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IDENTIFICATION OF ECONOMIES OF SCOPE IN A STOCHASTIC PRODUCTION ENVIRONMENT

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ABSTRACT

This paper extends the definition of economies of scope to multioutput firms that face an uncertain production environment. Identification of economies of scope in this environment, however, requires separability assumptions on the technology. These identification restrictions are demonstrated in the paper, and for each identification restriction the definition of economies of scope is generalized to the case of uncertain production and risk aversion.

Key Words: Production Risk, Multioutput, Production, Identification.

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Rita E. Curtis and Camilo Sarmiento*

INTRODUCTION

Under certainty, the economic basis for multioutput firms is economies of scope (Panzar and Willig). An input joint technology (Hall) underpins economies of scope, which exists if the sum of the costs from producing multiple outputs individually exceeds the cost of producing the same outputs jointly. The existence of shared inputs across outputs or fixed costs are common sources of economies of scope (Panzar and Willig, Gorman, Panzar) while the absence of economies of scope in production is associated with a cost function that is strongly separable with respect to output.

To date, economies of scope has not been defined under production uncertainty. Seminal research on stochastic technologies instead initially focused on separating preferences from the technology for the single output case using an ex ante cost function. For example, Pope and Chavas show that modeling all moments of the distribution of output will separate preferences from the technology while Chambers and Quiggin show that modeling all states of nature achieve the same result.

In order to define economies of scope for stochastic technologies, it is necessary to first extend the definition of the single output stochastic technology to joint and nonjoint stochastic technologies. Given properties of a multioutput stochastic technology, this paper then demonstrates that the single output approach for separating preferences from the technology is not sufficient for identification of properties of a multioutput technology using an ex ante cost function. In particular, if the effect of the stochastic factor in the production environment is correlated across outputs (e.g., the effect of weather in multiple crops), stochastic dependence results and it may not be possible to identify input nonjointness in an ex ante cost function and, thus, the definition of economies of scope is not generally identified in a stochastic production environment.

Given the identification problem, the paper explores restrictions on stochastic technologies that allow for both common random effects and identification of economies of scope. For example, structure from multiplicative risk and a restricted form of multioutput Just-Pope production function are shown to identify economies of scope in an *ex ante* cost function. More generally, if input usage depends only on observed output and the own moments of the joint distribution of output, then economic of scope can be identified. However, if input usage is not sufficiently represented by this information (e.g., covariances are also needed) then identification in an *ex ante* cost function fails.

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For different identification restrictions, the main propositions of the paper parallel Panzar and Willig's results on economies of scope under certainty, i.e., sub-additivity of the *ex ante* cost functions associated with each output is a sufficient condition for multioutput firms in a stochastic production environment. Importantly, both the sub-additivity conditions and the identification restrictions on an *ex ante* cost function are testable hypotheses.

GENERAL DEFINITIONS OF JOINT AND NONJOINT STOCHASTIC TECHNOLOGIES

In a deterministic production environment, a multioutput technology can be defined by

$$V(\mathbf{y}) = \{\mathbf{x}\text{: } T(\mathbf{y}, \mathbf{x}) \leq 0\} \tag{1a}$$

where $V(\mathbf{y})$ is the set of inputs that fulfill the feasibility criterion defined by the transformation function $T(\cdot)$. In particular, if $\mathbf{x} \in V(\mathbf{y})$, then the output vector $\mathbf{y} = (y_1,\ldots y_m) \in Q$ can be produced by inputs, $\mathbf{x} = (x_{11},\ldots x_{1n},\ldots x_{m1},\ldots x_{mn})$ where $\mathbf{x}$ denotes total input usage and x_{ik} denotes the k^{th} input used in the production of the i^{th} output.

A technology is input nonjoint if total input usage across outputs is additive, i.e., $x_k = \sum_{i=1}^{m} x_{ik}$ for all k =1,...n inputs. That is, (1a) can be represented as

$$V(\mathbf{y}) = \sum_{i=1}^{m} V(y_i) \tag{1b}$$

where

$$V(y_i) = \{\mathbf{x}_i\text{: } T(y_i, \mathbf{x}_i) \leq 0\},$$

and $\mathbf{x}_i = (x_{i1},\ldots x_{in})$.

