

Variable Growth Impacts on Optimal Market Timing in All-Out Production Systems

Jay R. Parsons, Dana L. Hoag, W. Marshall Frasier, and Stephen R. Koontz¹

Abstract

This paper addresses the economic impacts of growth variability on market timing decisions in an all-in, all-out production system. Marketing decisions based on the pen average are determined to be different than those based on the entire distribution of output levels. A case study data set of 350 swine provides verification of our theoretical construct.

Paper presented at the Western Agricultural Economics Annual Meetings, Long Beach, CA, July, 2002.

Copyright 2002 by Jay Parsons, Dana Hoag, W. Marshall Frasier, and Stephen Koontz. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided this copyright notice appears on all such copies.

¹ Jay Parsons (jay.parsons@OptimalAg.com) is a graduate research assistant, Dana Hoag (dhoag@ceres.agsci.colostate.edu) is professor, Marshall Frasier (mfrasier@ceres.agsci.colostate.edu) and Stephen Koontz (skoontz@ceres.agsci.colostate.edu) are associate professors at Colorado State University.

Introduction

The notion that the output from a production process can vary is not a new one. This is especially true as it applies to agricultural production where numerous factors such as weather and genetics jointly determine the final outcome. Yield variations are especially pertinent in the livestock industry where we typically see entire pens of animals marketed at one time based on the average size in the pen. Ideally, to be entirely confident about these marketing decisions, the entire range of the data should be understood (Pringle, 2000). Averages mask this information. Information that might return more than it costs to collect.

Previous research on the optimal slaughter weight of livestock has focused on feeding strategies, genetics, and pricing systems. For instance, it has been shown that there are higher profits per hog for leaner gilts relative to the fatter barrows and that the gilts pay more marketed through a component pricing system while the barrows pay more in a live weight pricing system (Boland, Preckel, and Schinckel, 1993). Other studies have shown that feed prices and animal replacement costs are important in determining the optimal market weight (Chavas, Kliebenstein, and Crenshaw, 1985), have examined how producers might modify their feeding decisions to best respond to changes in input and output prices (Crabtree, 1977), and used gain isoquants to establish decision rules for optimal rations through various growing phases (Heady, Sonka, and Dahm, 1976).

In general, past research has focused on establishing decision rules based on a representative animal from the group. This may be appropriate in industries like poultry where variability has been reduced to minimal levels in recent years. However, these same decision rules may be sub-optimal for heterogeneous animals such as cattle, where there are frequent calls to improve quality and consistency (Smith et al. 1995 and NCBA). Grid marketing and

complicated sorting systems (i.e. Brethour, 1989) that use ultrasound to identify individual animal traits show that the beef industry understands that economic losses can occur when pens are sold based on average animal traits.

The objective of the present paper is to present a model that accounts for the distribution of the animals in the market timing decision. For example, when a pen is marketed, variability in animal growth results in some animals being over-finished, while others have not yet reached their full economic potential. The impact of this distribution on the optimality conditions is explored through a thorough analysis of the marginal curves resulting from the production process.

Swine production provides the application focus of the present paper but the methods extend to other species. By choosing swine as our application focus, we are able to utilize extensive data sets available from university researchers to test our model. However, as it turns out, the market timing for swine within their production cycle places limits on the economic value associated with a full account of the output distribution. The value of our model is not so much in its application to the swine industry as presented in the present paper as it is in the theoretical construct itself. Specifically, the notion that decreasing marginal returns result in a situation where the average output level is not the basis from which to compute the average marginal value product for a group of animals. A model accounting for the entire distribution of output levels provides a more accurate assessment of the marginal value associated with continuing to feed a pen of animals. In doing so, this may lead to situations where market timing decisions based on the average output level are significantly less than optimal.

This paper extends previous research in two ways. First, whereas previous research has focused on decision rules as they pertain to a representative animal for a given group, we are

considering the entire distribution of animals. Therefore, the decision rules developed in this paper are a better representation for the full economic potential of all-in, all-out pen marketing practices. Second, by developing this model, we present a framework to explore the impact of production variability on any production situation characterized by a simultaneous termination of the production process across multiple producing units. In doing so, we make it possible to better assess the impact of practices such as tightening the genetic line or employing a sophisticated sorting regime on the potential profits of an all-in, all-out production system.

Theoretical Model

The first step in developing the theoretical model is the determination of an appropriate production function. The use of a Gompertz sigmoidal curve to describe potential growth in swine has proved useful (Whittemore, p. 56). The curve to give weight W_t at time t is given by

$W_t = Ae^{-be^{-kt}}$ where A is the upper asymptotic weight, k is a growth constant, and b is a time scale parameter. However, Parks (p. 35) points out that this form makes the determinations of A and k biased. Therefore, we follow the suggestion of Parks and use the following modification of the Gompertz function as a model for potential growth.

$$W_t = A \left(\frac{W_o}{A} \right) e^{-kt} \quad (1)$$

where W_o is defined to be the initial weight and t is defined to be the time that has elapsed since the initial weight was observed. Then, as $t \rightarrow \infty$, $W_t \rightarrow A$ and at $t = 0$, $W_t = W_o$. The parameter $k > 0$ serves as a shape parameter that influences the slope, curvature, and point of inflection of the sigmoidal curve.

