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# Estimating the Potential Gains from Mergers.

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**Abstract:** We introduce simple production economic models to estimate the potential gains from mergers. We decompose the gains into technical efficiency, size (scale) and harmony (mix) gains, and we discuss alternative ways to capture these gains. We propose to approximate the production processes using the non-parametric Data Envelopment Analysis (DEA) approach, and we use the resulting operational approach to estimate the potential gains from merging agricultural extension offices in Denmark.

**Contents:** 1. Introduction, 2. Literature, 3. Production Models, 4. Measures of Merger Gains, 5. Decomposing Merger Gains, 6. Alternative Decompositions, 7. The Danish Agricultural Extension Services, 8. Final Remarks, References.

**Keywords:** Data Envelopment Analysis, Management, Organization, Mergers.

## 1. Introduction

There are frequent reports of mergers and takeovers in the business press. It seems that mergers play an important role in the restructuring of many sectors. There is also a considerable theoretical literature on the pros and cons of mergers, and a number of studies trying to evaluate the effects of actual mergers ex post.

There are many reasons for merging. The internal or *organizational* reasons include the possibility of exploiting economies of scale, economies of scope, risk sharing,

scarce managerial skills, etc. The external or *market* oriented reasons include the possibility of gaining market power via size or scope or the facilitation of collusive behavior. There are also many obstacles to mergers, including the possible conflicts between different business cultures and public policies directed against the exercise of market power.

The aim of this paper is to focus on the *potential production economic effects* of mergers and in particular to discuss ways of quantifying these. We deviate from previous papers by estimating the potential gains a priori rather than the realized gains ex post. We deviate also by using a multiple inputs multiple outputs production model, as opposed to a more aggregate cost model. Lastly, we deviate by developing a framework where the potential gains can be decomposed and related to different strategic possibilities, viz. improvement of efficiency in individual firms, exchange of inputs and outputs via inter-firm markets, and genuine full scale merger.

We model the multiple inputs multiple outputs production process using an activity analysis or so-called *Data Envelopment Analysis* (DEA) approach. This approach is easy to use and it has proved to be a flexible and powerful tool in a large number of empirical studies. A particular advantage is that it does not require prices on inputs or outputs. Our specific methodological contribution is to show how the effects of mergers can be captured and decomposed by DEA models.

In the application, we use our approach to evaluate the potential gains from mergers of *agricultural extension offices* in Denmark.

The outline of the paper is as follows. In Section 2, we relate our approach to some existing literature. In Section 3, we introduce a class of production models and we review how they can be estimated using DEA. In Section 4, we propose aggregate measures of the potential gains from a merger, and in Section 5, we discuss how these measures can be decomposed into technical efficiency, size and harmony effects. Some alternative decompositions and the pros and cons of our proposed decomposition are discussed in Section 6. The application is discussed in Section 7, and final remarks are given in Section 8.


## 2. Literature

The general economic literature emphasizes how mergers may affect costs and competition, cf. e.g. Perry and Porter(85) and Farrell and Shapiro(90). A recent issue is the strategic value of an early merger, cf. Nilssen and Sørgaard(1998).

The production economic effects of mergers are related to the cost aspect. This includes the technical efficiency of production, the (dis) economies of scale and the (dis) economies of scope. The efficiency of alternative production plans and the economies of scale have received considerable attention in the Data Envelopment Analysis (DEA) literature, cf. the references below. Less focus has been given to the *economies of scope*, an exception being Färe, Grosskopf and Lovell (1994, Sec.10.4). Economies of scope prevail, if joint production is cheaper than separate production.

With two products, this means that $C(y_1,y_2) < C(y_1, 0)+C(0, y_2)$, where $C(y_1, y_2)$ is the minimal costs of producing products 1 and 2 in the amounts $y_1$ and $y_2$. A key feature of most examples is the use of some sharable or quasi-public input in production, say a technological improvement which can be used in one area without affecting its use in another area. In this paper, we use the scope idea in a somewhat broader way. We do not require the merged units to initially produce different types of products, just different product mixes. To emphasize this, we shall talk about economy of harmony or mix.

A related line of literature is concerned with estimating the potential gains from *resource reallocations.* An early paper combining this question with DEA models is Lewin and Morey (1981). They discuss the decomposition of inefficiency in a hierarchical organization into what can be attributed to inefficiencies in the production units with given resources and the misallocation of resources among the units at different levels of the organization. A recent contribution is Bännlund, Chung, Färe and Grosskopf(1998). They estimate the potential gains from allowing certain inputs (pollution permits) to be traded among the firms of an industry. This is done by comparing the profit under an existing distribution of the permits to the profit that is possible when pollution permits can be reallocated. Our approach is related to this approach, except that we do not have a single relevant output like profit (and therefore use Farrell type proportional changes to capture gains). Additionally, we distinguish the gains associated with reallocation among similarly sized firms, called the harmony effect below, and the gains that are available by changing the scale of the firms, called the size effect below.

Kao and Yang(1989) use a DEA model to analyze the reorganization of the national forest districts in Taiwan. The need for fewer and larger districts gives rise to four alternative district-plans. The new districts are merged from the old ones, and the expected performance of the new districts are evaluated by comparing the aggregation of the constituent districts to a district production model based on the original ones. This is similar to the approach we use here to measure the overall potential gain from a merger. The aim is different, however, since Kao and Yang(1989) seek districts that are similar in terms of efficiency. They do so to provide a fair basis for subsequent competition and comparison, and they do not consider alternative ways of accomplishing this. (See Bogetoft(1997) for some similar design considerations in a formal agency setting involving DEA.) We seek a reorganization that maximizes the potential gains and we consider alternative ways to accomplish this ranging from learning from peer units to genuine mergers.

Our approach is also related to the notion of the *structural efficiency* of an industry. According to Farrell(1957,p.262) structural efficiency is "the extent to which an industry keeps up with the performance of its own best firms" and it can be measured by comparing the horizontal aggregation of the industry's firms with the frontier constructed from its individual firms. This is similar to the way we measure the aggregate gain from a merger. A related approach is the average unit approach suggested by Försund and Hjalmarsson(1979). In this approach, the structural efficiency is estimated by taking the average of each type of input and each type of output and by measuring the associated average unit's distance to the frontier. This is similar to our concept of harmony used to capture the mix effects of a merger, except

that we correct for individual inefficiencies first.

It is relevant to observe - as does Farrell(1957,p.261) - that the structural efficiency generally falls short of the average individual efficiency. The reason is the "curvature" of the frontier or more precisely the convexity of the production possibility set. We shall emphasize this below. Here, we just note that this is not a problem in our estimation of the potential gains from a merger. In fact, it is precisely the ability of an average unit to save more inputs or produce more outputs that we consider to be an important source of gains from a merger, which we then call the harmony effect. Caution is called for, however, in ex post studies. Often the cost aspects of a merger is evaluated by comparing the average pre-merger efficiency with the post-merger efficiency of the new unit, cf. e.g. Akhaven, Berger and Humphrey(1997), Chapin and Schmidt(1999) and the references herein. In this case the positive effects of a merger may well be underestimated simply because the target for the merged unit is more demanding than the targets for its constituent parts. Put differently, even though this measure indicates increased slack, the merger may be advantageous from the point of view of production economics. Our harmony index emphasizes this.

Lastly, we note that there are technical resemblances between the merger issue and the role of aggregation in production theory. There is a large literature on the aggregation of variables and the separability of production processes. There are also a few papers explicitly linking these issues to the efficiency measurement problem, cf. Färe and Lovell(1988). Such studies may give conditions on the technology under which the different merger effects can be excluded, see also Bogetoft(1998b). We hope to pursues this issue in later research.

It is clear that one cannot predict or prescribe mergers based solely on the potential market economic or production economic gains. Mergers involve several issues that are hard to capture in a formal model, including the similarity of the business cultures in the merged units. Subsequent to the application described below, several managers contacted us to discuss their merger possibilities. They consistently emphasized the importance of such additional factors.

