Forecasting interregional freight flows by gravity models. Utilising OLS-, NLS- estimations and Poisson-, Neural Network- specifications.

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Abstract

In this paper we compare four different specifications of gravity models for inter regional freight flow prediction. The most used specification with OLS estimation is compared with a model where errors are assumed to be Poisson distributed and a model similar to this but with errors assumed to be normally distributed, namely Non-linear Least Square (NLS). We also compare these with a Feed Forward Back Propagation Neural Network. (NN) Data consists of freight flows between Norwegian counties. The attribute describing the nodes is population and distance in kilometres is used as a proxy for costs on transport links. Since we here only are interested in inter regional flows intra regional flows are excluded. Results are also compared with an earlier study by Bergkvist and Westin (1997) were also intraregional data were used. Performance measures used here shows that OLS compared to Poisson, NLS and Neural Network specifications will produce worse predictions. However, the question on how to compare performance is not indisputable and of great importance since different measures can produce quite different results, not just in scale but also in ranking. When non-linear models are used the lack of a simple easily interpretable R-square measure as in linear regression is evident. We therefore use different measures of performance and discuss their pros and cons.

JEL classification: C45, R41

Keywords: Gravity model, Transportation, Freight flows, Spatial interaction, OLS, Poisson-regression, Neural network.

1. INTRODUCTION

Gravity models[1] have traditionally been used to estimate flows of goods and people between nodes in networks. Variables, functional form and estimation methods have not changed much since those currently used produce reasonable results which are hard to improve upon without increasing costs c.f. Haynes, K.E. and A.S. Fotheringham (1984) or Sen, A. and T.E. Smith (1995). However, these results are far from perfect and when used as support in infrastructure investment analysis even small errors on flow forecasts can produce costly mistakes. The need for improved forecasting models is thus still present.

However, methods do not just differ in “pure” forecasting error for example caught by a performance measure such as root mean square error (RMSE), they also differ in qualitative aspects. Differences that may not be noticed unless one uses several performance measures preferably also combined with different kind of residual plots. The type of qualitative error may play a different role in different types of investment projects. If the method is good at forecasting small flows but worse at large it may still be very useful when the actual link is not so heavily used. Sometimes though one may suspect that the investment in mind may change flows dramatically and hence brings up the need for a model that can handle non-linearities and qualitative changes off flows in a proper way.

The availability and quality of data is also something that affects a method’s performance. In this kind of data zero flows between some or several nodes is common and thus creates problems for an estimation method such as OLS (values have to be logged), the distance measure inside a node/region may also have been set to an arbitrary (e.g. average distance) number which arises the question if using them will improve or just confuse the information available. In Bergkvist and Westin (1997) both these problems are present and the zero flow problem dealt with. There OLS-, non linear (NLS) – and Poisson- regression are compared with a feed forward backpropagation neural network (NN) as a tool for forecasting freight

[1] The name emanates from physichs and refers to that the attraction between bodies is related to their mass and distance. Which also is ”true” for the Gravity models here used.

flows in- and between Norwegian counties. There only RMSE was used to measure performance and the neural network performed best.

Neural networks have quite recently been discovered as a useful tool in regional science and research have concentrated on examinig the possibilities of existing neural network paradigms in this new area. Earlier, Nijkamp et al. (1996) compared logit and NN in the case of transport mode choice while telecommunication flows where analysed by Fischer and Sucharita (1994), Bergkvist and Westin (1997) in their turn compare NN with other methods as a tool to forecast road transport flows between and in Norwegian counties.

The contribution here would be to further explore the possibilities of NN and compare it with OLS and NLS-estimation as well as with a Poisson model. We also perform an indirect sensitivity test of results from Bergkvist and Westin (1997) since we use the same data except for in-county flows. Moreover we compare different performance measures and evaluate their pros and cons.

The paper is structured as follows. In section two, the gravity model and different ways to specify and estimate it are discussed. Our data is described in section three and results are presented in section four. Conclusions and comments are then presented in the last section.

2. ESTIMATION OF GRAVITY MODELS

The most common formulation of the gravity mode is: (c.f. Sen, A. and T.E. Smith (1995)).

$$X_{rs} = AO_r^{\alpha} D_s^{\beta} \exp(\lambda c_{rs}) \qquad (1)$$

In (1), the flow between nodes r and s is a function of the attributes of the nodes given by O_r and D_s while affinity between nodes are given by c_{rs}. Parameters to be estimated are A, α, β and λ. For estimation purposes we will make assumptions about the error term and how it enters in equation (1). In (2) we assume that ε_{rs} is normally distributed and $E(\varepsilon_{rs})=0$. And by this we will get NLS.

$$X_{rs} = AO_r^{\alpha} D_s^{\beta} \exp(\lambda c_{rs}) + \varepsilon_{rs} \qquad (2)$$

In (3b) logs are taken of (3a) and we assume that $\ln(\varepsilon_{rs})$ is normally distributed and $E(\ln(\varepsilon_{rs}))=0$. Thus we can use OLS to estimate the now linear model.

$$X_{rs} = AO_r^{\alpha} D_s^{\beta} \exp(\lambda c_{rs}) \varepsilon_{rs} \qquad (3a)$$

$$\ln X_{rs} = \ln A + \alpha \ln O_r + \beta \ln D_s + \lambda c_{rs} + \ln(\varepsilon_{rs}) \qquad (3b)$$

This is, as mentioned earlier, impossible when flows (X_{rs}) are zero. A way to handle this without having to add desinformation, aggregate or lose information would be to use something like NLS or Poisson regression. Were the choice would be made upon theoretical and empirical considerations. Both these approaches are also used since it a priori is hard to actually tell which model that will best suit such demands since the models have to be estimated before things such as residuals can be examined (Sen, A. and T.E. Smith (1995)). If the dependent variable (X_{rs}) is assumed to be Poisson distributed we get the Poisson regression model as specified in (4).

$$E[X_{rs} | O_r, D_s, c_{rs}] = AO_r^{\alpha} D_s^{\beta} \exp(\lambda c_{rs}) \qquad (4)$$

One question is if the Poisson distribution gives a better performance compared to the normal distribution assumptions in (2) and (3b).