Given output as a function of time as described in equation (1), we can then derive the marginal physical product with respect to time as a function of time

$$\text{MPP}(t) = \frac{\partial W_t}{\partial t} = -k \cdot \ln\left(\frac{W_o}{A}\right) \cdot A \cdot \left(\frac{W_o}{A}\right)^{e^{-kt}} \cdot e^{-kt} \quad (2)$$

or as a function of weight

$$\text{MPP}(W_t) = \frac{\partial W_t}{\partial t} = -k \cdot W_t \cdot \ln\left(\frac{W_t}{A}\right). \quad (3)$$

Note that for all $t < \infty$, we have $0 < W_t < A$. Therefore, the MPP is always positive.

Also, the second derivative

$$\frac{\partial^2 W_t}{\partial t^2} = \frac{\partial \text{MPP}(W_t)}{\partial t} = k^2 \cdot W_t \cdot \ln\left(\frac{W_t}{A}\right) \cdot \left[1 + \ln\left(\frac{W_t}{A}\right)\right]$$

is negative precisely when

$$1 + \ln\left(\frac{W_t}{A}\right) > 0$$

or

$$W_t > A \cdot e^{-1}. \quad (4)$$

Therefore, the production function (1) is characterized by positive but diminishing marginal returns with respect to time whenever relationship (4) holds.

Furthermore, to analyze the concavity of the MPP curve, we calculate

$$\frac{\partial^3 W_t}{\partial t^3} = \frac{\partial^2 \text{MPP}(W_t)}{\partial t^2} = -k^3 \cdot W_t \cdot \ln\left(\frac{W_t}{A}\right) \cdot \left[1 + 3 \cdot \ln\left(\frac{W_t}{A}\right) + \left[\ln\left(\frac{W_t}{A}\right)\right]^2\right]$$

which is negative precisely when

$$1 + 3 \cdot \ln\left(\frac{W_t}{A}\right) + \left[\ln\left(\frac{W_t}{A}\right)\right]^2 < 0$$

or

$$0.0729 \cdot A \cong A \cdot e^{\left(\frac{-3-\sqrt{5}}{2}\right)} < W_t < A \cdot e^{\left(\frac{-3+\sqrt{5}}{2}\right)} \cong 0.6825 \cdot A. \quad (5)$$

Therefore, under the assumption of a constant output price P_w , the marginal value product curve given by

$$\text{MVP}(W_t) = P_w \cdot \text{MPP}(W_t) = -k \cdot P_w \cdot W_t \cdot \ln\left(\frac{W_t}{A}\right) \quad (6)$$

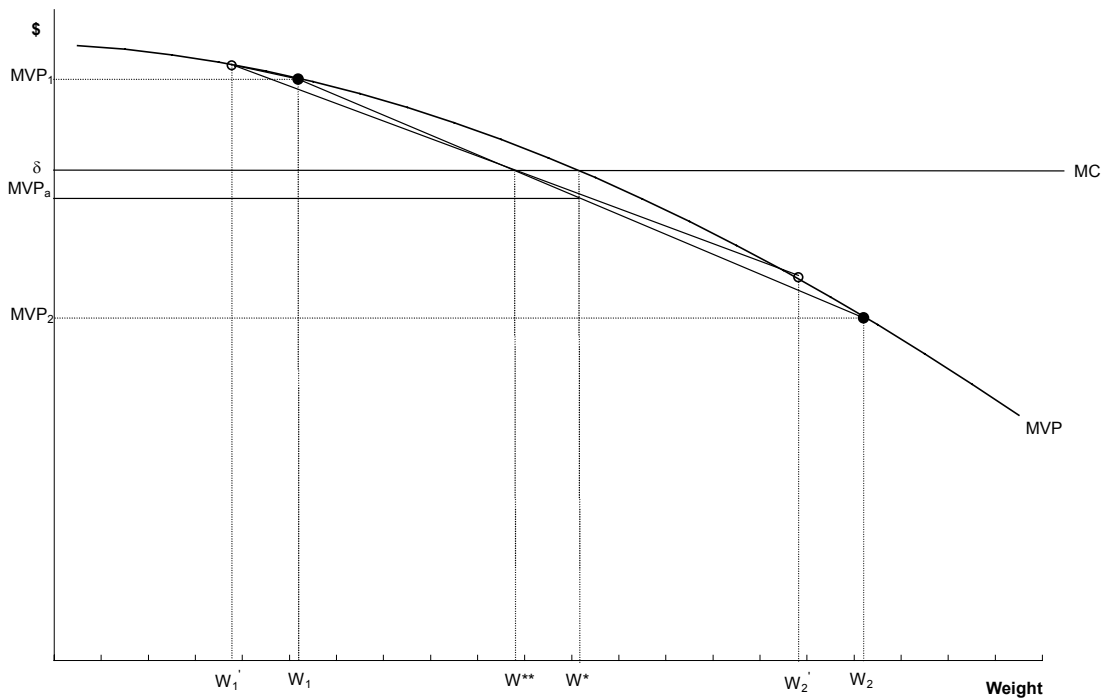
is concave over the weight regions indicated by relationship (5) and convex otherwise.