## 3. Production Models

Consider the case where each of n Decision Making Units (DMUs), $i \in I=\{1, 2, \ldots, n\}$, transforms p inputs to q outputs. Let $x^i = (x_1^i, \ldots, x_p^i) \in \Re_0^p$ be the inputs consumed and $y^i = (y_1^i, \ldots, y_q^i) \in \Re_0^q$ the outputs produced in DMU$^i$, $i \in I$. Also, let T be the production possibility set

$$T = \{(x,y) \in \Re_0^{p+q} | \quad x \text{ can produce } y\}$$

and let $x \rightarrow P(x)$ and $y \rightarrow L(y)$ be the associated production and consumption correspondences

$$P(x) = \{y | (x,y) \in T\} \qquad L(y) = \{x | (x,y) \in T\}$$

4

Some regularity assumptions are usually imposed on T. Classical assumptions are that for all $x'$, $x'' \in \Re_0^p$ and $y'$, $y'' \in \Re_0^q$ we have

A1 *disposability*: $(x',y') \in T$ and $x'' \geq x'$ and $y'' \leq y' \Rightarrow (x'',y'') \in T$

A2 *convexity*: T convex

A3 *s-return to scale*: $(x', y') \in T \Rightarrow k(x', y') \in T$ for $k \in K(s)$

where s = "crs", "drs", "vrs", or "irs" corresponding to constant, decreasing, varying or increasing return to scale, and where $K(crs) = \Re_0$, $K(drs) = [0,1]$, $K(vrs) = \{1\}$, and $K(irs) = [1, +\infty)$ respectively. A less common but very relevant assumption here is the replicability or (super) additivity assumption that for all $x'$, $x'' \in \Re_0^p$ and $y'$, $y'' \in \Re_0^q$

A4 *additivity*: $(x',y') \in T$ and $(x'',y'') \in T \Rightarrow (x'+x'',y'+y'') \in T$

From an applied point of view, we believe that the additivity assumption has advantages over the scaling and the convexity assumptions typically adhered to in microeconomic textbooks. The appeal of the additivity assumption is straightforward. If one DMU produces y' using x' and another produces y" using x', a unit with inputs x'+x" should be able to produce at least y'+y", since it can simply operate as two independent divisions imitating the original ones. The convexity assumption lacks this "peer group" or "proved by way of examples" rationale. A convex combination is an addition of potentially artificial units derived by down-scaling feasible ones.

Given a technology, efficiency has to do with the ability to reduce inputs without affecting outputs or to increase outputs without requiring more inputs. In the case of multiple inputs and outputs, the efficiency of a DMU, say DMU, is often measured by the so-called Farrell(1957) measures

$$E^i = Min\{E \in \Re_0 \mid (Ex^i, y^i) \in T\} \quad or \quad F^i = Max\{F \in \Re_0 \mid (x^i, Fy^i) \in T\}$$

where $E^i$ is the maximal contraction of all inputs and $F^i$ is the maximal expansion of all outputs that are feasible in T.

In many applications, the underlying production possibility set T is unknown. The Data Envelopment Analysis (DEA) approach can be used to model and evaluate productive units in such cases. For a text-book introduction to DEA, see Charnes, Cooper, Lewin and Seiford (1994). Assuming that $x^i = (x_1^i, ..., x_p^i) \in \Re_0^p$ are the inputs actually consumed and $y^i = (y_1^i, ..., y_q^i) \in \Re_0^q$ are the outputs actually produced in DMU, $i \in I$, the DEA approaches estimate T from the observed data points and evaluate the observed productions relative to the estimated technology.

The estimate of T, the empirical reference technology T* with correspondences P*(.) and L*(.), is constructed according to the *minimal extrapolation* principle: T* is the smallest subset of $\Re_0^{p+q}$ that contains the actual production plans $(x^i, y^i)$, $i \in I$, and

satisfies certain technological assumptions specific to the given approach.

The (relative) efficiency of DMU$^j$ may then be measured in input or output space by using the Farrell-measures above with T* substituted for T.

Different DEA models invoke different assumptions about the technology. The original <u>c</u>onstant <u>r</u>eturns to <u>s</u>cale (crs) DEA model proposed by Charnes, Cooper and Rhodes (1978, 1979) assumes A1, A2 and A3(crs) while the <u>d</u>ecreasing <u>r</u>eturns to <u>s</u>cale (drs) and (local) <u>v</u>ariable <u>r</u>eturns to <u>s</u>cale (vrs) models developed by Banker (1984) and Banker, Charnes and Cooper (1984) appeal to A1, A2 and A3(drs) and A1, A2 and A3(vrs), respectively. It is easy to see, cf. e.g. the references above, that A1, A2 and A3(s) lead to the empirical reference technology

$$T^*(s) = \{(x, y) \in \Re_0^{p+q} \,|\, \exists\, l \in \Re_0^n : x \geq \textstyle\sum_i \lambda^i x^i,\ y \leq \textstyle\sum_i \lambda^i y^i,\ \lambda \in \Lambda(s)\}$$

where $\Lambda(crs) = \Re_0^n$, $\Lambda(drs) = \{\lambda \in \Re_0^n \,|\, \sum_i \lambda^i \leq 1\}$ and $\Lambda(vrs) = \{\lambda \in \Re_0^n \,|\, \sum_i \lambda^i = 1\}$.

The assumptions A1-A3 have been relaxed in the <u>f</u>ree <u>d</u>isposability <u>h</u>ull (fdh) model used by Deprins, Simar and Tulkens (1984), and the <u>f</u>ree <u>r</u>eplicability <u>h</u>ull (frh) model briefly proposed in Tulkens(1993). The fdh model invokes only A1 and T*(fdh) therefore has the structure above with $\Lambda(fdh) = \{\lambda \in \Re_0^n \,|\, \sum_i \lambda^i = 1,\ \lambda^i \in \{0, 1\}\ \forall i\}$. The frh model presumes A1 and the additivity assumption A4 such that T*(frh) has the structure above with $\Lambda(frh) = \{\lambda \in \Re_0^n \,|\, \lambda^i\ \text{integer}\ \forall i\}$. DEA models partially relaxing the convexity assumptions are suggested in Bogetoft (1996) and Petersen (1990).

We note that DEA provides an inner approximation of the underlying production possibility set. The efficiency estimates are therefore optimistic and the potentials input savings and output expansions are underestimated. This applies also to the merger gains we shall estimate below. They are in general downwards biased. When we decompose the gains to identify alternative ways to capture the gains, the bias persist, but since it affects all estimates, the relative attractiveness of the different organizational remedies are not systematically affected - except perhaps for a particular underestimation of the size effect due to relatively few large units, cf. the discussion in section 7.


## 4. Measures of Potential Merger Gains

Let us assume that it makes "organizational sense" to merge the J DMUs, i.e. the DMUs with indexes $j \in J \subseteq \{1, 2, ..., n\}$. In our application we merge DMUs that are close in a geographic sense since here proximity to customers is crucial. In other cases, it may be of greater importance to have the same owners or to have similar organizational cultures in order for a merger to be meaningful.

The merged unit is denoted DMU$^J$. Direct pooling of the inputs and outputs gives a unit which has used $\sum_{j \in J} x^j$ to produce $\sum_{j \in J} y^j$. This corresponds to having a completely decentralized organization where the decentralized units correspond to the J-units.

A radial input based measure of the *potential overall gains from merging* the J-DMUs is therefore

$$(P_1) \qquad E^J = \text{Min}\{E \in \mathfrak{R}_0 \mid ( E[\textstyle\sum_{j \in J} x^j], \textstyle\sum_{j \in J} y^j ) \in T \}$$

$E^J$ is the maximal proportional reduction in the aggregated inputs $\sum_{j \in J} x^j$ that allows the production of the aggregated output profile $\sum_{j \in J} y^j$. If $E^J < 1$, we can save by merging. If $E^J > 1$, the merger is costly.

Similarly, an output based measure of the potential overall gains from merging the J-DMUs could be

$$(P_2) \qquad F^J = \text{Max} \{F \in \mathfrak{R}_0 \mid ( \textstyle\sum_{j \in J} x^j, F[\textstyle\sum_{j \in J} y^j] ) \in T \}$$

$F^J$ is the maximal proportional expansion of the aggregate output $\sum_{j \in J} y^j$ that is feasible in a (merged) unit with aggregate input $\sum_{j \in J} x^j$. If $F^J > 1$, we can gain by merging. If $F^J < 1$, the merger is costly.

If we insert a DEA estimate of the production possibility set we get the following operational measures of the potential merger gains

$$(P_3) \qquad
\begin{aligned}
& \text{Min} \quad E^J \\
& E^J, \lambda \\
& \text{s.t.} \quad E^J[\textstyle\sum_{j \in J} x^j] \;\geq\; \textstyle\sum_{i \in I} \lambda^i x^i \\
& \qquad\quad\; [\textstyle\sum_{j \in J} y^j] \;\leq\; \textstyle\sum_{i \in I} \lambda^i y^i \\
& \qquad\quad\; \lambda \in \Lambda(k)
\end{aligned}$$

and

$$(P_4) \qquad
\begin{aligned}
& \text{Max} \quad F^J \\
& F^J, \lambda \\
& \text{s.t.} \quad [\textstyle\sum_{j \in J} x^j] \;\geq\; \textstyle\sum_{i \in I} \lambda^i x^i \\
& \qquad\quad\; F^J[\textstyle\sum_{j \in J} y^j] \;\leq\; \textstyle\sum_{i \in I} \lambda^i y^i \\
& \qquad\quad\; \lambda \in \Lambda(k)
\end{aligned}$$

We observe that neither of these programs may have feasible solutions. In such cases we define $E^J = +\infty$ and $F^J = -\infty$, respectively. Intuitively, the programs may be infeasible for two reasons. Firstly, the merged unit may be large and the return to scale properties may not favor large units. This may be the case in the drs, vrs and fdh model. Secondly, the merged unit may involve an input mix that is not very "powerful" or an output mix that is "hard" to produce. This may be the case in the fdh model. We shall return to the scaling and mixture effects of mergers in greater detail below.