These more traditional methods will be compared to a NN, in this case a feed forward backpropagation neural network. In doing this beside from leaving assumptions about the probability distribution we also leave some assumptions about the functional form of the gravity model and say that $X_{rs} = f(O_r, D_s, c_{rs})$ and specify $f(O_r, D_s, c_{rs})$ as e.g.,

$$X_{rs} = \sum_{i=1}^{M} w_i \frac{e^{z_i} - e^{-z_i}}{e^{z_i} + e^{-z_i}}; \qquad z_i = w_{Oi} O_r + w_{Di} D_s + w_{ci} c_{rs} \qquad (5)$$

This is one structure this type of neural network can have. In Bergkvist and Westin (1997) exactly this was used with M=2, in this paper we found that a more complex form gave better results. The general structure can be seen in Figure 15 in Appendix. Each circle in the hidden layers defines a neuron or processing element and contains the summation and output function defined in (6), in the output layer the inputs are just summed together linearly. In (6) x_j is the current output of neuron j in layer l, M^l is the number of neurons in layer l-1and w_{ij} is the weight/parameter to the jth neuron in layer l from the ith neuron in layer l-1. The number of hidden layers and neurons can be arbitrarily chosen but the number of hidden layers are typically not more than one to three.

$$x_j^l = \frac{e^{\sum_{i=1}^{M^l} w_{ji}^l x_i^{l-1}} - e^{-\sum_{i=1}^{M^l} w_{ji}^l x_i^{l-1}}}{e^{\sum_{i=1}^{M^l} w_{ji}^l x_i^{l-1}} + e^{-\sum_{i=1}^{M^l} w_{ji}^l x_i^{l-1}}} \tag{6}$$

The estimation is not as straightforward for the neural network as for the other methods. This since there exists some free parameters which have to be chosen by the researcher. No analytical results are available as guidance, so finding these parameters is mostly a trial-and-error process. The weights/parameters w:s and x:s are estimated so that the Root Mean Square Error (RMSE) is minimised, for any given M^l. The parameters M^l and l are free to be set by the researcher after evaluation of performance has been done, there are also parameters in the gradient descent search algorithm such as step length which to be set by the researcher.

For a more detailed description of the feed forward back propagation NN, see e.g. Rumelhart et al. (1986).

3. THE FREIGHT DATA

Data is from Norway and put was into a flow matrix by the Norwegian transport institute in Oslo (T∅I). They consists of road??? freight flows in whole tons of general cargo between nineteen Norwegian counties from the year 1988. We use population size in each region as a measure of attraction between nodes. This should be seen as a proxy for potential demand in the nodes with the assumption that income is evenly distributed among citizens. It is also not a very unrealistic assumption regarding that Norway is a highly developed welfare state. It is also reasonable to look upon size of population as production potential and hence a greater population would induce a greater supply and increased transports from that node. This would then give rise to two effects from population growth in the Origin node. Increasing export and flows as supply increases and decreasing export as internal demand increase. As cost of transportation or friction between counties distance in kilometres is used.

Total number of observations, i.e. flows on links, are 340, which includes flows between counties and zero flows.

Observations are randomly divided into two sets of size 115 and 225 which becomes the test and training set respectively. We do this to be able to evaluate the performance of the different estimators. It is of extra importance for a NN since these are able to fit themselves so good to data, it is actually that they can use so many parameters in estimation that they totally fit or even overfit the data. This will give poor forecasting when forecasting out of the estimation sample and hence we have to use a test set not previously "seen" by the NN to calibrate the net and evaluate performance.

Table 1. Descriptive statistics of the Norwegian freight flows NN II.

Set	Whole set N=340				Train set N=225				Test set N=115			
Var.	X	O	D	c	X	O	D	c	X	O	D	c
Mean	35.65	218475	218475	824.56	31.22	210297	222174	830.6	44.32	234476	211238	812.75
Std.Dev	74.94	104011	104011	690.58	60.16	99389	107942	686.72	97.32	111219	95896	701
Min	0	74654	74654	41	0	74654	74654	50	0	74654	74654	41
Max	628	451099	451099	2831	432	451099	451099	2831	628	451099	451099	2777

Descriptive statistics regarding the three sets are presented in Table 1. As can be seen from there are not very big differences between the train and test set. An indication that the random sampling and division into two sets gave an even result.

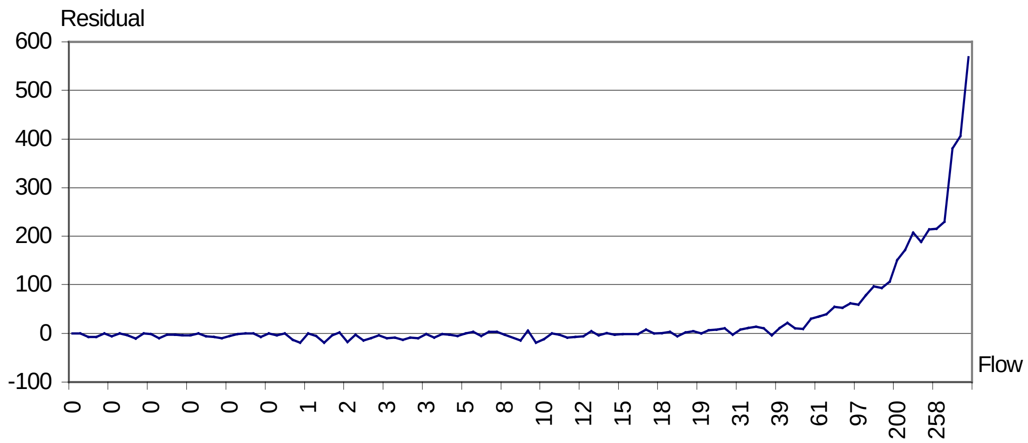
4. RESULTS FROM ESTIMATION AND FORECASTING

The estimators are primary evaluated using the RMSE measure defined in (7). The purpose here is however not just to arrive at one number or measure of performance why we will use also other measures of fit and moreover present residual plots so that more qualitative conclusions can be drawn. Residual plots will be of two kinds, the absolute size of the residual (*e*) and it's relative size to the actual flow[2]. The relative residual is obviously not defined for zero flows why these cannot be presented.

It is also interesting to see if the models capture different parts of the observations to the same extent. It is for example possible that a model performs differently on in-sample data than in an out-of-sample forecast. Therefore we also show residual plots for the training sets as well for the test sets. The primary interest is however on the out-of-sample performance why residuals for the training sets are kept in the Appendix. Residual plots from OLS estimations in Figure 1,

Figure 2 and Figure 16 show that this estimator has the largest deviations for all sets and also in both relative and absolute measures. This estimator also show similar performance for both test and train set, absolute *e* increase with actual flow size and both set also contain negative *e*:s, especially for smaller flows.