Jensen's Inequality

To maximize profits from the production of a single animal, we simply feed the animal until the marginal value product equals the marginal cost. Let the unit of time t be days and start with a simplified assumption that the marginal cost is represented by a constant δ that captures the daily cost of feeding the animal. Since Ostwald (1883), physicists and chemists have been studying differential equations of the type in equation (3) (Parks). That is, the rate of change in output W with respect to the independent variable t is uniquely related to the value of W at that t . Nelder (1962) was among those to argue that this is more likely to lead to natural laws of nature than differential equations of the form expressed in equation (2). The argument is that more fundamental information can be gained by comparing marginal products at the same value of output W than at the same value of input t . We adopt this concept in using equation (3) together with a constant output price P_w to produce figure 1 where the marginal value product is expressed as a function of weight.

For a single animal with the marginal curves depicted in figure 1, the profit maximizing weight to terminate production is represented by W^* where $MVP=MC$. Now, assume there are two animals in a pen that are to be marketed together. Let their weights be represented by W_1 and W_2 with $W^* = (W_1 + W_2)/2$. By Jensen's Inequality (Mittlehammer, p. 120), we know that the average of the marginal value products (MVP_a) will be less than the marginal value product of the average weight ($MVP(W^*) = \delta$) over any concave region of the marginal value product curve. In the case of maximizing profits for the pen marketed together, it is the average of the marginal value products that we wish to equate to the constant δ representing the average of the marginal costs. Therefore, as figure 1 indicates, with two animals in the pen, profits are maximized by shifting the market weight to the left. The result is a lower market weight for each animal (W_1' and W_2' , respectively) and a lower average weight W^{**} .

Figure 1



The magnitude of the shift and its subsequent effect on market timing decisions will be influenced by two things, the curvature of the marginal value product curve and the distribution of the animal weights. In (5), we determined that, under the assumption of a constant output price, the MVP curve (6) would be concave when the weight is between 0.07294 and 0.68254. Whitemore (p. 6) points out that, “prime meat is found from pigs slaughtered between 30% and 60% of mature size.” One can conclude that, in the case of swine, it is likely that the MVP curve will be in the latter stages of concavity around the profit maximizing market weight.

In terms of the effect of the distribution, we can expect all symmetric distributions lying within the concave region to behave similar to the two animals depicted in figure 1. Obviously, the larger the standard deviation of the distribution, the larger the difference between δ and MVP_a . Thus, the degree of dispersion will affect the magnitude of the shift from W^* to W^{**} . If the distribution is asymmetric, we can expect W^* to lie closer to either W_1 or W_2 where W_1 and W_2 represent the minimum and maximum weights, respectively, in the distribution. Thus, an asymmetric distribution will likely decrease the difference between δ and MVP_a . If the distribution is not contained within the concave region of the MVP curve, then we can expect to see a further decrease in the difference between δ and MVP_a with the possibility existing that δ could be less than MVP_a .

Increasing Marginal Costs

With regards to marginal cost, we have limited ourselves to estimating the daily cost of feed. In figure 1, we naively assumed this constant at δ . This served its purpose as a simplifying assumption in the above exposition but, in reality, the daily cost of feeding an animal grows with the size of the animal. One of the “laws of animal science” is the long held belief that to

maintain body weight, animals should be fed in proportion to their “metabolic” body size $W^{0.75}$ (Parks; Kleiber). Therefore, a function of the form $F = aW^{0.75}$ seems appropriate where F is daily feed intake, W is the weight of the animal, and a is some constant.