A general sufficient condition for the merger to be (weakly) advantageous is that the technology satisfies the *J-additivity* condition

$$\textstyle\sum_{j \in J} T \subseteq T$$

In such cases it is possible to produce $\sum_{j\in J}y^j$ using $\sum_{j\in J}x^j$ given that it was possible to produce $y^j$ using $x^j$, $j\in J$.

We note that the J-additivity condition $\sum_{j\in J}T \subseteq T$ is equivalent to either of the conditions

$$\sum_{j\in J}L(y'^j) \subseteq L(\sum_{j\in J}y'^j)$$

$$\sum_{j\in J}P(x'^j) \subseteq P(\sum_{j\in J}x'^j)$$

for arbitrary $(x'^j,y'^j)\in T$, $j\in J$. A sufficient condition for the merger of the J-DMUs to be weakly advantageous is of course that either of these conditions hold for the given values of $(x^j,y^j)$, $j\in J$.

Since merger advantages are often expressed in cost terms, we further note that the first of these conditions implies *subadditivity* of the cost function $C(y,w) = \text{Min}\{wx \mid x\in L(y)\}$ for all possible input prices $w\in \Re_0^p$

$$\sum_{j\in J}C(y'^j,w) \geq C(\sum_{j\in J}y'^j,w)$$

and that the latter implies *superadditivity* of the revenue function $R(x,p) = \text{Max}\{py \mid y\in P(x)\}$ for all possible output prices $p\in \Re_0^q$

$$\sum_{j\in J}R(x'^j,p) \leq R(\sum_{j\in J}x'^j,p)$$

Under convexity assumptions, we have by duality theory that the latter conditions are in fact equivalent to the former, c.f. also Färe, Grosskopf and Lovell(1994,ch.10.4.)

So far, we have discussed J-additivity (or in convex cases, cost subadditivity) as a means of avoiding merger programs $(P_1)$ and $(P_2)$ with no feasible solutions. It suffices to ensure, however, not only that the programs have finite solutions, but that $E^J \leq 1$ and $F^J \geq 1$, i.e. that we get weak gains as opposed to losses from mergers. Moreover, if we require weak gains for arbitrary mergers, the additivity condition is not only sufficient, it is a necessary condition as well. The latter only requires a free disposability assumption. We record these observations as a proposition.

**Proposition 1**
Consider a technology satisfying free disposability A1. We then have
- Additivity A4 is a necessary and sufficient condition for $(P_1)$ and $(P_2)$ to have (feasible) solutions with $E^J \leq 1$ and $F^J \geq 1$ for arbitrary mergers. The crs and frh technologies satisfy these conditions.
- For convex technologies, subadditive cost functions and superadditive revenue functions are necessary and sufficient conditions to have (feasible solutions) with $E^J \leq 1$ and $F^J \geq 1$ for arbitrary mergers. The crs technology satisfies these conditions

**Proof:**

We first show sufficiency of A4. By induction, the additivity assumption A4 implies the J-additivity assumption $\sum_{j\in J} T \subseteq T$. Therefore, when it is possible to produce $y^j$ using $x^j$, $j\in J$, it is also possible to produce $\sum_{j\in J}y^j$ using $\sum_{j\in J}x^j$. This implies $E^J \leq 1$ and $F^J \geq 1$ and in particular that we get feasible solutions to $(P_1)$ and $(P_2)$.

We next show the necessity of A4. Let $(x^1,y^1)$ and $(x^2,y^2)$ be arbitrary points in T. By assumption, the merger programs now give $E^J \leq 1$ and $F^J \geq 1$ for J={1,2}. This means than $(E^J(x^1 + x^2), y^1 + y^2)$ (and $(x^1 + x^2, F^J(y^1 + y^2))$ is T and by A1, we get $(x^1 + x^2, y^1 + y^2)$ in T. Therefore, T is additive.

The crs model is a free disposable A1 model and it is additive A4, since it satisfies A2 and A3(crs) which implies A4. (Indeed, the crs model can alternatively be defined by A1, A3(crs) and A4). The frh is defined by A1 and A4 directly.

The second bullet in the proposition follows by the equivalence under convexity of additivity in the technology and subadditivity (superadditivity) of the costs (revenue) functions. Combining with the first part of the proposition, we now get the second part.

<div align="right">Q.E.D.</div>

We note that the free-disposability assumption is not needed for the sufficiency parts, only for the necessity proofs. Moreover, weak free disposability in the sense that $(x, y) \in T \Rightarrow (ax, y)\in T$ for $a \geq 1$ and $(x, y) \in T \Rightarrow (x, by)\in T$ for $b \leq 1$ will do. For simplicity, we have ignored theses extensions in the proposition

## 5. Decomposing Merger Gains

Our measures of the potential overall merger gains, $E^J$ and $F^J$, encompass several effects. In this section, we decompose the overall effect into technical efficiency, scale and mix effects and we discuss the organizational relevance of this decomposition.

Some or all of the units in J may be technically inefficient and this may be captured in $E^J$ and $F^J$. A merger may bring in new management which may facilitate the elimination of such inefficiencies. However, it is also possible to reduce technical inefficiencies through other means, e.g. by imitating the better performers, sometimes referred to as the peer units. To avoid compounding the effects, therefore, it is useful to adjust the overall merger gains for the *technical efficiency effect*. To do so, we can project the original units to the production possibility frontier and use the projected plans as the basis for evaluating the remaining gains from the merger.

Thus, for example, we may project $(x^j,y^j)$ into $(E^j x^j,y^j)$ for all $j\in J$, where $E^j = E^{\{j\}}$ is the standard efficiency score for the single DMU, and use the projected plans $(E^j x^j,y^j)$, $j\in J$, as the basis for calculating the *adjusted overall gains* from the merger

$(P_5)$      $E^{*J} = Min\{E\in\Re_0 \,|\, ( E[\sum_{j\in J}E^j x^j], \sum_{j\in J} y^j ) \in T\}$

The output based measure $F^J$ can be adjusted in a similar manner, but we shall restrict

ourselves to input based measures from hereon.

If we insert a DEA estimate of the production possibility set, we get the following operational measure of the adjusted overall gains $E^{*J}$

$$\begin{aligned}
&\text{Min} \quad E \\
&E, \lambda \\
&\text{s.t.} \quad E\left[\sum_{j \in J} E^j x^j\right] \geq \sum_{i \in I} \lambda^i x^i \\
&\qquad\quad \left[\sum_{j \in J} y^j\right] \leq \sum_{i \in I} \lambda^i y^i \\
&\qquad\quad \lambda \in \Lambda(k)
\end{aligned}$$
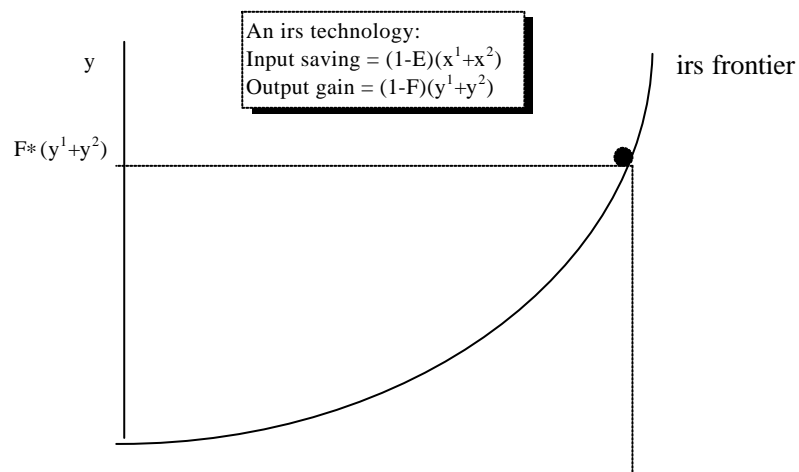
Letting $T^J = E^J/E^{*J}$ we get

$$E^J = T^J * E^{*J}$$

where $T^J \in [0,1]$ indicates what can be saved by individual adjustments in the different units in J.

Since the individual units can be projected to the frontier in many ways, there are many possibilities to construct merger measures that are adjusted for technical inefficiencies at the individual level. In the $E^{*J}$ program ($P_5$), we could for example use output based projections of the individual units instead of the input based projections, we could supplement the proportional reductions with non-proportional slack adjustments, or we could introduce non-radial projections.

