

Process Innovations in a Duopoly with Two Regions*

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Abstract

We extend previous models of duopolies by introducing regions. This analysis highlights how incentives to conduct process R&D are affected by increasing regional distance, and the effect that agglomeration (in terms of population) has on two firms producing a high- and low-quality good respectively. We find that, under reasonable assumptions, an increase in transport costs (regional distance), raises the incentive to conduct process R&D for the high-quality good, while the reverse is true for the low-quality good. Transport costs generally lower production. We interpret this result to arise because the high-quality good can more easily regain (some) market output, due to its high quality, which gives an impetus for process R&D. The second result is that an increase in agglomeration in the high-quality region, lowers the incentive to conduct process R&D for the high-quality good, while the opposite is true for the low-quality good. This seems consistent with a view of spatial product life-cycles where process R&D is increasingly moved to 'peripheral' regions as agglomerative tendencies continue in high-quality output regions.

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JEL Classification: R3, O31, L13, C72

1 Introduction

This paper extends previous work on duopoly models with price and research decisions, by introducing a regional dimension. These models were pioneered by authors Mussa and Rosen (1978); Shaked and Sutton (1982). It was extended by Bonanno and Haworth (1998) who examined the incentives for conducting

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process R&D and product R&D under Bertrand and Cournot duopoly respectively, where the two firms produced high and low quality goods respectively. Bertrand duopoly is usually claimed to depict a situation of more intense competition, because firms choose prices, and generally lead to higher output. That paper found that more process innovations (process R&D) were optimally undertaken under Cournot competition than Bertrand competition. Furthermore, when comparing the behaviour of the high-quality good to that of the low-quality good, it was found that whenever there is a difference, the high-quality producer was more likely to conduct process R&D. The reverse result held for the low-quality good.

We alter the model by Bonanno and Haworth (1998) to include regional distance, which is simply proxied by a unit transport cost inferred on the consumers. To make the analysis tractable, we have homogenous unit costs of production, where our predecessors instead had higher unit costs for the high-quality good. We analyse Bertrand-competition and examine the effects that an increase in distance between regions has on the incentive to conduct process R&D. We also examine what effect agglomerations have on the incentive to innovate, by increasing the share of the population in the high-quality region. This is intended to capture dynamic aspects of the product-life cycle, where population becomes more concentrated.

2 The Model

There are two regions, each with one firm producing a good. Region H (high) produces the high quality good and region L (low) the low quality good. The quality level of the low quality good is set at 1. We have two types of consumers, one in each region. Each consumer has an income E that can be spent consuming only one unit of one good. If it is used for consumption, the utility is equal to $E - p - t + \theta k$, where p is the good's price, k its quality and θ the taste parameter. Lower values of θ reflect a preference of variety goods. For consumers with $\theta > \theta_*$, where $\theta_* \in (0, 1)$ is some threshold value, demand is positive. If there is no consumption the utility is E . t are fixed per unit transport costs, which is zero if the good is produced in the home region but larger than zero if produced in the other region. This parameter constitutes the main difference over models of previous authors (Mussa and Rosen, 1978; Bonanno and Haworth, 1998). Consumers taste parameters, θ , are uniformly distributed over the interval $(0, 1]$. Assume initially, that there are N consumers in each region. This implies that for a value $x \in (0, 1]$ there will be $2xN$ consumers of preference x or lower.

We now derive the demand function in each region.

2.1 Demand

Consider consumption of the low quality good. For which θ is the consumer indifferent between consuming it and not consuming at all? This happens when

$$E = E - p_L + \theta \quad (1)$$

Implying that $\theta_0^L = p_L$ (the superscript denoting the region). Furthermore we can define the level θ for which a consumer is indifferent between purchasing the low quality good and the high quality good.

$$E - p_L + \theta = E - p_H - t + \theta k_H \quad (2)$$

which implies $\theta_1^L = \frac{p_L - p_H - t}{1 - k_H}$.

