A Study of Dynamic Relationship between Housing Values and Interest Rate in the Korean Housing Market

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Seungryul Ma (Daegu University in Korea)

Abstract

The goal of this study is to identify the long-term relationship between housing value and interest rate in the Korean housing market, using the Cointegration Test and Spectral Analysis. The result shows the long-term negative (–) equilibrium relationship between housing values and interest rate. Moreover, the Granger Causality Test for confirming the short-term dynamic relationship between these variables notes the one-way causality from interest rate to the change rate of housing and the transfer function model certifies concretely the causal structure of this relationship. The result of this study suggests that the interest rate adjustment policy in the Korean housing market can work very effectively and it will contribute to forecast the change of future housing values hereafter.

Keywords: Dynamic relationship; Housing value; Interest rate; Cointegration and spectral analysis; Long term equilibrium

1. Introduction

Housing values in the Korean housing market zoomed over from the second half of 2001 to 2002. Many scholars said that the major cause that housing value rises suddenly is due to the unprecedented low interest rate for such period. Therefore, in order to stabilize the housing value, the interest rate should be increased. However some scholars said that the ascent of interest rate implies the recovery of economy and it results in the increase of housing demand. And increased housing demand also can be a cause of rise of housing values. Therefore, this study aims to identify the long-term equilibrium relationship between housing values and interest rate and the causal structure between these two time series data. The results of this study will be contribute to identify that the interest rate policy works effectively in the Korean housing market and to improve the accuracy of forecasting of housing value.

The review of literature is described in the chapter 2. The data and analysis methodology will be noted in the chapter 3. The result of analysis is interpreted and is suggested in the chapter 4. Finally the conclusion and policy implication will be described in the chapter 5.

2. Theoretical Review of relationship between housing values and interest rates

There are several studies on the relationship between interest rate and macroeconomic variables. Bernanke and Blinder (1992) note that the federal fund rate (the interest rate on Federal funds) is very sensitive to the moving of macroeconomic variables and the interest rate is an important explanatory variable (indicator) in monetary policy actions. Stone and Ziemba (1993) argue the fluctuation of interest rate in Japan is the primary cause of the increase of the property value including land value for 1985-1989 years and its decrease for 1990-1992. On the other hand, Quan and Titman (1999) identify that the housing value is closely related with stock values in the long-term and also is influenced in GDP growth rate. Wheaton (1987) confirms the long-term cycle in the change of office building value and he (1999) also notes that its cycle of housing value is very different according to the housing type.

There are a few studies on the dynamic relationship between the housing value and interest rate in the Korean housing market. Ji (1999) identified the change of yield rate of bond preceded that of value of housing in the relationship of economic cycle between bond and housing. Yun (2001) confirmed that the change of housing value interacted with that of corporate bond in both ways through the Granger causality test that the quarter data from a first quarter of 1987 to the second one of 2001 is utilized. Using the Spectral Analysis Method, Ma (2002) noted that the common cycle of three years period is in existence in the time series data of interest rate, stock value, and business cycle in Korea and under these common cycles, they are closely preceded or followed each other. However, these studies focused on the short term period in the analysis of relationship between the housing and corporate bond in spite that its long their change is very important in the housing market. Furthermore, they did not consider the causal structure of the relationship between the change of housing value and that of interest rate.

This study focuses on the dynamic relationship between the value of housing and interest rate in the
long term aspect. It used the twelve years data from 1991, when the liberalization of interest rate and openness of financial market were begun, to 2002. At first, the long term relationship between the time series of housing and that of interest rate will be analyzed by the cointegration test in the time domain and the Spectral Analysis Method in the frequency domain. The spectral analysis method identifies the circulation cycle which is included in the macro economic variables. And it also is very helpful to find the close relationships among the time-series data under the common cycle. This study aims to identify the long-term relationship between the change of housing value and that of interest rate after the identification that the former is similar with the later or the common cycle is in existence. Hereafter this study tries to note the specific causal structure in two time-series through the transfer function test after the identification that the short term causality is in existence using the Granger causality test. The results of this study, which note the long term equilibrium relationship and causal structure between the housing values and interest rate, will contribute to identify the effectiveness of interest rate policy in the Korean housing market. And they also is expected to improve the accuracy in forecasting of housing values.