Assuming that individual technical inefficiencies have been dealt with, we are left with the two most interesting production economic effects of a merger.

One is the *scaling* or *size effect*. A merger leads to a unit that operates at a large scale. This may or may not be advantageous, depending on the scale properties of the underlying technology. Figure 1 below illustrates a case with positive size effect.



y

An irs technology:
Input saving = (1-E)(x$^1$+x$^2$)
Output gain = (1-F)(y$^1$+y$^2$)
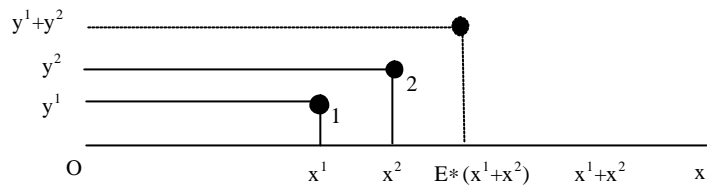
irs frontier

F*(y$^1$+y$^2$)

10

Figure 1. The Size Effect of Merging

The other main effect of a merger is that it leads to other input and output mixes. This may be advantageous by taking us into more "productive" directions of the product space. We shall refer to this as the *harmony, scope* or *mixture effects*. Figure 2 illustrates a case with positive harmony effect.
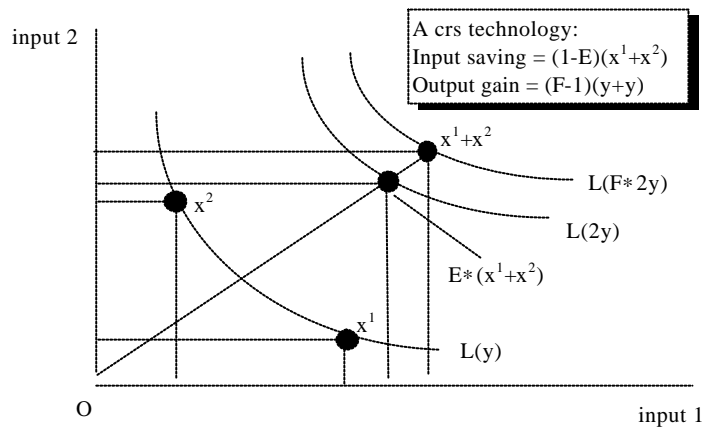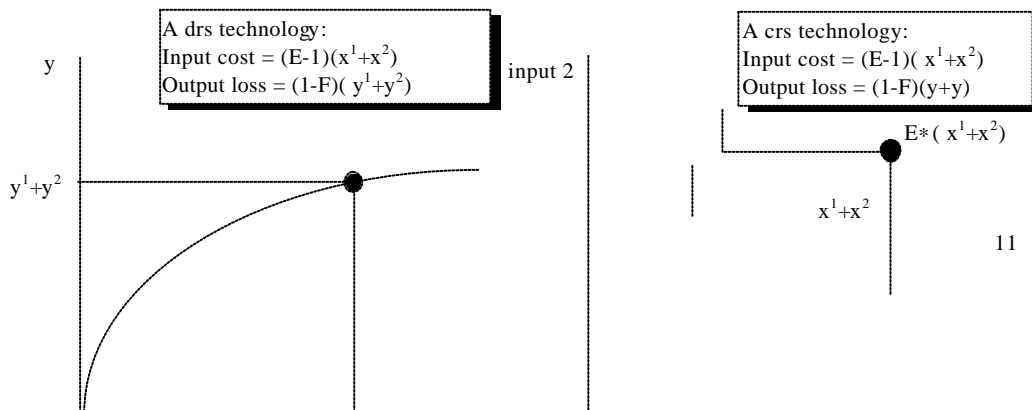
Figure 2. The Harmony Effect of Merging

Without further assumptions about the technology, we cannot put signs on the size and the harmony effects. We have already illustrated cases where they are positive. Negative size and harmony effects are illustrated in Figure 3. A case of opposing effects is illustrated in Figure 4 below.
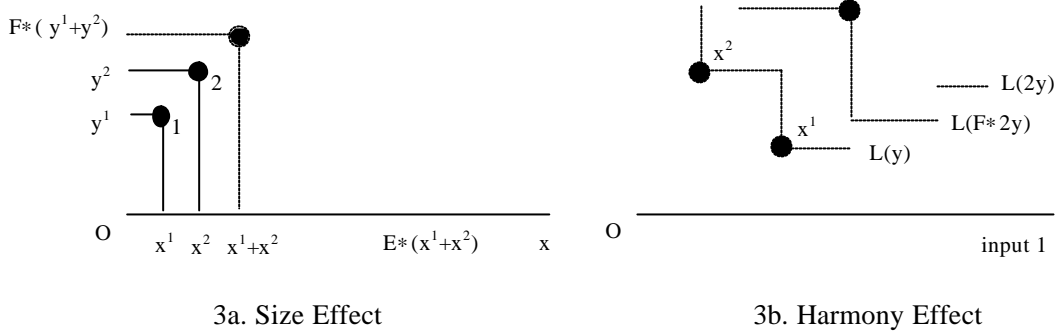
11

F*( $y^1+y^2$ ) 

$y^2$ ... 2

$y^1$ ... 1

O ... $x^1$ ... $x^2$ ... $x^1+x^2$ ... E*($x^1+x^2$) ... x

$x^2$

... L(2y)

... L(F*2y)

$x^1$ ... L(y)

O ... input 1

3a. Size Effect | 3b. Harmony Effect

Figure 3.  Negative Size and Harmony Effects

We note that both the size and harmony effects are reflected in the additivity condition $\sum_{j\in J} T \subseteq T$, or equivalently, the conditions $\sum_{j\in J} P(x^j) \subseteq P(\sum_{j\in J} x^j)$ or $\sum_{j\in J} L(y^j) \subseteq L(\sum_{j\in J} y^j)$. Positive size effects correspond to strict inclusions with proportional input (or output) profiles, while positive mixture effects corresponds to strict inclusions with input (or output) profiles pointing in different directions.

In Figures 1-3, it was easy to distinguish the size and harmony effects, since we assumed constant return to scale (leaving no room for size effects) when we illustrated the harmony effect and we assumed a single input single output technology (leaving no room for harmony effects), when we illustrated the size effects. In general, however, it is less obvious how to distinguish the two effects. We shall return to some of the ambiguities below. First, however, we outline a decomposition which we find natural and useful.

We propose to capture the *harmony gains* by examining how much of the average input could have been saved in the production of the average output, i.e. by the measure $H^J$

$(P_6) \qquad H^J = \text{Min}\{H\in\Re_0 \mid ( H[|J|^{-1}\sum_{j\in J}E^j x^j], |J|^{-1}\sum_{j\in J}y^j ) \in T \}$

where $|J|$ is the number of elements in J. We look at the average input and average output, since we do not want the expansion of size to come into play yet. Using the average is most relevant if the units in J are not too different in size to begin with. If the sizes differ considerably, we may be picking up scale effects, e.g. if some units are larger than and some are smaller than the "most productive scale size" as defined by Banker(1984). Note that $H^J<1$ indicates a savings potential due to improved harmony, while $H^J >1$ indicates a cost of harmonizing the inputs and outputs.

As previously, we may insert a DEA estimate of the production possibility set and hereby get an operational linear programming measure of the potential harmony gains

Min   H
H, $\lambda$
s.t.    $H[|J|^{-1}\sum_{j\in J}E^j x^j] \geq \sum_{i\in I} \lambda^i x^i$
$[|J|^{-1}\sum_{j\in J}y^j ] \leq \sum_{i\in I} \lambda^i y^i$

12

$$\lambda \in \Lambda(k)$$

Next, we capture the *size gains* by asking how much could have been saved by operating at full scale rather than average scale, i.e. by the measure $S^J$

(P$_7$)     $S^J = \text{Min}\{S \in \mathfrak{R}_0 \mid ( S[H^J \sum_{j \in J} E^j x^j], \sum_{j \in J} y^j ) \in T \}$

The re-scaling is advantageous, $S^J < 1$, if we have economies of scale, and costly, $S^J > 1$, if the return to scale property does not favor larger units.

