

# Growth, Technological Interdependence and Spatial Externalities: Theory and Evidence

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## **Abstract**

This paper presents a theoretical model, based on the neoclassical growth literature, which explicitly takes into account technological interdependence among economies and examines the impact of location and neighborhood effects in explaining growth. Technological interdependence is supposed working through spatial externalities. The magnitude of the physical capital externalities at steady state, which is usually not identified in the literature, is estimated using a spatial econometric specification explaining the steady state income level. This spatially augmented Solow model yields a conditional convergence equation which is characterized by parameter heterogeneity. A locally linear spatial autoregressive specification is then estimated.

**KEYWORDS:** Conditional convergence, technological interdependence, spatial externalities, spatial autocorrelation, parameter heterogeneity, locally linear estimation

**JEL:** C14, C31, O4

# 1 Introduction

Why have some countries grown rich while others remain poor? This question is recurrent in the theoretical and empirical economic growth literature. Among the traditional stylized facts about country growth experiences over the last fifty years, one of them is that country growth rates appear to depend critically on the growth and income levels of other countries. Therefore, Klenow and Rodriguez-Clare (2005) give us four main facts reflecting this world wide interdependence. First, the growth slowdown that began in the mid-1970's was a world-wide phenomenon. It hit both rich and poor economies on every continent. Second, richer OECD countries grew much more slowly from 1950 to around 1980, despite the fact that richer OECD economies invested at higher rates in physical and human capital. Third, differences in country investment rates are far more persistent than differences in country growth rates. Finally, countries with high investment rates tend to have high levels of income more than they tend to have high growth rates.

These facts show the importance of global interdependence in explaining development and growth. Therefore, in this paper we argue that a model needs to include this global interdependence phenomenon in order to explain development and growth. Several models of economic growth emphasize the importance of international spillovers on growth rates as a major engine of technological progress. These international spillovers come from international trade and the role of foreign R&D (Coe and Helpman 1995, Eaton and Kortum 1996), or technology transfers (Barro and Sala-i-Martin 1997, Howitt 2000) or human capital externalities (Lucas 1993).

Moreover, in the recent literature, several papers show the importance of spatial effects on growth. In fact, it is difficult to believe that Belgium and Dutch or US and Canadian economic growth would ever significantly diverge, or that substantial productivity gaps would appear within Scandinavia. For example, Keller (2002) suggests that the international diffusion of technology is geographically localized, in the sense that the productivity effects of R&D decline with the geographic distance between countries.

This paper presents an augmented Solow model that includes technological interdependence among countries in order to take into account the neighborhood and locational effects on growth and convergence processes. Thus, we consider the Solow model including physical capital externalities as suggested by the Frankel-Arrow-Romer model (Arrow 1962, Frankel 1962 and Romer 1986) and spatial externalities in knowledge to model technological interdependence.

More specifically, in Section 2, we suppose that the technical progress depends on the stock of physical capital per worker, which is complementary with the stock of knowledge in the home country as in Romer (1986). It also depends

on the stock of knowledge in the neighboring countries which spills on the technical progress of the home country so as the countries are geographically close. This simple modeling strategy can be used to take into account both idea gaps and object gaps in economic development process (Romer 1993). A nation that lacks physical objects like factories and roads suffers from an object gap and a nation that lacks the knowledge used to create value in a modern economy suffers from an idea gap. These explanations are not mutually exclusive since a developing nation can suffer from both gaps at the same time. While the notion of an object gap highlights saving and accumulation as the neoclassical growth model, the notion of an idea gap directs attention to the patterns of interaction and communication between a country and the rest of the world.

Our model leads to an equation for the steady state income level as well as a conditional convergence equation characterized by parameter heterogeneity. Therefore, after presenting the database and the spatial weight matrix which is used to model the spatial connections between all the countries in the sample (Section 3), we estimate these equations and test the qualitative and the quantitative predictions of the model.

In Section 4, we estimate the effects of investment rate, population growth and location on the real income per worker at steady state using a spatial econometric specification. This estimation can be used to assess values of the structural parameters in the model. First, we estimate the share of physical capital ( $\alpha$ ) to be close to one third as expected. In fact, the estimated value of the capital share of GDP in the textbook Solow regression is overestimated (about 0.7). Two approaches are suggested in the literature to explain this value: first, as proposed by Mankiw, Romer and Weil (1992), human capital should be taken into consideration together with physical capital to achieve the commonly accepted value of one third for the capital share with a specification of the form  $Y = AK^{1/3}H^{1/3}L^{1/3}$ . This first approach has been largely developed in the theoretical as well as empirical literature. Second, as suggested by Romer (1986, 1987) among others, another way to raise the capital share from one third to two thirds is to argue that there are positive externalities to physical capital ( $\phi$ ). Using time series and cross-section regressions, he supports the claim that output takes the form  $Y = K^{\alpha+\phi}L^{1-\alpha-\phi}$  with a value for  $\alpha + \phi$  that is comprised in  $[0.7, 1]$  (Romer 1987). However, he cannot identify and hence estimate the value of physical capital externalities ( $\phi$ ) in the model he elaborates. In contrast, we show in this paper that in our model, we can indeed identify the parameter associated with physical capital externalities at steady state. We then estimate it and test for its significance. We find a value close to 0.15, which remains significant. Therefore we find evidence in favor of some physical capital externalities but these externalities are not strong

enough to generate endogenous growth. Finally, we assess the role played by technological interdependence in growth processes by estimating the parameter describing spatial externalities. It is highly significant with a value higher than 0.5. Therefore, in our opinion taking into account technological interdependence is fundamental to understand differences between income levels and growth rates in a world wide economy.

In Section 5, we estimate a spatial version of the conditional convergence model. In fact, several empirical papers have found strong evidence of convergence between economies after controlling for differences in steady states. Mankiw, Romer and Weil (1992) show that the neoclassical growth model with exogenous technological progress and decreasing returns for physical capital explains a major part of the differences in cross country per capita growth rates. This empirical evidence in favor of conditional beta-convergence is also confirmed by several other cross country empirical studies (Barro and Sala-i-Martin 1992, 1995). However, we show in this paper that technological interdependence leads to the spatial autocorrelation problem. In addition, Durlauf and Johnson (1995) have directly tested and rejected the hypothesis that the coefficients in these cross-country regressions are the same in different subsets of the sample of countries, highlighting the “spatial” heterogeneity problem. Our model takes into account both problems. Therefore, we first estimate a homogenous version of our spatially augmented conditional convergence model which yields a convergence speed close to 2% as generally found in the literature. However, we show that the technological interdependence generated by spatial externalities is very important in explaining the conditional convergence process. Finally, we estimate a spatial heterogeneous version of the local convergence model using the spatial autoregressive local estimation method (SALE) developed by LeSage and Pace (2004).

## 2 Theory

### 2.1 Technology and spatial externalities

In this section, we develop a neoclassical growth model with Arrow-Romer’s externalities and spatial externalities which implies an international technological interdependence in a world with  $N$  countries denoted by  $i = 1, \dots, N$ . Let us consider an aggregate Cobb-Douglas production function for country  $i$  at time  $t$  exhibiting constant returns to scale in labor and reproducible physical capital:

$$Y_i(t) = A_i(t)K_i^\alpha(t)L_i^{1-\alpha}(t) \tag{1}$$

with the standards notations:  $Y_i(t)$  the output,  $K_i(t)$  the level of reproducible physical capital,  $L_i(t)$  the level of labor and  $A_i(t)$  the aggregate level of technology:

$$A_i(t) = \Omega(t)k_i^\phi(t) \prod_{j \neq i}^N A_j^{\gamma w_{ij}}(t) \quad (2)$$