III Data and Analysis Method

1. Data

In order to identify the long term equilibrium between the housing values and interest rate, this paper uses the housing price index of Korean Citizen Bank as the substitution variable of real-estate values and the yield rate of 3 years maturity corporate bond published by the Korean Bank. The time series of growth rate of housing price (HRATE) is generated by the percent change rate of the housing price index from the same month of the previous year. Each time series data should be standardized due to the difference of measure units. Standardized time series data can be summarized as Table 1.

<table>
<thead>
<tr>
<th>Table 1: Mean value and standard deviation of Time Series Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>CB3</td>
</tr>
<tr>
<td>HRATE</td>
</tr>
</tbody>
</table>

Each variable is defined as follows.

CB3 ≡ the yield rate of 3 years maturity corporate bond
HRATE ≡ the growth rate of housing price
ZCB3 ≡ standardized CB3
ZHRATE ≡ standardized HRATE

![Figure 1: The Trend of Each Time Series Data (1991.01 - 2002.12 (12 years))]
\( \chi_t \sim I(d) \). If we define the relationship of cointegration in the time series analysis aspect, time series of each composition of \((nx1) Vector \chi_t\) are the process \(I(1)\) and \((nx1) Vector \ a\), which is non-zero, is existed. In case that \(Z_t = a'\chi_t\), the linear combination of \(\chi_t\), is stationary \((Z_t \sim I(0))\), it is said that vector \(\chi_t\) is cointegrated and \(a\) is cointegrating vector. In summary, cointegration is the circumstance that the linear combination of each non-stationary variable is stationary. Considering the spurious regression even if the composition series of \(\chi_t\) follows the process \(I(1)\) and the cointegration is existed, in case that \(VAR(p)\) model is deducted, there is possibility to lose an important information on the long-term relationship among variables. Therefore, this paper adopts the vector error correction model (VECM). This grasps the long-term equilibrium between the change of wage series and that of inflation ones through the cointegration test.²

2) Spectral Analysis

When circulation cycle (cycle length or period) of analysis target time series was known well, harmonic analysis can be enforced in order to model the cyclic component of time series. General expression for harmonic analysis is expressed as follows.

\[
\chi_t = \mu + R \cos(\omega t + \phi) + \epsilon_t, \quad \text{Equation (1)}
\]

\(\chi_t\) = observation value of \(\chi\) in time \(t\)
\(\mu\) = mean value of time series, \(R\) = amplitude
\(\phi\) = phase: Distance to the first peak in \(t = 0\)
\(\omega = 2\pi / \tau\) =angular frequency
\(\tau\) = period or cycle length
\(t\) = observed time (\(t = 0, 1, 2, ..., N\))
\(\epsilon_t\) = white noise (mean value: 0, variance: \(\sigma^2\))

In order to make an easy to estimate the parameters, equation (1) can be expressed as follows, including the sine and cosine terms.

\[
\chi_t = \mu + A \cos(\omega t) + B \sin(\omega t) + \epsilon_t, \quad \text{Equation (2)}
\]

In case that the graph cannot explain the regularity of circulation cycle through visual analysis of time series, harmonic analysis cannot be employed directly, in other words, we cannot execute the harmonic analysis if we do not know the circulation cycle in advance. And then we can adopt the periodogram analysis because it is analysis method that is consisted of many set of harmonic analysis and spectral analysis is analysis method that amends the periodogram analysis by the smoothing method. If we use periodogram analysis and spectral analysis, we can identify the approximate values of one or some circulation cycles that is explained by the section of bigger variance in the time series data. After identifying the circulation cycle, we enforce harmonic analysis again using these circulation cycle and can do the modeling of the periodic components. Periodogram model can be expressed by the sum of periodic components of \(N/2\) \((i = 1, 2, 3, ..., N/2\) ) frequencies like equation (3).