Figure 1 Residual plot for the test set with OLS estimation. Residuals versus actual flows .



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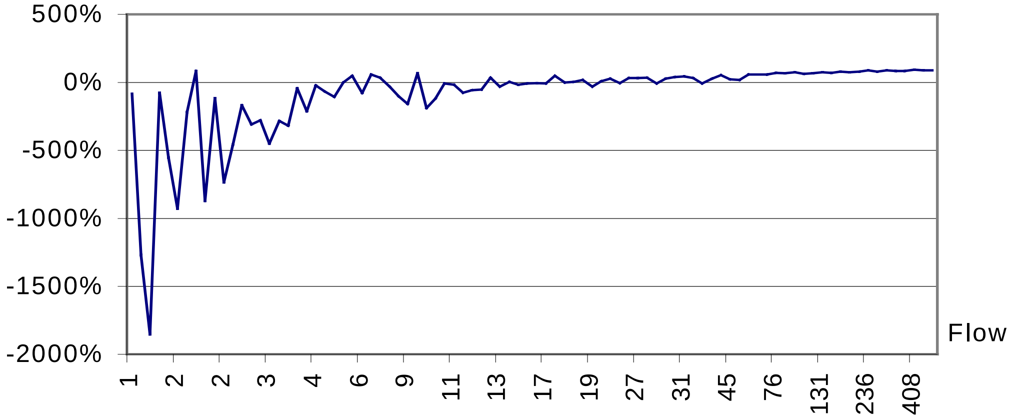
[2] $e = X_{rs} - \hat{X}_{rs}$; relative $e = \left(X_{rs} - \hat{X}_{rs}\right) / X_{rs}$

Figure 2 Relative size of residual compared to actual flow size plotted against flow. Test set with OLS estimation.



The performance of the NLS estimator showed in Figure 3, and Figure 17 exhibits a different behaviour compared to OLS, it has the second smallest absolute deviation for the train set and the test set. It also shows a more irregular pattern for large flows than OLS, the absolute error does not either increase as smooth with increasing flow size as for OLS and it also exhibits some large relative deviations for greater flows.

Figure 3 Residual plot for the test set with NLS estimation. Residuals versus actual flows .

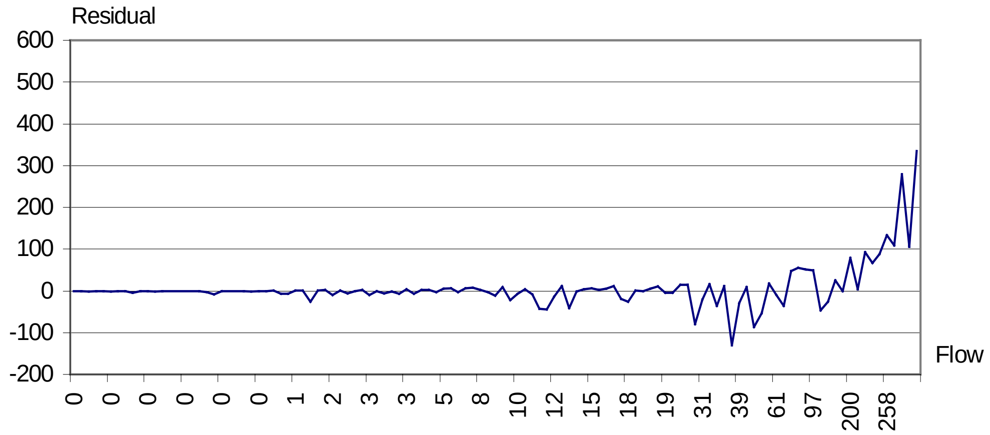
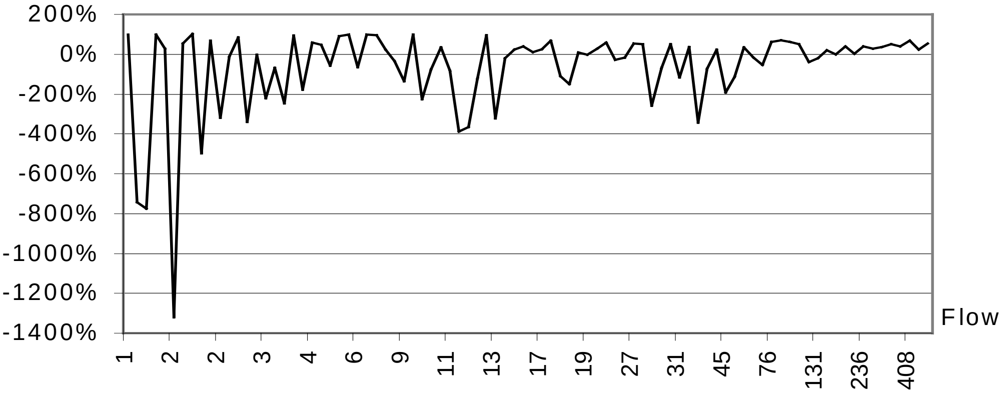


Figure 4 Relative size of residual compared to actual flow size plotted against flow. Test set with NLS estimation.



Poisson regression in Figure 5, and Figure 18 gives a result very close to NLS but in general a little larger absolute and relative deviation for most observations.

Figure 5 Residual plot for the test set with a Poisson specification. Residuals versus actual flows .

