Decomposition & Synergy: a Study of the Interactions And Dependence Among the 5 Brazilian Macro Regions

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Abstract

The methodology originally developed by Sonis, Hewings, and Miyazawa (1997) and in a exploratory way applied by Guilhoto, Hewings, and Sonis (1998) to an interregional input-output table at the level of 2 regions for the year of 1992 for the Brazilian economy is now expanded and discussed more thoroughly when applied to an interregional table at the level of the 5 macro regions of the Brazilian economy (constructed by the author for the year of 1995). The methodology used in this work is based on a partitioned input-output system and exploits techniques of the Leontief inverse through the nature of the internal and external interdependencies giving by the linkages, which allows to classify the types of synergetic interactions within a preset pair-wise hierarchy of economic linkages sub-systems. The application of the above methodology is done by taking into consideration the vector of final demand and the vector of gross output such that it is possible to estimate the contribution of each interaction to the total production in the productive process of each region.

The results for the Brazilian economy show that in terms of the productive structure: a) the North region has practically no relation with the Northeast region and vice-versa; b) while the South region has some impact on the production of the North region, the inverse is not true; c) despite the fact that the demands from the Central West region have some impact on the production of the other regions, the production in the Central West region has its relations concentrated with the Southeast and South regions; c) the South and Southeast regions show to be the most important regions in the system.

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I. Introduction

The methodology originally developed by Sonis, Hewings, and Miyazawa (1997), which classifies the types of synergetic interactions and allows to examine the structure of the trading relations among the regions, and in a exploratory way applied by Guilhoto, Hewings, and Sonis (1998) to an interregional input-output table at the level of 2 regions for the year of 1992 for the Brazilian economy is now expanded and discussed more thoroughly when applied to an interregional table at the level of the 5 macro regions (North, Northeast, Central West, Southeast, and South) of the Brazilian economy (constructed by the author for the year of 1995)

This work is organized in the following way: a) the theoretical background will be presented in the next section; b) the third section will present the results for the Brazilian economy; and c) some final remarks will be made in the last section.

II. Theoretical Background

This methodological section will be divided into two parts: a) in the first one it is made reference to the theory originally developed for the two regions case; and b) in the second it is showed how this theory can be extended to the *n* regions case.

II.1. The Two Regions Case

A complete description for the 2 regions case is presented in Sonis, Hewings, and Miyazawa (1997), which is the basis for this section.

Consider an input-output system represented by the following block matrix, A, of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad (1)$$

where A_{11} and A_{22} are the quadrat matrices of direct inputs within the first and second regions, and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

The building blocks of the pair-wise hierarchies of sub-systems of intra/interregional linkages of the block-matrix Input-Output system are the four matrices $A_{11,} A_{12}, A_{21}$ and A_{22}, corresponding to four basic block-matrices:

$$A_{11} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix} \quad A_{21} = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix} \quad A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \qquad (2)$$

This section will usually consider the decomposition of the block-matrix (1) into the sum of two block-matrices, such that each of them is the sum of the block-matrices (2) $A_{11,} A_{12}, A_{21}$ and A_{22}. From (1) 14 types of pair-wise hierarchies of economic sub-systems can be identified by the decompositions of the matrix of the block-matrix A (see Figure 1 and Table 1).

Consider the hierarchy of Input-Output sub-systems represented by the decomposition $A = A_1 + A_2$. Introducing the Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system.

The possibilities for the A_1 matrix are presented in Table 1. Also, Figure 1 shows the schematic representation of the possible forms of the A_1 matrices.

Consider the hierarchy of input-output sub-systems represented by the decomposition $A = A_1 + A_2$ and their Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system. If f is the vector of final demand and x is the vector of gross output, then it is possible to generate the decomposition of gross output into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. Such decomposition is important for the empirical analysis of the structure of actual gross output and the contribution that the relations among the regions have to the total gross output.

While 14 types of pair-wise hierarchies of economic linkages have been developed (Figure 1 and Table 1), it is possible to suggest a typology of categories into which these types may be placed. The following characterization is suggested:

1. backward linkage type (VI, IX): power of dispersion

2. forward linkage type (V, X): sensitivity of dispersion

3. intra- and inter- linkages type (VII, VIII): internal and external dispersion

4. isolated region vs. the rest of the economy interactions style (I, XIV, IV, XI)

5. triangular sub-system vs. the interregional interactions style (II, XIII, III, XII).

By viewing the system of hierarchies of linkages in this fashion, it will be possible to provide new insights into the properties of the structures that are revealed.