For Region H , similar to the above we write

$$E = E - p_L - t + \theta \quad (3)$$

Implying $\theta_0^H = (p_L + t)$. In addition, the indifference parameter θ_1^H is derived from

$$E - p_L - t + \theta = E - p_H + \theta k_H \quad (4)$$

which implies $\theta_1^H = \frac{p_L - p_H + t}{1 - k_H}$. These results imply that $\theta_0^H > \theta_0^L$ and $\theta_1^H > \theta_1^L$.

Now, total demand for the low quality good is the sum of the number of consumers falling within the range $\theta_0 < \theta < \theta_1$ for both regions. $\theta_1 < \theta < 1$ is the range of the parameter for which consumers prefer the high quality good. This implies the demand functions D_L and D_H for each type of good respectively

$$D_H = (1 - \theta_1^L)N + (1 - \theta_1^H)N = \left(\frac{1 - k_H + p_H - p_L}{1 - k_H} \right) 2N \quad (5)$$

$$D_L = (\theta_1^L - \theta_0^L)N + (\theta_1^H - \theta_0^H)N = \left(\frac{2p_L k_H - 2p_H + t(k_H - 1)}{(1 - k_H)} \right) N \quad (6)$$

2.2 Production and equilibria

We assume a constant returns to scale production function. Costs are given by

$$C_i = cq_i \quad (7)$$

where c is the per unit cost (assumed to be the same for both firms) and q_i production. We also assume that demand is positive when price equals cost, that prices and quantities are always positive and profits ≥ 0 .

The profit conditions are for the goods are in the Bertrand-Nash competitive situation:

$$\begin{aligned} \Pi_H(p_H, p_L) &= (p_H - c) \left(\frac{1 - k_H + p_H - p_L}{1 - k_H} \right) 2N \\ \Pi_L(p_H, p_L) &= (p_L - c) \left(\frac{2p_L k_H - 2p_H + t(k_H - 1)}{(1 - k_H)} \right) N \end{aligned} \quad (8)$$

Firms choose price to maximize profits which gives us

$$\begin{aligned} p_H &= \frac{(4k_H^2 - 4k_H + t - tk_H + 6ck_H)}{2(4k_H - 1)} \\ p_L &= \frac{(c + 2ck_H + t - tk_H + k_H - 1)}{(4k_H - 1)} \end{aligned} \quad (9)$$

Quantities produced are then

$$\begin{aligned} q_H &= \frac{(4k_H - 2c - t)N}{(4k_H - 1)} \\ q_L &= \frac{(1 - 2c - t)k_H N}{2(4k_H - 1)} \end{aligned} \quad (10)$$

The last expression requires $2c + t < 1$ for positive production. *Equilibrium* profit levels are

$$\begin{aligned} \pi_H(p_H, p_L) &= \frac{(2c + t - 4k_H)^2 (k_H - 1) N}{2(4k_H - 1)^2} = \frac{q_H^2 (k_H - 1)}{2N} \\ \pi_L(p_H, p_L) &= \frac{2k_H (2c + t - 1)^2 (k_H - 1) N}{(4k_H - 1)^2} = \frac{8q_L^2 (k_H - 1)}{N} \end{aligned} \quad (11)$$

What happens with prices, quantities and profits if the regions were more distant? It can easily be seen that prices, quantities and profits are all lowered since $k_H > 1$, and we assume that production and profits are positive before increasing t .