Following Chambers and Quiggin, the multioutput stochastic technology corollary to (1) may be represented for each realization of the state of nature s as

$$V(\mathbf{y}_s) = \{\mathbf{x}\text{: } T_s(\mathbf{y}, \mathbf{x}) \leq 0\}. \tag{2a}$$

and

$$V(\mathbf{y}_s) = \sum_{i=1}^{m} V(y_{is}) \tag{2b}$$

where

$$V(y_{is}) = \{\mathbf{x}_i: T_s(y_i, \mathbf{x}_i) \leq 0\},$$

where s = {1,…r} are states of nature, $\mathbf{y_s} = (y_{1s},\ldots y_{ms})$ is the vector of output produced in state s and other variables are defined as above. Hence, as in the certainty case, the technology in (2b) is input nonjoint if total deterministic input usage across outputs is additive for all states of nature, i.e., $x_k = \sum_{i=1}^{m} x_{ik}$ for all k = 1,…n inputs.

Each state of nature in (2) has an associated probability function conditional on the deterministic input usage, $P(\mathbf{y_s}|\mathbf{x})$, that satisfies the properties of a probability space, i.e.,

$$P(\mathbf{y_s}|\mathbf{x}) \geq 0 \text{ and } \sum_{s=1}^{r} P(\mathbf{y_s}|\mathbf{x}) = 1. \tag{3}$$

The corresponding marginal probability of y_{is} is captured in the probability space

$$P_i(y_{is}|\mathbf{x}) \geq 0 \text{ and } \sum_{s=1}^{r} P_i(y_{is}|\mathbf{x}) = 1. \tag{4}$$

Consistent with (2b), the marginal probability of y_{is} in a stochastic input nonjoint technology is

$$P_i(y_{is}|\mathbf{x}) = P_i(y_{is}|\mathbf{x}_i). \tag{5}$$

This condition implies that inputs allocated to output j, where $j \neq i$, do not affect the distribution function of output i.

Conserving the property of stochastic dependence in the definition of input nonjointness is relevant in agricultural applications and other resource based industries. For example, stochastic factors such as rainfall are not allocable across outputs but, importantly, may affect both the input joint and input nonjoint technology in a similar manner. Alternatively stated, the effect of the stochastic factor is correlated across outputs for both joint and nonjoint technologies.[1] Mathematically, if $y_{is} = y_{is}(\mathbf{x}_i)$ and $y_{js} = y_{js}(\mathbf{x}_j)$ and s is a state of nature that affects both y_{is} and y_{js} then $\mathrm{Cov}(y_{is}, y_{js}) \neq 0$ regardless of whether (5) holds. Indeed, the definition of an input nonjoint technology in (2b) and (5) is sufficiently general to allow for common random effects, i.e., $P(\mathbf{y_s}|\mathbf{x}) \neq \prod_{i=1}^{m} P_i(y_{is}|\mathbf{x_i})$.

[1] Important agricultural economics literature incorporates the fact that the effect of weather may be correlated across crops.

EX ANTE COST FUNCTIONS OF JOINT AND NONJOINT STOCHASTIC TECHNOLOGIES

To demonstrate the implications of stochastic dependence, consider a direct extension of the Pope and Chavas approach to separating preferences from the technology, which would define firm behavior as:

$$\max_{\mathbf{x}} \sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{k=1}^{n} w_k x_k] P[\mathbf{y_s} | \mathbf{x}, \eta] \qquad (6)$$

$$\text{s.t.} \quad \eta = F(\mathbf{x}), \text{ for } i = 1,\ldots,m,$$

where W is initial wealth; w_k is the price of input k, p_i is the price of output i; U is a von Newmann-Morgenstern utility function; and $F(\mathbf{x})$ is the vector of conditional expectations for each of the moments encompassed in η.[2] In particular, to illustrate the source of failure of identification of the property of nonjointness in an *ex ante* cost function, it suffices to consider a special case of (6), i.e.,

$$\max_{\mathbf{x}} \sum_{s=1}^{r} U[W + \sum_{i=1}^{2} p_i y_{is} - \sum_{k=1}^{n} w_k x_k)] P[\mathbf{y_s} | \mathbf{x}, \eta] \qquad (7a)$$

$$\text{s.t.} \quad E(y_i) = f_i(\mathbf{x}_1, \mathbf{x}_2), \ i = 1,2,$$

$$Var(y_i) = h_{ii}(\mathbf{x}_1, \mathbf{x}_2),$$

$$Cov(y_1,y_2) = h_{12}[(\mathbf{x}_1, \mathbf{x}_2)]$$

where $\eta = [E(y_1),E(y_2), Var(y_1), Var(y_2), Cov(y_1,y_2]$, η is treated as given;[3] $f_i(\mathbf{x}_1, \mathbf{x}_2) = \sum_{s=1}^{r} y_{is}P(\mathbf{y_s}| \mathbf{x}_1, \mathbf{x}_2)$; and $h_{ij}(\mathbf{x}_1, \mathbf{x}_2) = [\sum_{s=1}^{r} y_{is}y_{js}P(\mathbf{y_s}| \mathbf{x}_1, \mathbf{x}_2)] - f_i(\mathbf{x}_1, \mathbf{x}_2)f_j(\mathbf{x}_1, \mathbf{x}_2)$.