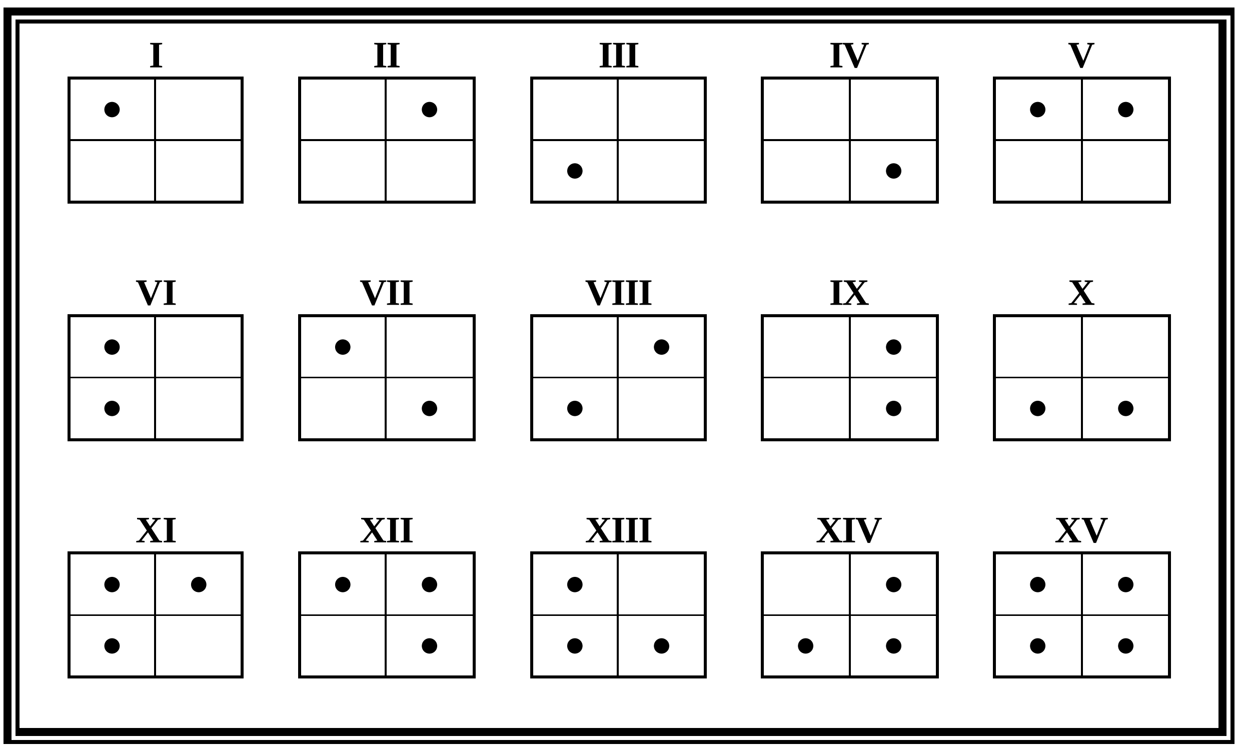

Figure 1

Schematic Representation of the Possible Forms of the A_1 Matrix – 2 Regions Case

Table 1

Taxonomy of Synergetic Interactions between Economic Sub-Systems

[Each entry presents a description of the structure and the corresponding form of the A_1 matrix]

I. *Hierarchy of isolated region versus the rest of economy*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}$
II. *The order replaced hierarchy of interregional linkages of second region versus lower triangular sub system*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}$
III. *The order replaced hierarchy of interregional linkages of first region versus upper triangular sub system*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}$
IV. *The order replaced hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$
V. *Hierarchy of forward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$
VI. *Hierarchy of backward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix}$
VII. *The hierarchy of intra- versus inter- regional relationships*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$
VIII. *The hierarchy of inter versus intra regional relationships*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$
IX. *Order replaced hierarchy of backward linkages*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & A_{22} \end{bmatrix}$
X. *Order replaced hierarchy of forward linkages*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XI. *The hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}$
XII. *The hierarchy of upper triangular sub system versus interregional linkages of first region*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$
XIII. *The hierarchy of lower triangular sub system versus interregional linkages of second region*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XIV. *Hierarchy of the rest of economy versus second isolated region*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

II.2. The n Regions Case

For the *n* regions case the number of decompositions increases dramatically as one increases the number of regions, such that from the 15 decompositions (including the whole system) for the 2 regions case, one goes to: a) 511 decompositions for the three regions case; b) 65,535 decompositions for the 4 regions; c) 33,554,431 decompositions for the 5 regions; and so on. In this way, the equation representation of the system for the *n* regions case becomes very complex, so what is presented here is a general idea of how the system works, as can be seen in a schematic way for the 5 regions case, as it is presented in Figure 2. From this figure one can see that in the 5 regions case one has 25 matrices. At first, one has to consider each matrix isolated, the next step is to consider the 25 matrices combined 2 at time, then 3 at time, and so forth, until one gets to the whole system. To measure the contribution of each combination for the production in the productive process one has to subtract from the result of the combination of *k* matrices all the possible lower level combinations of these matrices, e.g., the result of a set of 5 matrices must be subtracted from the results of all the possible combination of these five matrices at the level of 4, 3, 2, and 1 matrices.

The next section will present the results when the above methodology is applied to the interregional system of the 5 Brazilian macro regions.

III. An Application to the Brazilian Economy

In this section it is made first a general presentation of the main aspects of the five Brazilian macro regions and then it is made an analysis of the results derived from the application of the theory presented in section II.