2.3 Process innovations

We now introduce process innovations which are defined as reductions in unit cost. This is done by a two-stage procedure similar to D'Aspremont and Jacquemin (1988). The second stage maximizes profits conditional on research investments of both firms. In the first stage research is done. A research investment reduces unit cost of production by r . The cost of research is $c^r = \beta \frac{r^2}{2}$ with $\beta > 0$. This functional form is chosen to reflect diminishing returns of R&D. The exact value of β determines how fast diminishing returns sets in. We solve the problem by first considering the second stage, where research investments are treated as given. The profit equations are modified to

$$\begin{aligned} \tilde{\Pi}_H &= (p_H - c + r_H) \left(\frac{1 - k_H + p_H - p_L}{1 - k_H} \right) 2N - \beta \frac{r_H^2}{2} \\ \tilde{\Pi}_L &= (p_L - c + r_L) \left(\frac{2p_L k_H - 2p_H + t(k_H - 1)}{(1 - k_H)} \right) N - \beta \frac{r_L^2}{2} \end{aligned} \quad (12)$$

Firms choose price to maximize which gives us the following Nash-Bertrand solutions for prices

$$\tilde{p}_H = \frac{(4k_H^2 - 4k_H + t + 6ck_H - tk_H - 4k_H r_H - 2k_H r_L)}{2(4k_H - 1)} \quad (13)$$

$$\tilde{p}_L = \frac{(t + k_H + c + 2ck_H - tk_H - 2k_H r_L - r_H - 1)}{(4k_H - 1)} \quad (14)$$

with quantities

$$\tilde{q}_H = \frac{(4k_H^2 - 4k_H + t - tk_H + 2c - 2ck_H + 4k_H r_H - 2k_H r_L - 2r_H) N}{(4k_H - 1)(k_H - 1)}$$

$$\tilde{q}_L = \frac{2(2c - 2ck_H + t - tk_H + k_H - r_H - r_L + 2k_H r_L - 1) k_H N}{(4k_H - 1)(k_H - 1)}$$

and the equilibrium profit levels

$$\tilde{\pi}_H(p_H, p_L) = \frac{(4k_H - t - 2c + 2r_H + 2ck_H + tk_H - 4k_H r_H + 2k_H r_L - 4k_H^2)^2 N}{2(4k_H - 1)^2 (k_H - 1)} - \beta \frac{r_H^2}{2} \quad (15)$$

$$\tilde{\pi}_L(p_H, p_L) = \frac{2k_H N (r_H - t - k_H - 2c + r_L + 2ck_H + tk_H - 2k_H r_L + 1)^2}{(4k_H - 1)^2 (k_H - 1)} - \beta \frac{r_L^2}{2} \quad (16)$$

The first stage maximization problem is then to choose research to maximize the above profit levels. The equilibrium research values \tilde{r}_H and \tilde{r}_L for are quite long and are therefore put in the Appendix. It is there shown that for very reasonable assumptions about k_H, β and N , the incentive effect of an increase in t on optimal process R&D is more positive for the high-quality good than the low-quality good. We interpret this as the following. As distance increases between regions, it becomes more difficult to sell goods across regions. However, this is more likely to be countered with higher process R&D in the high-quality region, due to the easier selling-capability of the higher-quality good.

2.4 The effects of population agglomeration

From a spatial product life cycle perspective, it is commonly found that goods are developed in larger, urban regions, and then, as they mature, production is typically transferred to smaller ones (Karlsson, 1999; Duranton and Puga, 2001).¹ Hence, it should be interesting to see what effect changing sizes of regions have on the incentive for process innovation. We analyse this by letting

¹The original observation (Vernon, 1966) concerns the observation that production moves from developed to less developed countries.

the host region for the high-quality good have more than half of the total population and see what effect an increase in its share have on the incentive to innovate. Rewriting the population sizes gives us

$$\begin{aligned} N_H &= \alpha 2N \\ N_L &= (1 - \alpha) 2N, \quad 1/2 < \alpha < 1 \end{aligned} \quad (17)$$