The function describing the aggregate level of technology  $A_i(t)$  of any country  $i$  depends on three terms. First, as in the Solow model (Solow 1956, Swan 1956), we suppose that a part of technological progress is exogenous and identical to all countries:  $\Omega(t) = \Omega(0)e^{\mu t}$  where  $\mu$  is its constant rate of growth. Second, we suppose that each country's aggregate level of technology increases with the aggregate level of physical capital per worker  $k_i(t) = K_i(t)/L_i(t)$  available in that country<sup>1</sup>. The parameter  $\phi$ , with  $0 < \phi < 1$ , describes the strength of home externalities generated by the physical capital accumulation. Therefore, we follow Arrow's (1962) and Romer's (1986) treatment of knowledge spillover from capital investment and we assume that each unit of capital investment not only increases the stock of physical capital but also increases the level of the technology for all firms in the economy through knowledge spillover. However, there is no reason to constrain these externalities within the barriers of the economy. In fact, we can suppose that the external effect of knowledge embodied in capital in place in one country extends across its border but does so with diminished intensity because of spatial friction generated by distance or border effect for instance. This idea is modeled by the third term in equation (2). The particular functional form we assume for this term in a country  $i$ , is a geometrically weighted average of the stock of knowledge of its neighbors denoted by  $j$ . The degree of international technological interdependence generated by the level of spatial externalities is described by  $\gamma$ , with  $0 < \gamma < 1$ . This parameter is assumed identical for each country but the net effect of these spatial externalities on the level of productivity of the firms in a country  $i$  depends on the relative spatial connectivity between this country and its neighbors. We represent the technological interdependence between a country  $i$  and all the countries belonging to its neighborhood by the connectivity terms  $w_{ij}$ , for  $j = 1, \dots, N$  and  $j \neq i$ . We assume that these terms are non negative, non stochastic and finite; we have  $0 \leq w_{ij} \leq 1$  and  $w_{ij} = 0$  if  $i = j$ . We also assume that  $\sum_{j \neq i}^N w_{ij} = 1$  for  $i = 1, \dots, N$ .<sup>2</sup> The more a given country  $i$  is connected to its neighbors, the higher  $w_{ij}$  is and the more country

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<sup>1</sup>We suppose that all knowledge is embodied in physical capital per worker and not in the level of capital in order to avoid the scale effects (Jones, 1995).

<sup>2</sup>This hypothesis allows us to assume a relative spatial connectivity between all countries in order to underline the importance of the geographical neighborhood for economic growth. Moreover, it allows us to avoid spatial scale effects and then explosive growth.

$i$  benefits from spatial externalities. The spatial friction is then a function of these terms.

This international technological interdependence implies that countries cannot be analyzed in separation but must be analyzed as an interdependent system. Therefore, rewrite function (2) in matrix form:

$$A = \Omega + \phi k + \gamma W A \quad (3)$$

with  $A$  the  $(N \times 1)$  vector of the logarithms of the level of technology,  $k$  the  $(N \times 1)$  vector of the logarithms of the aggregate level of physical capital per worker and  $W$  the  $(N \times N)$  Markov-matrix with spatial friction parameters  $w_{ij}$ . We can resolve (3) for  $A$ , if  $\gamma \neq 0$  and if  $1/\gamma$  is not an eigenvalue of  $W$ <sup>3</sup>:

$$A = (I - \gamma W)^{-1} \Omega + \phi (I - \gamma W)^{-1} k \quad (4)$$

we can develop (4), if  $|\gamma| < 1$ , and regroup terms to obtain:

$$A = \frac{1}{1 - \gamma} \Omega + \phi k + \phi \sum_{r=1}^{\infty} \gamma^r W^{(r)} k \quad (5)$$

where  $W^{(r)}$  is the matrix  $W$  to the power of  $r$ . For a country  $i$ , we have:

$$A_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^\phi(t) \prod_{j=1}^N k_j^{\phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)}}(t) \quad (6)$$

The level of technology in a country  $i$  depends on its own level of physical capital per worker and on the level of physical capital per worker in its neighborhood. Replacing (6) in the production function (1) written per worker, we have finally:

$$y_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^{u_{ii}}(t) \prod_{j \neq i}^N k_j^{u_{ij}}(t) \quad (7)$$

with:  $u_{ii} = \alpha + \phi(1 + \sum_{r=1}^{\infty} \gamma^r w_{ii}^{(r)})$  and  $u_{ij} = \phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)}$ . The terms  $w_{ij}^{(r)}$  are the elements of the line  $i$  and the column  $j$  of the matrix  $W$  to the power of  $r$ , and  $y_i(t) = Y_i(t)/L_i(t)$  the level of output per worker. This model implies spatial heterogeneity in the parameters of the production function. However, we can note that if there is no physical capital externalities, that is  $\phi = 0$ , we have  $u_{ii} = \alpha$  and  $u_{ij} = 0$ , and then the production function is written as usually. This link between physical capital externalities and the heterogeneity

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<sup>3</sup>Actually  $(I - \gamma W)^{-1}$  exists if and only if  $|I - \gamma W| \neq 0$ . This condition is equivalent to:  $|\gamma| |W - (1/\gamma)I| \neq 0$  where  $|\gamma| \neq 0$  and  $|W - (1/\gamma)I| \neq 0$ .

in the parameters of the production function is very close to models with threshold effects due to these externalities studied by Azariadis and Drazen (1990) for example.

Finally, we can evaluate the social elasticity of income per worker in a country  $i$  with respect to all physical capital. In fact, from equation (7), it can be seen that when country  $i$  increases its own stock of physical capital per worker, it obtains a social return of  $u_{ii}$ , whereas this return increases to  $u_{ii} + \sum_{j \neq i}^N u_{ij} = \alpha + \frac{\phi}{1-\gamma}$  if all countries simultaneously increase their stocks of physical capital per worker.<sup>4</sup> In order to warrant the local convergence and then avoid explosive or endogenous growth, we suppose that there is decreasing social return:  $\alpha + \frac{\phi}{1-\gamma} < 1$ .<sup>5</sup> This hypothesis will be tested in section 4.2.

## 2.2 Capital accumulation and steady state

As in the textbook Solow model, we assume that a constant fraction of output  $s_i$  is saved and that labor exogenously grows at the rate  $n_i$  for a country  $i$ . We suppose also a constant and identical annual rate of depreciation of physical capital for all countries, denoted by  $\delta$ . The evolution of output per worker in the country  $i$  is governed by the fundamental dynamic equation of Solow:

$$\dot{k}_i(t) = s_i y_i(t) - (n_i + \delta) k_i(t) \quad (8)$$

where the dot on a variable represents its derivative with respect to time. Since the production function per worker is characterised by decreasing returns, equation (8) implies that the physical capital-output ratio of country  $i$ , for  $i = 1, \dots, N$ , is constant and converges to a balanced growth rate defined by  $\dot{k}_i(t)/k_i(t) = g$ , or:  $[k_i/y_i]^* = s_i/(n_i + g + \delta)$  or in other words:<sup>6</sup>

$$k_i^* = \Omega^{\frac{1}{(1-\gamma)(1-u_{ii})}}(t) \left( \frac{s_i}{n_i + g + \delta} \right)^{\frac{1}{1-u_{ii}}} \prod_{j \neq i}^N k_j^{* \frac{u_{ij}}{1-u_{ii}}}(t) \quad (9)$$

As the production technology is characterized by externalities across countries, we can observe how the physical capital per worker at steady state depends on the usual technological and preference parameters but also on physical capital per worker intensity in neighboring countries. The influence of the spillover effect will be greater the larger the externalities generated by the physical capital accumulation,  $\phi$ , and the coefficient  $\gamma$  that measures the strength of technological interdependence.

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<sup>4</sup>See appendix for the proof.

<sup>5</sup>See appendix for the proof.

<sup>6</sup>The balanced growth rate is  $g = \frac{\mu}{(1-\alpha)(1-\gamma)-\phi}$



In order to determine the equation describing the real income per worker of country  $i$  at steady-state, rewrite the production function in matrix form:  $y = A + \alpha k$ , and substitute  $A$  by its expression in equation (4) to obtain:

$$y = (I - \gamma W)^{-1} \Omega + \alpha k + \phi (I - \gamma W)^{-1} k \quad (10)$$

premultiplying both sides by  $(I - \gamma W)$ , we have:

$$y = \Omega + (\alpha + \phi)k - \alpha \gamma W k + \gamma W y \quad (11)$$

Rewrite this equation for economy  $i$ :

$$\ln y_i^*(t) = \ln \Omega(t) + (\alpha + \phi) \ln k_i^*(t) - \alpha \gamma \sum_{j \neq i}^N w_{ij} \ln k_j^*(t) + \gamma \sum_{j \neq i}^N w_{ij} \ln y_j^*(t) \quad (12)$$

Finally, introducing the equation of capital-output ratio at steady-state in logarithms for  $i = 1, \dots, N$  in equation (12), we obtain the real income per worker of country  $i$  at steady-state:<sup>7</sup>

$$\begin{aligned} \ln y_i^*(t) &= \frac{1}{1 - \alpha - \phi} \ln \Omega(t) + \frac{\alpha + \phi}{1 - \alpha - \phi} \ln s_i - \frac{\alpha + \phi}{1 - \alpha - \phi} \ln(n_i + g + \delta) \\ &- \frac{\alpha \gamma}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln s_j + \frac{\alpha \gamma}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\ &+ \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln y_j^*(t) \end{aligned} \quad (13)$$