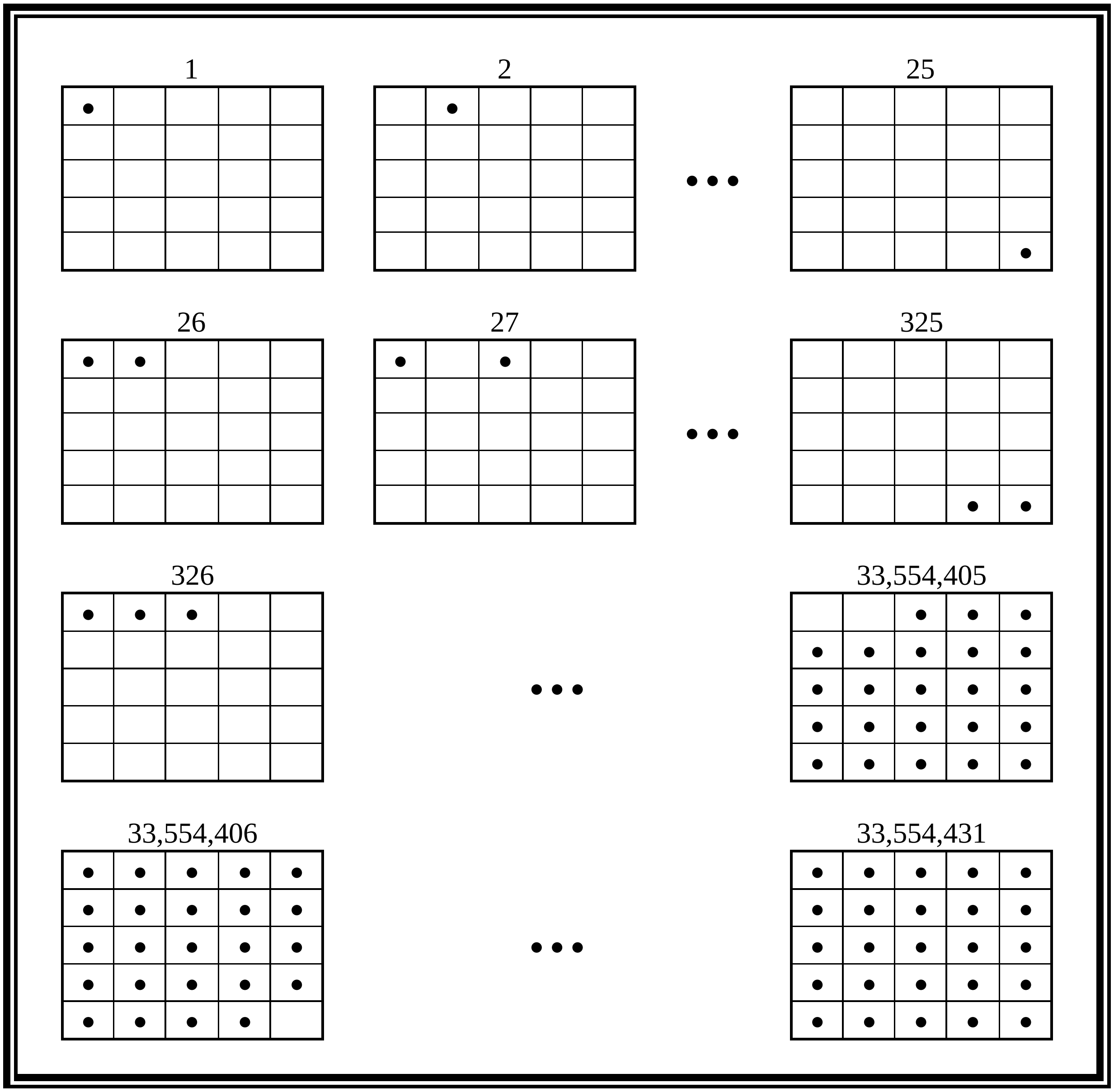

Figure 2

Schematic Representation of the Possible Forms of the A_1 Matrix – 5 Regions Case

III.1 The Brazilian Macro Regions

According to the classification of Brazilian Institute of Geography and Statistics (IBGE) the Brazilian Economy is divided into 5 macro regions, see Figure 3: a) North (6 States); b) Northeast (9 States); c) Central West (4 States and the Federal District); d) Southeast (4 States); and e) South (3 States).

The overall size of the Brazilian territory is 8,511,996 Km^2 of which 45.25% belongs to the North region, 18.25% to the Northeast, 18.85% to the Central West, 10.85% to the Southeast, and 6.76% to the South. However the economic and population distribution do not follow the geographical distribution, as can be seen in Table 2.

Figure 3

Map of Brazil and Its 5 Macro Regions

Table 2

Main Economical and Geographical Characteristics of the Brazilian Macro Regions

	Size		Population (1996)		Urban Population	GDP 1995
	km^2	Share (%)	Number (1,000)	Share	%	Share (%)
North	3,851,560	45.25	11,288	7.19	62.36	5.27
Northeast	1,556,001	18.28	44,767	28.50	65.21	13.62
Central West	1,604,852	18.85	10,501	6.69	84.42	7.25
Southeast	924,266	10.85	67,001	42.66	89.29	56.97
South	575,316	6.76	23,514	14.97	77.22	16.89
Brazil	8,511,996	100.00	157,070	100.00	78.36	100.00

Source: IBGE (1997a and 1997b), Considera and Medina (1998).

Having 45.25% of the Brazilian territory the North region has only 7.19% of the Brazilian population and the smallest number peoples living per km^2, it also has the smallest share of population living in the cities (62.36%) and the smallest share in the Brazilian GDP (5.27%). The most developed regions in Brazil are the Southeast and the South region. The Southeast region has a share of 56.97% of the Brazilian GDP with 42.66% of its population and 10.85% of the territory, while the South region has a share of 16.89% in the Brazilian GDP with 6.76% of the territory and 14.97% of the population. The Southeast region is the most industrialized region in Brazil, while the South region is the one more closed to the Mercosur countries which is the region that due to the continental size of Brazil could be the one to get the most benefits from the Mercosur integration. The Central West region has been an important region for Brazil in terms of agriculture, mainly because of the favorable type of land that this region has, an it has a reflex in its share in the population (6.69%) and GDP (7.25%) of Brazil. The Northeast region has serious problems of draught and in the beginning of the formation of the Brazilian State it used to be it most important region, this region has 18.28% of the Brazilian territory, 28.50% of its population and 13.62% of its GDP, recently oil extraction and processing has been one of the most growing business in the region and with the openness of the Brazilian economy a lot of industries have been installing they production units in the region (in part due to the fiscal incentives giving by the various levels of the state).