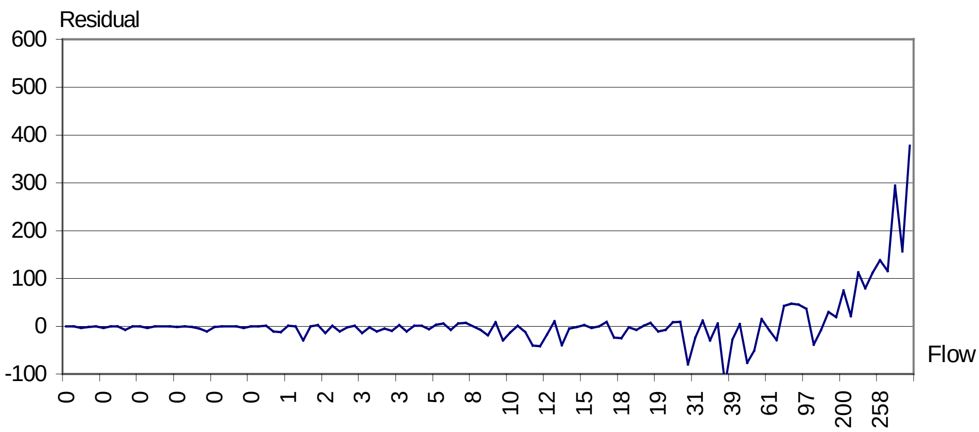
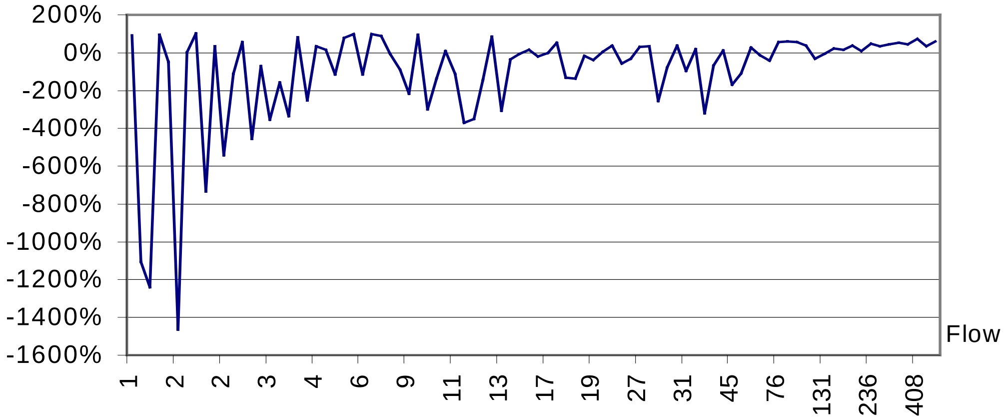


Figure 6	Relative size of residual compared to actual flow size plotted against flow. Test set with Poisson regression.



The neural network performance in Figure 7, and Figure 19 also that shows a similar performance pattern as NLS and Poisson. The difference is mainly in magnitude, where the neural network has the lowest visible max-deviation in all tables.

Figure 7	Residual plot for the test set with a BP-NN specification. Residuals versus actual flows .

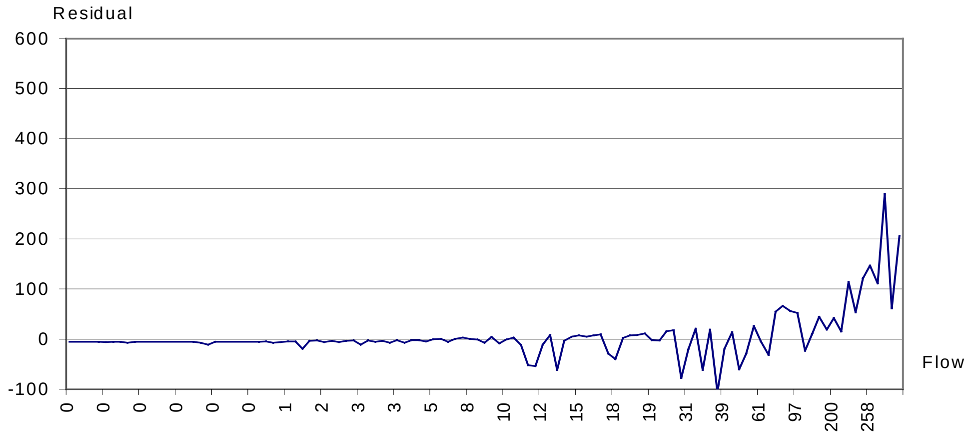
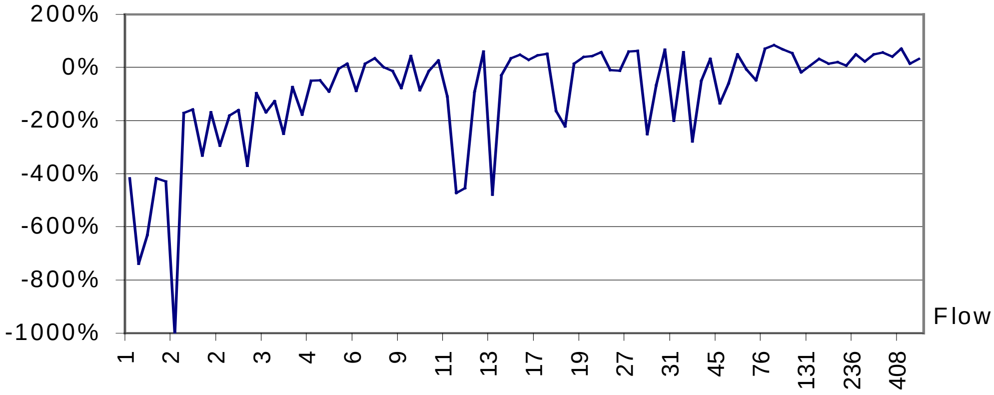


Figure 8 Relative size of residual compared to actual flow size plotted against flow. Test set with BP-NN specification.



Besides the graphical evaluation made from the figures we also compare some different numerical measures. But the result obtained from these are first of all dependent on estimation method and it's objective function. OLS, NLS and NN estimations has the same objective. Namely to minimise the squared some of residuals (MSE) which ii the same as minimising the Root Mean Square Error (RMSE) shown in equation (7). RMSE is also chosen to be one of our numerical measures of evaluation. The Poisson regression has a different objective function, here the maximum likelihood as specified in equation (8) is used, but hence not used to evaluate other estimators.

$$MSE = \sum_{i=i}^{N} \left(x_i - \hat{x}_i \right)^2$$

This is of course an advantage for the estimators that uses the RMSE, which is one more reason that other performance measures will be used as well.

$$RMSE = \left[\frac{1}{N} \sum_{i=i}^{N} \left(x_i - \hat{x}_i \right)^2 \right]^{\frac{1}{2}} \quad \Leftrightarrow \quad \sqrt{MSE} \qquad (7)$$

In Table 2-Table 5 results from the estimations are given.

$$\max \ln L = \max \sum_i [-\exp(\ln A + \alpha \ln O_{ir} + \beta \ln D_{is} + \lambda c_{irs}) + x_i(\ln A + \alpha \ln O_{ir} + \beta \ln D_{is} + \lambda c_{irs}) - \ln x_i!] \qquad (8)$$

As mentioned before there is not only one solution for the NN, but we only present the final result of our simulations. This is not by any means guaranteed to be the optimal solution and should therefore only be considered as our best try.