\[
\chi_t = \mu + \sum_i (A_i \cos(\omega_i t) + B_i \sin(\omega_i t)) + \epsilon_t, \quad \text{Equation (3)}
\]

\(\cos(\omega_i t)\) = cosine function of each frequency \(\omega_i\) in all points (\(t = 0, 1, 2, \ldots\)),
collection of \(\omega_i\) is \(2\pi i / N\) in \(i = 1, 2, 3, ..., N/2\)
\(\sin(\omega_i t)\) = sine function of each frequency \(\omega_i\) in all points (\(t = 0, 1, 2, \ldots\))

However, the above periodogram analysis has a defect that sampling error is big. In order to solve it, this paper makes smooth the sample spectrum which is a corrected periodogram analysis. Moreover, cross-spectral analysis is implemented for getting the information on the relationship of two series.

Through cross-spectral analysis, we can get the information on the time series data in the N/2 frequency bands. The level of relationship in certain frequency band can be identified by the statistic values of the squared coherence ($S_{xy}(\omega)^2$) and the phase relationship or time lag can be confirmed by the statistic values of the phase ($\phi_{xy}(\omega)$). The coherency in bivariate cross spectral analysis displays the percentage of shared variance between two series in certain frequency among N/2 frequencies and it is a similar to $R^2$ in regression analysis. Moreover, phase presents the timing between the peak of series x and that of series y and it also is similar to the time lag in the time domain.

And harmonic analysis will be applied to the common cycle which is identified by the spectral analysis and it will display the fitted cycle in the graph. This result will clearly shows the relationships between two series. The cross-covariance ($C_{x,y,r}$) of time series x, y in lag r can be presented as follows.

$$C_{x,y,r} = \frac{1}{n} \sum_{t=0}^{n-1-r} x_t y_{t-r}, \quad |r| < n \quad \text{equation (4)}$$

We can get the cross-periodogram ($I_{x,y}(w)$) through Fourier transformation of cross-covariance function.

$$I_{x,y}(w) = \frac{1}{2\pi} \sum_{|r|<n} C_{x,y,r} e^{-iwr} \quad \text{equation (5)}$$

If we smooth the above cross-periodogram, we can get the smoothed cross-spectrum and the complex numbers which are composed of cross-periodogram and cross-spectrum are used in measuring the coherence and phase of both series in each frequency. Through the utilization of each smoothed spectrum and the smoothed cross-spectrum of series x, y, the values of squared coherence ($s_{x,y}(w)^2$) can be deduced as follows,

$$s_{x,y}(w)^2 = \frac{g_{x,y}(w)^2}{g_{x,x}(w)g_{y,y}(w)} \quad \text{equation (6)}$$

$g_{x,y}(w) = $ the smoothed cross-spectrum  
$g_{x,x}(w) = $ the smoothed spectrum of series x  
$g_{y,y}(w) = $ the smoothed spectrum of series y

The value of phase ($\phi_{xy}(w)$) of cross-spectrum can be measured by the part of imaginary number (Im) and real number (Re) of cross-spectrum as follows.

$$\phi_{x,y}(w) = \arctan \left[ \frac{\text{Im} g_{x,y}(w)}{\text{Re} g_{x,y}(w)} \right] \quad \text{equation (7)}$$

### 3) Transfer Function Model

In order to identify the short-term causality between movements of both series which is found by the cointegration test and spectral analysis, after identifying the direction of movement by the Granger causality, inherent causal structure which is existed in variables is identified by the transfer function model.\(^4\) If we can identify the movement of unitary direction, the transfer function model is appropriate

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\(^3\) The part of imaginary number (Im) of cross-spectrum is quadrature spectrum and that of real number (Re) is cospectrum.