III.2. The Productive Relation among the Regions

Using a set of interregional input-output tables built by the author at the level of 22 sectors for the year of 1995 for the 5 Brazilian macro regions (North (N), Northeast (NE), Central West (CW), Southeast (SE), and South (S)), the methodology presented in section II is applied, and the results are presented in this section.

Due to computational problems, i.e., the computer resources available to the author were not enough to carry out the estimations directly at the 5 regions level, the estimations were carried in the following way: a) first, it was considered each region against all the others aggregated; and b) then, the results for the five regions where derived from the results obtained from five four regions cases where two regions were aggregated.

It was necessary to derive the five regions case from the four regions case due to computer time requirements. In the 4 regions case the computer resources required are considerable, the time to estimated all the 65,535 combinations on a 120 MHz Pentium computer (used by the author) would be more than one week. Fortunately, in practical terms, the combinations of 1, 2, 3, 4, and 5 matrices generates more than 99.90% of production explanation for a given region, which allows to take the remaining explanation as a residual of all the other combinations (even in this case the computer takes more than 6 hours to generate the results for each interregional system of 4 regions).

To aggregate the 5 regions into 4 it was taken into consideration the geographic localization of the regions as well as their economic relations, resulting into 5 combinations: a) N+NE, CW, SE, S; b) N+CW, NE, SE, S; c) NE+CW, N, SE, S; d) N, NE, CW+SE, S; and e) N, NE, CW, SE+S. The results for the 4 regions case are presented in the appendix.

Below it is made an analysis of the results for the 2 regions and 5 regions cases. The results for the 2 regions case allow on the one hand a first view of how each region interacts with the rest of the economy and on the other hand permits to see the importance of each interaction to generated the production in each region. The 5 regions case will give more emphasis on the analysis of the importance of the links among the regions to the production generated into each region.

III.2.1. The 2 Regions Case (One Region Against all the others)

Starting from the isolated regions (block matrices) and then adding the interactions among them it is possible to measure how each interaction adds to the total production. These results are presented in Table 3 and in Figure 3 for each of the 2 regions case, i.e., one region against the rest of Brazil.

The results show that decomposition **I**, that measures the contribution of the production inside the region to the total production in the productive process, is the most important element in all of the 5 Brazilian regions, however it presents the highest values in the most developed regions, Southeast (84.52%) and South (76.86%). For the Northeast region it represents 73.12%, it also shows that the North (68.44%) and the Central West (64.33%) are the regions more dependents on the other regions for their productive process.

The most important decompositions for the region 1 (isolated Brazilian region), in the 2 regions case, are decompositions **I, II, V, IX,** and **XII**, which are related with the matrices A_{11} , A_{12} , and A_{22} (Table 2 and Figure 3), this meaning that the inputs that each Brazilian region buys from the rest of the economy has practically no impact over its production. From the data one has that the inputs that the rest of the economy buys from a given region (A_{12}) represents from 12.15% (South) to 27.32% (North) of the production in this region, while the production relations inside the rest of Brazil (A_{22}) represents from 2.72% (South) to 8.12% (North) of the production in this region.

Giving the size of the Brazilian economy and the importance of the Southeast and South regions economy, for region 2 (the Rest of Brazil), in the 2 regions case, one has that the most important decompositions are the decompositions **III, IV, VI, X,** and **XIII**, which are related with the matrices A_{22} , A_{21} , and A_{11} (Table 2 and Figure 3). A closer look at the data also shows that with the exceptions of the cases where the Southeast and the South regions are taken isolated the relations inside the rest of Brazil economy (A_{22}) responds for around 97% of the production in the productive process.

Table 2
Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of *$(x_1\text{-}f)$* in *x*

North and Rest of Brazil

	North					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	60.24	60.24	-	-	-	-	-	-	-	-
II	16.34	-	16.34	-	-	-	-	-	-	-
III	-	-	-	-	-	0.80	-	-	0.80	-
IV	-	-	-	-	-	97.88	-	-	-	97.88
V	5.40	2.70	2.70	-	-	-	-	-	-	-
VI	-	-	-	-	-	0.20	0.10	-	0.10	-
VII	-	-	-	-	-	-	-	-	-	-
VIII	0.25	-	0.12	0.12	-	0.05	-	0.03	0.03	-
IX	13.44	-	6.72	-	6.72	-	-	-	-	-
X	-	-	-	-	-	0.73	-	-	0.37	0.37
XI	0.11	0.04	0.04	0.04	-	0.02	0.01	0.01	0.01	-
XII	4.00	1.33	1.33	-	1.33	-	-	-	-	-
XIII	-	-	-	-	-	0.17	0.06	-	0.06	0.06
XIV	0.14	-	0.05	0.05	0.05	0.11	-	0.04	0.04	0.04
XV	0.08	0.02	0.02	0.02	0.02	0.04	0.01	0.01	0.01	0.01
Total	100.00	64.33	27.32	0.23	8.12	100.00	0.17	0.08	1.40	98.35

