On the Dynamics of Net versus Gross Multipliers

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Abstract

Industries often promote their interests by arguing that they have a big impact on the rest of the economy. To substantiate such claims usually some measure of size (employment or value added) is labelled the direct effect and is then multiplied with the corresponding (gross) multiplier to calculate what is labelled the total impact of the sector or project at hand. To avoid double-counting impacts and to solve the endogenous/exogenous mix-up involved the *net multiplier* concept was introduced (Oosterhaven and Stelder, 2002).

Both the standard (gross) multiplier and the new net multiplier are essentially static concepts. When applied in a dynamic setting the question of stability rises. The stability of the gross multipliers from the standard input-output model is based on the stability of its input coefficients. The stability of net multipliers is also based on the stability of its additional exogenous demand/total output ratios, which are unstable by definition. This note will argue that this property should not be seen as a vice but as an additional virtue of the net multiplier concept.

In a closed economy, assuming fixed input price ratios, the stability of the input coefficients is a technological feature. In an open regional or national economy, with growing exogenous demand, gross multiplier stability also implies the absence of import substitution. This is unlikely whenever the growth of exogenous demand is substantial. The net multiplier concept forces the user to consider not only import substitution but also export substitution explicitly. Depending on the relative size of import versus export substitution, the net multiplier may either rise or fall, whereas the gross multiplier only rises when the economy grows.

From this, the paper argues that using net multipliers is more appropriate than using gross multipliers not only in a static setting but also in a dynamic setting, that is when judging the relative importance of industries is the issue.

Keywords

Input-output analysis, net multipliers, import substitution, export substitution.

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C67, L91, R15

1. INTRODUCTION

Arguments for state aid and state intervention in favour of certain sectors of industry are often based on their assumed economic importance for the region or nation at hand. Mostly, the arguments are not primarily based on the own size and the direct impact of the sector or project at hand, but on its assumed *indirect* importance for the regional or national economy (cf. Oosterhaven, Eding and Stelder, 2001). To substantiate such claims consultants, but also academics, traditionally multiply direct employment or some other kind of size indicator with a sector- or project-specific employment or value added multiplier. The result is presented as an estimate of the total impact, direct plus indirect plus induced plus whatever other impacts one can think of (see in the case of plant closedowns, Cole, 1988, discussed in Jackson, Madden & Bowman, 1997, and Oosterhaven, 2000). The main problem with this traditional approach is the claim of each and every sector to be economically more important than its own share in total emp1oyment or value added indicates. Naturally, this can not be true. When the claims of all sectors in an economy are added an (implicit) estimate of the total size of the economy will result that is many times larger than its actual size.

Section 3 presents a new concept, the *net multiplier*. When this net multiplier is applied to all sectors, the estimated total of all impacts will aggregate to the correct size of the whole economy.

2. THE SIN OF EXAGGERATING SECTORAL IMPACTS

The *direct* economic importance of a certain sector can, seemingly easily, be measured by some kind of size indicator, preferably by its direct contribution to the gross regional or gross national product (GRP or GNP), or else by its direct contribution to total regional or national employment. Even this simple way of measuring the direct impact is not without problems, as will become clear soon.

The line of reasoning and the calculation of the *indirect* economic importance of a sector normally starts with making an inventory of its relations with other actors in the economy. It is of importance to make a sharp distinction between the concept of relation or linkage on the one hand and the concept of effect or impact on the other. The difference is that the first concept is causally neutral whereas the second concept implies causality. Clearly, when the issue of the indirect economic importance of a certain sector is concerned, one is or at least one should be interested in causality.

A sector may have large backward and forward linkages, but that does not tell us whether that sector is (passively) receiving impulses from other sectors or (actively) sending impulses to other sectors. Consequently, the existence of large forward and backward linkages, without further information, does not imply that the sector at hand can be considered a key sector for regional of national development, despite the fact that this is regularly done in empirical development economics (see e.g. Schultz, 1977). To be labelled a *key sector,* a second criterion has to be satisfied. The sector at hand, besides (1) having *large linkages* to pass on growth impulses, also needs (2) to *generate* these growth *impulses* itself. Thus, the growth in this sector needs to be considered as (largely) exogenous from the rest of the economy (see Beyers, 1976, and Oosterhaven, 1981, ch. 5, for a further discussion).

If we look at the standard Leontief model, final demand for sectoral outputs **f** is exogenous, and the causality of the model runs as follows. Any change in final as well as total demand for sectoral outputs is matched, without supply constraints, by endogenous sectoral production **x**. Sectoral production, in its turn, determines the endogenous intermediate demand for sectoral outputs **A x** as well as the endogenous demand for primary inputs, such as value added and imports. The model solution for total value added v is the following:

$$v = \mathbf{v_c}'\,\mathbf{x} = \mathbf{v_c}'\,(\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f} = \mathbf{v_c}'\,\mathbf{L}\,\mathbf{f} \qquad (1)$$

where $\mathbf{v_c}'$ represents a row with value added coefficients, **A** the matrix with intermediate input coefficients, and **L** the Leontief-inverse (see Oosterhaven, 1981, ch. 2, or Miller & Blair, 1985, ch. 2, for details).