Table 2. Root Mean Square Error (RMSE) for the different methods .

	Estimator			
Data set	OLS	NLS	Poisson	NN
Train set	24	18	19	29
Test set	92	53	57	47

Table 2 confirms that the NLS model performed best on the training set while the neural network performed best on the test set. The neural net has the most even performance. If we measure the performance by use of the standard R^2 (equation (9)) measure as in Table 3, the result remains the same. But by looking closer at equations (7) and (9) we see that it basically is the same measure but with a different scaling. It's interpretation is however not as in the linear case and numbers cannot be interpreted as percent explained where one reason to this is that it in a non-linear case not necessarily is bounded to be –1 and 1.

Table 3. R^2 standard for the different methods .

	Estimator			
Data set	OLS	NLS	Poisson	NN
Test set	0,11	0,70	0,65	0,76

$$R^2 = 1 - \sum_{i=1}^{N} (x_i - \hat{x}_i)^2 / \sum_{i=1}^{N} (x_i - \overline{x})^2 \qquad (9)$$

Another measure is the R^2_{Corr}

Table 4 and equation (10) (see Cameron (1994)) which is bounded by 0 and 1 but shares, amongst other disadvantage, the disadvantage of not being interpretable as percent explained. No change in ranking occurs here either.

Table 4. R^2_{Corr} for the different methods .

	Estimator			
Data set	OLS	NLS	Poisson	NN
Test set	0,56	0,76	0,74	0,81

$$R^2_{Corr} = \left[\sum_{i=1}^{N} (x_i - \bar{x})(\hat{x}_i - \bar{\hat{x}}) \right]^2 / \sum_{i=1}^{N} (x_i - \bar{x})^2 \sum_{i=1}^{N} (\hat{x}_i - \bar{\hat{x}})^2 \qquad (10)$$

The measurement in Table 5 is different from others since it does not try to measure some sum of residuals but rather to examine how big share of the relative residual that is above a certain range. Here we also experience a change in ranking as Poisson regression show the smallest amount of big errors and neural networks the largest. But as we have seen from residual plots are the largest relative errors for small flows why one has to consider what is most important, Overall small absolute or relative errors, or perhaps some mix.

Table 5. Share predicted with a relative error greater than 20%.

	Estimator			
Data set	OLS	NLS	Poisson	NN
Test set	86,1	80,9	79,1	87

The conclusion is that standard residual deviance based performance measures show that NN has he best performance but that differences are small and when measured in a little different way ranking can be completely changed. It is hence of great importance to carefully choose a performance measure that address the problem at hand.

Table 6. Parameters of OLS, NLS and Poisson regressions .

Parameter	OLS	NLS	Poisson
Constant	-10,71 (-2,63)	4,55*(*)	4,34*(*)
Population Origin	0,60 (2,64)	0,48 (*)	0,50*(*)
Population Dest.	0,57 (2,60)	0,77*(*)	0,79*(*)
Km	-0,002 (-7,67)	-0,0054*(*)	-0,0045*(*)

t-values in parenthesis, (*) asymptotic significance at 95% level, * bootstrapped significance at 95% level.

Parameters from estimation can be found in Table 6 and they are significant at a 95% level for all estimators and means of deriving them except for the asymptotic t-value for the Population Origin parameter in NLS. For the Neural network no such values were possible to obtain since the software[3] here used had no such functions implemented. Values are quite similar in size which supports the findings that estimators are quite similar in behaviour and performance.

If we compare results here to those obtained in Bergkvist and Westin (1998) from Table 7 we see that the NN is also here the best performing estimator but that NLS completely fails on the test set. The difference in data sets is that we now have omitted inter regional flows which before were present and whose distance all were set to 30 kilometres. Which created some very large flows with a constant distance and hence made the data more "badly" behaving. Which may be a reason why the NN's performance is so very much better since it is more flexible than other estimators.

Table 7. Root Mean Square Error (RMSE) for the different methods inter- and intra-regional flows .

	Estimator			
Data set	OLS	NLS	Poisson	NN
Train set	838	168	408	574
Test set	520	177194	443	341

Due to the more complicated structure of the NN here used it is very hard and would be of little use to derive elasticities analytically. It is however possible to get elasticities numerically by varying input and measure the output change. Analytical and numerical elasticities can be found in Table 8. The elasticities reported for the different variables in the NN are calculated as other variables are held constant around their mean. However, if we look at the plots of elasticities for the NN in Figure 9-Figure 11 we see that the NN is not by any mean constant elastic. It even shows a different functional form for different input variables. Looking at Origin Population in Figure 9 we see that the elasticity first decrease and then starts to

[3] Neuralworks explorer

increases with population size which could be a sign that increasing population consumes greater shares of potential export and that agglomeration effects and increasing returns-to scale eventually creates greater supply and also again better export prospects. The effect of increasing population in the destination node is not ambiguous, an increasing demand for goods increases the demand for goods and hence freight flows increase. But also here elasticity increase why there may exist similar effects as in the Origin node. That income is not a linear function of population but rather that a greater population eventually make it possible to exploit economics of scale so that income may increase at a higher rate than population. Figure 11 is even more interesting since it's shape could be due to how substitution possibilities exist for lorry freights on different distances. As distance increase, train and air transportation becomes more and more an alternative to lorries, the demand for lorries get more and more elastic. Then for certain kind of transport there exist really no alternative means of transportation why elasticity starts to go towards zero and stays there for the really long but necessary transports.

Table 8 Analytical** and numerical* elasticites .

	OLS**	NLS**	Poisson**	BP-NN*
Population Origin	0,60	0,48	0,50	-0,004 - 0,006
Population Dest.	0,57	0,77	0,79	0,05 - 0,35
Km	-0,002	-0,0054	-0,0045	-1,7 - 0

Figure 9 Elasticities for Population Origin related to Population size.

