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Classical models of urban population density. The case of Barcelona Metropolitan Area.

ABSTRACT: urban density analysis examines the spatial distribution of population (in gross or net terms) within urban areas. This analysis provides a measure of concentration that is helpful when examining the urban structure. In this paper, we study sixteen classical functional forms concerning the relationship between density-distance for the case of Barcelona Metropolitan Area.
Keywords: population density, functional form, metropolitan area, subcentre.

1. Introduction.

It is important for planners to understand urban population distributions. This paper surveys classical models of urban population density, and empirical evidence for Barcelona Metropolitan Area is presented. The main contribution is the estimation of sixteen previous functional forms for one same area. This analysis allows us the comparison of the results of different models.

The urban density analysis is useful for understanding present population distributions and service needs. Public transportation and other services (schools and hospitals, for example) are well suited to this type of analysis. Planners can test various scenarios as they strive to control and design the future density structure of residential areas. This fact is especially important in the study area because the growth of the population density in the second half of the century has been less usual than other Western Europe metropolitan areas

The case of the population distribution in Barcelona Metropolitan Area is different from other European Metropolitan Regions. After two decades (1960-1980) of strong immigration from the rural areas of south Spain, the population of Barcelona municipality grew from 1 557 863 to 1 754 900. While for eleven municipalities around the central city the population grew from 571 088 to 1 413 603.

The absence of some form of planning in these years had a special effect for the population distribution. The urban structure is irregular and the population is concentrated in small areas. The result of this particular process is a special case of suburbanization that combines high-density areas within a large industrial area. This fact can be seen in one of the most important industrial region of the European Union, the fifth metropolitan region by volume of industrial employment (Trullén, 1997).

From the family of urban density analysis, the monocentric density model is a simple spatial model that relates urban population density to distance from the centre of the city, usually defined as the central business district (CBD). These models can be considered the classical models of urban population density. What is the sense of the classical term? For us a classical model is a functional form of the density that includes only one exogenous variable: distance. The theoretical basis for the classical approach is derived from the work of von Thünen. Von Thünen's model of agricultural land use was later applied to an urban situation by Alonso (1964) and developed in mathematical form by Muth (1961, 1969) and Mills (1970, 1972)[1].

In this paper, the selection process of the models for the metropolitan area and for each one of the eleven subcentres defined is based on two alternative methods. The first one consists in using the Box-Cox transformation to analyse the simplest forms involving only one exogenous variable, lineal, semi-log and double-log. The second one consists of the validation of the model used when no limitation of the functional form is assumed. In this second strategy, by means of the usual statistical tools and the selection statistic AIC the results obtained are compared. In addition, Golfeld and Quandt's test is performed for heteroscedasticity, Ramsey's for linearity, and Jarque and Bera's for disturbance normality. In section 2, the sixteen functional forms for the density-distance relation are introduced. Section 3 analyses the information available for the estimation in the case of Barcelona Metropolitan Area and its eleven subcenters. In section 4, results are presented and finally, in section 5, the conclusion is discussed.

2. The functional forms.

The monocentric urban density analysis has received considerable attention form two disciplines: urban geography and regional science. This has been approached both theoretically and empirically. The classic study by Colin Clark (1951) has led to an extensive body of literature dealing with empirical implementations for a wide range of metropolitan areas and cities, in different countries and at different times. In this work, we analyse the classical econometric models of urban density. Some of them have been used in studies about traffic planning and some others in theoretical models on housing market. Quantitative geography has also attempted to model the urban population density. We analyse sixteen functional forms that originate both in theoretical models and in empirical observation and we introduce in the analysis a new functional form more general. Some of these functions have been used in studies about traffic planning, for example Tanner (1961) and Smeed (1963), and some others in theoretical models on housing market (Muth, 1969). Quantitative geography has also attempted to model the urban population density (Stewart, (1947), Newling (1969, 1971)). The generalisation of the functional form and the comparison of results are due to Casetti (1973), McDonald and Bowman, (1976), Kau and Lee, (1976a, 1976b), Zielinski (1979), Anselin and Can (1986) and Smith (1997). McDonald and Bowman estimate ten functional forms with data on sixteen cities, and they compare the results with the mean standard error, with the determination coefficient, and with the prediction of the total city population. Kau and Lee generalise the functional form by following the technique Box-Cox on data of forty cities. Zielinski uses the determination coefficient to evaluate ten functional forms, estimated for seven cities. Anselin and Cain, compare five forms for a city, following the contrast of McKinnon, White y Davidson (1983). Reviews of this literature are due to Thrall (1988), McDonald (1989), Smith (1997) and Wang and Zhou (1999). The functional form of urban population density is not unique and this fact forces the use of a selection process in each analysed case.