This spatially augmented Solow model has the same qualitative predictions as the textbook Solow model about the influence of the own saving rate and the own population growth rate on the real income per worker of a country  $i$  at steady-state. First, the real income per worker at steady state for a country  $i$  depends positively on its own saving rate and negatively on its own population growth rate. Second, it can also be shown that the real income per worker for a country  $i$  depends positively on saving rates of neighboring countries and negatively on their population growth rates. In fact, although the sign of the coefficient of the saving rates of neighboring countries is negative, each of those saving rates ( $\ln s_j$ ) positively influences its own real income per worker at steady state ( $\ln y_j^*(t)$ ) which in turn positively influences the real income per worker at steady state for country  $i$  through spatial externalities and global

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<sup>7</sup>Note that when  $\gamma = 0$ , we have the model elaborated by Romer (1986) with  $\alpha + \phi < 1$  and when  $\gamma = 0$  and  $\phi = 0$ , we have the Solow model.

technological interdependence. The net effect is indeed positive as can also be shown by computing the elasticity of income per worker in country  $i$  with respect to its own rate of saving  $\xi_s^i$  and with respect to the rates of saving of its neighbors  $\xi_s^j$ . We then obtain respectively:<sup>8</sup>

$$\xi_s^i = \frac{\alpha + \phi}{1 - \alpha - \phi} + \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left( \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \quad (14)$$

and:

$$\xi_s^j = \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left( \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \quad (15)$$

These elasticities help us to better understand the effects of an increase of the saving rate in a country  $i$  or in one of its neighbors  $j$  on its income per worker at steady state. First, we note that an increase of the saving rate in a country  $i$  leads to a higher impact on the real income per worker at steady state than in the textbook Solow model because of technological interdependence modeled as a spatial multiplier effect which represents the knowledge diffusion. Furthermore, an increase of the saving rate of a neighboring country  $j$  positively influences the real income per worker at steady state in the country  $i$ .

We can also compute the elasticity of income per worker with respect to the depreciation rate for country  $i$  denoted by  $\xi_n^i$ , and for neighboring countries  $j$ , denoted  $\xi_n^j$ :

$$\xi_n^i = -\frac{\alpha + \phi}{1 - \alpha - \phi} - \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left( \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \quad (16)$$

and:

$$\xi_n^j = -\frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left( \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \quad (17)$$

In section 4.2, we will test these qualitative and quantitative predictions of the spatially augmented Solow model.

## 2.3 Conditional Convergence

As the textbook Solow model, our model predicts that income per worker in a given country converges to that country's steady state value. Rewriting the fundamental dynamic equation of Solow (8) including the production function (7), we obtain:

$$\frac{\dot{k}_i(t)}{k_i(t)} = s_i \Omega^{\frac{1}{1-\gamma}}(t) k_i^{-(1-u_{ii})}(t) \prod_{j \neq i}^N k_j^{u_{ij}}(t) - (n_i + \delta) \quad (18)$$

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<sup>8</sup>See appendix for details

The main element behind the convergence result in this model is also diminishing returns to reproducible capital. In fact,  $\partial(\dot{k}_i(t)/k_i(t))/\partial k_i(t) < 0$  since  $u_{ii} < 1$ . When a country increases its physical capital per worker, the rate of growth decreases and converges to its own steady state. However, an increase of physical capital per worker in a neighboring country  $j$  increases the firm's productivity of the country  $i$  because of the technological interdependence. We have:  $\partial(\dot{k}_i(t)/k_i(t))/\partial k_j(t) > 0$  since  $u_{ij} > 0$ . Physical capital externalities and technological interdependence only slow down the decrease of marginal productivity of physical capital, therefore the convergence result is still valid under the hypothesis  $\alpha + \frac{\phi}{1-\gamma} < 1$ , in contrast with endogenous growth models where marginal productivity of physical capital is constant. This hypothesis is tested in section 4.2.<sup>9</sup>

In addition, our model makes quantitative predictions about the speed of convergence to steady state. As in the literature, the transitional dynamics can be quantified by using a log linearization of equation (18) around the steady state, for  $i = 1, \dots, N$ :

$$\begin{aligned} \frac{d \ln k_i(t)}{dt} &= -(1 - u_{ii})(n_i + g + \delta) [\ln k_i(t) - \ln k_i^*] \\ &+ \sum_{j \neq i}^N u_{ij}(n_i + g + \delta) [\ln k_j(t) - \ln k_j^*] \end{aligned} \quad (19)$$

We obtain a system of differential linear equations whose resolution is too complicated to obtain clear predictions. However, imposing the following hypotheses about the relations between the gaps of countries with respect to their own steady state:

$$\ln k_i(t) - \ln k_i^* = \theta_j [\ln k_j(t) - \ln k_j^*] \quad (20)$$

$$\ln y_i(t) - \ln y_i^* = \Theta_j [\ln y_j(t) - \ln y_j^*] \quad (21)$$

the speed of convergence is given by:<sup>10</sup>

$$\frac{d \ln y_i(t)}{dt} = \frac{\mu}{1 - \gamma} - \lambda_i [\ln y_i(t) - \ln y_i^*] \quad (22)$$

with:

$$\lambda_i = \frac{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j} (n_j + g + \delta)}{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j}} - \sum_{j=1}^N u_{ij} \frac{1}{\Theta_j} (n_j + g + \delta) \quad (23)$$

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<sup>9</sup>See appendix for details

<sup>10</sup>See appendix for details

These hypotheses postulate that the gap of the country  $i$  in respect to its own steady state is proportionate to this same gap for the neighboring country  $j$ . Therefore, if  $\Theta_j = 1$ , countries  $i$  and  $j$  are in the same distance in respect to their steady state. If  $\Theta_j > 1$  (resp.  $\Theta_j < 1$ ) then the country  $i$  is farther (resp. closer) to its own steady state than the country  $j$ . The relative gap between countries in respect to their steady state influences the value of the speed of convergence. In fact,  $\partial\lambda_i/\partial\Theta_j = u_{ij}(n_j + g + \delta)/\Theta_j^2 > 0$ , and the speed of convergence is high if the country  $i$  is far from its own steady state as the true speed of convergence of the textbook Solow model (Barro and Sala-i-Martin, 1995). Moreover, the speed of convergence is high if the neighboring country  $j$  is close to its own steady state. So, there is a strong spatial heterogeneity in our model since the speed of convergence of the country  $i$  is a function of parameters  $w_{ij}$  representing spatial interactions and a function of the distance of the neighboring countries in respect to their own steady state. When there is no physical capital externalities ( $\phi = 0$ ), the heterogeneity of the speed of convergence reduces to that of the textbook Solow model:  $\lambda_i = -(1 - \alpha)(n_i + g + \delta)$ . Therefore, we have the same link between physical capital externalities and spatial heterogeneity as the one we obtained with the production function. The solution for  $\ln y_i(t)$ , subtracting  $\ln y_i(0)$ , the real income per worker at some initial date, from both sides, is:

$$\begin{aligned} \ln y_i(t) - \ln y_i(0) &= (1 - e^{-\lambda_i t}) \frac{\mu}{1 - \gamma \lambda_i} \frac{1}{\lambda_i} - (1 - e^{-\lambda_i t}) \ln y_i(0) \\ &+ (1 - e^{-\lambda_i t}) \ln y_i^* \end{aligned} \quad (24)$$

The model predicts convergence since the growth of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady-state. Rewrite equation (24) in matrix form:  $G = DC - Dy(0) + Dy^*$  where  $y(0)$  is the  $(N \times 1)$  vector of the logarithms of initial level of real income per worker,  $y^*$  is the  $(N \times 1)$  vector of the logarithms of real income per worker at steady-state,  $C$  is the  $(N \times 1)$  vector of constant,  $D$  is the  $(N \times N)$  diagonal-matrix with the  $(1 - e^{-\lambda_i t})$  terms and  $G$  is the  $(N \times 1)$  vector of the growth's rates of real income per worker. Introducing equation (13) in matrix form:  $y^* = (I - \rho W)^{-1} \left[ \frac{1}{1 - \alpha - \phi} \Omega + \frac{\alpha + \phi}{1 - \alpha - \phi} S - \frac{\alpha \gamma}{1 - \alpha - \phi} W S \right]$ , where  $\rho = \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi}$  and  $S$  is the  $(N \times 1)$  vector of logarithms of saving rate divided by the effective rate of depreciation, premultiplying both sides by the inverse of  $D(I - \rho W)^{-1}$  and rearranging terms we obtain:

$$\begin{aligned} G &= D \left( C + \frac{1}{1 - \alpha - \phi} \Omega \right) - Dy(0) + \rho DW y(0) + \frac{\alpha + \phi}{1 - \alpha - \phi} DS \\ &- \frac{\alpha \gamma}{1 - \alpha - \phi} DWS + \rho DW D^{-1} G \end{aligned} \quad (25)$$