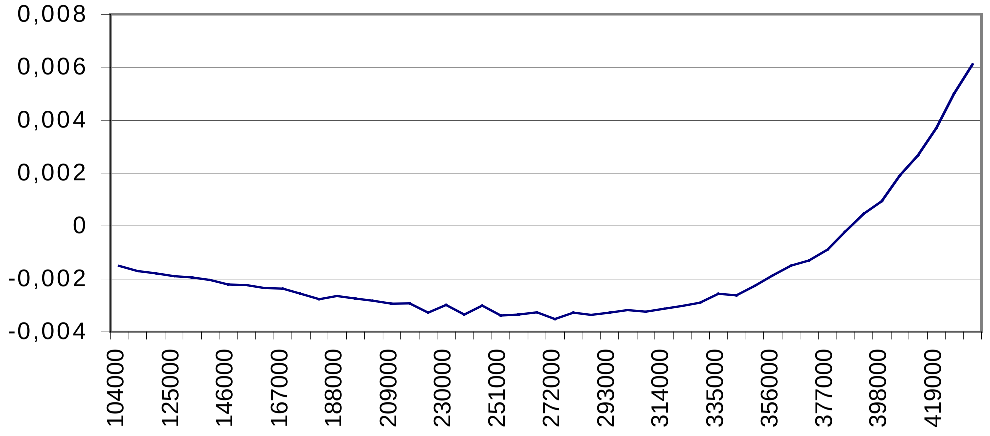

0,008
0,006
0,004
0,002
0
-0,002
-0,004
104000
125000
146000
167000
188000
209000
230000
251000
272000
293000
314000
335000
356000
377000
398000
419000

Figure 10	Elasticities for Population Destination related to Population size.

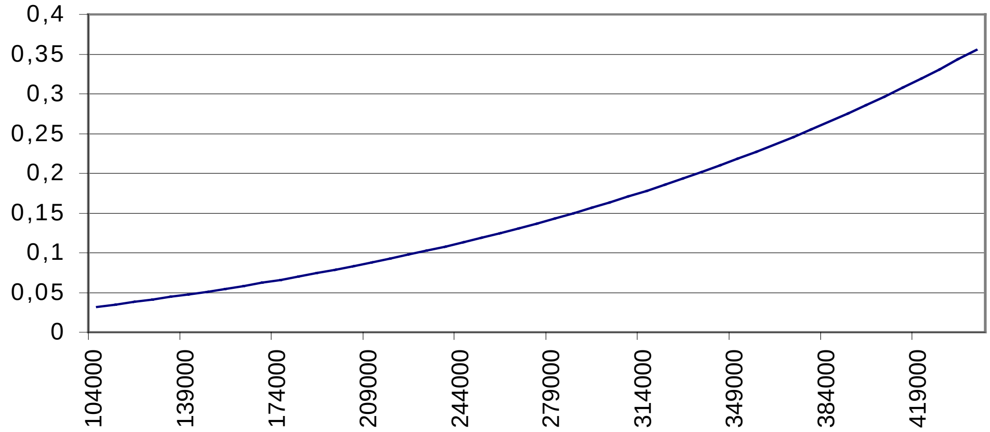
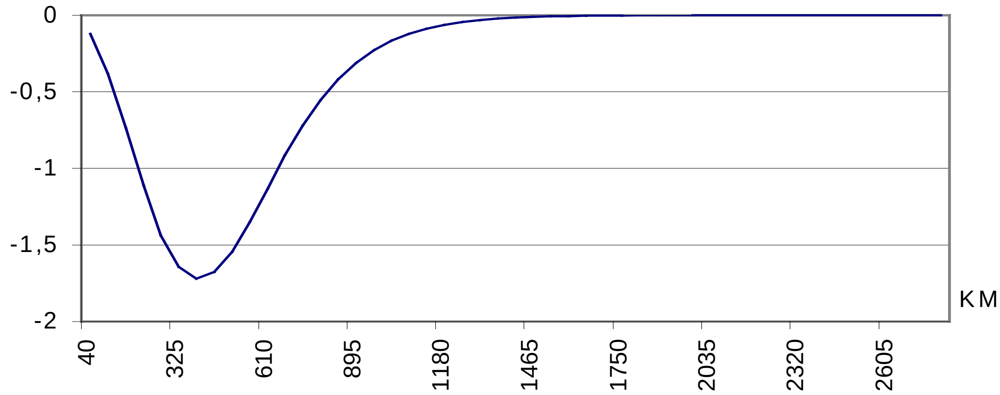


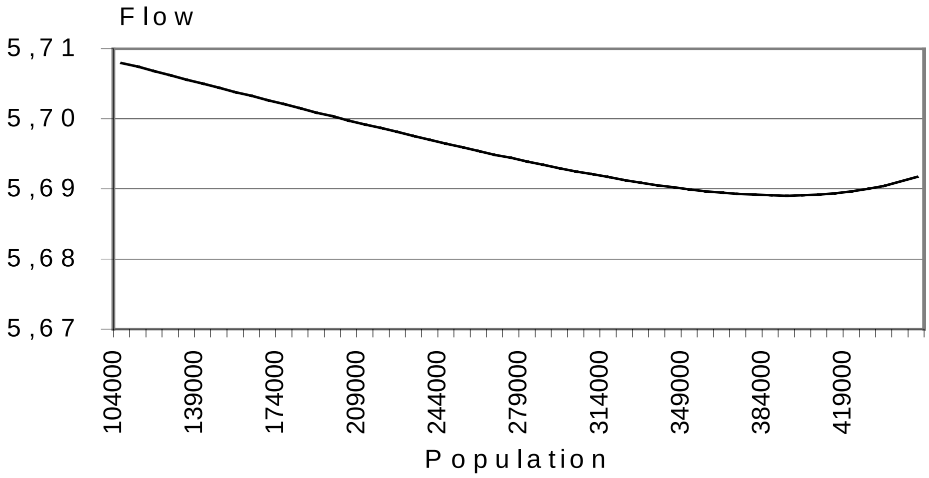
Figure 11	Elasticities for Kilometers related to distance in Kilometers.



In Figure 14 we have plotted how the flow changes as one variable at the time is varied between it's min- and max values and others held constant at their means.

