alpha dx. \quad (22)$$

The budget constraint for the park-and-ride system is

$$2n \int_0^m P_x \bar{y} dx = mF. \quad (23)$$

Substituting (23) into (21), the total transportation cost is given by

$$C_{PR} + C_A = m [F + 2\pi cG(m)] - \frac{2n}{1+\alpha} \int_0^m (P_x + \alpha cx^\alpha) \bar{y} dx.$$

As long as each market is segmented, i.e., $\bar{y} < \pi x/n$, the planner can reduce the sum of transportation costs by increasing n . Therefore, at the optimum, each market must be overlapped, i.e., all households use the park-and-ride system. ■

Proposition 9 At the optimum, the number of railway firms and the railway fair are given by

$$\begin{aligned} n^* &= [2(1+\alpha)]^{-\frac{1}{1+\alpha}} n^c, \\ P_x^* &= [2(1+\alpha)]^{-\frac{1}{1+\alpha}} P_x^c. \end{aligned}$$

Proof. The sum of transportation costs is given by

$$\begin{aligned} C &= \int_0^m \int_0^{\bar{A}} 2\pi x P_x + 2n \int_0^{\frac{\pi x}{n}} cy^\alpha dy dx \\ &= \int_0^m \left[2\pi x P_x + \frac{2nc}{1+\alpha} \left(\frac{\pi x}{n} \right)^{\alpha+1} \right] dx. \end{aligned} \quad (24)$$

The budget constraint is

$$\int_0^m 2\pi x P_x dx - mnF = 0, \quad (25)$$

and the participation constraints are

$$c \left(\frac{\pi x}{n} \right)^\alpha + P_x \leq cx^\alpha, \quad (26)$$

for $x \in (0, m)$.

Substituting (25) into (24),

$$C = m \left[nF + \frac{2c}{1+\alpha} \frac{\pi^{1+\alpha}}{n^\alpha} G(m) \right]. \quad (27)$$

Minimizing the total costs (27) with respect to n gives the optimal number of railways:

$$n^* = \pi \frac{2\alpha c G(m)^{\frac{1}{1+\alpha}}}{(1+\alpha)F}. \quad (28)$$

Comparing (28) with (13), the ratio is given by $[2(1+\alpha)]^{-\frac{1}{1+\alpha}}$. Substituting (28) into (25),

$$\int_0^m x P_x dx = \frac{mF}{2} \frac{2\alpha c G(m)^{\frac{1}{1+\alpha}}}{(1+\alpha)F}.$$

Infer that $P_x = Acx^\alpha$. As the LHS is $AcmG(m)$, $A = (\alpha/(1+\alpha))^{\frac{1}{1+\alpha}} (F/(2cG(m)))^{\frac{\alpha}{1+\alpha}}$. Therefore,

$$P_x^* = \frac{\alpha}{1+\alpha} \frac{F}{2cG(m)^{\frac{1}{1+\alpha}}} c x^\alpha. \quad (29)$$

Comparing (29) with (14), the ratio is given by $[2(1+\alpha)]^{-\frac{1}{1+\alpha}}$. ■

Denote the ratio by a function $g = [2(1+\alpha)]^{\frac{1}{1+\alpha}}$. Logarithmic differentiation gives $g'/g = [1 - \ln 2(1+\alpha)]/(1+\alpha)^2$. The ratio is increasing in $\alpha \in (0, e/2 - 1)$, and decreasing in $\alpha \in (e/2 - 1, \infty)$. Further note that $g(0) = g(1) = 2$. To the extent that the marginal cost of automobile is decreasing ($\alpha < 1$), the ratio belongs to $(2, e^{e/2})$. The optimal number of railway firms is at the most a half of the monopolistically competitive one. Besides, to the extent that the marginal cost is increasing ($\alpha > 1$), the ratio belongs to $(1, 2)$ and the optimal number of railway firms is relatively large. Interestingly, within the range of $\alpha \in (0, e/2 - 1)$, the number of railway firms tends to decrease when the cost performance of automobile is worsened.

Substituting (28) and (29) into (26), the operation constraint for the social optimum is

$$F \leq 2\alpha(1+\alpha)^{\frac{1}{\alpha}} (1+2\alpha)^{-\frac{1+\alpha}{\alpha}} cG(m) \equiv \bar{F}^*. \quad (30)$$

Lemma 10 $\bar{F}^* \geq \bar{F}^c$ if $\alpha \leq 1$.

Proof. Comparing (30) with (15),

$$\frac{\bar{F}^*}{\bar{F}^c} = \frac{1}{2}(1+\alpha)^{\frac{1}{\alpha}}.$$

The proof completes because the RHS is a decreasing function of α . ■

A possible commuter system depends on (i) railway cost F , (ii) automobile cost α (and c), and (iii) city size m .

- In a larger city, the operation constraints are relaxed.
- If $0 < \alpha < 1$, then $\bar{F}^c < \bar{F}^* < \bar{F}^m$.
- If $\alpha > 1$, then $\bar{F}^* < \bar{F}^c < \bar{F}^m$.

3 Conclusion

Although the results are interesting, our model is still a prototype of urban transportation network. Thus, there are many ways to extend the results of this paper.

First, though lot size is fixed for simplicity in this paper, endogenizing it will provide much more flavor of spatial economics as well as reality. Second, we do not consider congestion in automobile network. Since the congestion control by tax is important subject recently, it is better to incorporate the congestion-related phenomena into our park-and-ride setting. Third, we had better generalize the form of cost functions of transportation, namely fixed and marginal costs as well. Finally, while we assume a single CBD, say monocenter, in this model, there are several CBDs in reality, e.g., Tokyo, Paris, etc. In such multicentric cities, the effects of park-and-ride might be different from those in monocentric city.

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