Northeast and Rest of Brazil

	Northeast					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	68.24	68.24	-	-	-	-	-	-	-	-
II	8.82	-	8.82	-	-	-	-	-	-	-
III	-	-	-	-	-	1.20	-	-	1.20	-
IV	-	-	-	-	-	96.28	-	-	-	96.28
V	4.84	2.42	2.42	-	-	-	-	-	-	-
VI	-	-	-	-	-	0.49	0.25	-	0.25	-
VII	-	-	-	-	-	-	-	-	-	-
VIII	0.22	-	0.11	0.11	-	0.08	-	0.04	0.04	-
IX	10.23	-	5.12	-	5.12	-	-	-	-	-
X	-	-	-	-	-	1.10	-	-	0.55	0.55
XI	0.34	0.11	0.11	0.11	-	0.04	0.01	0.01	0.01	-
XII	6.85	2.28	2.28	-	2.28	-	-	-	-	-
XIII	-	-	-	-	-	0.42	0.14	-	0.14	0.14
XIV	0.19	-	0.06	0.06	0.06	0.24	-	0.08	0.08	0.08
XV	0.28	0.07	0.07	0.07	0.07	0.15	0.04	0.04	0.04	0.04
Total	100.00	73.12	18.99	0.35	7.53	100.00	0.44	0.17	2.30	97.09

Table 2 (Continued)

Central West and Rest of Brazil

	Central West					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	63.53	63.53	-	-	-	-	-	-	-	-
II	15.29	-	15.29	-	-	-	-	-	-	-
III	-	-	-	-	-	0.85	-	-	0.85	-
IV	-	-	-	-	-	97.10	-	-	-	97.10
V	6.82	3.41	3.41	-	-	-	-	-	-	-
VI	-	-	-	-	-	0.40	0.20	-	0.20	-
VII	-	-	-	-	-	-	-	-	-	-
VIII	0.08	-	0.04	0.04	-	0.10	-	0.05	0.05	-
IX	9.70	-	4.85	-	4.85	-	-	-	-	-
X	-	-	-	-	-	0.83	-	-	0.41	0.41
XI	0.08	0.03	0.03	0.03	-	0.05	0.02	0.02	0.02	-
XII	4.33	1.44	1.44	-	1.44	-	-	-	-	-
XIII	-	-	-	-	-	0.37	0.12	-	0.12	0.12
XIV	0.08	-	0.03	0.03	0.03	0.21	-	0.07	0.07	0.07
XV	0.08	0.02	0.02	0.02	0.02	0.10	0.02	0.02	0.02	0.02
Total	100.00	68.44	25.11	0.11	6.34	100.00	0.36	0.16	1.74	97.73

Southeast and Rest of Brazil

	Southeast					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	80.68	80.68	-	-	-	-	-	-	-	-
II	6.41	-	6.41	-	-	-	-	-	-	-
III	-	-	-	-	-	8.43	-	-	8.43	-
IV	-	-	-	-	-	76.05	-	-	-	76.05
V	5.22	2.61	2.61	-	-	-	-	-	-	-
VI	-	-	-	-	-	5.58	2.79	-	2.79	-
VII	-	-	-	-	-	-	-	-	-	-
VIII	0.34	-	0.17	0.17	-	0.47	-	0.23	0.23	-
IX	3.30	-	1.65	-	1.65	-	-	-	-	-
X	-	-	-	-	-	4.87	-	-	2.44	2.44
XI	0.70	0.23	0.23	0.23	-	0.37	0.12	0.12	0.12	-
XII	2.64	0.88	0.88	-	0.88	-	-	-	-	-
XIII	-	-	-	-	-	3.10	1.03	-	1.03	1.03
XIV	0.24	-	0.08	0.08	0.08	0.63	-	0.21	0.21	0.21
XV	0.47	0.12	0.12	0.12	0.12	0.50	0.13	0.13	0.13	0.13
Total	100.00	84.52	12.15	0.60	2.72	100.00	4.07	0.69	15.38	79.85

Table 2 (Continued)

South and Rest of Brazil

	South					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	72.04	72.04	-	-	-	-	-	-	-	-
II	10.57	-	10.57	-	-	-	-	-	-	-
III	-	-	-	-	-	2.96	-	-	2.96	-
IV	-	-	-	-	-	90.52	-	-	-	90.52
V	6.96	3.48	3.48	-	-	-	-	-	-	-
VI	-	-	-	-	-	1.69	0.85	-	0.85	-
VII	-	-	-	-	-	-	-	-	-	-
VIII	0.18	-	0.09	0.09	-	0.21	-	0.11	0.11	-
IX	6.02	-	3.01	-	3.01	-	-	-	-	-
X	-	-	-	-	-	2.36	-	-	1.18	1.18
XI	0.27	0.09	0.09	0.09	-	0.16	0.05	0.05	0.05	-
XII	3.58	1.19	1.19	-	1.19	-	-	-	-	-
XIII	-	-	-	-	-	1.43	0.48	-	0.48	0.48
XIV	0.15	-	0.05	0.05	0.05	0.39	-	0.13	0.13	0.13
XV	0.23	0.06	0.06	0.06	0.06	0.28	0.07	0.07	0.07	0.07
Total	100.00	76.86	18.54	0.29	4.31	100.00	1.44	0.36	5.82	92.38

Source: Estimated by the author

North

	N	RB
N	64.33	27.32
RB	0.23	8.12

Rest of Brazil

	N	RB
N	0.17	0.08
RB	1.40	98.35

Northeast

	NE	RB
NE	73.12	18.99
RB	0.35	7.53

Rest of Brazil

	NE	RB
NE	0.44	0.17
RB	2.30	97.09

Central West

	CW	RB
SE	68.44	25.11
RB	0.11	6.34

Rest of Brazil

	CW	RB
SE	0.36	0.16
RB	1.74	97.73

Southeast

	SE	RB
SE	84.52	12.15
RB	0.60	2.72

Rest of Brazil

	SE	RB
SE	4.07	0.69
RB	15.38	79.85

South

	S	RB
S	76.86	18.54
RB	0.29	4.31

Rest of Brazil

	S	RB
S	1.44	0.36
RB	5.82	92.38

Source: Table 2

Figure 3

Schematic Representation of the Results for the 2 Regions Case

In general, for the Brazilian case one has that the size of the regional economy really has an impact on the results, the North and the Central West regions being the more open economies, the South and the Southeast regions being the more closed ones and the Northeast region being in a middle condition among the other regions. In the next section when it will be taking into consideration the relation among the five regions it will be possible to see how each region has its production in the productive process related with the production on the other regions.