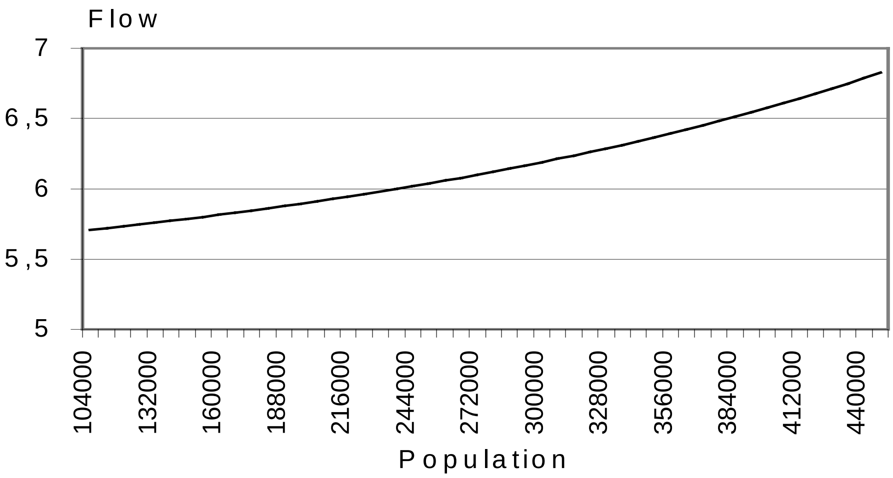
Figure 12 show what elasticities indicate, namely that the flow first decrease as population increase and eventually starts to increase first as population reaches a certain level. Here a population over 350 000 is needed.

Figure 12 Relation between Origin Population size and flow size.



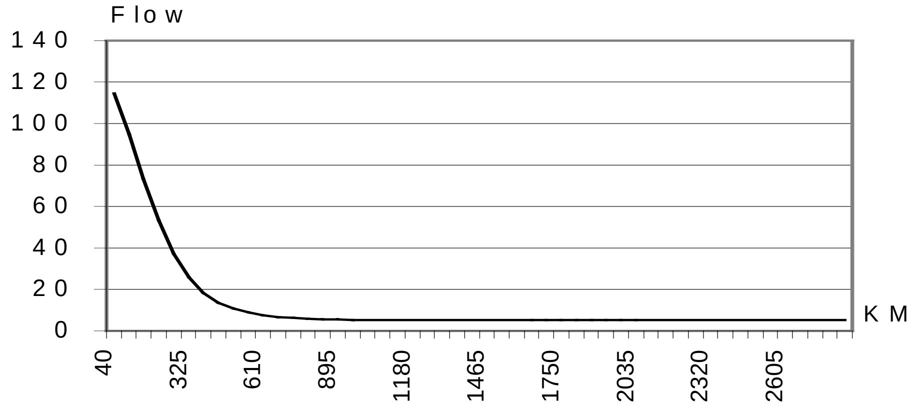
For population at the destination in Figure 13 we get an almost linear relation between population and flow size, which is as we would expect from investigating elasticities.

Figure 13 Relation between Destination Population size and flow size.



The relation between kilometres and flow in Figure 14 is that flow decreases sharply for distances up to about 300 kilometres and that the decrease then slows down an finally the flow starts to get totally insensitive for distance. From elasticities we also know that the decrease accelerates up to 300 kilometres.

Figure 14 Relation between distance in kilometres and flow size.



5. FINAL COMMENTS

Here we have compared four different estimation methods and their ability to predict freight flows between Norwegian counties. Amongst these the neural network compared best in terms of root mean square error. Differences are nevertheless small and if the measurement of performance is changed to one that is more sensitive to shares of larger deviation estimators such as NLS and Poisson regression performed better. It is hence crucial to know what kind of errors one wants to minimise before choosing evaluation method and estimator. For measurements based on residual deviation we get the ranking that NN is best closely followed but NLS and Poisson regression and OLS a little more behind. This is similar to results in Bergkvist and Westin (1997) but there NLS failed completely in out-of-sample forecasting and the differences between Poisson and NN was larger. This probably since data was less “well-behaving” and therefor caused greater problems for the more linear estimators.

We also investigated elasticities and all of them are constant elastic with similar sensitivity except for the NN which is not constant elastic and also shows different functional form on elasticities for different explanatory variables. An interesting property since plots of elasticities are possible to interpret in a economic consistent way.

NN performs well but has it's drawbacks in estimation time and also the time consuming process that is necessary to find a good enough topology and find reasonable free parameters which have to be set by the researcher. Poisson is here more straightforward to use and also shows reasonable performance on data used here and in Bergkvist and Westin (1997). The NN however give more information due to it's ability to treat inputs in a much more individual way.

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APPENDIX

Figure 15 Structure of a neural network.