The representation of (7a) under an input nonjoint stochastic technology is

$$\max_{\mathbf{x}} \sum_{s=1}^{r} U[W + \sum_{i=1}^{2} p_i y_{is} - \sum_{k=1}^{n} w_k x_k)] P[\mathbf{y_s} | \mathbf{x}, \eta] \qquad (7b)$$

---

[2] As in the certainty case, the unconditional input demands can be determined from (6) by first maximizing over $\mathbf{x} = (x_1,\ldots x_n)$ conditional on the moments on the distribution of output, η, and then maximizing with respect to η.

[3] Analogously to the certainty case, η is given in the optimization.

s.t. $E(y_i) = f_i(\mathbf{x}_i)$, i = 1,2;

$Var(y_i) = h_{ii}(\mathbf{x}_i)$; and

$Cov(y_1,y_2) = h_{12}[(\mathbf{x}_1, \mathbf{x}_2)]$

where the restriction that separates preferences from the technology in an *ex ante* cost function is

$$P(\mathbf{y}_s | \mathbf{x}, \eta) = P(\mathbf{y}_s | \eta) \quad (8)$$

where $\eta = [E(y_1),E(y_2), Var(y_1), Var(y_2), Cov(y_1,y_2)]$ is fixed in the optimization. To show this separation, note that the first order conditions of the optimization problem in (7b) given (8) are:

$$\phi w_k = [\lambda_1(\partial h_{11}(\mathbf{x}_i)/\partial x_{ik}) + \lambda_2(\partial h_{22}(\mathbf{x}_i)/\partial x_{ik}) + \lambda_3(\partial h_{12}(\mathbf{x}_i)/\partial x_{ik})],\ \forall\ k; \quad (9a)$$

$$E(y_i) = f_i(\mathbf{x}_i),\ i = 1,2; \quad (9b)$$

$$Var(y_i) = h_{ii}(\mathbf{x}_i); \quad (9c)$$

$$Cov(y_1,y_2) = h_{i2}[(\mathbf{x}_1, \mathbf{x}_2)], \quad (9d)$$

where $\phi = \sum_{s=1}^{r}[\partial U(W_{bs})/\partial W_{bs}]P[\mathbf{y}_s|\eta]$ for $W_{bs} = U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{k=1}^{n} w_k x_k)]$. Yet, the first order conditions in (9) are alternatively represented by normalizing the Lagrange multiplier and input prices as:

$$w_k / w_1 = [(\partial h_{11}(\mathbf{x}_i)/\partial x_{ik}) + (\lambda_2/\lambda_1)(\partial h_{22}(\mathbf{x}_i)/\partial x_{ik}) + (\lambda_3/\lambda_1)(\partial h_{12}(\mathbf{x}_i)/\partial x_{ik})]/ \quad (10a)$$

$$[(\partial h_{11}(\mathbf{x}_i)/\partial x_{i1}) + (\lambda_2/\lambda_1)(\partial h_{22}(\mathbf{x}_i)/\partial x_{i1}) + (\lambda_3/\lambda_1)(\partial h_{12}(\mathbf{x}_i)/\partial x_{i1}],\ \forall\ k \neq 1;$$

$$E(y_i) = f_i(\mathbf{x}_i),\ i = 1,2, \quad (10b)$$

$$Var(y_i) = h_{ii}(\mathbf{x}_i), \quad (10c)$$

$$Cov(y_1,y_2) = h_{i2}[(\mathbf{x}_1, \mathbf{x}_2)]. \quad (10d)$$

Assuming that the conditional moments for all given input allocations are strictly convex sets in (10), it follows that explicit solutions for the optimal conditional input demands exist and equal

$$x_{ik} = x_{ik}[\mathbf{w}, E(y_1), var(y_1), E(y_2), var(y_2), Cov(y_1,y_2)];$$

and

$$x_k = \sum_{i=1}^{m} x_{ik}\,[\mathbf{w}, E(y_1), var(y_1), E(y_2), var(y_2), Cov(y_1,y_2)]\ .$$