III.2.2. The 5 Regions Case

The results for the 5 regions case are presented in Figure 4 which is derived from the data presented in the Appendix, estimated for 5 cases of 4 regions.

When comparing the results presented in this section with the results of the previous section one has that with minor differences (probably due to rounding problems) the sum of the partial results are the same as the aggregated result, which give us confidence in the results obtained in this section and at the same time validate the analysis in the previous section.

Taking a closer look at the relations among the 5 Brazilian macro regions it is clear the importance of the Southeast and the South region for the Brazilian economy. Also, it is possible to identify a set of at most 6 relations that responds for more than 97% of the production in the productive process in a given region.

For the North region the most important relations are the relations inside itself (64.27%), the sales that it makes to the Central West (1.68%), Southeast (17.60%), and South (7.01%) regions, and the relations inside the Southeast (4.97%) and the South (1.64%) regions.

In the Northeast region the most important relations are the relations inside itself (73.03%), the sales that it makes to the Central West (0.98%), Southeast (12.76%), and South (4.03%) regions, and the relations inside the Southeast (4.91%) and the South (1.41%) regions.

The results for the Central West region show that the most strong links for this region are with the Southeast and the South regions such that the relations inside itself represents 68.41%, while the sales to the Southeast and the South regions represent respectively 20.42% and 3.46%, also the relations inside the Southeast region represents 4.65%.

North

	N	NE	CW	SE	S	
N	64.27	0.49	1.68	17.60	7.01	91.05
NE	0.01	0.18	0.00	0.04	0.01	0.24
CW	0.00	0.01	0.34	0.12	0.02	0.49
SE	0.19	0.21	0.15	4.97	0.47	5.99
S	0.03	0.06	0.03	0.44	1.64	2.20
	64.50	0.95	2.20	23.17	9.15	99.97

Northeast

	N	NE	CW	SE	S	
N	0.13	0.00	0.00	0.05	0.01	0.19
NE	0.81	73.03	0.98	12.76	4.03	91.61
CW	0.00	0.01	0.29	0.08	0.02	0.40
SE	0.12	0.28	0.19	4.91	0.48	5.98
S	0.02	0.06	0.03	0.24	1.41	1.76
	1.08	73.38	1.49	18.04	5.95	99.94

Central West

	N	NE	CW	SE	S	
N	0.06	0.00	0.00	0.01	0.01	0.08
NE	0.00	0.17	0.00	0.04	0.01	0.22
CW	0.32	0.83	68.41	20.42	3.46	93.44
SE	0.06	0.18	0.09	4.65	0.28	5.26
S	0.02	0.02	0.01	0.13	0.79	0.97
	0.46	1.20	68.51	25.25	4.55	99.97

Southeast

	N	NE	CW	SE	S	
N	0.20	0.01	0.00	0.06	0.01	0.28
NE	0.01	0.51	0.01	0.14	0.04	0.71
CW	0.01	0.00	0.40	0.13	0.02	0.56
SE	1.67	2.53	1.89	84.49	6.02	96.60
S	0.02	0.05	0.04	0.24	1.49	1.84
	1.91	3.10	2.34	85.06	7.58	99.99

South

	N	NE	CW	SE	S	
N	0.12	0.00	0.00	0.03	0.01	0.16
NE	0.01	0.32	0.00	0.07	0.02	0.42
CW	0.00	0.01	0.25	0.11	0.01	0.38
SE	0.05	0.10	0.07	3.39	0.22	3.83
S	0.86	1.95	1.16	14.41	76.82	95.20
	1.04	2.38	1.48	18.01	77.08	99.99

Shares of Main Relations

	N	NE	CW	SE	S
N. of Matrices	6	6	4	6	5
% Prod.	97.17	97.12	96.94	98.09	97.73

Source: Figures A.1 to A.5 in the Appendix

Figure 4

Schematic Representation of the Results for the 5 Regions Case

Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x
North, Northeast, Central West, Southeast, and South

The results for the Southeast region that is the less dependent on the other regions show that the relations inside itself represents 84.49% of the production in the productive process, while the sales to the North, Northeast, Central West and the South regions represent respectively 1.67%, 2.53%, 1.89%, and 6.02% of that production, also the relations inside the Southeast region represents 1.49%.

For the South region the most important relations are the relations inside itself (76.82%), the sales that it makes to the Northeast (1.95%), Central West (1.16%), and Southeast (14.41%) regions, and the relations inside the Southeast (3.39%) region.

An overview of the relations among the regions, in the productive process, shows that: a) the North region has practically no relation with the Northeast region and vice-versa; b) while the South region has some impact on the production of the North region, the inverse is not true; c) despite the fact that the demands from the Central West region have some impact on the production of the other regions, the production in the Central West region has its relations concentrated with the Southeast and South regions; c) the South and Southeast regions show to be the most important regions in the system.