Whittemore (p. 589) points out that most empirical estimates of feed intakes of pigs of various weights involve pigs growing positively. He suggests a value for a between 0.09 and 0.11 when the weight units are measured in kilograms and the pigs are being fed under commercial conditions. Adopting the lower bound and converting to English units leaves us with a naive but practical formula

$$F = 0.20W^{0.75} \quad (7)$$

to represent pounds of daily feed intake, F , as a function of weight, W . If we assume a constant positive feed price P_f per pound of feed, then the marginal cost with respect to time, representing the cost of feeding the animal another day, can be written as a function of weight

$$MC(W_t) = P_f \cdot F = 0.20 \cdot P_f \cdot W_t^{0.75}. \quad (8)$$

Examining the characteristics of the marginal cost curve, we first note the obvious that (8) is positive for all positive values of W_t . Second, we note that the marginal cost with respect to time is monotonically increasing since

$$\frac{\partial[MC(W_t)]}{\partial t} = -0.15 \cdot P_f \cdot k \cdot W_t^{0.75} \cdot \ln\left(\frac{W_t}{A}\right) > 0$$

for all $0 < W_t < A$. Finally, we analyze the concavity of the marginal cost curve by calculating

$$\frac{\partial^2[MC(W_t)]}{\partial t^2} = 0.15 \cdot P_f \cdot k^2 \cdot W_t^{0.75} \cdot \ln\left(\frac{W_t}{A}\right) \cdot \left[1 + 0.75 \cdot \ln\left(\frac{W_t}{A}\right)\right]$$

which is positive when

$$1 + 0.75 \cdot \ln\left(\frac{W_t}{A}\right) < 0$$

or

$$W_t < A \cdot e^{-\frac{4}{3}} \cong 0.2636 \cdot A. \quad (9)$$

Therefore, the marginal cost curve is convex whenever relationship (9) holds and concave otherwise. Applying Whittemore's observation from above, it is then likely that the marginal cost curve will be concave over the weight regions in which marketing of swine occurs. Jensen's Inequality then presents us with a situation where we can expect the average of the marginal costs to be less than the marginal cost of the average.

Figure 2 depicts our situation with two animals weighing W_1 and W_2 , respectively. With both the MC and MVP curves being concave over the applicable region, we can expect to have a situation where $MVP_a < MVP(W^*)$ and $MC_a < MC(W^*)$ where

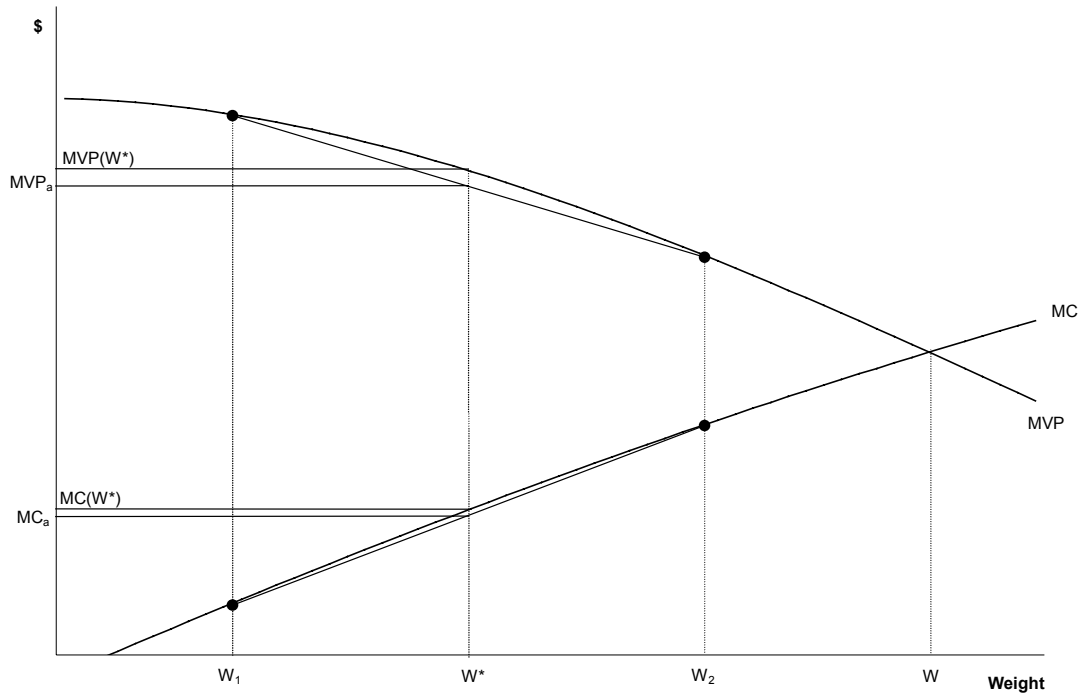
$$MVP_a = 0.5 MVP(W_1) + 0.5 MVP(W_2)$$

and

$$MC_a = 0.5 MC(W_1) + 0.5 MC(W_2).$$

The net effect this has on the marginal profit will be determined by the relative curvature of the two curves over the applicable region.

Figure 2



Intuitively, we might expect the situation as it is depicted in figure 2 where the curvature of the MVP curve is more pronounced than that for the MC curve. Then, the average marginal profit at W^* ,

$$\pi_a = MVP_a - MC_a \quad (10)$$

would be less than the marginal profit of the average,

$$\pi(W^*) = MVP(W^*) - MC(W^*). \quad (11)$$

This would lead us to the conclusion that the average marginal profit would reach zero prior the weight W at which the marginal profit of the average is zero. As in the case of constant marginal costs explored earlier, we can expect profits for this pen of two animals with increasing marginal costs to be maximized at an average market weight somewhere to the left of W . However, the counter balancing effect of a concave marginal cost curve will make that shift less pronounced than the shift from W^* to W^{**} indicated in figure 1.