Finally, we can rewrite this equation for a country  $i$ :

$$\begin{aligned}
\ln y_i(t) - \ln y_i(0) &= \Delta_i - (1 - e^{-\lambda_i t}) \ln y_i(0) \\
&+ (1 - e^{-\lambda_i t}) \frac{\alpha + \phi}{1 - \alpha - \phi} \ln s_i - (1 - e^{-\lambda_i t}) \frac{\alpha + \phi}{1 - \alpha - \phi} \ln(n_i + g + \delta) \\
&+ (1 - e^{-\lambda_i t}) \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln y_j(0) \\
&- (1 - e^{-\lambda_i t}) \frac{\alpha\gamma}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln s_j \\
&+ (1 - e^{-\lambda_i t}) \frac{\alpha\gamma}{1 - \alpha - \phi} \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\
&+ \frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} (1 - e^{-\lambda_i t}) \sum_{j \neq i}^N \frac{1}{(1 - e^{-\lambda_j t})} w_{ij} [\ln y_j(t) - \ln y_j(0)] \quad (26)
\end{aligned}$$

with  $\Delta_i$  the constant equal to:  $\Delta_i = (1 - e^{-\lambda_i t}) \left( \frac{\mu}{1 - \gamma} \frac{1}{\lambda_i} + \frac{1}{1 - \alpha - \phi} \Omega \right)$ . The growth rate of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady-state. More specifically, the growth rate of real income per worker depends positively on its own saving rate and negatively on its own population growth rate. Moreover, it depends also, in the same direction, on the same variables in the neighboring countries because of technological interdependence. We can observe that the growth rate is higher the larger the initial level of income per worker and the growth rate in neighboring countries. Finally, the last term of the equation (26) show that the rate of growth of a country  $i$  depends on the rate of growth in its neighboring countries weighted by their speed of convergence and by the spatial friction terms. In section 5, we will test the predictions of the spatially augmented Solow model. We will see how technological interdependence may influence growth and then conditional convergence.

### 3 Data

Following the literature on empiric growth, we draw our basic data from the Heston, Summers and Aten (2002) Penn World Tables (PWT), which contain information on real income, investment and population (among many other variables) for a large number of countries. The PWT data have recently been revised and we use the 6.1 version, which extends the data through 2000 for many countries. In this paper, we use a sample of 91 countries over the 1960-1995 period. These countries are those of Mankiw et al. (1992) non-oil sample

for which we have the data, so we have excluded 7 countries.

We measure  $n$  as the average growth of the working-age population (ages 15 to 64). For this, we have compute the number of workers as Caselli (2004):  $RGDPCH \times POP/RGDPW$ , where  $RGDPCH$  is real GDP per capita computed with the chain method,  $RGDPW$  is real-chain GDP per worker and  $POP$  is the total population. Real income per worker is measured by the real GDP computed with the chain method divided by the number of workers. Finally, the saving rate  $s$  is measured as the average share of gross investment in GDP as Mankiw et al. (1992).

The Markov-matrix  $W$  defined in equation (3) corresponds to the so called spatial weight matrix commonly used in spatial econometrics to model spatial interdependence between regions or countries (Anselin 1988; Anselin and Bera 1998; Anselin 2001). More precisely, each country is connected to a set of neighboring countries by means of a purely spatial pattern introduced exogenously in  $W$ . The elements  $w_{ii}$  on the diagonal are set to zero whereas the elements  $w_{ij}$  indicate the way the country  $i$  is spatially connected to the country  $j$ . In order to normalize the outside influence upon each country, the weight matrix is standardized such that the elements of a row sum up to one. For the variable  $x$ , this transformation means that the expression  $Wx$ , called the spatial lag variable, is simply the weighted average of the neighboring observations.

Various matrices are considered in the literature: a simple binary contiguity matrix, a binary spatial weight matrix with a distance-based critical cut-off, above which spatial interactions are assumed negligible, more sophisticated generalized distance-based spatial weight matrices with or without a critical cut-off. The notion of distance is quite general and different functional forms based on distance decay can be used (for example inverse distance, inverse squared distance, negative exponential etc.). The critical cut-off can be the same for all regions or can be defined to be specific to each region leading in the latter case, for example, to  $k$ -nearest neighbors weight matrices when the critical cut-off for each region is determined so that each region has the same number of neighbors.

It is important to stress that the connectivity terms  $w_{ij}$  should be exogenous to the model to avoid the identification problems raised by Manski (1993) in social sciences. This is the reason why we consider pure geographical distance, more precisely great circle distance between capitals, which is indeed strictly exogenous; the functional forms we consider are simply the inverse of squared distance, which can be interpreted as reflecting a gravity function, and the negative exponential of squared distance to check for the robustness of the results. The general term of the first matrix  $W1$  is defined as follows in standardized

form  $[w1_{ij}]$ :

$$w1_{ij} = w1_{ij}^* / \sum w1_{ij}^* \quad \text{with} \quad w1_{ij}^* = \begin{cases} 0 & \text{if } i = j \\ d_{ij}^{-2} & \text{otherwise} \end{cases} \quad (27)$$

The general term of the second matrix  $W2$  is defined as follows in standardized form  $[w2_{ij}]$ :

$$w2_{ij} = w2_{ij}^* / \sum w2_{ij}^* \quad \text{with} \quad w2_{ij}^* = \begin{cases} 0 & \text{if } i = j \\ e^{-2d_{ij}} & \text{otherwise} \end{cases} \quad (28)$$

where  $d_{ij}$  is the great circle distance between country capitals.<sup>11</sup>

## 4 Impact of saving, population growth and location on real income

### 4.1 Specification

In this section, we follow Mankiw et al. (1992) in order to evaluate the impact of saving, population growth and location on real income. Taking equation (13), we find that the real income per worker along the balanced growth path, at a given time ( $t = 0$  for simplicity) is:

$$\begin{aligned} \ln \left[ \frac{Y_i}{L_i} \right] &= \beta_0 + \beta_1 \ln s_i + \beta_2 \ln(n_i + g + \delta) \\ &+ \theta_1 \sum_{j \neq i}^N w_{ij} \ln s_j + \theta_2 \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\ &+ \rho \sum_{j \neq i}^N w_{ij} \ln \left[ \frac{Y_j}{L_j} \right] + \varepsilon_i \end{aligned} \quad (29)$$

where  $\frac{1}{1-\alpha-\phi} \ln \Omega(0) = \beta_0 + \varepsilon_i$ , for  $i = 1, \dots, N$ , with  $\beta_0$  a constant and  $\varepsilon_i$  a country-specific shock since the term  $\Omega(0)$  reflects not just technology but also resource endowments, climate, institutions for instance, and then it may

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<sup>11</sup>The great-circle distance is the shortest distance between any two points on the surface of a sphere measured along a path on the surface of the sphere (as opposed to going through the sphere's interior). It is computed using the equation:

$$d_{ij} = \text{radius} \times \cos^{-1}[\cos |long_i - long_j| \cos lat_i \cos lat_j + \sin lat_i \sin lat_j]$$

where radius is the Earth's radius, lat and long are respectively latitude and longitude for  $i$  and  $j$ .

differ across countries. We suppose also that  $g + \delta = 0.05$  as used in the literature since Mankiw et al. (1992) and Romer (1989). We have finally the following theoretical constraints between coefficients:  $\beta_1 = -\beta_2 = \frac{\alpha + \phi}{1 - \alpha - \phi}$  and  $\theta_2 = -\theta_1 = \frac{\alpha\gamma}{1 - \alpha - \phi}$ . Equation (29) is our basic econometric specification in this section.

In the spatial econometrics literature, this kind of specification, including the spatial lags of both endogenous and exogenous variables, is referred to as the spatial Durbin model (see Anselin and Bera, 1998), we have in matrix form:

$$y = X\beta + WX\theta + \rho Wy + \varepsilon \quad (30)$$

here  $y$  is the  $(N \times 1)$  vector of logarithms of real income per worker,  $X$  the  $(N \times 3)$  matrix including the constant term, the vectors of logarithms of investment rate and the logarithms of physical capital effective rates of depreciation.  $W$  is the  $(N \times N)$  spatial weight matrix,  $WX$  represents the spatially lagged exogenous variables<sup>12</sup> and  $Wy$  the endogenous spatial lag variable.  $\beta' = [\beta_0 \ \beta_1 \ \beta_2]$ ,  $\theta' = [\theta_1 \ \theta_2]$  and  $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$  is the spatial autocorrelation coefficient.  $\varepsilon$  is the  $(N \times 1)$  vector of errors supposed identically and normally distributed so that  $\varepsilon \sim N(0, \sigma^2)$ .