\(^4\) In case that the non-causal system which the feedback conditions between output series $\{ x_t \}$ and input series $\{ y_t \}$ be in existence, the transfer function model might be not adequate. Therefore we
in analyzing the causal structure which is inherent in variables. In this case, the transfer function model is the model that analyzes the dynamic relationship between input time series and output time series (see Park and Heu (1999), Choi (2002), and Yaffee and Mcgee (2000)). The transfer function model of single input single output generally is presented as follows.

\[ y_i = v(B)x_i + n_i \]  \hspace{1cm} \text{equation (8)}

but, \[ v(z) = \sum_{j=-\infty}^{\infty} v_j z^j \]

where, \( y_i \) = output time series, \( x_i \) = input time series, \( B \) = backshift operator, \( v(z) \) = transfer function (coefficient \( v_j \) of transfer function is impulse response weight in time lag \( j \)).

\( n_i \) = noise term which satisfied the ARMA model.

The transfer function model which satisfies stability and causality is presented as follows.

\[ y_i = v_0x_i + v_1x_{i-1} + v_2x_{i-2} + \ldots + n_i \]
\[ = v(B)x_i + n_i \]  \hspace{1cm} \text{equation (9)}

but, \[ v(z) = \sum_{j=-\infty}^{\infty} v_j z^j, \sum_{j=-\infty}^{\infty} |v_j| < \infty \]

However, we assume that input time series \( \{x_i\} \) and noise time series \( \{n_i\} \) is independent each other. And in this paper we eventually handle the transfer function model which is a rational function type because the transfer function \( v(z) \) has infinite number of impulse reaction weight.

\[ y_i = v(B)x_i + n_i \]  \hspace{1cm} \text{equation (10)}

\[ v(z) = \frac{\omega_j(z)}{\delta_j(z)} z^b \]

but, \[ \omega_j(z) = \omega_0 - \omega_1 z - \ldots - \omega_j z^j \]
\[ \delta_j(z) = 1 - \delta_1 z - \ldots - \delta_j z^j \]
\( b \) = the delay parameter

This transfer function \( v(z) \) is called as the transfer function which the degree is \((b, r, s)\). If the input time series is not a white noise process in the process of identification of transfer function \( v(z) \), it should be prewhitened, and this whitening model filters the output time series \( \{y_i\} \). We obtain the sample impulse reaction weight function using the prewhitened inference test and identify the degree \((b, r, s)\) of transfer function model by the Box and Jenkins Method (1976). ARIMA model of noise time series can be presented as follows.

\[ \phi(B)n_i = \theta(B)v_i \]  \hspace{1cm} \text{equation (11)}

\{\nu_i\} : white noise process

should identify the movement of unitary direction through the Granger causality test.
Based upon the above analysis, transfer function model, which is a type of rational number function, of single input single out is expressed as follows.

\[ y_t = \frac{w_t(B)}{\delta(B)} x_{t-h} + \frac{\theta(B)}{\phi(B)} v_t \]  

equation (12)

We assume that output time series \( \{y_t\} \), input time series \( \{x_t\} \), and noise time series \( \{v_t\} \) are normal when we identify the transfer function model, which is a type of rational function. Therefore, if they are not normal time series, they should be changed into the normal time series before discriminating transfer function model.

### Analysis result

#### 1. Unit root test

This study implements the cointegration test in order to grasp the existence of long-term equilibrium between the change rates of real-estate values (ZHRATE) and interest rate (ZCB3) time series. To do that, we assume that these data are non-stationary and, in order to change them into the stationary series, the number of differentials (d) is needed should be same. Unit root test on each time series is as follows.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF statistics</th>
<th>PP statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZHRATE</td>
<td>-3.2681</td>
<td>-1.6754</td>
<td>1%: -3.4767, 5%: -2.8815</td>
</tr>
<tr>
<td>ZCB3</td>
<td>-1.7641</td>
<td>-1.7228</td>
<td>10%: -2.5773</td>
</tr>
</tbody>
</table>

Note 1. The result is generated by the model which has only y-intercept.