The corresponding DEA based operational measure of the size gains is

$$
\begin{aligned}
&\text{Min} \quad S \\
&S, \lambda \\
&\text{s.t.} \quad S[H^J \sum_{j \in J} E^j x^j] \geq \sum_{i \in I} \lambda^i x^i \\
&\qquad\quad \sum_{j \in J} y^j \leq \sum_{i \in I} \lambda^i y^i \\
&\qquad\quad \lambda \in \Lambda(k)
\end{aligned}
$$

Using these definitions, we have

$$E^{*J} = H^J * S^J$$

and by $E^J = T^J * E^{*J}$ we get our *basic decomposition*

$$E^J = T^J * H^J * S^J$$

This corresponds to a decomposition of the basic merger index $E^J$ into a technical efficiency index $T^J$, a harmony index $H^J$ and a size index $S^J$. The technical efficiency measure $T^J$ captures what can be gained by making the individual units efficient. The remaining potentials to save, $E^{*J}$, are created by the harmony effect, $H^J$, and the size effect, $S^J$.

Figure 4 below illustrates the decomposition in a case with positive harmony effect and negative size effect.



A drs technology:
Harmony saving $= (1-H)(x^1+x^2) > 0$
Scale saving $= (H - H*S)(x^1+x^2) < 0$
Net saving $= (1-E)(x^1+x^2) = (1-H*S)(x^1+x^2)$

13

$2L(y)$

$\frac{1}{2}x^1 + \frac{1}{2}x^2$

$H^*(\frac{1}{2}x^1 + \frac{1}{2}x^2)$
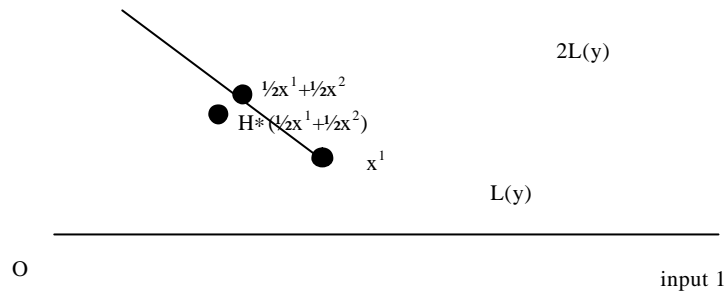
$x^1$

$L(y)$

O

input 1
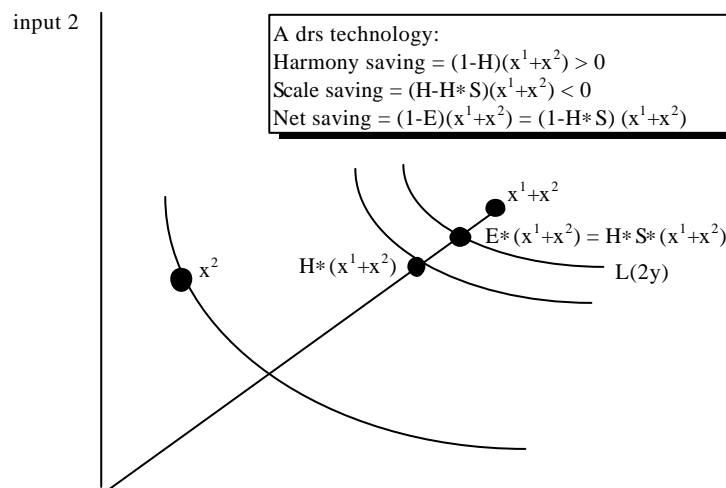
Figure 4.  Positive Harmony and Negative Size Effects

The decomposition of the potential gains from merging DMUs into a technical efficiency measure, a harmony measure and a size measure is important because full scale mergers are typically not the only organizational option available and it may be that alternative organizational changes may be easier to implement. In particular, we suggest that the following may guide the *organizational restructuring*:

*Low technical efficiency measure $T^J$:*
One could let the inefficient DMUs learn from the practices and procedures of the more efficient ones. If the problem is not a lack of skills but rather motivation, one could improve the incentives, e.g. by using relative performance evaluation and yardstick competition based on the technical efficiency measures, cf. Bogetoft(1994,95,97,2000). Of course, if the problem is scarcity of management talent, it may still be necessary to make a genuine merger to transfer control to the more efficient administrative teams and hereby improve the managerial efficiency (X-efficiency).

*Low harmony measure $H^J$:*
One could consider reallocating the inputs and outputs among the DMUs to create more "powerful" input mixes and more easily produced output mixes. This can be done a) inside a hierarchy, b) by long term contracts or perhaps c) by creating a market for key inputs and outputs, cf. also Brännlund, Chung, Färe and Grosskopf(1998). In the next section, we will formalize the relationship between the harmony measure and the reallocation problem.

*Low size measure $S^J$:*
In this case, full scale mergers may be the only alternative. If we need large amounts of fixed capital, highly specialized staff, long run-lengths or simply a critical mass to obtain sufficient returns from scale, it may be relevant to merge. Also, and perhaps most importantly, this may be relevant if the reallocation through contracts or a market are associated with too many transaction costs to make it attractive, cf. the general discussion of the size of the firm in the industrial organization literature, e.g. Tirole(1988).

14

## 6. Alternative Decompositions

The decomposition developed above gives only one - natural we believe - way to define and distinguish between technical efficiency, the size and the harmony effects.

A similar decomposition is possible in the output space, but it will not in general lead to the same quantitative measures of the different effects.

Also, there is typically some possibility of *substituting between the harmony and size effects.* To show this, consider the following modified definition of the harmony effect

$$(P_8) \qquad H_\alpha^J = Min\{H \in \Re_0 \,|\, ( H[\alpha\sum_{j \in J}E^j x^j], \, \alpha\sum_{j \in J}y^j \,) \in T\}$$

where $\alpha \in [0,1]$ is a scalar that defines the activity level at which we calculate the harmony gains. Above, we used $\alpha = |J|^{-1}$, i.e. we used a simple mean size to determine the harmony gains. We could still define the size effects as in $(P_6)$ except that we should use $H_\alpha^J$ instead of $H$. Now, by varying $\alpha$, we get different values for the harmony and size effects. It is straightforward to prove the following simple lemma.

**Lemma 1** $H_\alpha^J$ is independent of, weakly increasing in, weakly decreasing in or non-monotonic in $\alpha \in [0,1]$ when T is a constant return to scale (crs), decreasing return to scale (drs), increasing return to scale (irs) or varying return to scale (vrs) technology.

**Proof:** Assume first that all programs have feasible solutions. Now, since $(H_\alpha^J [\alpha\sum_{j \in J}E^j x^j], \, \alpha\sum_{j \in J}y^j \,) \in T$, so is $(H_\alpha^J [\alpha'\sum_{j \in J}E^j x^j], \, \alpha'\sum_{j \in J}y^j \,) \in T$ for $\alpha' \in \Re_0$ when T is a crs technology, for all $\alpha' \in [0, \alpha]$ when T is drs technology, and for all $\alpha' \in [\alpha, +\infty)$ when T is a irs technology, respectively. This implies that $H_\alpha^J$ decreases when $\alpha$ reduces in a drs technology and that $H_\alpha^J$ decreases when $\alpha$ increases in an irs technology. In the crs case, it shows that $H_\alpha^J$ weakly decreases when $\alpha$ varies and, since this holds for arbitrary $\alpha$, it implies that $H_\alpha^J$ is constant. In a vrs technology, we get a non-monotonic $H_\alpha^J$. For small values, it decreases, then it is constant and than it increases in $\alpha \in [0,1]$. The constancy occurs when $(\alpha\sum_{j \in J}E^j x^j, \, \alpha\sum_{j \in J}y^j \,)$ is projected into a most productive scale size plan, cf. Banker(1984). Cases where the program may not be feasible follows similarly.

<div align="right">Q.E.D.</div>

Lemma 1 emphasizes the possibility of making alternative decompositions, i.e. it emphasizes the indefiniteness of the decomposition. If we choose a low value of $\alpha$ in a drs technology, we assign some size effect to the harmony component. In the irs case, a low value of $\alpha$ would lead us to assign some of the harmony effect to the size component. We leave it to future research to determine more general technological properties that are necessary for the decomposition to be independent of $\alpha$.

It is worthwhile also to compare our resulting size effect with the *traditional measure of scale efficiency* $S^\#(x,y)$ of some unit (x,y)

$$S^\#(x,y) = E(x,y;crs) / E(x,y;vrs)$$

where E(x,y;crs) is the unit's Farrell based input measure assuming constant return to scale and E(x,y;vrs) is the Farrell based measure assuming variable return to scale. The traditional interpretation is that the underlying technology is infact a vrs technology, and that the scale efficiency therefore measures the loss from not operating at the most productive scale size, cf. e.g. Banker(1984) and Banker, Charnes and Cooper(1984). This measure basically assumes that it is possible to adopt optimal scale size under vrs and it measures the losses from not adapting hereto. We could of course use a similar approach here and let the harmony effect be

$$E(\sum_{j\in J}E^j x^j, \sum_{j\in J}y^j ;crs)$$

and the size effect be

$$S^{\#}(\sum_{j\in J}E^j x^j, \sum_{j\in J}y^j) = E(\sum_{j\in J}E^j x^j, \sum_{j\in J}y^j ; crs) / E(\sum_{j\in J}E^j x^j, \sum_{j\in J}y^j ; vrs)$$

This would in most cases give results that differ from the harmony and size measures we propose. The alternative harmony effect would be larger, i.e. $H^{\#}$ would be smaller, as the technology we use to evaluate $H^{\#}$ would be larger.