In order to simplify the study of urban population density and makes the comparisons of its results easier to understand, Colin Clark (1951) proposed two general hypotheses: (i) in all cities, excluding a business and commercial area, there are densely populated areas, which decrease when moving away from the center, and

(ii) in most of the cities, as time passes the density decreases in the central areas and increases in the suburbs, thus producing a territorial expansion of the city. The first empirical evidence that population density falls with increasing distance from the city center (assumed to be at the center of CBD) is credited to this work.

• Clark (1951) claims that the urban population density can be correctly described by means of the negative exponential function:

$$D(x) = D_0 e^{bx} \qquad (1)$$

where x stands for the distance to the center measured in length units, $D(x)$ stands for the resident population density per surface unit, $D_0>0$ and $b<0$.

• Stewart (1947), a geographer, had already suggested a lineal relationship between density and distance:

$$D(x) = D_0 + bx \qquad (2)$$

• Tanner (1961) and Smeed (1963) proposed two new functional forms in their studies on city traffic. Their contributions are based on two special cases of the gamma quadratic function. The Tanner model assumes:

$$D(x) = D_0 e^{cx^2} \qquad (3)$$

with $D_0>0$ and $c<0$.

• In Smeed's case the relationship is

$$D(x) = D_0 x^e \qquad (4)$$

with $D_0>0$ and $e<0$.

• Aynvarg (1968), a soviet geographer, introduced a new functional form:

$$D(x) = D_0 e^{(bx)x^e} \qquad (5)$$

with $D_0>0$, $b\neq 0$ and $e<0$.

After these first contributions Newling (1969, 1971) developed empirically two (linear and semi-log) quadratic forms of the negative exponential density function. He applied the concept of allometry to the density of cities and deduced rules for intraurban allometric growth in the United States, showing mathematically that increasing density has a depressive effect on the rate of growth. Using the 1950 census data for forty-six Standard Metropolitan Areas (SMAs) and Urbanized Areas (UAs), Newling applied the parameter estimates developed by Muth (1961). Newling showed a strong correlation between density and rate of growth. He determined for a critical density (32 000 persons per square mile) above which the rate of growth is negative and below which the rate of growth is positive. He suggested that the calculation of critical densities for other cities may be important for the field of planning and there may be an optimum urban density.

• Newling (1969) modified the work of previous researchers with two quadratic forms of exponential model, indicating a population density crater surrounding the central business district. The logarithmic transformation of Newlig's modification produces a partial upside-down U-shaped curve:

$$D(x) = D_0 e^{(bx+cx^2)} \qquad (6)$$

In later works, the same author (Newling, 1971) suggests the possibility of an intrinsically lineal functional form through a polynomial of degree two:

$$D(x) = D_0 + bx + cx^2 \qquad (7)$$

both with $D_0>0$, $b\neq 0$ and $c<0$.

The existence of a crater of residential population density makes, intuitively, sense because CBD's in North American cities are for the most part non-residential. This assumption is not clear in European cities. The difference between Clark and Newling's models suggest a dynamic interpretation, Clark's model reflects the

North American city in its early stages and the Newling model portrays the city in a later developmental stage. The central density reduces over time and the peak density shifts outward from the city center.

The research during the two first decades after Clark's work is characterised by the accumulation of additional empirical evidences supporting two models: Clark's and Tanner´s model. Along the same lines, we can point out the works of Berry, Simmons and Tennant (1963), Latham and Yates (1970). Some years later, density gradients for employment and firms have been introduced (Mills 1972; Kemper and Schemenner 1974). In all the studies, the functions are estimated by OLS, and no contrast for the econometric validation of the results is performed. This accumulation of empirical evidence reveals another fact: the relationship between city size and density gradient. It was demonstrated empirically that in the United States, smaller cities have a steeper density gradients and are more compact than larger cities are, whereas Asian cities of the same size have steeper density gradients and are more compact (Berry et al. 1963). Evidence collected for cities in Europe, North America and Australia indicated that as cities grow, they exhibit decreased density and diminished compactness. Winsborough (1965) associated the change in central density with a change in transportation technology resulting from increased ownership of automobiles over time.

• McDonald and Bowman (1976) in their work to test the usefulness of some alternative gross population density functions for urbanised areas proposed two new functional forms:

$$D(x) = D_0 (x_R - x)^b \tag{8}$$

with $D_0>0$, $b>0$ and x_R radius of the urbanised area, and:

$$D(x) = D_0 e^{(ax+b/x)} \tag{9}$$

with $D_0>0$, $a<0$ and $b>0$.