The demand equations become

$$\begin{aligned} D_H &= (1 - \theta_1^L)(1 - \alpha) 2N + (1 - \theta_1^H) \alpha 2N = \\ &= \frac{2(p_L - p_H + 2t\alpha - t + k_H - 1)N}{(k_H - 1)} \end{aligned} \quad (18)$$

$$\begin{aligned} D_L &= (\theta_1^L - \theta_0^L)(1 - \alpha) 2N + (\theta_1^H - \theta_0^H) \alpha 2N = \\ &= \frac{2(p_H - k_H p_L - t\alpha + t - t\alpha k_H)N}{(k_H - 1)} \end{aligned}$$

The profit equations are

$$\begin{aligned} \bar{\Pi}_H &= (p_H - c + r_H) \left(\frac{2(p_L - p_H + 2t\alpha - t + k_H - 1)N}{(k_H - 1)} \right) 2N - \beta \frac{r_H^2}{2} \\ \bar{\Pi}_L &= (p_L - c + r_L) \left(\frac{2(p_H - k_H p_L - t\alpha + t - t\alpha k_H)N}{(k_H - 1)} \right) N - \beta \frac{r_L^2}{2} \end{aligned} \quad (19)$$

Solving of the second stage Bertrand-Nash game gives us

$$\begin{aligned} \bar{p}_H &= \frac{(t - t\alpha - 2k_H + 3ck_H - 2tk_H + 3t\alpha k_H - 2k_H r_H - k_H r_L + 2k_H^2)}{(4k_H - 1)} \\ \bar{p}_L &= \frac{(t + k_H + c + 2ck_H - r_H - 2t\alpha k_H - 2k_H r_L - 1)}{4k_H - 1} \end{aligned} \quad (20)$$

quantities

$$\begin{aligned} \bar{q}_H &= \frac{2N(t - t\alpha + c - 2k_H - r_H - ck_H - 2tk_H + 3t\alpha k_H + 2k_H r_H - k_H r_L + 2k_H^2)}{(4k_H - 1)(k_H - 1)} \\ \bar{q}_L &= \frac{2k_H N(2c + t + k_H - r_H - r_L - 2ck_H - 2t\alpha k_H + 2k_H r_L - 1)}{(4k_H - 1)(k_H - 1)} \end{aligned} \quad (21)$$

and equilibrium equations

$$\bar{\pi}_H(p_H, p_L) = \frac{4N^2 (t\alpha - t - c + 2k_H + r_H + ck_H + 2tk_H - 3t\alpha k_H - 2k_H r_H + k_H r_L - 2k_H^2)^2}{(4k_H - 1)^2 (k_H - 1)} - \beta \frac{r_H^2}{2} \quad (22)$$

$$\bar{\pi}_L(p_H, p_L) = \frac{2N^2 k_H (r_H - t - k_H - 2c + r_L + 2ck_H + 2t\alpha k_H - 2k_H r_L + 1)^2}{(4k_H - 1)^2 (k_H - 1)} - \beta \frac{r_L^2}{2} \quad (23)$$

Solving of the first stage gives expressions for optimal process R&D showed in the appendix. The numerical results, using the same numerical parameter values as in the previous example, suggest that optimal process R&D is negatively affected by an increase in the regional size of the high-quality region, while the increase in size of the high-quality region has a positive effect on the optimal process R&D size in the low-quality region.

3 Discussion

Our results seem to suggest that as agglomeration forces become more pronounced, process R&D is increasingly moved to 'peripheral' regions. This seems consistent with a view of spatial product life-cycles where process R&D is increasingly moved to 'peripheral' regions as agglomerative tendencies continue in high-quality output regions. We plan to extend this paper to study the role for product R&D in the two regions, where agglomeration becomes more pronounced. Another extension would be to conduct a comparative study of the Cournot and Bertrand situations respectively. In a longer perspective other competitive forms could naturally be studied as well.