The difficulty with this alternative approach, however, is that the size effect is hard to interpret. If the merged unit is below most productive scale size, cf. Banker(1984), the scale effect would be what can be gained by increasing the size of the unit. However, this is not possible as we have already included all units. If the merged unit is above most productive scale size, the scale measure would capture what can be gained by reducing the size of the merged unit. However, this is not possible when we are only considering a given merger. The scale effect $S^{\#}$ would therefore ignore the underlying problem of matching the inputs and outputs of a given set of units. The measure would implicitly presume that it is possible to adapt via a market or via a large set of units. (Similar caveats on the interpretation of scale efficiency can be raised in many applications although it – to the best of our knowledge – has not been an issue in the literature.)

The introduction of modified harmony indexes is only one way to vary the decomposition. A more fundamental alteration would be to calculate harmony and scaling effects without presuming technical efficiency. Organizationally, this may be very relevant since it may be easier to reallocate resources than to change the internal culture, tradition and routines and hereby the technical efficiency. The difficulty of such a *reversed decomposition* is that the rates of substitution needed to evaluate the size and harmony gains, are only well defined on the frontier. Still, by making proper assumptions about the technology, methodologically sound "off-the-frontier-reallocation-gains" may be calculated. Some initial work along these lines is reported in Bogetoft and Färe(1999).

We close this discussion of alternative ways to decompose the overall gains by taking a closer look at the harmony measure H in cases of convex, free disposable technologies. This also supports the organizational interpretations of H that we have used above.

Consider what can be saved – after individual learning - by mere reallocations of inputs and outputs, of resources and obligations, among the units in J. Let the new input and output combinations after reallocation be $(x^{*j}, y^{*j})$ for $j \in J$. If we measure the saving potential in proportion to best practice usage, the savings from pure reallocations can be determined by solving the following *pure reallocation problem*:

$(P_8)$   $\quad h^J = \text{Min} \quad h$
$\qquad\qquad h, (x^{*j}, y^{*j}), j \in J$
$\qquad\qquad \text{s.t} \quad h^J[\sum_{j \in J}E^j x^j] \quad \geq \quad \sum_{j \in J}x^{*j},$
$\qquad\qquad\qquad\qquad \sum_{j \in J}y^j \quad \leq \quad \sum_{j \in J}y^{*j}$
$\qquad\qquad\qquad (x^{*j}, y^{*j}) \in T \text{ all } j \in J$
$\qquad\qquad\qquad h \in \mathfrak{R}_0$

The choice variables in this program are the contraction factor h and the new input and output combinations $(x^{*j}, y^{*j})$ for $j \in J$. The constraints are that we must have enough inputs to produce at least the old aggregate output and that all production plans after reallocation must be feasible in T.

Now, as our next proposition shows, the saving from pure reallocations is equal to the harmony measure.

**Proposition 2**
If T is free disposable and convex, A1 and A2, we have that $H=h^J$, i.e. the harmony index captures what can be gained by pure reallocations.

**Proof:**
The proof first shows $H \geq h^J$ and next $H \leq h^J$

To show that $H \geq h^J$ we tentatively consider

$$(x^{*k}, y^{*k}) = (H^J[|J|^{-1}\sum_{j \in J}E^j x^j], |J|^{-1}\sum_{j \in J}y^j ), k \in J$$

By the definition of H, we have
$$(x^{*k}, y^{*k}) \in T, \quad k \in J$$
Moreover
$$\sum_{j \in J}x^{*j} = H^J\sum_{j \in J}E^j x^j,$$
so the first constraint in the h-program is fulfilled for $h \geq H$. The second constraint is fulfilled because
$$\sum_{j \in J}y^{*j} = |J||J|^{-1}\sum_{j \in J}y^j = \sum_{j \in J}y^j.$$
In summary, we have found a feasible solution to the h program and we can conclude that $h^J \leq H^J$ as desired.

We will now show that $H \leq h^J$. Consider an arbitrary solution $(x^{*k}, y^{*k})$ $k \in J$ to the $h^J$ program. By definition of $h^J$ we have

$$h^J [\sum_{j \in J}E^j x^j] \quad \geq \quad \sum_{j \in J}x^{*j}$$

17

$$\sum_{j \in J} y^j \leq \sum_{j \in J} y^{*j}$$
$$(x^{*j}, y^{*j}) \in T \text{ all } j \in J$$

It follows that

$$h^J |J|^{-1} [\sum_{j \in J} E^j x^j] \geq |J|^{-1} \sum_{j \in J} x^{*j}$$
$$|J|^{-1} \sum_{j \in J} y^j \leq |J|^{-1} \sum_{j \in J} y^{*j}$$
$$(|J|^{-1} \sum_{j \in J} x^{*j}, |J|^{-1} \sum_{j \in J} y^{*j}) \in T$$

where the last inclusion is a consequence of $(x^{*j}, y^{*j}) \in T$ all $j \in J$ and T being convex. Now, using free disposability, it follows that $(h^J |J|^{-1} [\sum_{j \in J} E^j x^j], |J|^{-1} \sum_{j \in J} y^j) \in T$. Using the definition of $\check{H}$, $H^J = \text{Min}\{H \in \Re_0 \mid ( H[|J|^{-1} \sum_{j \in J} E^j x^j], |J|^{-1} \sum_{j \in J} y^j ) \in T\}$, we hereby get $H^J \leq h^J$ as desired.

Q.E.D.


Proposition 2 gives a rationale for defining the mix effect as H. Also, it formalizes the interpretation of H as what can be gained by pure reallocation among the units in J. This also gives a rationale for our organizational interpretations and recommendations from the last section. A low H shows that there are large potentials to gain from pure reallocation among the units – a genuine merger is not necessary.

Note that H is a measure of allocative inefficiency that is derived in an "incomplete market". There are no given prices and only a few agents between which the reallocations most take place and be balanced. In contrast, the traditional production economic notion of allocative efficiency, i.e. the ratio of cost efficiency to technical efficiency, presumes that market prices are given and hereby that the input choice of any given unit can take place without being concerned about the wider market effects.


## 7. The Danish Agricultural Extension Services

In Denmark, agricultural advisory services are provided by 71 extension offices. (Two of these offices, numbered 52 and 47, actually share advisors, and they are therefore treated as one, numbered office 47, in this study.) These offices serve different geographical regions. The offices are co-operatives owned by the farmers in the corresponding regions. The regions have a long tradition of cooperation. The regional organizations are part of a national organization which operates a supra-advisory office, The National Agricultural Advisory Centre at Skejby, from which the local offices can buy standardized computer-programs, expert-help, etc.

In this section, we are going to evaluate these offices. We expect to find high relative efficiency levels because of the similarity of the technologies and the widespread cooperation of the offices. At the same time, we expect potential gains from mergers, in particular from the harmony effect. The reason being that the farmers' demand for extension services may change relatively fast due to new market conditions and environmental regulations while the qualifications and structures of the extension offices may adopt more slowly due to union restrictions, recruitment difficulties, etc.

The data for this study includes a rather detailed description of all the activities in the economic sections of the 71 offices in the year Oct. 1994-Sept. 1995. Due to confidentiality clauses, we cannot reveal the names or location of these offices. For the purpose of this analysis, we have aggregated the information into a description of the production process in terms of 4 inputs and 4 outputs, namely

Inputs:
HELABOR: Number of employees with higher education (academics)
LELABOR: Number of employees with lower education (technicians etc)
EDB:           Computation costs
BUILDCOST:   Office rent and other costs

Outputs:
ECOACC:       Number of external accounts (financial statements) produced
PRODACC:      Number of internal accounts produced
BUDGETACC:  Number of budgets produced
TOTOACC:       Number of other services produced, e.g. subsidy applications

We note that the data does not capture the ultimate extension output, namely improved farm performance. The National Agricultural Advisory Centre is presently developing quality measurement techniques that could improve the output description used here.

Summary statistics for the 4 inputs and 4 outputs are provided in Table 1 below. In the period concerned, 1 US$ was approximately 7 Dkr.