The associated *ex ante* cost function is

$$C[\mathbf{w}, E(y_1), var(y_1), E(y_2), var(y_2), Cov(y_1,y_2)].$$

The information in preferences, ϕ, is therefore filtered out by conditioning the optimization problem of maximizing utility if (7) holds.[4] Different from the certainty case, however, equations that solve x_{ik} for all k in the first order conditions in (10) cannot be solved independently for each i because of $Cov(y_1,y_2)$ in (10d). Identification of input nonjointness in an *ex ante* cost function may thus fail even if preferences are separated from technology. This may occur in the presence of stochastic factors such as rain or frost.

More generally, any *ex ante* cost function defined in terms of the covariance matrix of the joint distribution of output (and any other moments) does not normally contain structure that identifies an input nonjoint stochastic technology. In addition, failure of identification using the Chambers and Quiggin approach under a general stochastic technology is shown in Appendix 1. The next sections derive restricted stochastic technologies in which input nonjointness is identified in an *ex ante* cost function.

IDENTIFICATION OF ECONOMIES OF SCOPE

USING SEPARABILITY RESTRICTIONS ON RISK

Case 1. A special case of (8) maps the probability of observed output $\mathbf{y}_s$ directly from the expected value of output and the state of nature associated with the observed output. That is,

$$P[\mathbf{y}_s | \mathbf{x}, E(\mathbf{y})] = P[\mathbf{y}_s | E(\mathbf{y})]. \qquad (11)$$

The structure in (11) only holds if

$$\mathbf{y_s} = E(\mathbf{y}) + \varepsilon$$

$$= f(\mathbf{x}) + \varepsilon$$

where $E(\varepsilon) = E(\varepsilon\ |\mathbf{x}) = 0$. Therefore, (11) represents multiplicative risk (see Pope and Chavas).

[4] To separate preferences from technology, the variance-covariance matrix of the joint distribution of output needs to be included in an *ex ante* cost function if given the mean the joint distribution of output, input affect the variance-covariance matrix of the distribution.

Given (11), solving

$$\max_{\mathbf{x}} \sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{k=1}^{n} w_k x_k)] P[\mathbf{y_s} | \mathbf{x}, E(\mathbf{y})]$$

s.t. $E(y_i) = f_i(\mathbf{x}_i), \forall\ i,$

where $P[\mathbf{y_s}|\ \mathbf{x}, E(\mathbf{y})] = P[\mathbf{y_s}|\ E(\mathbf{y})]$, then reduces to

$$w_k / w_1 = [\partial f_i(\mathbf{x}_i)/\partial x_{ik}]/[\partial f_i(\mathbf{x}_i)/\partial x_{il}],\ \forall\ i \text{ and } k \neq 1; \text{ and}$$

$$E(y_i) = f_i(\mathbf{x}_i),\ \forall i.$$

Multiplicative risk not only separates preferences from technology but also provides a structure in which input nonjointness can be identified from the *ex ante* cost function. In particular, provided that $E(y_i) = f_i(\mathbf{x}_i)$ is defined on a strictly convex set for all i, the conditional input demands may be uniquely solved from the first order conditions as

$$x_{ik} = x_{ik}[\mathbf{w}, E(y_i)],\ \forall\ i, k,$$

and, thus,

$$x_k = \sum_{i=1}^{m} x_{ik}\ [\mathbf{w}, E(y_i)].$$

The associated *ex ante* cost function is

$$\sum_{k=1}^{n} w_k x_k = \sum_{k=1}^{n} \sum_{i=1}^{m} w_k x_{ik}[\mathbf{w}, E(y_i)] = \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i)] \qquad (12)$$

where C_i is the *ex ante* cost function of output i . If the technology in (12) is nonjoint,

$$C[\mathbf{w}, E(\mathbf{y})] = \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i)];$$

if the technology is joint,

$$C[\mathbf{w}, E(\mathbf{y})] \leq \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i)]. \qquad (13)$$

Proposition 1. Given (11), a sufficient condition for multioutput plants is the sub-additivity condition of (13) (see Appendix 2 for proof).