• The generalisation of the functional form for the density gradient is due to Kau and Lee (1976). Starting from Clark's model and relying on Box and Cox (1964) work, they propose two general functional forms in order to describe the relationship between the population density and the distance to the city center:

$$\frac{D(x)^{\lambda} - 1}{\lambda} = \beta_1 + \beta_2 x \tag{10}$$

and

$$\frac{D(x)^{\lambda} - 1}{\lambda} = \beta_1 + \beta_2 \frac{x^{\lambda} - 1}{\lambda} \tag{11}$$

where β_1 and β_2 are the regression parameters; λ are the functional form parameters.

When λ =1, the density in form (10) is regressed on distance (Stewart's model). If λ approaches zero, the dependent variable is the natural logarithm of the density (Clark's model). Similarly, (11) will reduce to the linear form when λ is equal to one, and to a double logarithmic form (Smeed's model) when λ approaches zero. Using data on U.S. urban areas in 1970, Kau and Lee found that λ exceeded zero in 23 out of 40 cases; the density function is between exponential and linear in almost 50% of the cases.

• Frankena (1978), in his paper proposed two functional forms, polynomial and exponential:

$$D(x) = D_0 + bx + cx^2 + dx^3 \tag{12}$$

$$D(x) = D_0 e^{bx+cx^2+dx^3} \tag{13}$$

his results indicated in both cases: $D_0>0$, $b<0$, $c>0$ $d<0$.

• The last classical models of urban population density belong to Zielinski (1979). In his work he compared some functions (Stewart, Clark, Tanner, Smeed, Aynvarg, Newling) and introduced two new functional forms:

$$D(x) = D_0 e^{(bx+cx^2)x^e} \qquad (14)$$

and

$$D(x) = D_0 e^{(cx^2)x^e} \qquad (15)$$

with $D_0>0$, $b>0$, $c<0$, $e<0$.

• A new more general functional form is introduced in this analysis. It is possible to join the functions of Zielinski (15) and Frankena (13) and generalise the density-distance relationship. This function has never been used before:

$$D(x) = D_0 e^{(bx+cx^2+dx^3)x^e} \qquad (16)$$

In table 1 all the functional forms of classical models of urban population density and the relationships between the functions are show.

TABLE 1. FUNCTIONAL FORM OF URBAN POPULATION DENSITY FUNCTIONS

$D(x)=D_0 \exp(bx+cx^2+dx^3)x^e$
$\ln D(x)=\beta_1+\beta_2\ln x+\beta_3 x+\beta_4 x^2+\beta_5 x^3$

e=0

$D(x)=D_0 \exp(bx+cx^2+dx^3)$
$\ln D(x)=\beta_1+\beta_2 x+\beta_3 x^2+\beta_4 x^3$
(Frankena 1, 1978)

d=0

$D(x)=D_0 \exp(bx+cx^2)x^e$
$\ln D(x)=\beta_1+\beta_2\ln x+\beta_3 x+\beta_4 x^2$
(Zielinski 1, 1979)

e=0 c=0

b=0

$D(x)=D_0 \exp(bx+cx^2)$
$\ln D(x)=\beta_1+\beta_2 x+\beta_3 x^2$
(Newling 1, 1969)

$D(x)=D_0 \exp(cx^2)x^e$
$\ln D(x)=\beta_1+\beta_2\ln x+\beta_3 x^2$
(Zielinski 2, 1979)

$D(x)=D_0 \exp(bx)x^e$
$\ln D(x)=\beta_1+\beta_2\ln x+\beta_3 x$
(Aynvarg, 1968)

e=0

b=0

c=0

b=0

e=0

$D(x)=D_0 \exp(bx)$
$\ln D(x)=\beta_1+\beta_2 x$
(Clark, 1951)

$D(x)=D_0 \exp(cx^2)$
$\ln D(x)=\beta_1+\beta_2 x^2$
(Tanner, 1961)

c=0

$D(x)=D_0 x^e$
$\ln D(x)=\beta_1+\beta_2\ln x$
(Smeed, 1963)

$\beta_4=0$ $\beta_3=0$

$D(x)=\beta_1+\beta_2 x+\beta_3 x^2+\beta_4 x^3$
(Frankena 2, 1978)

$D(x)=\beta_1+\beta_2 x+\beta_3 x^2$
(Newling 2, 1971)

$D(x)=\beta_1+\beta_2 x$
(Stewart, 1947)

$D(x)=D_0(x_R - x)^b$
$\ln D(x)=\beta_1+\beta_2\ln(x_R - x)$
McDonald-Bowman 1,1976)

$D(x)=D_0 \exp(ax+b/x)$
$\ln D(x)=\beta_1+\beta_2 x+\beta_3(1/x)$
(McDonald-Bowman 2,1976)

$D(x)^\lambda-1/\lambda=\beta_1+\beta_2 x$
(Kau-Lee 1,1976)

$D(x)^\lambda-1/\lambda=\beta_1+\beta_2(x^\lambda-1/\lambda)$
(Kau-Lee 2,1976)

3. Data and study area.

Catalonia is a region located in north-east of Spain (NUTS 2 in the UE classification). Barcelona Metropolitan Area (BMA) has a population of 4.062.780 (1998), the central city has 1.266.991 (see figure 1). The BMA has 128 municipalities (see figure 2), and the central city is divided in 10 districts with a population of 150 560 (1998 average). The total number of units for the estimation the density functions in the BMA case is 138. As said in the introduction, the most important changes in the population distribution are located in the municipalities around the central city. These areas are in the urban literature called subcentres.