Noting that  $\beta$  and  $\theta$  can be expressed as:

$$\beta = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'y \quad (31)$$

$$\theta = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'Wy \quad (32)$$

we can write the concentrated log-likelihood function for this model as shown in (33) where  $C$  denotes an inessential constant:

$$\ln(L) = C + \ln |I - \rho W| - \frac{n}{2} \ln(e_1'e_1 - 2\rho e_2'e_1 + \rho^2 e_2'e_2) \quad (33)$$

with  $e_1 = y - \tilde{X}\beta$ ,  $e_2 = Wy - \tilde{X}\theta$  and  $\tilde{X} = [X \ WX]$ . Given a value of  $\rho$  that maximizes the concentrated likelihood function (say  $\hat{\rho}$ ), we compute estimates for  $\beta$  and  $\theta$  using:

$$\hat{\zeta} = (\beta - \hat{\rho}\theta) = \begin{bmatrix} \hat{\beta} \\ \hat{\theta} \end{bmatrix} \quad (34)$$

Finally, an estimate of  $\sigma_\varepsilon^2$  is calculated using:

$$\hat{\sigma}^2 = (y - \hat{\rho}Wy - X\hat{\zeta})'(y - \hat{\rho}Wy - X\hat{\zeta})/n \quad (35)$$

<sup>12</sup>In practice, the spatially lagged constant is not included in  $WX$ , since there is an identification problem for row-standardized  $W$  (the spatial lag of a constant is the same as the original variable).



## 4.2 Results

In the first column of table 1, we estimate the textbook Solow model by OLS. Our results about its qualitative predictions are essentially identical to those of Mankiw et al. (Table 1, p. 414 of their article), since the coefficients on saving and population growth have the predicted signs and are strongly significant. But, as also underlined by Bernanke et al. (2003) with the recent vintage of PWT, the overidentifying restriction is rejected (the  $p$ -value is 0.038). The estimated capital share remains close to 0.6 as in Mankiw et al. (1992). It is therefore too high.

However, we claim that the textbook Solow model is misspecified since it omits variables due to technological interdependence and physical capital externalities. In fact, the econometric specification of our theoretical model is, in matrix form:

$$y = \frac{\alpha}{1-\alpha}S + \frac{\phi}{1-\alpha}(I - \gamma W)^{-1}k^* + (I - \gamma W)^{-1}\varepsilon \quad (36)$$

with  $S$  the  $(N \times 1)$  vector of logarithms of investment rate divided by the effective rate of depreciation. Therefore the error term in the Solow model contains omitted information since we can rewrite it:

$$\varepsilon_{Solow} = \frac{\phi}{1-\alpha}(I - \gamma W)^{-1}k^* + (I - \gamma W)^{-1}\varepsilon \quad (37)$$

We also note the presence of spatial autocorrelation in the error term even if there is no physical capital externalities (i.e.  $\phi = 0$ ), and then the presence of technological interactions between all countries through the inverse spatial transformation  $(I - \gamma W)^{-1}$ . Furthermore, it is straightforward to show that OLS lead to biased estimators if the endogenous spatial lag variable is omitted as in the textbook Solow model.

[Table 1 around here]

In the subsequent columns of table 1, we estimate the spatially augmented Solow model for the two spatial weight matrices  $W1$  and  $W2$  using maximum likelihood.<sup>13</sup> Many aspects of the results support our model. First, all the coefficients have the predicted signs and the spatial autocorrelation coefficient,  $\rho$ , is highly positively significant. Second, the joint theoretical restriction  $\beta_1 = -\beta_2$  and  $\theta_2 = -\theta_1$  is not rejected since the  $p$ -value of the  $LR$ -test is 0.455 for the  $W1$  matrix and 0.311 for the  $W2$  matrix. Third, the  $\alpha$  implied by the coefficient in the constrained regression is very close to one-third for the two

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<sup>13</sup>James LeSage provides a function to estimate the spatial Durbin model in his Econometrics Toolbox for Matlab (<http://www.spatial-econometrics.com>)

matrices. The  $\phi$  estimate is about 0.15 to 0.18 and weakly significant. More specifically, we can test the absence of physical capital externalities represented by  $\phi$ . In fact, if  $\phi = 0$  in the specification (29), we have:

$$\begin{aligned} \ln \left[ \frac{Y_i}{L_i} \right] &= \beta'_0 + \beta'_1 \ln s_i + \beta'_2 \ln(n_i + g + \delta) \\ &+ \theta'_1 \sum_{j \neq i}^N w_{ij} \ln s_j + \theta'_2 \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\ &+ \gamma \sum_{j \neq i}^N w_{ij} \ln \left[ \frac{Y_j}{L_j} \right] + \varepsilon_i \end{aligned} \quad (38)$$

with  $\beta'_1 = -\beta'_2 = \frac{\alpha}{1-\alpha}$ ,  $\theta'_2 = -\theta'_1 = \frac{\alpha\gamma}{1-\alpha}$  hence  $\theta'_1 + \beta'_1\gamma = 0$  and  $\theta'_2 + \beta'_2\gamma = 0$ . Specification (38) is the so-called constrained spatial Durbin model which is formally equivalent to a spatial autoregressive error model written in matrix form:

$$y = X\beta' + \varepsilon_{Solow} \quad \text{and} \quad \varepsilon_{Solow} = \gamma W\varepsilon_{Solow} + \varepsilon \quad (39)$$

where  $\beta' = [\beta'_0 \ \beta'_1 \ \beta'_2]$  and  $\varepsilon_{Solow}$  is the same as before with  $\phi = 0$ . Hence, we have the textbook Solow model with spatial autocorrelation in the errors terms. Estimation results by maximum likelihood using  $W1$  and  $W2$  are presented in table 2. We can test the non-linear restrictions with the common factor test (Burrige, 1981). We only weakly reject these restrictions and then the null hypothesis  $\phi = 0$  and we conclude that there are some physical capital externalities.

The  $\gamma$  estimate is higher than 0.5 which shows the strong technological interdependence between countries and the importance of neighborhood in the determination of the real income. However, these externalities are not enough to generate endogenous growth since the value of  $\alpha + \frac{\phi}{1-\gamma}$  is below 1 and close to 0.6 or 0.7. We obtain lower results than those obtained by Romer (1987) about the importance of physical capital externalities and social return since he finds an elasticity of output with respect to physical capital close to unity.

[Table 2 around here]

A last result of our model is of interest. Indeed, it is well known that the neoclassical model fails to predict the large differences in income observed in the real world. The calibrations of Mankiw (1995) indicate that the Solow model, with reasonable differences in rates of saving and population growth, can explain incomes that vary by a multiple of slightly more than two. However, there is much more disparity in international living standards than the

neoclassical model predicts since it varies by a multiple of more than ten. These calculations have been made with an evaluation of the elasticities of real income per worker with respect to the saving rate and to the effective rate of depreciation which are approximately 0.5 and -0.5. Mankiw (1995) notes that we can obtain better predicted real income per worker differences with higher elasticities. Our model predicts that the saving rate and population growth have higher effects on real income per worker because of physical capital externalities and technological interdependence.

In order to compute these elasticities of real income per worker at steady state with respect to the saving rate and the effective rate of depreciation, we can rewrite equations (14 and 15) in matrix form:<sup>14</sup>

$$\Xi = \beta_1 I + (\beta_1 \rho + \theta_1) W (I - \rho W)^{-1} \quad (40)$$

Therefore, from estimations reported in table 1, we obtain a  $(91 \times 91)$  matrix  $\Xi$  with direct elasticities on the main diagonal and off-diagonal terms representing cross-elasticities. On a column  $j$ , we have the effects of an increase of the saving rate  $s_j$  of the country  $j$  on all countries. Of course, because of the  $w_{ij}$  terms, the effect is more important for closer countries. On a line  $i$ , we have the effects of an increase of the saving rate of each country in the neighborhood of country  $i$  on its real income per worker. We note also that the sum of each line is identical for all countries. This propriety, coming from the Markov propriety of  $W$ , means that an identical increase of the saving rate in all countries will have the same effect on their real income per worker at steady state.