From (1) it is clear that the value added multipliers $\mathbf{v_c}'\ \mathbf{L}$ may only be multiplied with exogenous final demand **f** and *not* with endogenous total output **x**. When the latter is done, this unavoidably leads to the over-estimation of the importance of the sector at hand. The reason is that (1) assumes that the intermediate part of total output (**A x**) is endogenously determined by the size of (mainly the other sectors') total output. Multiplying the total of **x** with the value added multipliers $\mathbf{v_c}'\ \mathbf{L}$ results in double counting the endogenous part (**x – f**). When calculating the importance of Schiphol airport for Dutch national employment, for instance, BCI/NEI (1997, table 5.1) forget that part of the backward employment effect actually occurs within the aviation industry that was already included in the direct (platform-tied) employment at Schiphol that was assumed exogenous, but is not, at least not entirely. As a consequence, that part of backward employment is in fact double counted.

Things get even more wrong when ad hoc estimates of causal forward effects are added to the so-called direct effect, which total is then multiplied with standard employment or value added multipliers in order to estimate the so-called backward impacts of a certain industrial complex of project. This procedure easily leads to triple counting of effects. Besides the above mentioned double counting of part of the direct effect with part of the backward effects, this will also lead to double counting of part of both the direct and the backward effects with the forward effects. When evaluating the economic impact of a rail freight line from the port of Rotterdam to the Ruhr-area, for instance, Knight Wendling (1992) added the backward effects of the Rotterdam port industries on inland freight transport to the forward effects of the freight line on the Rotterdam port industries. The principal reason for all this over-estimation and double counting is simple: multipliers are used outside the context of the model from which they are derived. The simple remedy "don't do it", however, is too naive. Practitioners will continue to need simple devices like multipliers, which they will unavoidably want to simply multiply with total direct employment or value added.

3. THE REMEDY: USE NET MULTIPLIERS

Thus, the standard input-output multipliers may only be multiplied with (exogenous) final demand. When they are multiplied with *total* sectoral value added or employment, they in fact operate as "gross multipliers" since this misuse will result in over-estimating the economic importance of the sector at hand. Therefore, we will use the new label *net multiplier* to indicate any multiplier that may rightfully be multiplied with total sectoral output, value added or employment without resulting in an overestimation of that sector's economic importance.

More precisely, in the case of total sectoral output, we will define the *Type I net total output multipliers* as $\mathbf{i}' \ (\mathbf{I} - \mathbf{A})^{-1} \ \langle \mathbf{f_c} \rangle$, with $\mathbf{i}' \ (\mathbf{I} - \mathbf{A})^{-1}$ being the standard total output multipliers and $\langle \mathbf{f_c} \rangle$ a diagonal matrix with the fractions of total sectoral output that may rightfully be considered exogenous (i.e. f_j / x_j).

In the case of value added and employment multipliers, the corresponding net multipliers need of course to be multiplied with total sectoral value added or employment, and not with total output. This means that the “gross” value added and employment multipliers first need to be standardised (cf. Oosterhaven, 1981, ch. 4; Miller and Blair, 1985, ch. 4) before the corresponding net multiplier can be formulated. This leads to the following definition of the *Type I net value added multipliers*:

$$\boldsymbol{\mu}_\mathbf{I}' = \mathbf{v_c}' \ (\mathbf{I} - \mathbf{A})^{-1} \ \langle \mathbf{v_c} \rangle^{-1} \ \langle \mathbf{f_c} \rangle \qquad (2)$$

In (2) $\langle \mathbf{f_c} \rangle$ is again the diagonal matrix with the sectoral final output ratios (of Type I) that secures the net character of the multipliers, while $\langle \mathbf{v_c} \rangle^{\mathbf{-1}}$ represents the diagonal inverse of the sectoral value added ratios that secures the standardisation with respect to sectoral value added.

When the standard Leontief model is extended with endogenous household consumption expenditures (cf. Oosterhaven, 1981, ch. 6, or Batey, 1985), the *Type II net value added multipliers* are defined as:

$$\boldsymbol{\mu}_{\mathbf{II}}' = \mathbf{v_c}' \, (\mathbf{I - A - Q})^{\mathbf{-1}} \, \langle \mathbf{v_c} \rangle^{\mathbf{-1}} \, \langle \mathbf{f_c}^{*} \rangle \qquad (3)$$

In (3) q_{ij} from **Q** indicates the endogenous consumption expenditures on products from sector i paid for from incomes earned in sector j per unit of output in sector j, while $\langle \mathbf{f_c}^* \rangle$ now represents what may be called the Type II final output ratios, which are derived from $\mathbf{f^* = f - Qx}$. Since consumption is now also endogenous, only $\mathbf{f - Qx}$ remains as *exogenous* final demand.