The unit root test shows that ZCB3 time series is non-stationary because ADF and PP statistics does not reject null hypothesis which unit root exists. ADF statistics of ZHRATE time series does not reject the null hypothesis in 1% level and it is non-stationary series but ADF statistics reject the null hypothesis in 5% level. We can conclude that the ZHRATE time series is non-stationary because its trend is non-stationary type and sample auto-correlation function (SACF) is also non-stationary series.

![Sample auto-correlation function of HRATE time series](image)

In order to convert each non-stationary time series into normalcy time one, the first differential should be implemented on the original time series and the first differentiated time series can be defined as follows.

\[
DZCB3 \equiv \text{the first 3 years expiration that is serene return of corporate bond}
\]

\[
DZHRATE \equiv \text{the first real-estate values fluxion that is calm}
\]

The trends of the first differentiated time series from 1991.01 to 2002.12 is more stationary than that of original series like the next figures.
In order to confirm the existence of unit root on the above first differentiated series, ADF and PP tests are implemented and their results are as follows.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF statistics</th>
<th>PP statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DZHRATE</td>
<td>-2.9799</td>
<td>-6.7737</td>
<td>1%: -3.4767, 5%: -2.8815</td>
</tr>
<tr>
<td>DZCB3</td>
<td>-4.6375</td>
<td>-9.8941</td>
<td>10%: -2.5773</td>
</tr>
</tbody>
</table>

Note 1. The result is generated by the model which has only y-intercept.

The first differentiated series are stationary because the null hypothesis that the unit root exists in analyzed time series is rejected in 1 % level (exceptionally ADF test statistics rejects the null hypothesis in 5 % level. Now we can analyze the long-term equilibrium through the co-integration because total original time series are confirmed as the time series of I(1) that the number of differentiation is d=1.

2. The result of cointegration test

In order to decide the lag of variables which are used in the cointegration test, at first, we inferred the VAR (p) model using the original time series which is not differentiated and confirmed the lag which generated the smallest values of AIC (Akaike Information Criterion) and SIC (Schwartz Information Criterion) as follows.

<table>
<thead>
<tr>
<th>Lag</th>
<th>AIC</th>
<th>SIC</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 1</td>
<td>-0.9407</td>
<td>-0.8169</td>
<td></td>
</tr>
<tr>
<td>P = 2</td>
<td>-1.1580</td>
<td>-0.9518</td>
<td></td>
</tr>
<tr>
<td>P = 3</td>
<td>-1.2423</td>
<td>-0.9535</td>
<td>0</td>
</tr>
<tr>
<td>P = 4</td>
<td>-1.1999</td>
<td>-0.8287</td>
<td></td>
</tr>
<tr>
<td>P = 5</td>
<td>-1.1695</td>
<td>-0.7157</td>
<td></td>
</tr>
</tbody>
</table>

Based upon the above estimation, Both AIC and SIC can select the lag: p=3 (p=2 in case of the differentiated variables). The type of time series of the change rate of housing value (HRATE) and interest rate (CB3) is not the deterministic linear trends like Figure 1. Therefore, this study implements the cointegration test using the model which has a restricted constant in only cointegration relationship and its results are as follows.

<table>
<thead>
<tr>
<th>Model</th>
<th>Null hypothesis</th>
<th>λ_{max} Statistics</th>
<th>Critical value</th>
<th>λ_{max} Statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZHRATE</td>
<td>H0: r = 0</td>
<td>28.1389***</td>
<td>5%: 19.96</td>
<td>23.9444***</td>
<td>5%: 15.67</td>
</tr>
</tbody>
</table>

Note 1. Null hypothesis is that the number of cointegration vector is smaller than that of r or is equal to that of r.  
2. This study analyzes the model which has a restricted constant in only cointegration relationship.  
3. Critical values (critical values) is critical value of Osterwald-Lenum (1992).