In average, the elasticity of real income per worker in respect to the saving rate is about 0.9 for the  $W1$  matrix and 0.84 for the  $W2$  matrix. In the same way, in average, the elasticity of real income per worker in respect to the effective rate of depreciation is about -1.65 for the  $W1$  matrix and -1.69 for the  $W2$  matrix. We have also all results about cross elasticities indicating effects of saving rates or population growth rates of neighboring countries on real income per worker of the country under study.<sup>15</sup> Therefore, these values of elasticities provide a much better explanation about the differences between countries' real income per worker. In fact, physical capital externalities, technological interdependence and more generally neighborhood effects, explain these income inequalities between countries since they imply higher elasticities.

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<sup>14</sup>We focus here on the elasticities of income in regard with the saving rate. The elasticities of income in regard with the effective depreciation rate are symmetric.

<sup>15</sup>All results are available from the authors upon request

## 5 Impact of saving, population growth and location on growth

We estimate now the predictions about conditional convergence of our spatially augmented Solow model in two polar cases. First, we suppose, as Mankiw et al. (1992) and Barro and Sala-i-Martin (1992), that the speed of convergence is identical for all countries and we refer to this case as the homogenous model. Second, we estimate a model with complete parameter heterogeneity and we refer to this case as the heterogenous model.

### 5.1 Homogenous model

In this section, we follow Mankiw et al. (1992) and Barro and Sala-i-Martin (1992, 1995) in order to estimate equation (26): we first assume that the speed of convergence is homogenous and so identical for all countries:  $\lambda_i = \lambda$  for  $i = 1, \dots, N$ . Rewrite equation (26), dividing by  $T$  in both sides, in the following form:

$$\begin{aligned}
 \frac{[\ln y_i(t) - \ln y_i(0)]}{T} &= \beta_0 + \beta_1 \ln y_i(0) + \beta_2 \ln s_i + \beta_3 \ln(n_i + g + \delta) \\
 &+ \theta_1 \sum_{j \neq i}^N w_{ij} \ln y_j(0) + \theta_2 \sum_{j \neq i}^N w_{ij} \ln s_j \\
 &+ \theta_3 \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\
 &+ \rho \sum_{j \neq i}^N w_{ij} \frac{[\ln y_j(t) - \ln y_j(0)]}{T} + \varepsilon_i \tag{41}
 \end{aligned}$$

where  $\beta_0$  is a constant,  $\beta_1 = \frac{(1-e^{-\lambda T})}{T}$ ,  $\beta_2 = -\beta_3 = \frac{(1-e^{-\lambda T})}{T} \frac{\alpha + \phi}{1 - \alpha - \phi}$ ,  $\theta_1 = \frac{(1-e^{-\lambda T})}{T} \frac{\gamma(1-\alpha)}{1 - \alpha - \phi}$ ,  $\theta_3 = -\theta_2 = \frac{(1-e^{-\lambda T})}{T} \frac{\alpha\gamma}{1 - \alpha - \phi}$  and  $\rho = \frac{\gamma(1-\alpha)}{1 - \alpha - \phi}$ . In matrix form, we have also a non-constrained spatial Durbin model which is estimated in the same way as the model in the section 4.2.

In the first column of table 3, we estimate a model of unconditional convergence. This result is identical to many previous authors about the failure of income to converge (De Long, 1988, Romer, 1987 and Mankiw et al. 1992). The coefficient on the initial level of income per worker is slightly positive and non significant. Therefore, there is no tendency for poor countries to grow faster on average than rich countries.

[Table 3 around here]

We test the convergence predictions of the textbook Solow model in the second column of table 3. We report regressions of growth rate over the period 1960 to 1995 on the logarithm of income per worker in 1960, controlling for investment rate and growth of working-age population. The coefficient on the initial level of income is now significantly negative; that is, there is strong evidence of convergence. The results also support the predicted signs of investment rate and working-age population growth rate. However, it is well-known in the literature that the implied value of  $\lambda$ , the parameter governing the speed of convergence is much smaller than the prediction of the textbook Solow model or the 2% per year found by Barro and Sala-i-Martin. Indeed, our results give a value of  $\lambda = 0.0076$  which implies a half-life of about 91 years.

Once again, we claim that the textbook Solow model is misspecified since it omits variables due to technological interdependence and physical capital externalities. Therefore, as in Section 4.2, the error terms of the Solow model contain omitted information and are spatially autocorrelated.

Note that spatial effects have received less attention in the literature although major econometric problems are likely to be encountered if they are present in the standard  $\beta$ -convergence framework, since statistical inference based on OLS will then be flawed. The first study we are aware of that takes up the issue of location and growth explicitly is DeLong and Summers (1991). Likewise, Mankiw (1995) points out that multiple regression in the standard framework treats each country as if it were an independent observation. Temple (1999) in his survey on the new growth evidence also draws attention to error correlation and regional spillovers though he interprets these effects as mainly reflecting an omitted variable problem. Despite these observations, the appropriate econometric treatment of spatial effects is often neglected in the macroeconomic literature. Sometimes it is handled by straightforward use of regional dummies or border dummy variables (Chua 1993, Barro and Sala-i-Martin 1995, Ales and Chua 1997, Easterly and Levine 1997). Nevertheless a few recent empirical studies apply the appropriate spatial econometric tools as Conley and Ligon (2002), Ertur et al. (2005), Moreno and Trehan (1997).

In table 4, we estimate the spatially augmented Solow model for the two spatial weight matrices  $W1$  and  $W2$ . Many aspects of the results support this model. First, all the coefficients are significant and have the predicted signs. The spatial autocorrelation coefficient  $\rho$  is highly positively significant which shows the importance of the role played by technological interdependence on the growth of countries. Second, the coefficient on the initial level of income is significantly negative, so there is strong evidence of convergence after controlling for those variables that the spatially Solow model says determine the

steady state. Third, the  $\lambda$  implied by the coefficient on the initial level of income is about 1.5% to 1.7% which is closer to the value usually found about the speed of convergence in the literature.

[Table 4 around here]

Finally, in table 5, we test the absence of physical capital externalities since  $\phi = 0$  implies a spatial Durbin model in constrained form and then a spatial autoregressive error model. Using the same approach as in Section 4.2, we now strongly reject the null hypothesis  $\phi = 0$  and we conclude that there are indeed physical capital externalities.

[Table 5 around here]

## 5.2 Heterogenous model

In some recent papers, Durlauf (2000, 2001) and Brock and Durlauf (2001) draw attention on the assumption of parameter homogeneity imposed in cross-section growth regressions. Indeed, it is unlikely to assume that the parameters that describe growth are identical across countries. Moreover, evidence of parameter heterogeneity has been found using different statistical methodologies such as in Canova (1999), Desdoigts (1999), Durlauf and Johnson (1995). Each of these studies suggests that the assumption of a single linear statistical growth model that applies to all countries is incorrect.

From the econometric methodology perspective, Islam (1995), Lee, Pesaran and Smith (1997) or Evans (1998) have suggested the use of panel data to address this problem but this approach is of limited use in empirical growth contexts, because variation in the time dimension is typically small. Some variables as for example political regime do not vary by nature over high frequencies and some other variables are simply not measured over such high frequencies. Moreover high frequency data will contain business cycle factors that are presumably irrelevant for long run output movements. The use of long run averages in cross sectional analysis has still a powerful justification for identifying growth as opposed to cyclical factors. Durlauf and Quah (1999) underline also that it might appear to be a proliferation of free parameters not directly motivated by economic theory.

The empirical methodology we propose takes into account the spatial heterogeneity embodied in our spatially augmented Solow model. Reconsider

equation (26), dividing by  $T$  in both sides:

$$\begin{aligned}
\frac{[\ln y_i(t) - \ln y_i(0)]}{T} &= \beta_{0i} + \beta_{1i} \ln y_i(0) + \beta_{2i} \ln s_i + \beta_{3i} \ln(n_i + g + \delta) \\
&+ \theta_{1i} \sum_{j \neq i}^N w_{ij} \ln y_j(0) + \theta_{2i} \sum_{j \neq i}^N w_{ij} \ln s_j \\
&+ \theta_{3i} \sum_{j \neq i}^N w_{ij} \ln(n_j + g + \delta) \\
&+ \rho_i \sum_{j \neq i}^N w_{ij} \frac{[\ln y_j(t) - \ln y_j(0)]}{T} \tag{42}
\end{aligned}$$

with  $\beta_{0i} = \frac{(1-e^{-\lambda_i T})}{T} \left( \frac{g}{1-\gamma} \frac{1}{\lambda_i} + \frac{1}{1-\alpha-\phi} \Omega \right)$ ,  $\beta_{1i} = \frac{(1-e^{-\lambda_i T})}{T}$ ,  $\beta_{2i} = -\beta_{3i} = \frac{(1-e^{-\lambda_i T})}{T} \frac{\alpha+\phi}{1-\alpha-\phi}$ ,  $\theta_{1i} = \frac{(1-e^{-\lambda_i T})}{T} \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$ ,  $\theta_{3i} = -\theta_{2i} = \frac{(1-e^{-\lambda_i T})}{T} \frac{\alpha\gamma}{1-\alpha-\phi}$  and  $\rho_i = \frac{\gamma(1-\alpha)}{1-\alpha-\phi} \frac{(1-e^{-\lambda_i T})}{\Gamma_i}$ . The term  $\Gamma_i$  reflects the effects of the speed of convergence in the neighboring countries.