Proposition 1 states that (13) is a sufficient condition for the existence of multioutput plants. Paralleling the definition obtained by Panzar and Willig for economies of scope under certainty, economies of scope for the stochastic technology with no risk reducing (increasing) inputs requires sub-additivity of the *ex ante* cost functions associated with each output.

Case 2. This case considers weaker separability restrictions on how risk affects the technology but still enable economies of scope to be identified from the *ex ante* cost function in the presence of common random effects. In particular, following Just and Pope, this case allows for the existence of inputs that are risk reducing (increasing). To derive input demands conditional on the mean and variance of output, we consider the nonjoint case

$$\max_{\mathbf{x}} \sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{k=1}^{n} w_k x_k)] \;\; P[\mathbf{y_s}|\ \mathbf{x}, E(\mathbf{y}), \mathrm{diag}\boldsymbol{\Omega}] \tag{14}$$

$$\text{s.t.} \qquad E(y_i) = f_i(\mathbf{x}_i), \qquad \forall\ i.$$

$$\mathrm{var}(y_i) = f_i(\mathbf{x}_i), \qquad \forall\ i.$$

where diag$\boldsymbol{\Omega}$ is the diagonal components of the variance-covariance matrix, Ω, of the distribution of output; and $\mathrm{var}(y_i)$ represents the variance of output i conditional on input usage allocated to i.

Following the analysis of the previous section, if the density function of the objective function in (14) is not a function of input usage,

$$P(\mathbf{y_s}|\ \mathbf{x}, E(\mathbf{y}), \mathrm{diag}\boldsymbol{\Omega}) = P(\mathbf{y_s}|\ E(\mathbf{y}), \mathrm{diag}\boldsymbol{\Omega}), \tag{15}$$

i.e., the producer cannot affect the level of risk associated with a given level of expected output and its variance, then optimal choices from (14) are

$$x_{ik} = x_{ik}[\mathbf{w}, E(y_i), \mathrm{var}(y_i)]$$

and, thus, under input nonjointness

$$x_k = \sum_{i=1}^{m} x_{ik}[\mathbf{w}, E(y_i), \mathrm{var}(y_i)]$$

with the associated *ex ante* cost function defined as,

$$\sum_{k=1}^{n} w_k x_k = \sum_{k=1}^{n}\sum_{i=1}^{m} w_{ik}x_{ik}[\mathbf{w}, E(y_i), \mathrm{var}(y_i)] = \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i), \mathrm{var}(y_i)]. \tag{16}$$

Given (15),

$$C[\mathbf{w}, E(\mathbf{y}), \mathrm{diag}\Omega)] = \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i), \mathrm{var}(y_i)]$$

for the input nonjoint stochastic technology, and

$$C[\mathbf{w}, E(\mathbf{y}), \text{diag}\Omega)] \leq \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i), \text{var}(y_i)] \tag{17}$$

if the stochastic technology is joint.

Proposition 2. Given (15), a sufficient condition for multioutput plants is the sub-additivity condition in (17) (see Appendix 2 for proof).

Once again, paralleling the definition obtained by Panzar and Willig for economies of scope under certainty, Proposition 2 states that economies of scope requires sub-additivity of the *ex ante* cost functions associated with each output. In contrast to (11), (15) implies that input allocations to output i for a given level of expected output is able to affect the variance or riskiness of output j (for $i \neq j$).

Case 3. Restrictions similar to (11) and (15) but defined using other or additional higher moments of each output are also consistent with the sub-additivity condition of the *ex ante* cost function. That is, if

$$P(\mathbf{y_s}|\mathbf{x}, E(y_1),\ldots E(y_1^h),\ldots E(y_m),\ldots E(y_m^h)] = P(\mathbf{y_s}|E(y_1),\ldots E(y_1^h),\ldots E(y_m),\ldots E(y_m^h)], \tag{18}$$

where $E(y_j^h)$ is the h order moment of output j, then there are economies of scope if

$$C[w_1,\ldots w_n, E(y_1),\ldots E(y_1^h),\ldots E(y_m),\ldots E(y_m^h)] \leq \sum_{i=1}^{m} C_i[w_1,\ldots w_n, (y_i),\ldots E(y_i^h)]. \tag{19}$$

Proposition 3. Given (18), a sufficient condition for multioutput plants is the sub-additivity condition in (19). (See Appendix 2 for proof).

Consequently, (19) represents the most general structure that identifies economies of scope. The inability to identify economies of scope occurs if the separability restriction in (18) fails. This means that if an *ex ante* cost function requires inclusion of cross moments of the distribution of output, then it is not generally possible to identify economies of scope.