The result of cointegration test shows that both λ_{max} Statistics and λ_{max} Statistics have a cointegration
relationship in 1 % significant level. And the cointegration equation which is standardized on the change rate of housing value (HRATE) is as follows.

### Table 6: Cointegration equation

<table>
<thead>
<tr>
<th></th>
<th>ZHRATE = 0.892 - 1.1891*ZCB3</th>
<th>ADF test</th>
<th>PP test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.2285) (0.3068)</td>
<td>-3.6336***</td>
<td>-3.3397**</td>
</tr>
</tbody>
</table>

Note 1. ( ) is the standard error.
2. Unit root test is implemented on the residual time series.
3. **: 5% significant level, ***: 1% significant level

We can confirm that both time series of HRATE and that of CB3 are long-term negative (-) equilibrium relationship because the estimated coefficient present negative value (-) in the above cointegration equation. Moreover, we confirm that the type of residual time series (RESID1) is stationary like Figure 4. ADF and PP tests already showed this trend in the previous analysis.

![Figure 4: The change trend of residual time series of cointegration equation](image)

### 2. Spectral Analysis

In order to find the circulation cycle which exists internally in the time series of the change rate of housing value (ZHRATE) and interest rate (ZCB3), this study implemented the spectral analysis. To do that, we preliminarily removed the trend which existed in the original time series using the Hodrick-Prescott (HP) filter. The original time series and HP trend lines can be presented like Figure 5.

![Figure 5: The original time series and HP trend lines](image)

Each time series which the trend is removed by HP filter can be defined as follows.

- $\text{DEVZCB3} \equiv \text{the return rate of 3 years mature corporate bond which the trend is removed}$
- $\text{DEVZHRATE} \equiv \text{the change rate of housing value which the trend is removed}$

They can be presented like Figure 6.
According to the above figure, the change rate of housing value and interest rate show the reciprocal preceding and escorting movement under the very similar circulation cycle. The periodogram analysis using the above time series data shows the result such as Figure 7.

As a result of periodogram analysis, two times series generate the highest crests in the cycle of 36 months (3 years). Therefore, we can conclude that the common circulation cycle of 36 months (3 years) between two time series is existed. In order to confirm the relevancy between two series, twovariate cross spectral analysis such as coherency and phase analyses are implemented on this common circulation cycle and its results are presented like Figure 8.

Following the above analysis, the lag of ten month (=\(1.65/\pi\) x 18 month) between two crests in the 3 years common circulation cycle of the change rate of housing value and interest rate is existed because the movement between two series shows the phase value of -1.65. Based upon the phase statistics of 3 years common circulation cycle, the movement between two series can be presented like Figure 9.
In the cointegration analysis, we already confirmed that the long-term movement between interest rate and the change rate of housing values has negative (-) balanced relation. Therefore, we should observe that the preceding and escorting movement between two series under 3 years common cycle shows the lag of about 8 month between the peak of interest rate and that of change rate of housing values instead of that the lag of movement from the change rate of housing value to interest rate is 10 months. Based upon this perspective, we can note that the movement between two series the change rate of housing value will reach the peak in 8 months lag after that the interest rate reaches the peak (see the shadow section in Figure 9). Therefore we can note that the movement between two series directs from interest rate to the change rate of housing value under the reciprocal long-term negative (-) balanced relationship.