Table 1. Summary statistics of the 70 advisory offices

| Labor size | No. | econacc (acc. No.) | prodacc (acc. No.) | budgacc (acc. No.) | totoacc (acc. No.) | helabor (full-time) | lelabor (full-time) | EDB (100 kr) | Buildcost (100 kr) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | averages in groups | | | | |
| <10,00 | 7 | 188,43 | 75,57 | 32,43 | 78,71 | 2,09 | 4,37 | 1947,82 | 3577,17 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 10,00-19,99 | 27 | 431,70 | 240,33 | 67,85 | 165,63 | 5,75 | 9,73 | 4089,98 | 9239,44 |
| 20,00-29,99 | 16 | 660,44 | 362,94 | 107,13 | 266,31 | 10,28 | 14,08 | 6631,51 | 14813,25 |
| 30,00-39,99 | 10 | 807,20 | 456,50 | 128,90 | 364,00 | 12,40 | 19,98 | 8309,37 | 17227,83 |
| 40,00-49,99 | 2 | 1064,50 | 576,00 | 165,50 | 437,00 | 13,80 | 28,66 | 7431,58 | 17822,79 |
| 50,00-59,99 | 6 | 1235,67 | 520,33 | 166,83 | 576,83 | 17,60 | 39,42 | 9618,38 | 28314,19 |
| >60,00 | 2 | 1635,50 | 836,50 | 314,50 | 1182,00 | 27,75 | 44,55 | 19585,24 | 41861,53 |
| | | | | | | | | |
| Total | 70 | 44428,00 | 23337,00 | 7023,00 | 19623,00 | 646,79 | 1101,24 | 425005,93 | 973049,17 |
| Average | - | 634,69 | 333,39 | 100,33 | 280,33 | 9,24 | 15,73 | 6071,51 | 13900,70 |
| Min | - | 76,00 | 19,00 | 19,00 | 10,00 | 1,00 | 2,50 | 1189,85 | 1200,73 |
| Max | - | 1760,00 | 923,00 | 401,00 | 1216,00 | 31,00 | 49,60 | 20707,88 | 48189,33 |
| STDEV | - | 351,66 | 190,95 | 61,91 | 234,57 | 5,84 | 10,83 | 3560,40 | 9053,47 |

To investigate the efficiency of the individual offices, we initially calculated input based DEA scores for each of them. The efficiency distributions in both the crs and the vrs technologies are reported in Figure 5 below.
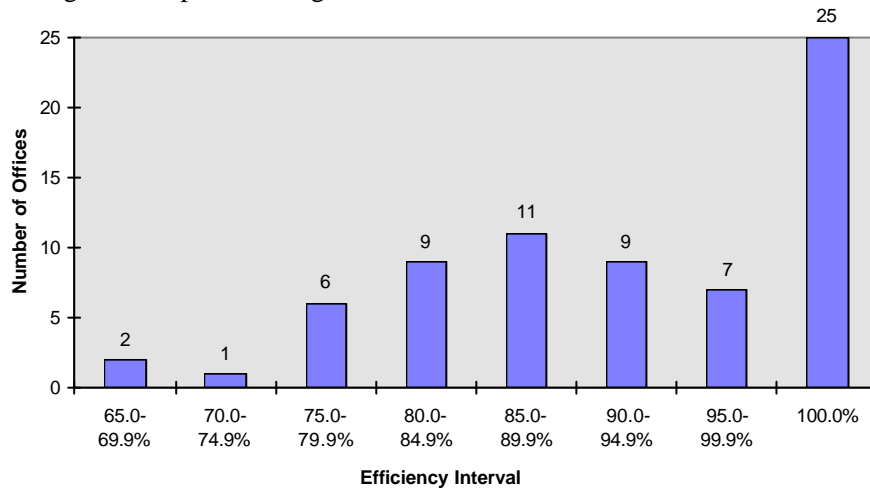


Figure 5a $E^j$ Efficiency Distribution of 70 Offices
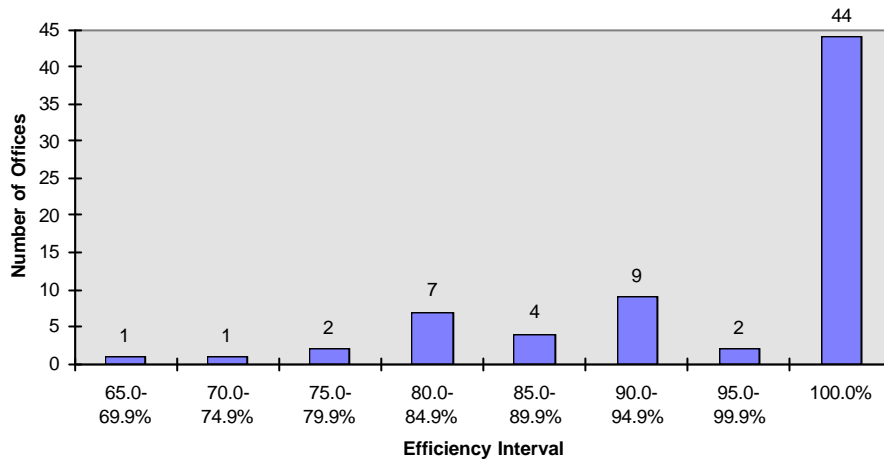in CRS Technology (mean = 91.25%, STDEV = 9.11%)

20

Figure 5b $E^j$ Efficiency Distribution of 70 Offices
in VRS Technology (mean = 94.98%, STDEV =7.87%)

We see that if the technology is modeled using a constant return to scale DEA model, the estimated average saving is 8.75%. In the varying return to scale model, the average saving potential is 5.02%. Informal comparison with other DEA based productivity studies show that these numbers are in no way extreme - given the number of DMUs and the number of inputs and outputs.

Many of the offices are located close to each other and a lot of the services are delivered via phone and computers. For the purpose of considering potential mergers, we have therefore examined what potentially could be saved by merging offices located within a driving distance of 50 kilometers (approximate 30 miles) from each other. This leads to a total of 458 possible mergers involving two or three offices. We have tested the merger gains from all of these combinations using both crs and vrs DEA-models as the basic production model. The distribution of the merger gains are reported in Table 2 below, where we have left out the cases with no potential gains.

Table 2 shows that there are considerable potential gains from merging. Assuming a crs technology and assuming that we have first corrected for individual inefficiencies, we see that 409 of the possible 458 $E^{*J}$ scores was less than 1. Furthermore, about 100 of these mergers generate a saving which is of approximately the same size, 8-10 %, as the average gains from the individual improvements reported above. Under a vrs technology, the gains from merging (as the gains from individual improvements) are considerably less, since the scale effects generally work against mergers.

Table 2. Distribution of merger efficiency measures (<100%) under CRS and VRS

| Efficiency | CRS technology | | VRS technology | |
|---|---|---|---|---|
| interval in % | $E^J$ | $E*^J$ | $E^J$ | $E*^J$ |
| 55,00-59,99 | 2 | 0 | 0 | 0 |
| 60,00-64,99 | 2 | 0 | 0 | 0 |
| 65,00-69,99 | 1 | 0 | 0 | 0 |
| 70,00-74,99 | 19 | 0 | 2 | 0 |
| 75,00-79,99 | 50 | 4 | 0 | 0 |
| 80,00-84,99 | 112 | 14 | 13 | 1 |
| 85,00-89,99 | 118 | 23 | 15 | 3 |
| 90,00-94,99 | 83 | 96 | 29 | 12 |
| 95,00-99,99 | 36 | 272 | 30 | 25 |
| Total | 423 | 409 | 89 | 41 |

To further examine the most promising mergers, Table 3 list the 25 mergers leading to the lowest $E^J$ scores under the crs assumption. Again, this illustrates that there are non-trivial potential gains from mergers in the Danish extension sector.

Finally, we list the 25 most promising mergers in a vrs technology in Table 4. In this case, the decompositions of the total gains also show that there is generally quite a bit to be gained in terms of the harmony effects. The size effect, however, generally works against the mergers. Only in the merger of the rather small offices 56 and 65, code A10, do we get a positive size effect.

We note that the negative size effect may be caused by the estimation procedure more than the production economic realities. Since we have relatively few large units, the production norms for large units - and hereby the estimated potential gains from creating such units via mergers - will be relatively small, due to a less precise estimate. In general, a smaller sample size - or a small sample size in a given region - leads to smaller production estimates when we use a minimal extrapolation estimation technique, cf. also Zhang and Bertels(1998).

Again, however, these numbers illustrate that the gains from mergers can be significant. The A13 merger, for example, suggest a potential gain of 15.88% from a merger even though the individual units cannot be improved. This gain is the result of a potential harmony improvement of 17.30% and a potential size loss of 1.72%.