In the next section some final remarks will be made.

IV. Conclusions

In this paper the methodology originally developed by Sonis, Hewings, and Miyazawa (1997) to a 2 regions case is extended to a *n* regions case and given a new dimension, such that it is possible to measure the contribution of each block matrix, that represents the relations among the regions, to the production in the productive process of a given region.

This methodology was applied to a set of interregional tables constructed by the author for 1995 for the 5 Brazilian macro regions. The results were derived for the 2 regions case, one region against the rest of the economy, as well as for the 5 regions case.

In general, the results for the Brazilian economy show that: a) the North region has practically no relation with the Northeast region and vice-versa; b) while the South region has some impact on the production of the North region, the inverse is not true; c) despite the fact that the demands from the Central West region have some impact on the production of the other regions, the production in the Central West region has its relations concentrated with the

Southeast and South regions; c) the South and Southeast regions show to be the most important regions in the system.

Despite the progress achieved in this paper, there are still some points left out that need further investigation, i.e.: a) applying the above methodology to a large set of data shows to be very demanding in terms of computer time, so there is a need for the construction of better algorithms of solution; b) when measuring the contribution of the synergy among a set of matrices, that represent the relations among the regions, it was given an equal importance to each matrix, if this is not the case what it is the right way to weight the contribution of each matrix to the final result of the synergy?; and c) what would be the right way to apply this methodology to measure how the relations among the regions have evolved through time and how this change has contributed to the growth of the regions.

V. References

Considera, C.M. and Medina, M.H.(1998). "PIB por unidade da federação: valores correntes . IPEA . Rio de Janeiro. December.

Guilhoto, J.J.M., G.J.D. Hewings, e M. Sonis (1998). "Synergetic Interactions Between 2 Brazilian Regions: An Application of Input-Output Linkages". *Proceedings of the Forty-Fifth North American Meeting of the Regional Science Association International*. Santa Fe, New Mexico, USA, November, 11-14.

IBGE (1997a). *Anuário Estatístico do Brasil 1996*, v. 56. Rio de Janeiro.

IBGE (1997b). *Contagem da população 1996*. Rio de Janeiro.

Sonis, M., G.J.D. Hewings, and K. Miyazawa (1997) "Synergetic Interactions Within the Pair-Wise Hierarchy of Economic Linkages Sub-Systems." *Hitotsubashi Journal of Economics*, 38, December.

V. Appendix (Results for the 4 Regions Case)

North + Northeast

	N+NE	CW	SE	S
N+NE	72.09	1.14	13.83	4.72
CW	0.01	0.30	0.10	0.02
SE	0.40	0.18	4.85	0.47
S	0.08	0.03	0.29	1.45

Central West

	N+NE	CW	SE	S
N+NE	0.23	0.00	0.05	0.02
CW	1.15	68.41	20.42	3.46
SE	0.21	0.09	4.65	0.28
S	0.04	0.01	0.13	0.79

Southeast

	N+NE	CW	SE	S
N+NE	0.72	0.01	0.20	0.06
CW	0.01	0.40	0.13	0.02
SE	4.20	1.89	84.48	6.02
S	0.07	0.03	0.24	1.49

South

	N+NE	CW	SE	S
N+NE	0.44	0.01	0.10	0.04
CW	0.01	0.25	0.11	0.01
SE	0.15	0.07	3.39	0.22
S	2.80	1.16	14.41	76.82

Source: Estimated by the author

Figure A.1

Schematic Representation of the Results for the 4 Regions Case

Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x
North+Northeast, Central West, Southeast and South

Table A.1

Contribution (%) of the Combination of 1, 2, 3, 4 , and 5 Block Matrices to the Production in each Region (North+Northeast, Central West, Southeast and South)

N. of Matrices	North+Northeast	Central West	Southeast	South
1	77.84	79.07	87.09	82.51
2	14.19	15.58	8.68	12.73
3	6.52	4.59	3.52	4.11
4	1.13	0.58	0.61	0.52
5	0.26	0.15	0.08	0.10
Residual	0.05	0.03	0.02	0.02
Total	100.00	100.00	100.00	100.00

Source: Estimated by the author

North + Central West

	N+CW	NE	SE	S
N+CW	68.13	0.72	19.42	4.64
NE	0.01	0.17	0.04	0.01
SE	0.22	0.17	4.73	0.34
S	0.04	0.04	0.24	1.07

Northeast

	N+CW	NE	SE	S
N+CW	0.42	0.01	0.13	0.03
CW	1.80	73.01	12.76	4.03
SE	0.31	0.28	4.91	0.48
S	0.05	0.06	0.24	1.41

Southeast

	N+CW	NE	SE	S
N+CW	0.60	0.01	0.20	0.04
NE	0.02	0.51	0.14	0.04
SE	3.56	2.52	84.49	6.02
S	0.06	0.05	0.24	1.49

South

	N+CW	NE	SE	S
N+CW	0.36	0.01	0.14	0.03
NE	0.01	0.32	0.07	0.02
SE	0.11	0.10	3.39	0.22
S	2.02	1.94	14.41	76.82

Source: Estimated by the author

Figure A.2

Schematic Representation of the Results for the 4 Regions Case

Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x

North+Central West, Northeast, Southeast and South

Table A.2

Contribution (%) of the Combination of 1, 2, 3, 4 , and 5 Block Matrices to the Production in each Region (North+Central West, Northeast, Southeast and South)