3. Causality analysis

This study analyzed the long term relationship between the change rate of housing value and interest rate through the cointegration test and spectral analysis, and noted the long term negative (-) equilibrium relation. Now we try to note that this long term movement direction from interest rate to the change rate of housing value can be applied to the short term relation, in other word, the short term causality exists between these two series or not. In order to model this relationship, this study adopts the transfer function model. Before estimating this model, Granger's test of causality is adopted to confirm the feedback circumstance which might be presented in these time series. Using the first differentiated time series data from 01.1991 to 12. 2002 (12 years), the Granger's test of causality is like Table 7.

<Table 7> Granger's test of causality (01.1991 - 12. 2002)

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Lag 1</th>
<th>Lag 2</th>
<th>Lag 3</th>
<th>Lag 4</th>
<th>Lag 5</th>
<th>Lag 6</th>
<th>Lag 7</th>
<th>Lag 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>DZHRATE does not Granger caused DZCB3</td>
<td>0.20</td>
<td>0.21</td>
<td>0.16</td>
<td>0.27</td>
<td>0.25</td>
<td>0.41</td>
<td>0.65</td>
<td>0.61</td>
</tr>
<tr>
<td>DZCB3 does not Granger cause DZHRATE</td>
<td>0.20</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The above result Granger' test of causality shows the uni-direction causality from interest rate to the change rate of housing value between two time series in short term as well as in long term. Based upon this result, this study estimate the transfer function model to verify specifically the dynamic relationship between this time series which is confirmed the direction of causal relation. In case of identifying the transfer function model which is the type of rational function, we assume that output series \( \{y\} \), input series \( \{x\} \), and noise series data \( \{n\} \) are normalcy. In order to filter the output series data in the process of identifying the transfer function \( \{\nu(z)\} \), at first, we should do the prewhitening of input series data. This study estimates ARIMA model for prewhitening of input series data (DZCB3) like Table 8.

<Table 8> The result of estimation of ARMA model

\[
(1 - 0.2128B + 0.1852B^2 + 0.1959B^3 + 0.1869B^4)(DZCB_t - 0.0172) = z_t
\]

\( \{z_t\} \) is the white noise series data which the mean is 0 and the variance is \( \sigma_z^2 \).
The factors of auto regression (AR) section as the filter of the prewhitening applies to the time series of DZHRATE which is output series and generate the output series data \( \{ w_t \} \) which is filtered. After generating the filtered output series, we calculate the cross correlation function \( \{ \rho_{zw}(k) \} \) and estimate the impact reaction weight function \( \{ \nu_k \} \) using them. After that, we note the type of the impact reaction weight function and identify the differential degree \( (b, r, s) \) of the transfer function model by the method of Box and Jenkins (1976). However, we can identify its differential degree \( (b, r, s) \) by noting the type of sample cross correlation function in the real analysis because the impact reaction weight function is in proportion to the sample cross correlation function. Sample cross correlation function and impact reaction weight function using it are like Figure 10.

**<Figure 10> Sample cross correlation function (CORR) and impact reaction weight function (IRF)**

Impact reaction function and cross correlation function (IRF and CORR) shows significantly the different big spike in lag 0 and 3, and the biggest spike in lag 5. After that the spike is gradually decreased in Figure 10. This study decides the differentiation level by the Box-Jenkins method which decided the differentiation level \( (b, r, s) \) of transfer function based upon the type of cross correlation function. We note that the estimated model which the level of differentiation are \( (3,1,2) \) is the fittest with the time series data of this study.

**<Table 9> The estimation result of transfer function model**

\[
ZDHRATE = -0.0258 + \frac{-0.1038 - 0.0695B^2}{1 - 0.8953B} DZCB3_{t-3} + \frac{1}{1 - 0.3414B + 0.2333B^2} \nu^\wedge_t
\]

\( \text{AIC} = -281.3432, \text{SIC} = -262.3545 \)

Pertmentow statistics (= chi square statistics) of residual \( \{ \nu^\wedge_t \} \) of noise time series matrix of estimated transfer function model are as follows.