To accommodate both spatial dependence and heterogeneity, we produce estimates using  $N$ -models, where  $N$  represents the number of cross-sectional sample observations, using the locally linear spatial autoregressive model in (42). The original specification was proposed by LeSage and Pace (2004) and labeled spatial autoregressive local estimation (SALE). This specification is for example used in Ertur et al. (2004) in the regional convergence context in Europe. We consider an extended version of this specification here as we also include spatially lagged exogenous variables and label it the local SDM model:

$$U(i)y = U(i)X\beta_i + U(i)WX\theta_i + \rho_i U(i)Wy + U(i)\varepsilon \tag{43}$$

where  $U(i)$  represent an  $N \times N$  diagonal matrix containing distance-based weights for observation  $i$  that assign weights of one to the  $m$  nearest neighbors to observation  $i$  and weights of zero to all other observations. This results in the product  $U(i)y$  representing an  $m \times 1$  sub-sample of observed GDP growth rates associated with the  $m$  observations nearest in location (using great circle distance) to observation  $i$ . Similarly, the product  $U(i)X$  extracts a sub-sample of explanatory variable information based on  $m$  nearest neighbors and so on. The local SDM model assumes  $\varepsilon_i \sim N(0, \sigma_i^2 U(i)I_N)$ .

The scalar parameter  $\rho_i$  measures the influence of the variable,  $U(i)Wy$  on  $U(i)y$ . We note that as  $m \rightarrow N$ ,  $U(i) \rightarrow I_N$  and these estimates approach the global estimates based on all  $N$  observations that would arise from the global SDM model. The local SDM model in the context of convergence analysis means that each region converges to its own steady state at its own rate (represented by the parameter  $\lambda_i$ ). Therefore, heterogeneity in both the level of

steady states and transitional growth rates toward this steady states is allowed. Estimation results are presented in Figures 1 to 8. Countries are ordered by continent and increasing latitude in each continent. The solid line in these figures display the corresponding parameters estimated in our spatially augmented Solow model and the dashed lines display the corresponding parameters estimated in the textbook Solow model.

[Figures 1 to 4 around here]

We note strong evidence in favor of parameter heterogeneity as Durlauf et al. (2001). This heterogeneity is furthermore linked to the location of the observations and is spatial by nature. The parameters for non spatially lagged variables have all the predicted signs. First in Figure 2, we note that the speed of convergence is high for European countries (especially for Belgium, Netherlands, France), and for USA, Canada and central American countries (Jamaica, Trinidad and Tobago, Panama...). However the speed of convergence is low for some south American countries and most of African and Asian countries. We note that it is very low for Japan and Republic of Korea, countries known for their high growth rates. However this can be due to the fact that the countries in their neighborhood are farther away from their steady states since the speed of convergence is positively linked to that gap. Second in Figure 3, the estimates of the saving rate are the highest for Asian countries, Peru in South America and some African countries. In Figure 4, we see that there is not any particular pattern for the estimates of the population growth rates.

[Figures 5 to 8 around here]

Estimates of the lagged saving rate has the predicted sign for all countries except for Mexico which could be a local outlier as well as Japan (Figure 5). The estimates of the population growth rate are relatively stable except for South America, Australia and New-Zealand (Figure 6). The impact of the lagged initial income level is strong in Africa and Europe while it is weaker for Asian countries (especially for Japan) and Southern American countries (Figure 7). The estimates of the lagged growth rate are positive for all countries, they are high for Asian countries and low for countries belonging to America (Figure 8).

[Figures 9 to 12 around here]



Local structural parameters can be recovered from the estimation of the constrained version of model (42) and they are displayed in Figures 9 to 12. In Figure 9, the income capital share is rather high, close to one half, for developing countries in Africa and Asia. In contrast, it is lower, close to one third, as expected for wealthier Northern countries. Physical capital externalities are lower for African, Asian and European countries than for USA and the whole American continent. These externalities are stronger for Japan, a result which appears consistent with its low convergence speed. Figure 10 displays spatial externalities which are indeed positive. In our model, this is strong evidence in favor of local technological interdependence. Again, we see that Mexico and Japan could be local outliers in Figures 9 to 12. Further research will have to treat these potential outliers by using robust Bayesian estimation methods for spatial models as proposed in LeSage (1997) and extended to local models in Ertur et al. (2004).

## 6 Conclusion

In this paper, we develop a neoclassical growth model which explicitly takes into account technological interdependence between countries under the form of spatial externalities. Actually, the stock of knowledge in one country is producing externalities that may cross over national borders and spill over into other countries with an intensity which is decreasing with distance. We simply refer in this paper to pure geographical distance. Its exogeneity is largely admitted and therefore represents its main advantage. However, a general distance concept related to economical, institutional, or sociological proximity could also be considered.

Our results have several implications: first, countries cannot be treated as spatially independent observations and growth models should explicitly take into account spatial interactions because of this technological interdependence. The predictions of our spatially augmented Solow model provide us with a better understanding of the important role played by geographical location and neighborhood effects in international growth and convergence processes. Second, our theoretical result shows that the textbook Solow model is misspecified since variables representing these effects are omitted.

Our estimation results support our model. All the estimated coefficients are significant with the predicted sign. The spatial autocorrelation coefficient is also positive and highly significant. In addition, our econometric model leads to estimates of structural parameters close to predicted values. The estimated capital share parameter is close to  $1/3$ , the estimated parameter for spatial externalities is close to  $1/2$  and shows the importance of technological inter-

actions in the economic growth process as well as in the world income distribution. Estimation of physical capital externalities shows that knowledge accumulation in the form of learning by doing also plays an important role in the economic growth process. Actually, we show that these externalities imply parameter heterogeneity in the conditional convergence equation. The spatial autoregressive local estimation method developed by LeSage and Pace (2004) allows estimation of local parameters reflecting the implied spatial heterogeneity.

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## Appendix 1

$$\begin{aligned} u_{ii} + \sum_{j \neq i}^N u_{ij} &= \alpha + \phi \left( 1 + \sum_{r=1}^{\infty} \gamma^r \sum_{j=i}^N w_{ij}^{(r)} \right) \\ &= \alpha + \phi \left( 1 + \gamma \sum_{j=i}^N w_{ij} + \gamma^2 \sum_{j=i}^N w_{ij}^{(2)} + \dots \right) \\ &= \alpha + \phi(1 + \gamma + \gamma^2 + \dots) \\ &= \alpha + \frac{\phi}{1 - \gamma} \end{aligned} \tag{44}$$

We can do this because of the Markov propriety of the  $W$  matrix. Indeed, the powers of the  $W$  matrix are also Markov matrices and then:  $\sum_{j=i}^N w_{ij} = \sum_{j=i}^N w_{ij}^{(2)} = \dots = 1$  for  $i = 1, \dots, N$ .