Empirical Application

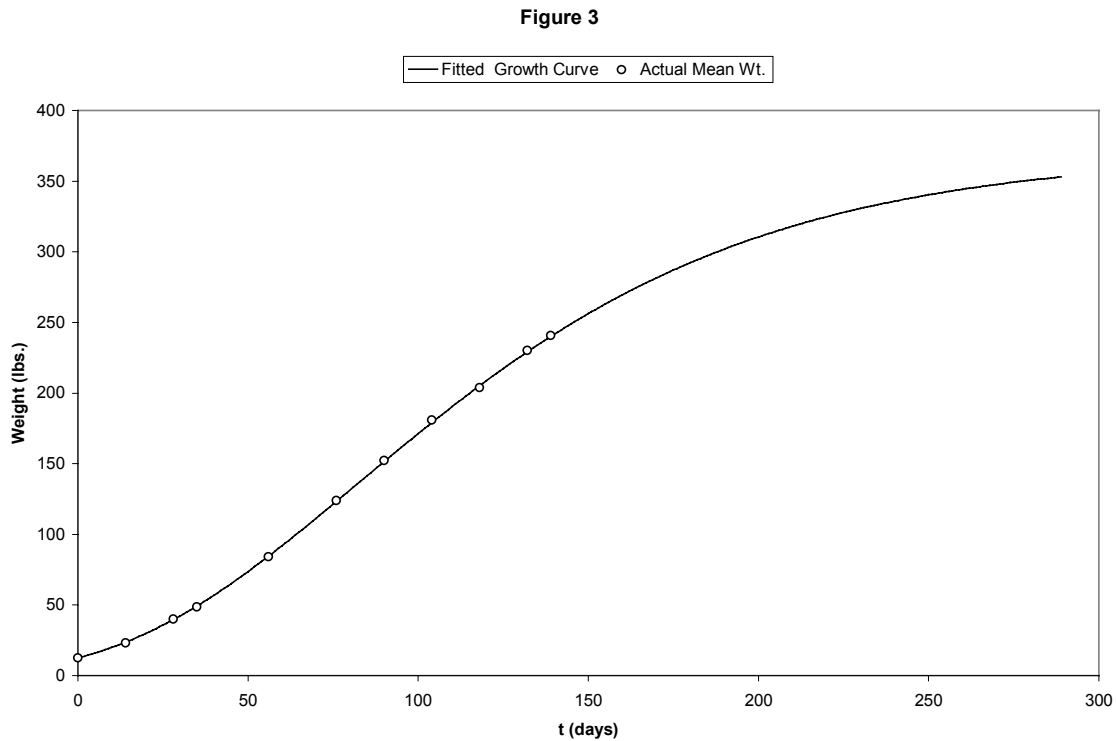
A panel data set consisting of twelve weight observations individually identified for 350 hogs every 1-3 weeks from 14 days of age to 171 days of age was obtained from Purdue University. The swine in the data set are all gilts taking part in a Purdue University study on antibiotic treatments. Two different genotypes are represented in the data and the pigs are divided into 32 pens of approximately 10-12 pigs per pen. At any point in time, each pen is receiving the same ration fed ad libitum. Exactly half of the animals are given an antibiotic treatment. However, the selection of the treatment animals is done by random draw at the beginning of the trial and again at the beginning of the finishing phase. Therefore, the animals fall into one of four categories concerning antibiotic treatments: (1) treatment in both the nursery and finishing phase, (2) treatment in the nursery and no treatment in finishing, (3) no treatment in the nursery and treatment in finishing, or (4) no treatment in either the nursery or finishing.

The data set is first analyzed as if one growth path existed for the entire set of 350 hogs. Our data set was plagued by a common problem in animal growth modeling. The fastest growing pigs were marketed prior to the twelfth weight observation resulting in a significant amount of missing data. Including all twelve observations to estimate our model parameters would downwardly bias the peak of the sigmoidal growth curve (Craig and Schinkel). Therefore, the group average from observation twelve was not used in the growth curve estimation.

Using the mean values for the entire group at each of the first eleven observations, we fitted a Gompertz growth curve (1) to the data. This resulted in the model

$$W_t = 370 \left(\frac{12.55}{370} \right) e^{-0.0148t} \quad (R^2 = 0.9999) \quad (1)'$$

as a representation of the growth path of the pen average. The fitted curve from equation (1)' is graphed along with the actual data of mean weights in figure 3.