Type I and Type II net *employment* multipliers are defined analogously by replacing $\mathbf{v_c}$ with $\mathbf{e_c}$, containing the sectoral employment/output ratios.

If we denote the extended Leontief inverse $(\mathbf{I - A - Q})^{\mathbf{-1}}$ in (3) with $\mathbf{L}^*$, standard input-output analysis tells us that the usual Type II multipliers based on $\mathbf{L}^*$ are larger than the usual Type I multipliers based on **L** (provided of course that $\mathbf{Q} > \mathbf{0}$). In case of the net multipliers, however, no such systematic relation can be found. This follows from the following important property.

Theorem 1. The output-weighted average of all sectoral *net* multipliers equals unity.

Proof. With $x = \mathbf{i' \, x}$, the economy-wide total output c.q. total input, this follows for the Type I net value added multipliers from:

$$\boldsymbol{\mu}_{\mathbf{I}}' \, (\mathbf{x} \, x^{-1}) = \mathbf{v_c}' \, \mathbf{L} \, \langle \mathbf{v_c} \rangle^{\mathbf{-1}} \, \langle \mathbf{f_c} \rangle \, \mathbf{x} \, x^{-1} = \mathbf{v_c}' \, \mathbf{L} \, \langle \mathbf{f} \rangle \, \langle \mathbf{v} \rangle^{\mathbf{-1}} \, \mathbf{x} \, x^{-1} = \mathbf{v}' \, \langle \mathbf{v} \rangle^{\mathbf{-1}} \, \mathbf{x} \, x^{-1} = 1$$

The proof for the Type II net multipliers $\boldsymbol{\mu}_{\mathbf{II}}'$ runs analogous with **L** and **f** replaced by $\mathbf{L}^*$ and $\mathbf{f}^*$. The proofs for the net employment multipliers runs analogous with $\mathbf{e_c}$ instead of $\mathbf{v_c}$. The proof for the net total output multipliers also runs analogous but without the $\mathbf{v_c}$ matrices and vectors.

The above theorem is, of course, precisely the reason for developing the concept of the net multiplier. As a consequence, the net multipliers avoid the double counting of impacts, as follows from the crucial next property.

Theorem 2. When each sectoral *net* multiplier is multiplied with its appropriate sectoral total and then summed over all sectors, the total for the whole economy will result.

Proof. For the Type II net value added multipliers this follows from:

$$\boldsymbol{\mu}_{II}'\,\mathbf{v} = \mathbf{v}_c'\,\mathbf{L}^*\,\langle\mathbf{v}_c\rangle^{-1}\,\langle\mathbf{f}_c^{ex}\rangle\,\mathbf{v} = \mathbf{v}_c'\,\mathbf{L}^*\,\langle\mathbf{f}^{ex}\rangle\,\langle\mathbf{v}\rangle^{-1}\,\mathbf{v} = \mathbf{v}'\,\langle\mathbf{v}\rangle^{-1}\,\mathbf{v} = \mathbf{v}'\,\mathbf{i} = v$$

The proof for the other net multipliers again runs analogous.

From Theorem 1 and Theorem 2 it naturally follows that sectors with multipliers smaller than one will be more dependent on other sectors than those other sectors are dependent upon them. The most extreme case being, of course, sectors that have a net multiplier equal to zero. Given the above, the interpretation of this extreme case is simple and precisely signifies why the concept of the net multiplier was developed. Such sectors have an (exogenous) final output equal to zero, which signifies that they are not able to generate exogenous growth impulses themselves. This does not imply that these sectors are not important, but it signifies that their growth is entirely dependent upon the impulses they receive from and through other sectors.

4. THE TIME DIMENSION

It may be pointed out that an exogenous final demand impulse will increase the final output ratio of the sector at hand (f_j / x_j) and thus its net multiplier. But, the indirectly increased output in all (other) sectors will result in decreases of their final output ratios and their net multipliers. In the new equilibrium both theorems will still hold. The fact that in equilibrium (most probably) the net multiplier of the primarily affected sector will increase and all others will decrease, is precisely what the net multiplier concept intends to pick up. The fact that the standard multipliers do not change again indicates that they are not the proper tool to use when measuring the importance of a sector for the regional or national economy, after a final demand impulse.

5. CONCLUSION

Both the theoretical discussion and the empirical illustration show that claims of economic importance are often misleadingly high. This article introduces the concept of the *net multiplier* to remedy this systematic upward bias. The most appealing aspect of this new multiplier is that double counting is avoided and when applied to all sectors the net multipliers neatly add up to the national economy. Another interesting property of the net multiplier is that it can be smaller than 1.00, which gives a numerical expression to the notion that certain sectors may be more dependent on the rest of the economy than the rest of the economy is on them. Nevertheless, the input-output *models* behind the net multiplier concept are the same as the models behind the standard multipliers. This means that they also ignore price-volume interactions.

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