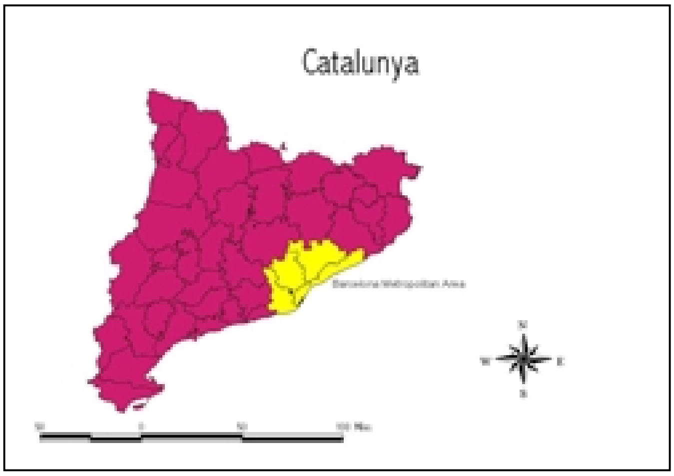

Figure 1.

Figure 2.

The definition of subcentres is not unique. Giuliano and Small (1991) present an excellent discussion about this point. Prior studies defined subcentres and documented their presence in various ways. Some authors arbitrarily define subcenter locations and then estimate density functions around these points, for example Bender and Hwang (1985). Others use centres as defined by a regional planning agency (Greene (1980) or Griffith (1981)). Still, others authors define subcentres as municipalities of a certain minimum size, for example Erickson (1986). In this work, this last definition is used, and the minimum size is a population of 50 000 inhabitants. In this situation, we have eleven municipalities. A summary of the characteristics of these eleven subcentres is shown in table 2.

Table 2. Subcentres characteristics.

Name	**Population (1998)**	**Area (Km^2)**	**Distance from the CBD of the central**	**Sample size (Number of census**

			city (Km)	tracts)
Hospitalet de Ll.	272.578	12,20	7,12	226
Badalona	218.725	13,56	8,89	156
Sabadell	189.404	36,11	17,05	130
Terrassa	158.063	31,77	23,77	113
S. Coloma de G.	133.138	4,95	6,89	99
Mataró	101.510	8,61	28,25	72
Cornellà de Ll.	84.927	6,05	9,02	70
S. Boi de Ll.	77.932	8,53	12,93	44
El Prat de Ll.	64.321	3,02	12,60	37
Granollers	51.873	9,67	23,14	35
Rubí	50.405	3,45	16,38	28

The selection of observational units for statistical estimation of functional forms of population density is most often constrained by the availability of data. Most of the more recent approaches are based on census tract data. The census tract areas are taken as a proxy variable of the residential land. Large parts of the works are based on the use of all census tracts of a metropolitan area. For example Frankena (1978) and Griffith (1981) for Toronto, Greene and Barnock (1978) for Baltimore, White (1977) for five European cities, Glickman and Ogury (1978) for 71 Japanese cities, and Alperovich (1983a, 1983b) for twenty Israeli cities

Some other works, however, use a random sample of about 40 to 50 census tract observations. Examples of this type of works are Kau and Lee (1976a, 1976b, 1977) and Kau, Lee and Chen (1983) (45 observations of 50 cities); Johnson and Kau (1980) (43 observations of 39 cities); and Anderson (1982) and Brueckner (1986) (50/70 observations for 30 cities)

In this present paper two kinds of data have been used: all census tract of each of the eleven subcentres that have been analysed (see table 2) and municipality area for the case of Barcelona Metropolitan Area. In the first case, the observations can be considered as a proxy variable of the residential land and the density is a net residential density. In the second case, the observations are a proxy of residential and industrial land and the population density can be considered a gross residential density. The functions have been estimated from the data sets of BMA for six periods (between 1975 and 1998) and eleven subcentres (municipalities with a population ranging from 50.000 to over 200.000) for one period (1991). The population data are taken from the Spanish Population Censuses.

The area of the census tracts has been measured with an electronically planimeter, and the distance from the municipality centre to the census tract with a ruler from each city map. The area and distance for the Barcelona Metropolitan Area has been measured with ArcView. The problem of determining the centre of a municipality has been studied by Alperovich (1982); the solution we adopt consists of taking as central census tract the one where the city government is located. In the central census tract of the eleven municipalities analysed the retail and business activities prevail. In this sense, the characteristics of the typical CBD are respected.