The growth curve parameters $A = 370$ and $k = 0.0148$ resulting from the estimation of (1)', combine to yield the marginal value product and marginal cost equations

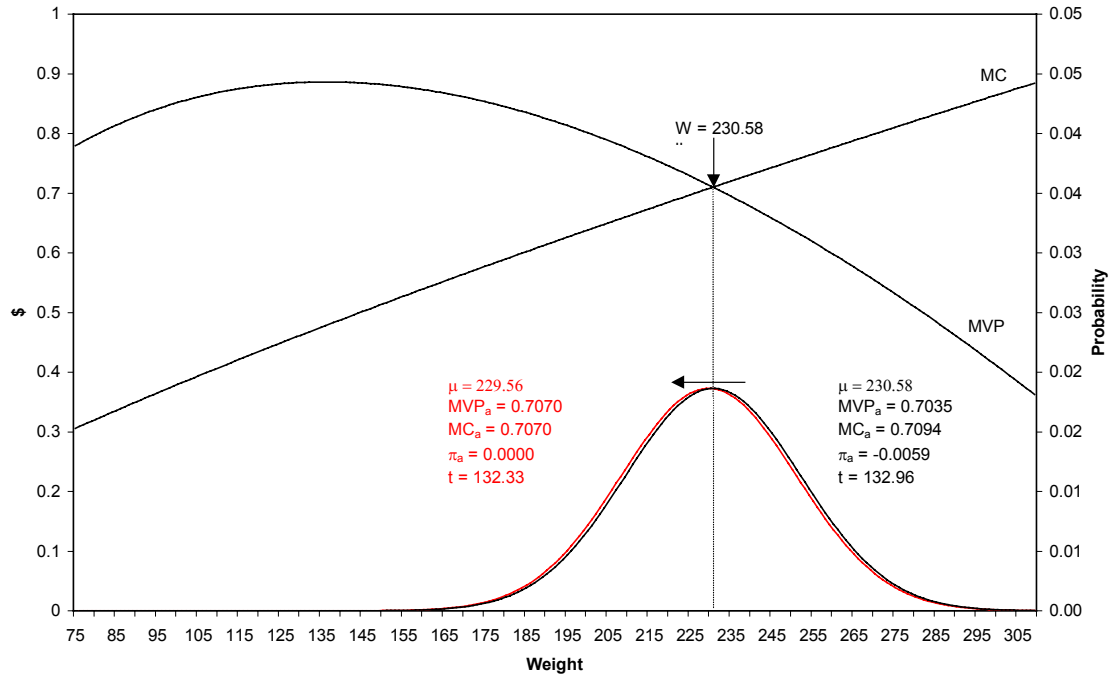
$$MVP(W_t) = -0.006512 \cdot W_t \cdot \ln \left(\frac{W_t}{370} \right) \quad (6)'$$

$$MC(W_t) = 0.012 \cdot W_t^{0.75} \quad (8)'$$

where the output price is assumed constant at $P_w = \$0.44$ per pound and the feed cost is assumed constant at $P_f = \$0.06$ per pound. These are graphed in figure 4. We can solve numerically for

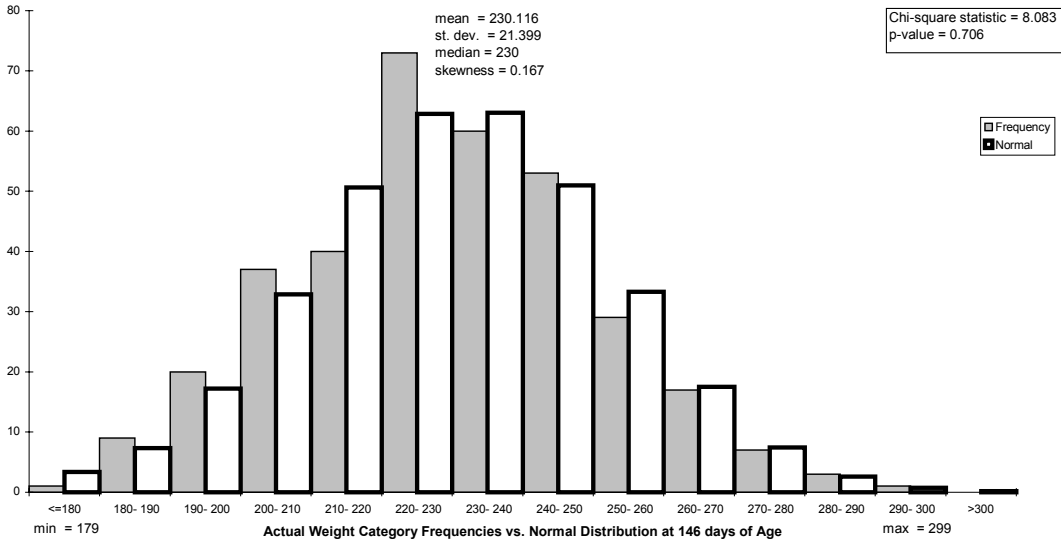
their point of intersection at $W = 230.58$ which represents the profit maximizing market weight for a single average animal. This corresponds to $t = 132.96$ or approximately 147 days of age.