**<Table 10> Peoteumaentout statistics of residual \( \{ \nu^\wedge_t \} \)**

<table>
<thead>
<tr>
<th>to lag</th>
<th>( \chi^2 ) statistics</th>
<th>DF</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.06</td>
<td>4</td>
<td>0.548</td>
</tr>
<tr>
<td>12</td>
<td>12.58</td>
<td>10</td>
<td>0.248</td>
</tr>
<tr>
<td>18</td>
<td>16.68</td>
<td>16</td>
<td>0.406</td>
</tr>
<tr>
<td>24</td>
<td>23.37</td>
<td>22</td>
<td>0.381</td>
</tr>
<tr>
<td>30</td>
<td>25.49</td>
<td>28</td>
<td>0.601</td>
</tr>
</tbody>
</table>
We can confirm that the transfer function model which is estimated from Permentow statistics of the above residual \( \{v_t\} \) is very suitable to the time series data. Moreover, in order to confirm that the residual \( \{v_t\} \) of noise series model is reciprocally independent with the input series data \( \{z_t\} \) which is passed through the prewhitening process, the cross correlation analysis between two series is implemented and its result is like Table 11.

<table>
<thead>
<tr>
<th>to lag</th>
<th>( \chi^2 ) Statistics</th>
<th>DF</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.01</td>
<td>4</td>
<td>0.734</td>
</tr>
<tr>
<td>11</td>
<td>4.53</td>
<td>10</td>
<td>0.920</td>
</tr>
<tr>
<td>17</td>
<td>9.44</td>
<td>16</td>
<td>0.894</td>
</tr>
<tr>
<td>23</td>
<td>10.95</td>
<td>22</td>
<td>0.975</td>
</tr>
<tr>
<td>29</td>
<td>12.53</td>
<td>28</td>
<td>0.995</td>
</tr>
</tbody>
</table>

According to above Pertmentow statistics, we can adopt the hypothesis that the residual \( \{v_t\} \) of noise series model is reciprocally independent of the input series data \( \{z_t\} \). Until now, we estimate the causal structure of the change rate of housing value and interest rate through using the transfer function model, and we can conclude that the estimated model is fitted very well to the these time series data.

Conclusion

This study implemented the cointegration test and spectral analysis using the time series data of the change rate of housing value and interest rate between 01.1991 and 12. 2002 and its results showed the long term negative (-) equilibrium relationship between two variables and confirmed the preceding and escorting relationship. After that, this study employed the Granger causality relation test in order to confirm the dynamic relation in short term, and its result showed the existence of uni-direction causality from the interest rate to the change rate of housing value. We also confirmed that the interest rate has the extrinsic causality on the housing value. Hereafter, in order to note the causal structure specifically, this study estimated the transfer function model and we confirmed that this estimated model is very suitable to the time series data of this study.

Consequently, the results of this study found the dynamic relationship of the close and negative (-) equilibrium- if the interest rate decrease and then the housing value increase, and if the interest rate increase and then the housing value decrease. This finding lessons that the interest rate policy is very effective in the housing market in Korea. Moreover, it also improves the accuracy of forecasting of housing value. And finally, this study will contribute to building the stabilization policy of housing values in the Korea.

Reference

Quan, Daniel C. And Sheridan Tilman, "Do Housing Values and Stock Values Move Together? An International Analysis ", Housing Economics Vol.27, No.2, 1999: 183-207

<SAS procedure>

Option nocenter ;
Data a1 ;
Infile 'C : \mortgage1\dhgrowth.csv' delimiter = ',' firstobs = 2 ;
Input yr dhouser dinfl dmcb3 dcb3 zdcb3 zdhrate zdinfl dzdhrate dzcb3 dzhrate ;
Data a2 ;
Set a1 ;
Proc arima ;
I var = dzcb3 ;
I var = dzhrate cross = (dzcb3) ;
E p = 2 inputs = (3$ (2)/(1) dzcb3) Method = ml plot ;
Run ;