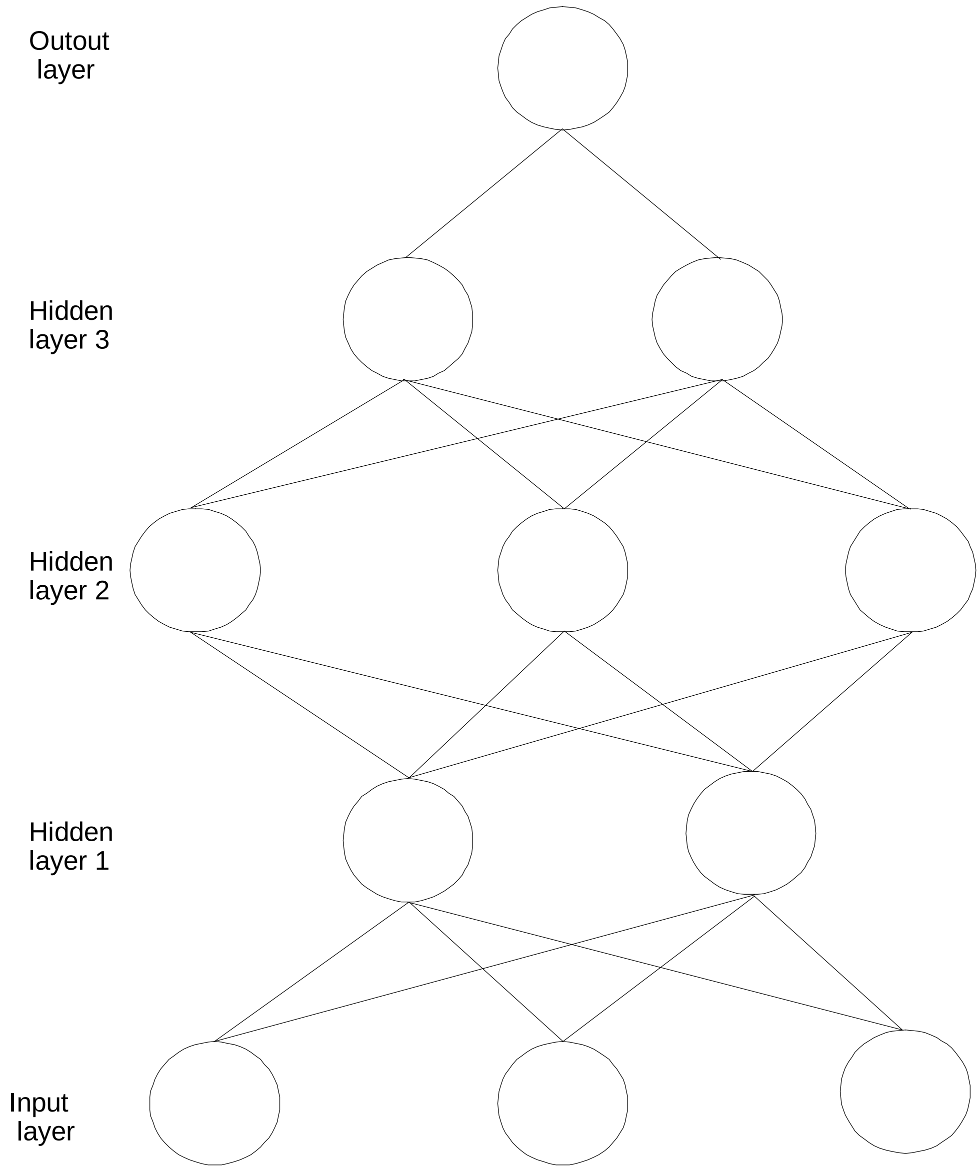

Figure 16 Residual plot for the training set with OLS estimation. Residuals versus actual flows.

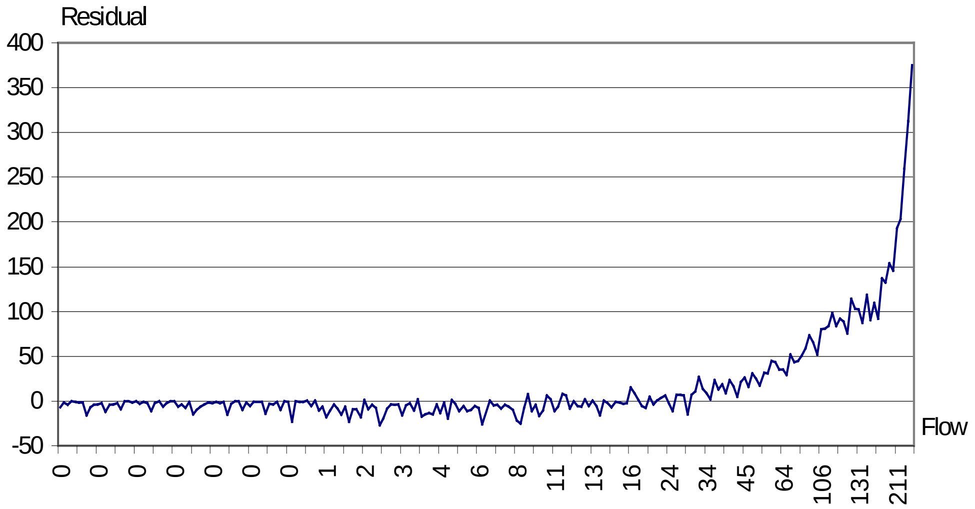

Figure 17 Residual plot for the training set with NLS estimation. Residuals versus actual flows.

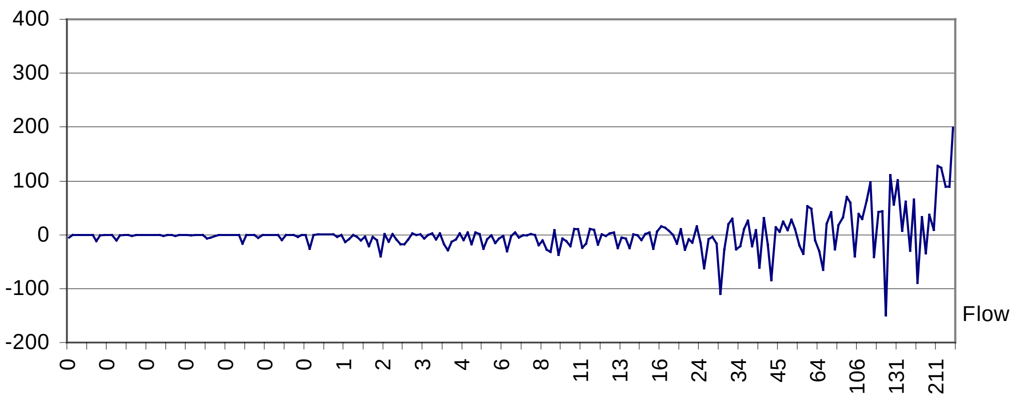

Figure 18 Residual plot for the train set with a Poisson specification. Residuals versus actual flows.

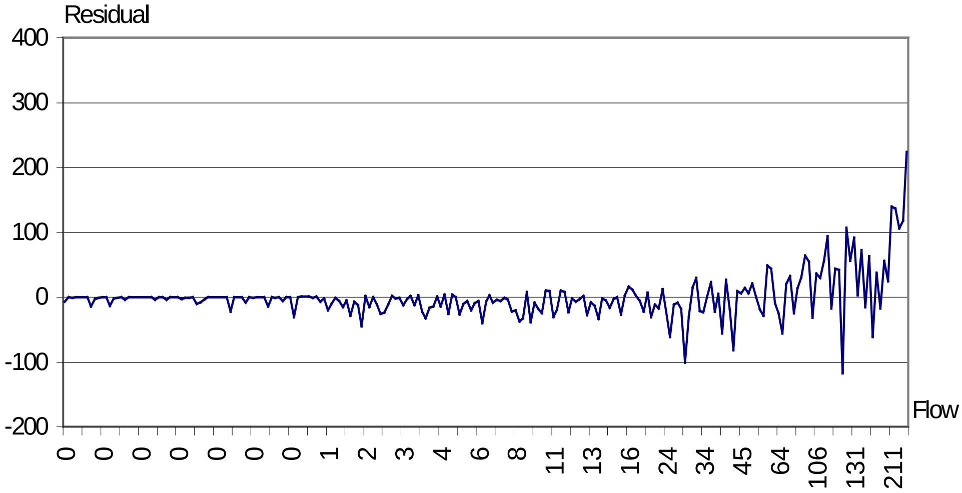

Figure 19 Residual plot for the training set with a BP-NN specification. Residuals versus actual flows.