Table 3. Merger efficiencies of the top 25 most promising mergers under CRS, in %

| Code | Merger | $E^J$ | $T^J$ | $E*^J$ ( $=H^J$ ) |
|------|--------|-------|-------|------------------|
| A1 | Office 46 and 49 | 58,20 | 68,61 | 84,83 |
| A2 | Office 46, 49 and 53 | 59,77 | 76,09 | 78,55 |
| A3 | Office 46, 49 and 50 | 62,03 | 75,95 | 81,67 |
| A4 | Office 46 and 53 | 62,87 | 79,44 | 79,14 |
| A5 | Office 33, 34 and 35 | 68,32 | 82,93 | 82,38 |
| A6 | Office 46, 49 and 51 | 70,05 | 79,87 | 87,71 |
| A7 | Office 02 and 06 | 70,34 | 74,74 | 94,11 |
| A8 | Office 46 and 47 | 71,37 | 84,13 | 84,83 |
| A9 | Office 46, 47 and 50 | 71,73 | 84,66 | 84,73 |
| A10 | Office 33, 37 and 39 | 71,84 | 88,75 | 80,95 |
| A11 | Office 49 and 53 | 72,28 | 76,36 | 94,66 |
| A12 | Office 68 and 71 | 72,30 | 73,45 | 98,43 |
| A13 | Office 33 and 34 | 72,32 | 81,22 | 89,04 |
| A14 | Office 46, 51 and 53 | 72,64 | 85,27 | 85,19 |
| A15 | Office 46 and 50 | 72,93 | 81,50 | 89,48 |
| A16 | Office 33 and 37 | 73,20 | 90,43 | 80,95 |
| A17 | Office 33, 37 and 40 | 73,45 | 89,67 | 81,91 |
| A18 | Office 33 and 35 | 73,49 | 91,16 | 80,62 |
| A19 | Office 34, 35 and 44 | 73,58 | 84,08 | 87,51 |
| A20 | Office 33, 37 and 44 | 74,09 | 91,20 | 81,24 |
| A21 | Office 03 and 06 | 74,45 | 81,25 | 91,63 |
| A22 | Office 04 and 06 | 74,76 | 77,14 | 96,92 |
| A23 | Office 02, 03 and 06 | 74,93 | 80,91 | 92,61 |
| A24 | Office 34 and 44 | 74,95 | 84,60 | 88,59 |
| A25 | Office 33, 39 and 40 | 75,07 | 91,90 | 81,69 |

Table 4. Merger efficiencies of the top 25 most promising mergers under VRS, in %

| Code | Merger combination | $E^J$ | $T^J$ | $E*^J$ | $H^J$ | $S^J$ |
|------|-------------------|-------|-------|--------|-------|-------|
| A1 | Office 46 and 49 | 73,88 | 84,41 | 87,53 | 83,29 | 105,09 |
| A2 | Office 68 and 71 | 74,59 | 75,42 | 98,90 | 97,72 | 101,21 |
| A3 | Office 56 and 57 | 80,38 | 85,62 | 93,88 | 93,39 | 100,52 |
| A4 | Office 57 and 71 | 81,07 | 76,87 | 105,46 | 99,58 | 105,90 |
| A5 | Office 28 and 32 | 81,68 | 81,95 | 99,67 | 94,08 | 105,94 |
| A6 | Office 57 and 68 | 82,13 | 83,35 | 98,54 | 96,73 | 101,87 |
| A7 | Office 64 and 67 | 82,52 | 84,19 | 98,02 | 90,44 | 108,38 |
| A8 | Office 12 and 31 | 82,61 | 91,97 | 89,82 | 87,27 | 102,92 |
| A9 | Office 56 and 64 | 83,31 | 85,99 | 96,88 | 89,23 | 108,57 |
| A10 | Office 56 and 65 | 83,38 | 96,04 | 86,82 | 89,61 | 96,89 |

| A11 | Office 12 and 32 | 83,95 | 88,52 | 94,84 | 94,55 | 100,31 |
|-----|-------------------|-------|--------|--------|-------|--------|
| A12 | Office 12 and 28 | 84,09 | 81,03 | 103,77 | 97,67 | 106,25 |
| A13 | Office 47 and 48 | 84,13 | 100,00 | 84,12 | 82,70 | 101,72 |
| A14 | Office 12, 31 and 32 | 84,35 | 92,99 | 90,71 | 87,10 | 104,14 |
| A15 | Office 28 and 29 | 84,43 | 80,76 | 104,54 | 96,89 | 107,90 |
| A16 | Office 28 and 31 | 85,00 | 87,26 | 97,41 | 91,86 | 106,04 |
| A17 | Office 57, 68 and 71 | 85,10 | 78,48 | 108,43 | 97,54 | 111,16 |
| A18 | Office 17 and 22 | 85,18 | 86,61 | 98,35 | 98,35 | 100,00 |
| A19 | Office 59 and 64 | 85,84 | 82,44 | 104,13 | 96,60 | 107,80 |
| A20 | Office 13 and 32 | 85,87 | 89,53 | 95,91 | 94,04 | 101,99 |
| A21 | Office 46 and 53 | 86,33 | 92,77 | 93,06 | 78,92 | 117,92 |
| A22 | Office 28, 31 and 32 | 86,39 | 88,90 | 97,18 | 88,74 | 109,51 |
| A23 | Office 17 and 19 | 86,77 | 86,18 | 100,68 | 94,01 | 107,09 |
| A24 | Office 22 and 23 | 87,42 | 93,16 | 93,84 | 89,18 | 105,23 |
| A25 | Office 12 and 17 | 87,54 | 82,93 | 105,56 | 94,03 | 112,26 |

Prior to the study period, there had been some re-organization of the advisory offices. Offices in several regions had in fact been merged. Subsequently, more offices have merged, including some of the combinations identified above. Since we cannot indicate the identity of the offices, we can also not reveal which offices have subsequently merged.

Needless to say, the empirical analysis above is intended to be illustrative. In a larger empirical study, more attention should be devoted to the details of the underlying production model.

It may for example be useful to include partial price information using assurance regions or similar, cf. eg. Charnes, Cooper, Lewin and Seiford (1994). In our application, limits on the salaries of academics (HELABOR) compared to technicians (LELABOR) could be relevant. Partial price information will effectively enlarge the production possibility set and hereby increase the overall gains from a merger as measured by $E$. However, it may also affect the relative contributions of technical efficiency, mix and scale effects and hereby the organizational diagnosis that one arrives at.

Another aspect of the modeling that deserves attention in full scale applications is controllability, i.e. the question of what are discretionary and non-discretionary variables. We have assumed here that all resources and products can be reallocated but in reality, some reallocations are easier than others. In our application, it may, for example, be relatively easy to exchange tasks that do not require farm visits, while it may be more difficult to actually exchange personal. Of course, controllability is related to the time horizon. In a short run perspective, less inputs and outputs can be controlled and the savings potentials from reorganization will be reduced.

For clarity of exposition, we have not covered such extensions of the basic ideas. We suggest however that numerous extensions will be straightforward given the large literature on alternative DEA models, and we leave the remaining extensions for future research.

## 8. Final Remarks

In this paper, we introduced simple non-parametric production models to compute the potential gains from merging Decision Making Units. We decomposed the gains into technical efficiency, size and harmony effects. A merger may force the units to perform more efficiently on an individual basis. It also affects the scale of operation which may or may not be advantageous, depending on the return to scale properties. Finally, it affects the mix of inputs available and outputs demanded. A merged unit faces a more balanced or harmonic input and output profile, which is typically advantageous.

The decomposition allows us to identify alternative means of improving performance. If the technical efficiency is low, gains are possible by learning the practices of peer units and by introducing incentive schemes to motivate efficiency. If the harmony index is low, improvements are possible by re-allocating resources, either within a hierarchy or through an inter-unit market for inputs and outputs. If the size index is low, a genuine merger may be called for to enable the optimal specialization, run-lengths etc.

The methodology was illustrated by computing gains from merging neighboring advisory offices in Denmark. We showed that considerable production economic gains from mergers can be expected. In many cases, the gains from individual improvements and from improved harmony effects were of the same order of magnitude.

There are numerous relevant extensions of the research reported here.

On the theoretical side, it is relevant to consider alternative decompositions and to identify technological regularities that suffice to make the decompositions unique. More generally, it is important to study what organizational changes to introduce in a post-productivity analysis and to discuss how the analysis could be tailored in the first place to support such changes, cf. also Bogetoft(2000).

On the applied side, our framework is particularly relevant in those cases where it is important to keep a multiple input multiple output description of the production process. This may be the case quite generally since a merger probably requires the units to match and complement each other in several dimensions. One area of possible application is environmental regulation. Farms and forests are subject to an increasing number of environmental restrictions in most countries. In Scandinavia, such constraints are often referred to as harmony constraints, since they concern the balance or harmony between different uses of the environment. The approach of this paper can be used to evaluate alternative designs of such restrictions, including the use of farm specific or tradable requirements. Further, it can be used to predict likely responses to such regulations. One way to meet the increasing number of constraints is to balance what one farm has in excess with what another farm lacks through a merger.

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