N. of Matrices	North + Central West	Northeast	Southeast	South
1	78.92	77.01	87.08	82.52
2	15.58	13.88	8.67	12.73
3	4.64	7.43	3.52	4.11
4	0.68	1.28	0.62	0.53
5	0.15	0.34	0.09	0.11
Residual	0.03	0.06	0.02	0.02
Total	100.00	100.00	100.00	100.00

Source: Estimated by the author

North

	N	NE+CW	SE	S
N	64.27	2.24	17.60	7.01
NE+CW	0.01	0.51	0.11	0.03
SE	0.19	0.36	4.97	0.47
S	0.03	0.09	0.44	1.64

Northeast + Central West

	N	NE+CW	SE	S
N	0.10	0.00	0.03	0.01
NE+CW	0.61	72.81	15.56	3.84
SE	0.09	0.38	4.69	0.39
S	0.02	0.07	0.20	1.16

Southeast

	N	NE+CW	SE	S
N	0.20	0.01	0.06	0.02
NE+CW	0.02	0.91	0.26	0.06
SE	1.67	4.42	84.49	6.02
S	0.02	0.08	0.24	1.49

South

	N	NE+CW	SE	S
N	0.12	0.00	0.03	0.01
NE+CW	0.01	0.59	0.16	0.04
SE	0.05	0.17	3.39	0.22
S	0.86	3.11	14.41	76.82

Source: Estimated by the author

Figure A.3

Schematic Representation of the Results for the 4 Regions Case

Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x
North, Northeast +Central West, Southeast and South

Table A.3

Contribution (%) of the Combination of 1, 2, 3, 4 , and 5 Block Matrices to the Production in each Region (North, Northeast +Central West, Southeast and South)

N. of Matrices	North	Northeast + Central West	Southeast	South
1	76.47	79.35	87.09	82.52
2	17.30	13.72	8.68	12.73
3	5.05	5.80	3.52	4.10
4	0.92	0.90	0.61	0.52
5	0.21	0.20	0.08	0.11
Residual	0.04	0.03	0.02	0.02
Total	100.00	100.00	100.00	100.00

Source: Estimated by the author

North

	N	NE	CW+SE	S
N	64.27	0.49	19.34	7.01
NE	0.01	0.18	0.04	0.01
CW+SE	0.19	0.22	5.52	0.49
S	0.03	0.06	0.47	1.64

Northeast

	N	NE	CW+SE	S
N	0.13	0.00	0.05	0.01
NE	0.81	73.03	13.82	4.03
CW+SE	0.12	0.29	5.40	0.49
S	0.02	0.06	0.28	1.41

Central West + Southeast

	N	NE	CW+SE	S
N	0.19	0.00	0.06	0.02
NE	0.01	0.47	0.14	0.04
CW+SE	1.54	2.37	87.56	5.82
S	0.02	0.04	0.27	1.42

South

	N	NE	CW+SE	S
N	0.12	0.00	0.03	0.01
NE	0.01	0.32	0.07	0.02
CW+SE	0.05	0.11	3.84	0.23
S	0.86	1.94	15.56	76.82

Source: Estimated by the author

Figure A.4
Schematic Representation of the Results for the 4 Regions Case
Contribution (%) of Each Block Matrix to the Total Share of $(x_1$-$f)$ in x
North, Northeast, Central West + Southeast and South

Table A.4
Contribution (%) of the Combination of 1, 2, 3, 4 , and 5 Block Matrices to the Production in each Region (North, Northeast, Central West + Southeast and South)

N. of Matrices	North	Northeast	Central West + Southeast	South
1	76.36	76.99	89.70	82.42
2	17.64	14.18	6.92	12.94
3	4.98	7.44	2.82	4.14
4	0.81	1.08	0.49	0.41
5	0.18	0.27	0.05	0.07
Residual	0.03	0.04	0.01	0.01
Total	100.00	100.00	100.00	100.00

Source: Estimated by the author

North

	N	NE	CW	SE+S
N	64.30	0.49	1.68	24.87
NE	0.01	0.18	0.00	0.04
CW	0.00	0.01	0.34	0.14
SE+S	0.22	0.27	0.19	7.23

Northeast

	N	NE	CW	SE+S
N	0.13	0.00	0.00	0.06
NE	0.81	73.08	0.98	17.02
CW	0.00	0.01	0.29	0.11
SE+S	0.14	0.35	0.23	6.78

Central West

	N	NE	CW	SE+S
N	0.06	0.00	0.00	0.02
NE	0.00	0.17	0.00	0.06
CW	0.32	0.83	68.40	23.69
SE+S	0.08	0.20	0.11	6.05

Southeast + South

	N	NE	CW	SE+S
N	0.19	0.00	0.00	0.08
NE	0.01	0.47	0.01	0.17
CW	0.00	0.01	0.37	0.15
SE+S	1.52	2.46	1.77	92.79

Source: Estimated by the author

Figure A.5

Schematic Representation of the Results for the 4 Regions Case

Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x
North, Northeast, Central West, and Southeast + South

Table A.5

Contribution (%) of the Combination of 1, 2, 3, 4 , and 5 Block Matrices to the Production in each Region (North, Northeast, Central West, and Southeast + South)

N. of Matrices	North	Northeast	Central West	Southeast + South
1	76.41	77.01	78.52	93.92
2	18.40	14.71	16.38	4.12
3	4.62	7.43	4.73	1.64
4	0.45	0.70	0.30	0.31
5	0.10	0.13	0.06	0.02
Residual	0.01	0.02	0.01	0.01
Total	100.00	100.00	100.00	100.00

Source: Estimated by the author