CONCLUSIONS

While Chambers and Quiggin's assertion that "duality theory applies exactly for stochastic technologies under the same assumptions required for it to apply to nonstochastic technologies" is true, this paper reveals an important distinction between stochastic and nonstochastic technologies. That is, while a well-behaved cost function may be defined for a stochastic technology with common random effects, this paper demonstrates that it is not generally possible to test for economies of scope using this *ex ante* cost function. Instead, separability restrictions must be imposed on the stochastic technology for economies of scope to be identified.

Three restricted specifications of a stochastic technology that enable economies of scope to be identified are defined in Section 4. For each specification, results obtained exactly parallel results obtained by Panzar and Willig for defining economies of scope using a nonstochastic technology. That is, sub-additivity of the individual *ex ante* cost function of each output is a sufficient condition for multioutput firms operating in a stochastic production environment.

Importantly, both the separability restrictions imposed for identification purposes of economies of scope and economies of scope itself are derived as testable hypotheses in this paper. This suggests that future applications involving multioutput stochastic technologies will not be a direct extension of the deterministic case. Instead, these analyses will need to provide evidence that the test for economies of scope is properly identified so that empirical tests of joint production can be properly executed. If the separability restrictions are rejected, it may not be possible to test for economies of scope. Given the importance attached to identifying joint production, this may limit the usefulness of dual cost function analyses in agriculture and other resource based industries subject to stochastic production.

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APPENDIX 1

Quiggin and Chambers show that the cost minimization problem subject to all states of nature is equivalent to the expected utility maximization problem subject to all states of nature. Quiggin and Chambers cost minimization problem under the presence of two outputs, two states of nature $\{s_1, s_2)$, and a stochastic input nonjoint technology is

$$\min_{\mathbf{x}} \sum_{k=1}^{n} w_k x_k$$

s.t. $y_{ij} = f_i(\mathbf{x}_i, s_j)$, for i = 1,2, and j = 1,2.

Solving the first order conditions, and given regularity assumptions in $f_i(\mathbf{x}_i, s_j)$, optimal conditional input demands for input k allocated to output 1 and each state of nature s_j obtains that

$$x_{1k} = x_{1k}[\mathbf{w}, y_{11}, y_{12}, s_1, s_2].$$

However, if the same states of nature affect output 2, then

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} s_1(w, y_{21}, y_{22}) \\ s_2(w, y_{21}, y_{22}) \end{bmatrix}.$$

Therefore,

$$x_{1k} = x_{1k}[\mathbf{w}, y_{11}, y_{12}, y_{21}, y_{22}].$$

Consequently, economies of scope are not generally identified under an unrestricted stochastic technology.

APPENDIX 2

Proof of Proposition 1. Under a monotonic utility function, if $C[w, E(y)] \leq \sum_{i=1}^{m} C_i[w, E(y_i)]$

then $U[W + \sum_{i=1}^{m} p_i y_{is} - C[\mathbf{w}, E(\mathbf{y})] \geq U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{i=1}^{m} C[\mathbf{w}, E(y_i)]$. Because a probability function is always nonnegative it also follows that

$$U[W + \sum_{i=1}^{m} p_i y_{is} - C[\mathbf{w}, E(\mathbf{y})]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y})] \geq U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{i=1}^{m} C[\mathbf{w}, E(y_i)]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y})]$$

and, thus,

$$\sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - C[\mathbf{w}, E(\mathbf{y})]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y})] \geq \sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{i=1}^{m} C[\mathbf{w}, E(y_i)]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y})].$$

Proof of Proposition 2. From the proof of Proposition 1, if $C[\mathbf{w}, E(\mathbf{y}), \text{diag}\Omega] \leq \sum_{i=1}^{m} C_i[\mathbf{w}, E(y_i), \text{var}(y_i)]$, then

$$\sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - C[\mathbf{w}, E(\mathbf{y}), \text{diag}(\Omega)]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y}), \text{diag}\Omega]$$

$$\geq \sum_{s=1}^{r} U[W + \sum_{i=1}^{m} p_i y_{is} - \sum_{i=1}^{m} C[\mathbf{w}, E(y_i), \text{var}(y_i)]\, P[\mathbf{y_s} \mid \mathbf{x}, E(\mathbf{y}), \text{diag}\Omega].$$

Proof of Proposition 3. Analogous to proofs of Propositions 1 and 2.