4. Empirical application.

We have first estimated the parameters of the Box-Cox transformation for the eleven subcentres (for one period) and the Metropolitan Area (for six periods, 1975-1998). The estimate of the values λ in (10) and (11) is obtained by an iterative process between -0.5 and 1.5 with intervals of length 0.1. The ML estimators for the eleven subcentres analysed indicate that the functional form parameters for 6 of the cities are significantly different from one, and in 9 of them the parameters are significantly different from zero at 0.05 level. In the case of the Metropolitan Area we rejected λ=1 and we did not rejected λ=0 for all periods.

These results suggest that for the subcentres (net density) the functional form varies from city to city and that the linear transformations are more appropriate in 9 cases. In the case of Metropolitan Area (gross density) we discard the linear (1) functional form. The Box-Cox technique does not seem to be appropriate to choose a functional form in cities where the null hypotheses on λ in (10) and (11) are rejected. A summary of the results is exposed in tables 3 and 4.

Table 3. Results of the estimate λ in (10) and (11).

λ estimates in (10)	Subcenters	λ estimates in (11)	Subcenters
λ≠0	9	λ≠0	9
λ≠1	6	λ≠1	6
λ=0	2	λ=0	2
λ=1	5	λ=1	5

Table 4. ML estimate for the parameters λ in (10) and (11).

Name	λ in (10)	λ in (11)
Hospitalet de Ll.	0,4*	0,4*
Badalona	0,3*	0,3*
Sabadell	0,5*	0,5*
Terrassa	0,3*	0,3*
S. Coloma de G.	0,3*	0,3*
Mataró	0,4*	0,4*
Cornellà de Ll.	0,3*	0,3*
S. Boi de Ll.	0,5*	0,6*
El Prat de Ll.	0,3	0.3
Granollers	0,3	0,4
Rubí	0,5*	0,5*
MBA	-0,1	0,1

Note:* indicates reject the hypothesis λ=0 based on 5% level of significance.

Although it is difficult to differentiate the semi-log from the double-log functional forms, the results indicate that for the gross density the logarithmic specifications dominate the linear specifications. In addition, the results would suggest that the functional form vary depending on the subcentres. This variation has enough significance to warrant empirical investigation for each particular case. This fact forces us to use a selection process for each subcentres to describe urban population density.

Since the Box-Cox technique does not allow us to select a model for each subcentres and the metropolitan area, we have proceeded to develop the second strategy, which consists of estimating the different alternative specifications. Afterwards, and for each of them, we perform a series of statistical contrasts with the objective of determining whether the disturbances satisfy the basic hypotheses of the regression model. Later, we consider the reformulation of statistical hypotheses for non-spherical models. In the process, several models of variance structure have been tested in order to obtain efficient estimators, something that requires applying GLS, which can be interpreted as OLS on a conveniently transformed data, or its equivalent WLS.

Once the variance structure is modified, we perform again the Golfeld and Quandt test. Except in some exceptional cases, the problem has been solved with the GLS estimate with variance structure $V(u_i)=\sigma^2 x_i$. In the subcenters with poorer results, we have tested some other structures to correct the problem, with negative results.

In the third stage by means of a selection statistic AIC are selected the most appropriate specifications among all spherical models.

In table 5 we show a selection of the estimates for the sixteen functionals forms for the subcentres and for each of six periods for Metropolitan Area. The figures 3 to 12 illustrate the selected models for each subcentres and Metropolitan Area. The criteria followed in the selection were the coefficients significance, the adjustment accuracy, the violation of the hypotheses, and for models with similar results, we used the AIC selection statistic.

5. Conclusion.

For two subcentres (Hospitalet de Ll. and Cornellà de Ll) it does not exist any model with coefficients significance. These subcentres are located in the south of the Metropolitan Area and they are very near. The urban density models proposed in the literature provide better results, in general (in terms of R^2 and with the contrasts) for the Metropolitan Area. The problem in this case is the non-normality of their disturbance. The selection statistic AIC is more suitable for the subcenters.

Tanner's model is selected in three subcentres: Badalona, Terrassa, and Granollers. Zielinski 1 and Zielinski 2 are selected in two subcentres. The value of R^2(adj.) in the subcentres indicates that the models used, with the distance as the only independent variable, can be misspecified, and the use of more explanatory variables in the case of net residential density is necessary. The no-normality and no-linearity problems arise in the most populated subcentres as well. In general, the most populated subcentres have poor results for the sixteen functional forms analysed.

The coefficient sign is positive for the most populated subcentres: Badalona and Sabadell. This is an assumption that is not considered by urban economy models: positive urban density gradient. The causes for this behaviour are historical and geographical, external to the behaviour of the economical activity of these subcentres.