Figure 4



Our data set contains an observation of the actual weights at 146 days of age. A Chi-square analysis provides strong evidence that we cannot reject the null hypothesis that these weights are normally distributed (figure 5). Analysis of weight data at 132 days of age and 153 days of age provided similar evidence of normally distributed weights (p-values of 0.903 and 0.174, respectively). Therefore, we will optimize under the assumption that the animal weights are normally distributed with a mean weight of W_t determined by model equation (1)' and a standard deviation of 21.4.

Figure 5



Profit maximization occurs when the average marginal profit, $\pi_a = MVP_a - MC_a$, is equal to zero. In other words, the optimization problem is to determine the mean weight W_t such that

$$\pi_a = \int_{W=W_o}^{W=A} \pi(W) \cdot N(W|W_t, \sigma) dW = 0 \quad (10)'$$

where $N(W|W_t, \sigma)$ is a normal probability distribution of W with a mean of W_t and a standard deviation of $\sigma = 21.4$. Dillon and Anderson (p. 142) point out that only if the probability distribution is of simple form, such as discrete or triangular, is an algebraic expression such as (10)' conveniently appraised. We were able to appraise it using the symbolic computational package called *Maple* and numerically find the W_t that made equation (10)' hold. However, we found it easier to analysis and conduct a sensitivity analysis on our results by converting the normal distribution in (10)' into a discrete distribution in an *Excel* spreadsheet. The results were identical, to three decimal places, to those obtained in *Maple*. The details of this conversion are

contained in Appendix A and the reader should assume that all results reported here are arrived at using the *Premium Solver for Excel*.

Results

Our results indicate that the optimum mean weight is indeed less than the 230.58 lbs. at which the marginal value product curve intersects the marginal cost curve. Figure 4 shows the implied shift to the left from a mean weight of 230.58 to a mean weight of 229.56 lbs. that is necessary to optimize profits for this group of 350 swine sold as one unit. We calculate $MVP(230.58) = MC(230.58) = 0.7101$, $MVP_a = 0.7035$, and $MC_a = 0.7094$ dollars per day at the point of intersection. The relationship,

$$MVP_a < MC_a < MVP(230.58) = MC(230.58) \quad (12)$$

indicates the concavity of both marginal curves with the marginal value product curve slightly more concave than the marginal cost curve. In fact, the closeness of MC_a to the value of $MC(230.58)$ indicates that the marginal cost curve is nearly linear. Most importantly, however, relationship (12) indicates the nonoptimality of feeding to a mean weight of 230.58 lbs. The fact that, at a mean weight of 230.58 lbs., we have $MVP_a < MC_a$ indicates that animals have been fed past the point of profit maximization. How far past is determined by solving equation (10)' for the optimum mean weight W_t .

When we solve equation (10)', we determine an optimum mean weight of 229.56 lbs. This produces the calculated values $MVP_a = MC_a = 0.7070$, $MC(229.56) = 0.7077$, and $MVP(229.56) = 0.7136$. Again, this indicates the relative concavity of the two curves. However, it also displays the difference between the actual marginal profit, $\pi_a = 0$, and the

perceived marginal profit, $\pi(229.56) = 0.7136 - 0.7077 = 0.0059$, indicated by the average animal.

The final task is to determine what difference this approximately one pound difference in mean weight makes in the market timing decision. Plugging a mean weight of $W_t = 229.56$ into equation (1)' and solving for t yields the optimal market timing of $t = 132.33$ days. This represents an approximately seven-tenths of day difference in the market timing obtained at a mean weight of 230.58 lbs. In other words, at $t = 132.33$ days, the marginal profit to be gained by feeding the pen of animals one more day is zero. Obviously, we would not expect the seven-tenths of a day difference between optimal market timing for the group and optimal market timing for the average animal to significantly impact profits. However, one can envision where relaxation of some of the restrictions of this model as it pertains to swine could lead to situations where this gap is more significant.

Sensitivity Analysis

Our baseline example for hogs turns out to show that market timing based on the average size is probably a sufficient decision rule. However, how would the market timing change for a pen that is more heterogeneous such as we commonly see with cattle or with smaller operations? One way to represent more heterogeneity is by expanding the variance in our model. In our hog example, we assumed the standard deviation was constant at 21.4. Table 1 summarizes the results if we assume the standard deviation is held constant at 15, 20, 25 or 30 lbs.