A Optimal Process R&D with equal-sized regions

The equilibrium values of R&D are shown below.

$$\tilde{r}_H = \left(\begin{array}{c} 4N\beta + \beta^2 + k_H(16N^2 - 28N\beta - 13\beta^2) \\ + k_H^2(60\beta^2 + 48N\beta - 64N^2) \\ + k_H^3(64N^2 + 16N\beta - 112\beta^2) + 64k_H^4(\beta(\beta - N)) \end{array} \right)^{-1} \quad (24)$$

$$\left(2N(2k_H - 1) \left(\begin{array}{c} 4\beta k_H - t\beta - 2c\beta - 8Nck_H \\ -4Ntk_H + 10c\beta k_H + 5t\beta k_H + 8Nk_H^2 \\ + 16Nck_H^2 + 8Ntk_H^2 - 20\beta k_H^2 \\ - 8c\beta k_H^2 - 4t\beta k_H^2 - 16Nk_H^3 + 16\beta k_H^3 \end{array} \right) \right)$$

$$\tilde{r}_L = \left(\begin{array}{c} 4N\beta + \beta^2 + k_H(16N^2 - 28N\beta - 13\beta^2) \\ + k_H^2(60\beta^2 + 48N\beta - 64N^2) \\ + k_H^3(64N^2 + 16N\beta - 112\beta^2) + 64k_H^4(\beta(\beta - N)) \end{array} \right)^{-1} \quad (25)$$

$$\left(4k_H N (2k_H - 1) \left(\begin{array}{c} 4N - 4Nc - 2Nt + \beta \\ -2c\beta - t\beta + 8Nck_H + 4Ntk_H \\ -8Nk_H - 5\beta k_H + 10c\beta k_H + 5t\beta k_H \\ + 4\beta k_H^2 - 8c\beta k_H^2 - 4t\beta k_H^2 \end{array} \right) \right)$$

We can first note that the denominator is the same for both expressions. To evaluate the effect of an increase in t , we have to make additional, informal, assumptions regarding the relationship between N and β . Suppose that the number of citizens in the economy are counted in (at least) the thousands. Secondly, given the fact that we have assumed $k_H > k_L = 1$, it seems reasonable to think that β should be of a reasonably similar order of magnitude. Furthermore, it doesn't seem reasonable to assume a quality difference higher than 4 times. Inserting $N = 1000\beta$, $k_H = 4$ and evaluating the denominator, we find that it is clearly larger than zero. This result would hold also with much less strict assumptions (k_H and/or N could be much smaller). Therefore, we assume that in what follows the denominator is positive. The effect of an increase in t can be shown by comparing the magnitude of the derivatives of the numerators. For the effect of an increase in t to have a larger effect on \tilde{r}_H than \tilde{r}_L we require that $\frac{d\tilde{r}_H}{dt} > \frac{d\tilde{r}_L}{dt}$. Given the assumption about the denominator, this is true if

$$2N\beta - 14N\beta k_H + 8N^2 k_H + 28N\beta k_H^2 - 16N\beta k_H^3 - 32N^2 k_H^2 + 32N^2 k_H^3 >$$

$$4N\beta k_H + 8N^2 k_H - 28N\beta k_H^2 + 56N\beta k_H^3 - 32N\beta k_H^4 - 32N^2 k_H^2 + 32N^2 k_H^3$$

It can easily be shown that this is true if and only if $k_H > 1$ which we have already assumed from the outset. We therefore conclude, that given reasonable assumptions of the parameter values, there is a larger incentive to conduct process R&D for firm H as the distance between the regions increases.