Other models not listed in Table 5 also reveal that, in general, the best models for all kind of subcentres and the Metropolitan Area are the simplest ones, i.e., those where the distance is given in a logarithmic, quadratic or linear expression.

DeBorger (1979) studied three urban areas in Belgium from 1900 to 1976 and observed that the central densities fell and the gradients flattened steadily over the time. The results for the Metropolitan Area from 1975 to 1998 indicate that the gradients flattened steadily over the time but for the central density, these results are not clear.

The Box-Cox transformation represents a possible way to select the functional form, but not the only one. This viewpoint is restricted to the choice among three functionals forms. Another method, the validation of the models and the selection by the statistic AIC, can contribute to the choice. In the case of the Barcelona Metropolitan Area and its subcentres it seems necessary to include another explanatory variable, apart from the distance, in order to model the urban density correctly. These variables should take into account some socio-economic characteristics of the population. In addition, further research is necessary to explain the possible effects of central subcentres densities, geographic restrictions and variation in transportation models on the functional form.

This paper has surveyed the classical models of urban population density that have appeared since 1947. There are several issues that require further research. On the one hand, it is possible to estimate more flexible functional forms proposed by Anderson (1982) and Alperovich (1997). On the other hand, it is necessary to introduce the spatial effects in the classical models of urban population density like in the Griffith (1981) and Anselin and Can (1986) works.

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Footnotes: [1] For a synthesis of a theoretical works see Fujita (1989) or Anas, Arnott and Small (1998).

Table 5. Selection of the estimate results of the 15 functional forms for BMA and his subcentres.

BMA and Subcentres	Functional Form	EM	β_1	β_2	β_3	β_4	S	F	R^2adj	AIC	J-B	Reset	GQ
BMA (1975)	Smeed	OLS	10.62* (25.89)	-1.44* (10.36)	--	--	1.24	106.22*	0.48	50.63	68.37*	2.81	1.34
BMA (1981)	Smeed	OLS	10.72* (26.67)	-1.43* (10.43)	--	--	1.22	108.85*	0.49	46.25	66.77*	2.22	1.31
BMA (1986)	Smeed	OLS	10.68* (27.19)	-1.40* (10.44)	--	--	1.19	109.06*	0.49	41.49	64.69*	2.17	1.40
BMA (1991)	Smeed	OLS	10.64* (28.86)	-1.34* (10.67)	--	--	1.11	113.98*	0.50	27.33	62.90*	1.33	1.36
BMA (1996)	Smeed	OLS	10.56* (30.04)	-1.26* (10.53)	--	--	1.06	111.05*	0.49	17.11	63.38*	0.43	1.11
BMA (1998)	Smeed	OLS	10.53* (30.40)	-1.24* (10.50)	--	--	1.05	110.32*	0.49	13.85	63.30*	0.42	1.10
Badalona	Tanner	OLS	9.94* (102.8)	0.11* (5.25)	--	--	0.73	27.58*	0.15	-83.87	13.14*	2.11	0.99
Sabadell	Kau-Lee 1	GLS	201.32* (12.2)	34.00* (2.62)	--	--	99.16	33.48*	0.20	10.99	1.13	0.57	1.14
Terrassa	Tanner	GLS	9.47* (90.9)	-0.06* (1.93)	-	-	0.86	979.34*	0.10	-63.82	33.55*	4.62*	2.76
S. Coloma de Gramenet	Zielinski 1	OLS	4.46* (2.05)	-2.33* (2.40)	13.10* (3.59)	-6.85* (4.69)	0.54	18.82*	0.36	-112.40	10.16*	0.09	1.71
Mataró	Newling 2	OLS	33688* (8.41)	16916* (3.119	-12585** (1.73)	-	19375.2	5.44*	0.11	-5.42	6.40*	2.69*	0.90
S. Boi de LL.	Zielinski 2	GLS	10.75* (38.3)	0.43* (2.29)	-0.60* (3.20)	-	0.76	585.73*	0.24	-15.29	3.29	4.28*	0.80

Table 5. Selection of the estimate results of the 15 functional forms for BMA and his subcentres. (continuation)

BMA and Subcentres	Functional Form	EM	β_1	β_2	β_3	β_4	S	F	R^2adj	AIC	J-B	Reset	GQ
El Prat de LL.	Zielinski 1	OLS	26.91* (4.62)	10.23* (2.99)	-20.82* (2.71)	4.38* (2.33)	0.72	4.89*	0.25	-19.29	2.11*	1.68	2.58
Granollers	Tanner	GLS	10.06* (67.2)	-0.44* (2.80)	--	--	0.82	726.62*	0.32	6.03	1.60	0.91	2.82
Rubí	Zielinski 2	OLS	12.02* (23.4)	1.18* (2.61)	-2.20* (4.39)	--	0.73	11.27*	0.44	-16.90	0.78	2.68	1.46