Table 1:

Standard Deviation	Optimal Mean Weight	$MVP_a = Mc_a$	t
15	230.076	0.7086	132.65
20	229.682	0.7075	132.41
25	229.229	0.7056	132.13
30	228.903	0.7018	131.93

Note that even with a standard deviation of 30 lbs., the difference in the optimal market timing of $t = 131.93$ and market timing determined by the average animal of $t = 132.96$ is only about a day. Also, note that the change in optimal market timing as the standard deviation moves from 20 to 25 lbs. is greater than the change in optimal market timing as the standard deviation moves from 25 to 30 lbs. This indicates the influence of the convex portion of the marginal value product curve as more of the weight distribution moves beyond 0.684 which is approximately 252 lbs. As the distribution widens, weights distributed in the convex portion of the curve will counter balance the influence of the weights distributed in the concave portion of the curve. This offsetting effect will limit the size of the downward shift made possible by an expanding standard deviation.

Summary and Conclusions

This research provides useful insight into the optimal market timing for pens of livestock. In the presence of decreasing marginal returns, the marginal value associated with the average output level is not representative of the average marginal value product for the pen. The degree of this separation is dependent upon the degree of concavity in the marginal value product curve and the degree of dispersion associated with the distribution of output levels existing in the pen. This separation may be partially offset by an analogous concavity in the marginal cost curve associated with the decreasing marginal increase in the cost of feeding a growing animal. The net effect can be expected to be such that the optimal market time for the pen taken as a whole arrives prior to the optimal market time for the average sized animal in the pen.

Our empirical application to swine verified our theoretical construct but provided an insignificant difference in the optimal market timing. Therefore, in the case of the swine

industry, one can conclude that marketing groups of hogs based on the group average appears to be an economically sound technique.

The insignificance of the differential in market timing for our baseline case study data is not totally unexpected. The swine industry has homogenized the genetics to the point that few distinguishable breeds exist in the feeding sector. Therefore, one would expect the average pig to be very representative of the group. Furthermore, the timing of the optimum market weight within the growth cycle of a pig is such that the concavity of the marginal curves is minimal. However, the theoretical construct of our model appears to be sound. Future research applying the principals of our model to more diverse production populations may likely yield significant insights into market timing decisions.

Appendix A

The normal distribution in equation (10)' was converted into discrete form by calculating

$$N_w(W_t, \sigma) = \int_{W=W-0.5}^{W=W+0.5} N(W|W_t, \sigma) dW$$

for $W = W_o$ to A . Then the average of the marginal value products and marginal costs can be calculated as

$$MVP_a(W_t) = \sum_{W=W_o}^{W=A} MVP(W) \cdot N_w(W_t, \sigma)$$

$$MC_a(W_t) = \sum_{W=W_o}^{W=A} MC(W) \cdot N_w(W_t, \sigma)$$

with the marginal profit

$$\pi_a(W_t) = MVP_a(W_t) - MC_a(W_t).$$

References:

- Boland, M.A., P.V. Preckel, and A.P. Schinkel. Optimal Hog Slaughter Weights Under Alternative Pricing Systems. *J. Agr. and Applied Econ.* 25(December, 1993): 148-163.
- Brethour, J.R. *Using Ultrasound Technology to Increase Cattle Feeding Profits*. Kansas Agricultural Experiment State Report of Progress No. 570: Kansas State University, 1989.
- Chavas, Jean-Paul, James Kliebenstein, and Thomas D. Crenshaw. Modeling Dynamic Agricultural Production Response: The Case of Swine Production. *Amer. J. Agr. Econ.* 67(1985): 636-646.
- Crabtree, J.R. Feeding Strategy Economics in Bacon Pig Production. *J. Agr. Econ.* 28(1977): 39-54.
- Craig, Bruce A. and Allan P. Schinkel. Nonlinear Mixed Effects Model for Swine Growth. *The Professional Animal Scientist.* 17(2001): 256-260.
- Dillon, John L. and Jock R. Anderson. 1990. *The Analysis of Response in Crop and Livestock Production*. Pergamon Press, Oxford, England.
- Heady, Earl O., Steven T. Sonka, and Fred Dahm. Estimation and Application of Gain Isoquants in Decision Rules For Swine Producers. *J. Agr. Econ.* 27(1976): 235-42.
- Kleiber, M. 1961. *The Fire of Life*. Wiley, New York, NY.
- Mittlehammer, Ron C. 1996. *Mathematical Statistics for Economics and Business*. Springer-Verlag, New York, NY.
- Parks, John R. 1982. *A Theory of Feeding and Growth of Animals*. Springer-Verlag, New York, NY.

Pringle, Lew. 2000. Operations Research: The Productivity Engine. *ORMS Today*. (June) 27(3): 28-31.

Smith, G.C., J.W. Savell, H.G. Dolezal, T.G. Field, D.R. Gill, D.B. Griffin, D.S. Hale, J.B. Morgan, S.L. Northcutt, and J.D. Tatum. *Improving the Quality, Consistency, Competitiveness and Market-Share of Beef: Executive Summary*. National Beef Quality Audit. Colorado State University, Oklahoma State University and Texas A&M University, December 1995.

Whittemore, Colin. 1993. *The Science and Practice of Pig Production*. Longman Scientific & Technical, Longman Group UK Ltd, Essex, England.