B Optimal Process R&D with different sizes of regions

The expressions for optimal process R&D are

$$\begin{aligned}
 r_H = & \left(\begin{array}{c} \beta^2 + 8N^2\beta + 32N^4k_H - 13\beta^2k_H \\ -60N^2\beta k_H - 128N^4k_H^2 + 128N^4k_H^3 \\ +60\beta^2k_H^2 - 112\beta^2k_H^3 + 64\beta^2k_H^4 \\ +128N^2\beta k_H^2 - 48N^2\beta k_H^3 - 64N^2\beta k_H^4 \end{array} \right)^{-1} \\
 & (-\beta - 8N^2 + 9\beta k_H + 32N^2k_H - 24\beta k_H^2 - 32N^2k_H^2 + 16\beta k_H^3)^{-1} \\
 & 8 \left(\begin{array}{c} c\beta + t\beta - t\alpha\beta - 2\beta k_H + 4N^2ck_H \\ +4N^2tk_H - 5c\beta k_H - 6t\beta k_H - 4N^2t\alpha k_H + 7t\alpha\beta k_H \\ +10\beta k_H^2 + 4c\beta k_H^2 + 8t\beta k_H^2 - 12t\alpha\beta k_H^2 - 4N^2k_H^2 \\ -8N^2ck_H^2 - 8N^2tk_H^2 + 8N^2t\alpha k_H^2 - 8\beta k_H^3 + 8N^2k_H^3 \end{array} \right) \\
 & (\beta + 8N^2 - 9\beta k_H - 32N^2k_H + 24\beta k_H^2 + 32N^2k_H^2 - 16\beta k_H^3) (2k_H - 1) N^2
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 r_L = & \left(\begin{array}{c} \beta^2 + 8N^2\beta + 32N^4k_H - 13\beta^2k_H - 60N^2\beta k_H \\ +60\beta^2k_H^2 + 128N^2\beta k_H^2 - 128N^4k_H^2 + 128N^4k_H^3 \\ -48N^2\beta k_H^3 - 112\beta^2k_H^3 + 64\beta^2k_H^4 - 64N^2\beta k_H^4 \end{array} \right)^{-1} \\
 & 4 \left(\begin{array}{c} \beta - 2c\beta - t\beta + 8N^2 - 8N^2c - 8N^2t\alpha - 5\beta k_H \\ +10c\beta k_H + 4t\beta k_H + 2t\alpha\beta k_H - 16N^2k_H \\ +16N^2ck_H + 16N^2t\alpha k_H + 4\beta k_H^2 - 8c\beta k_H^2 - 8t\alpha\beta k_H^2 \end{array} \right)
 \end{aligned} \tag{27}$$

The derivatives of these expressions with respect to α are

$$\begin{aligned}
 \frac{dr_H}{d\alpha} = & (-8N^2t\beta - 8k_HtN^2(4N^2 - 9\beta) + 16k_H^2tN^2(8N^2 - 13\beta) - 64k_H^3tN^2(2N^2 - 3\beta)) / \\
 & (\beta^2 + 8N^2\beta - k_H(13\beta^2 - 32N^4 + 60N^2\beta) + 4k_H^2(15\beta^2 - 32N^4 + 32N^2\beta) \\
 & - 16k_H^3(7\beta^2 - 8N^4 + 3N^2\beta) - 64k_H^4\beta(N^2 - \beta)) \tag{28}
 \end{aligned}$$

and

$$\begin{aligned}
 \frac{dr_L}{d\alpha} = & (32N^4tk_H - 8k_H^2tN^2(\beta + 16N^2) + 16k_H^3tN^2(3\beta + 8N^2) - 64N^2t\beta k_H^4) / \\
 & (\beta^2 + 8N^2\beta - k_H(13\beta^2 - 32N^4 + 60N^2\beta) + 4k_H^2(15\beta^2 - 32N^4 + 32N^2\beta) \\
 & - 16k_H^3(7\beta^2 - 8N^4 + 3N^2\beta) - 64k_H^4\beta(N^2 - \beta)) \tag{29}
 \end{aligned}$$

We find that the denominator is the same for both expressions. Analysing these using the sensible parameter values as in the former Appendix, the denominator is positive. The numerator is negative for the $\frac{dr_H}{d\alpha}$ case. This suggests that process R&D is negatively affected by increases in population in the high quality region. The numerator is positive in the low quality region suggesting the opposite result.

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