The first column indicates the name of the city, the second one the author who introduced the functional form between density and distance following the notation appearing in Table 1 and Table 2. The third one indicates the estimation method OLS or GLS; from the fourth to seventh the value of the cofficient and below the score t for the significance constrast in absolute value. In the eighth and the ninth the standard error of the regression and the value of the F statistic. The adjusted determination coefficient, the selection statistic AIC, the Jarque-Bera constrast value and Ramsey's specification value appear next. In the last column, the value of the F statistic for the Golfeld-Quandt heteroscedasticity test is placed.

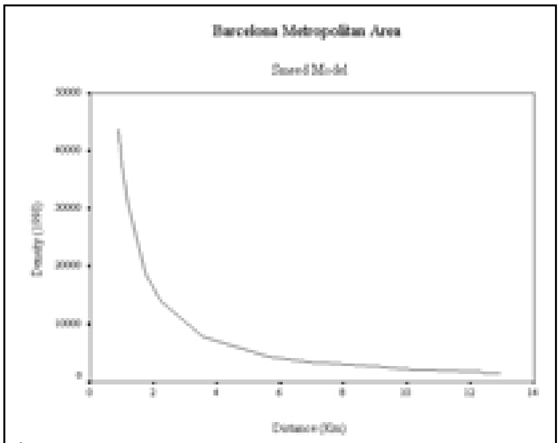

Fig 3

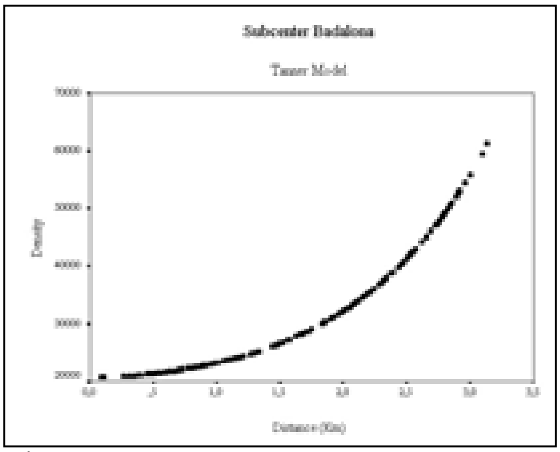

Fig. 4

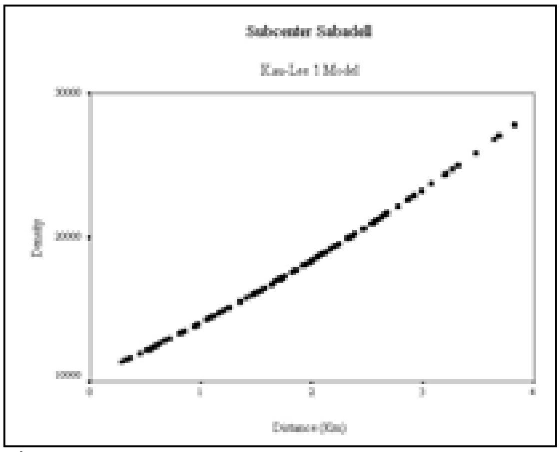

Fig. 5

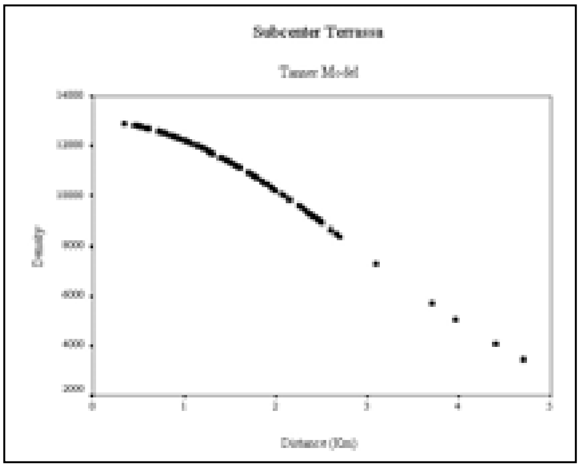

Fig. 6

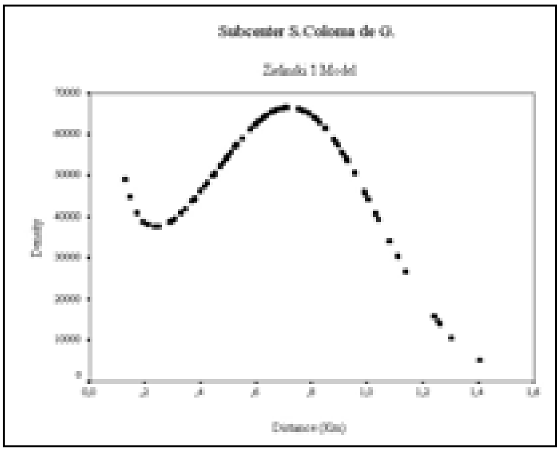

Fig. 7.

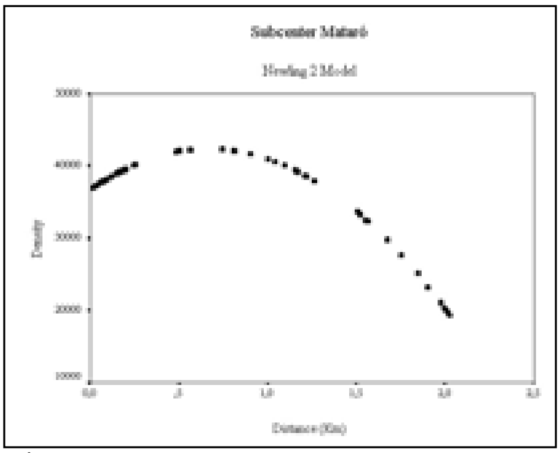

Fig. 8.

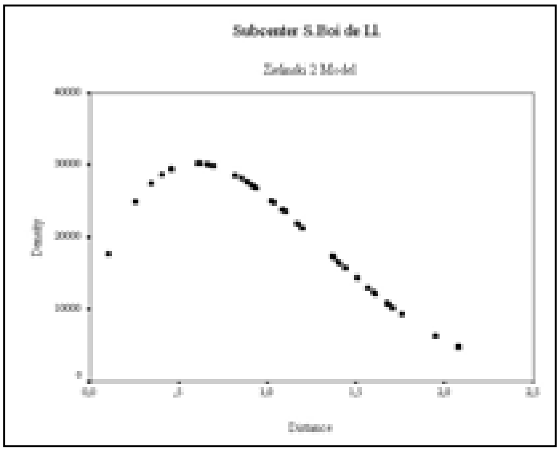

Fig. 9

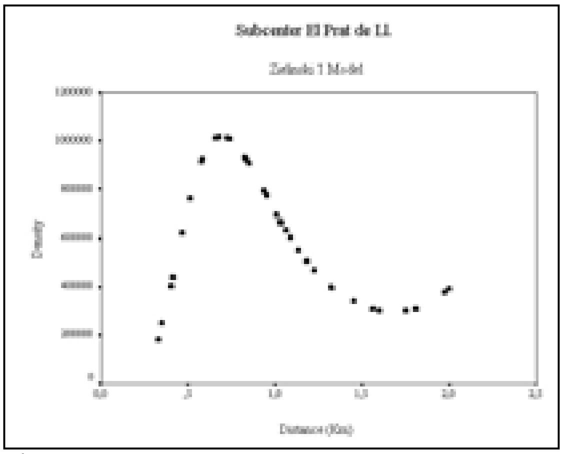

Fig. 10

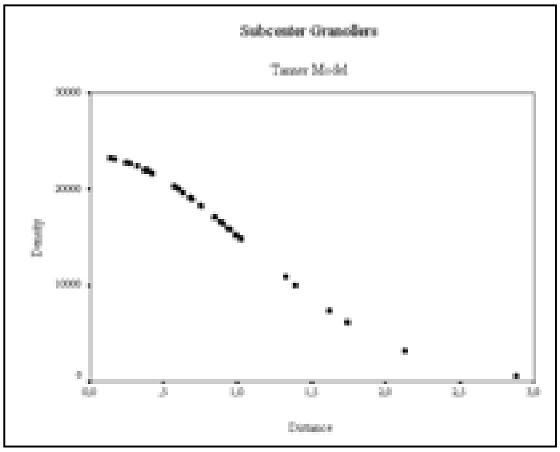

Fig. 11

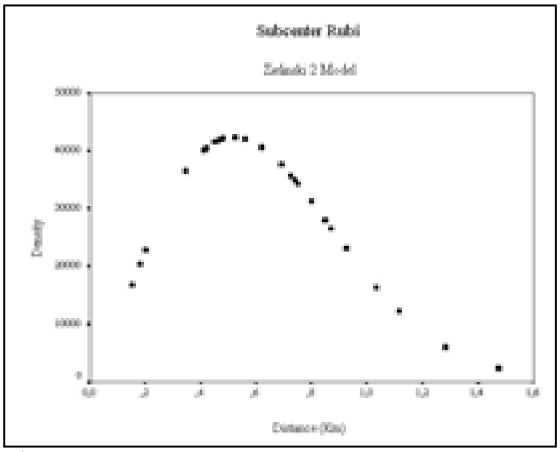

Fig. 12

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