## Appendix 2: Elasticities

Take equation (16) in matrix form:

$$y = \frac{1}{1 - \alpha - \phi} \Omega + \frac{\alpha + \phi}{1 - \alpha - \phi} S - \frac{\alpha\gamma}{1 - \alpha - \phi} WS + \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} Wy \quad (45)$$

where  $S$  is the  $(N \times 1)$  vector of logarithms of saving rates divided by the effective rate of depreciation. Subtracting  $\frac{(1-\alpha)\gamma}{1-\alpha-\phi}Wy$  in both sides, and pre-multiplying both sides by  $(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W)^{-1}$ , we obtain:

$$\begin{aligned} y &= \frac{1}{1 - \alpha - \phi} \left( I - \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} W \right)^{-1} \Omega \\ &+ \left( I - \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} W \right)^{-1} \left( \frac{\alpha + \phi}{1 - \alpha - \phi} I - \frac{\alpha\gamma}{1 - \alpha - \phi} W \right) S \end{aligned} \quad (46)$$

Deriving this expression in respect to the vector  $S$ , we obtain the expression of elasticities in matrix form:

$$\begin{aligned} \Xi &= \left( I - \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} W \right)^{-1} \left( \frac{\alpha + \phi}{1 - \alpha - \phi} I - \frac{\alpha\gamma}{1 - \alpha - \phi} W \right) \\ &= \left( I + \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} W + \left( \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} \right)^2 W^2 + \dots \right) \left( \frac{\alpha + \phi}{1 - \alpha - \phi} I - \frac{\alpha\gamma}{1 - \alpha - \phi} W \right) \\ &= \frac{\alpha + \phi}{1 - \alpha - \phi} I + \left( \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \right) \sum_{r=1}^{\infty} W^r \left( \frac{(1 - \alpha)\gamma}{1 - \alpha - \phi} \right)^r \end{aligned} \quad (47)$$

Finally, we can rewrite these expressions for each country  $i$  and we obtain the expressions in the text.



### Appendix 3: Local Convergence

In order to study the local stability of the system, rewrite equation (22) in matrix form:

$$\dot{\chi}(t) = J\chi(t) \quad (48)$$

where  $\chi(t)$  is the  $(N \times 1)$  vector of terms  $[\ln k_i(t) - \ln k_i^*]$  and  $J$  is the Jacobian matrix of the linearized system in the vicinity of the steady state:

$$J = -(1 - \alpha - \phi)diag(n_i + g + \delta) + \phi diag(n_i + g + \delta)(I - \gamma W)^{-1} \quad (49)$$

with  $diag(n + g + \delta)$  the diagonal matrix with general term  $(n_i + g + \delta)$ . We will show that the hypothesis  $\alpha + \frac{\phi}{1-\gamma} < 1$  implies the following relation for all lines  $j$  of the Jacobian matrix  $J$ :

$$|J_{ii}| > \sum_{j \neq i}^N |J_{ij}| \quad \text{for all } i = 1, \dots, N. \quad (50)$$

Proof:

$$\begin{aligned} & \alpha + \frac{\phi}{1-\gamma} < 1 \\ \Leftrightarrow & u_{ii} + \sum_{j \neq i}^N u_{ij} < 1 \\ \Leftrightarrow & \alpha + \phi + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} + \phi \sum_{j \neq i}^N \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < 1 \\ \Leftrightarrow & \phi \sum_{j \neq i}^N \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < (1 - \alpha - \phi) - \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \\ \Leftrightarrow & \sum_{j \neq i}^N \left| \phi \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} \right| < \left| -(1 - \alpha - \phi) + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \right| \quad \blacksquare \quad (51) \end{aligned}$$

Therefore, with the dominant negative diagonal theorem, the matrix  $J$  is d-stable and then the system is locally stable.

## Appendix 4: Convergence Speed

Introducing equation (22), for  $i = 1, \dots, N$ , in the production function (7) rewriting it in the following form:  $\frac{d \ln y_i(t)}{dt} = \frac{\mu}{1-\gamma} + u_{ii} \frac{d \ln k_i(t)}{dt} + \sum_{j \neq i}^N u_{ij} \frac{d \ln k_j(t)}{dt}$ , we obtain:

$$\begin{aligned} \frac{d \ln y_i(t)}{dt} &= \frac{\mu}{1-\gamma} + u_{ii}(n_i + g + \delta)([\ln y_i(t) - \ln y_i^*] - [\ln k_i(t) - \ln k_i^*]) \\ &+ \sum_{j \neq i}^N u_{ij}(n_j + g + \delta)([\ln y_j(t) - \ln y_j^*] - [\ln k_j(t) - \ln k_j^*]) \end{aligned} \quad (52)$$

Taking the following relation:

$$\sum_{j \neq i}^N u_{ij}(n_j + g + \delta)[\ln k_j(t) - \ln k_j^*] = \Delta_i \left[ u_{ii}[\ln k_i(t) - \ln k_i^*] + \sum_{j \neq i}^N u_{ij}[\ln k_j(t) - \ln k_j^*] \right] \quad (53)$$

we obtain, with the hypothesis (23), the expression of  $\Delta_i$ :

$$\Delta_i = \frac{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j} (n_j + g + \delta)}{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j}} \quad (54)$$

and then:

$$\begin{aligned} \frac{d \ln y_i(t)}{dt} &= \frac{\mu}{1-\gamma} + \sum_{j \neq i}^N u_{ij}(n_j + g + \delta)[\ln y_j(t) - \ln y_j^*] \\ &- \Delta_i[\ln y_i(t) - \ln y_i^*] \\ &= \frac{\mu}{1-\gamma} - \lambda_i[\ln y_i(t) - \ln y_i^*] \end{aligned} \quad (55)$$

with the hypothesis (24). We obtain finally the speed of convergence:

$$\lambda_i = \frac{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j} (n_j + g + \delta)}{\sum_{j=1}^N u_{ij} \frac{1}{\theta_j}} - \sum_{j=1}^N u_{ij} \frac{1}{\Theta_j} (n_j + g + \delta) \quad (56)$$

Table 1: Estimation results: Textbook Solow and spatially augmented Solow models

Model	TextBook Solow	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\ln y_i(1995)$	$\ln y_i(1995)$	$\ln y_i(1995)$
Obs. / Weight matrix	91	91 / (W1)	91 / (W2)
<i>constant</i>	4.651 (0.010)	0.988 (0.602)	0.530 (0.778)
$\ln s_i$	1.276 (0.000)	0.825 (0.000)	0.792 (0.000)
$\ln(n_i + 0.05)$	-2.709 (0.000)	-1.498 (0.008)	-1.451 (0.009)
$W \ln s_j$	-	-0.322 (0.079)	-0.372 (0.024)
$W \ln(n_j + 0.05)$	-	0.571 (0.501)	0.137 (0.863)
$W \ln y_j$	-	0.740 (0.000)	0.658 (0.000)
Restricted regression			
<i>constant</i>	8.375 (0.000)	2.060 (0.000)	2.908 (0.000)
$\ln s_i - \ln(n_i + 0.05)$	1.379 (0.000)	0.841 (0.000)	0.818 (0.000)
$W[\ln s_j - \ln(n_j + 0.05)]$	-	-0.284 (0.107)	-0.276 (0.088)
$W \ln y_j$	-	0.742 (0.000)	0.648 (0.000)
Test of restriction	4.427 (Wald) (0.038)	1.576 (LR) (0.455)	2.338 (LR) (0.311)
Implied $\alpha$	0.580 (0.000)	0.276 (0.016)	0.299 (0.031)
Implied $\phi$	-	0.180 (0.080)	0.151 (0.120)
Implied $\gamma$	-	0.557 (0.000)	0.508 (0.000)
$\alpha + \frac{\phi}{1-\gamma}$	-	0.683 (0.008)	0.606 (0.025)

Table 2: Spatial Autoregressive Error Model and non linear restrictions tests

Model	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\ln y_i(1995)$	$\ln y_i(1995)$
Obs. / Weight matrix	91 / (W1)	91 / (W2)
<i>constant</i>	6.483 (0.000)	6.708 (0.000)
$\ln s_i$	0.826 (0.000)	0.803 (0.000)
$\ln(n_i + 0.05)$	-1.692 (0.002)	-1.551 (0.004)
$\gamma$	0.829 (0.000)	0.738 (0.000)
Common factor test	5.927 (0.052)	4.216 (0.121)
Restricted regression		
<i>constant</i>	8.788 (0.000)	8.690 (0.000)
$\ln s_i - \ln(n_i + 0.05)$	0.841 (0.000)	0.809 (0.000)
$\gamma$	0.831 (0.000)	0.748 (0.000)
Test of restriction	2.342 (0.126)	1.846 (0.174)
Implied $\alpha$	0.457 (0.000)	0.447 (0.000)
Common factor test	6.693 (0.010)	3.723 (0.054)

Table 3: Unconditional convergence and the textbook Solow model

Model	Uncon. conv.	TextBook Solow
Dep. var.	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$
Obs. / Weight matrix	91	91
<i>const.</i>	-0.006 (0.718)	0.030 (0.359)
$\ln y_i(1960)$	0.002 (0.197)	-0.007 (0.000)
$\ln s_i$	—	0.021 (0.000)
$\ln(n_i + 0.05)$	—	-0.032 (0.008)
Implied $\lambda$	-0.002	0.008
Half-life	—	91.20

Table 4: Conditional convergence in the spatially augmented Solow model

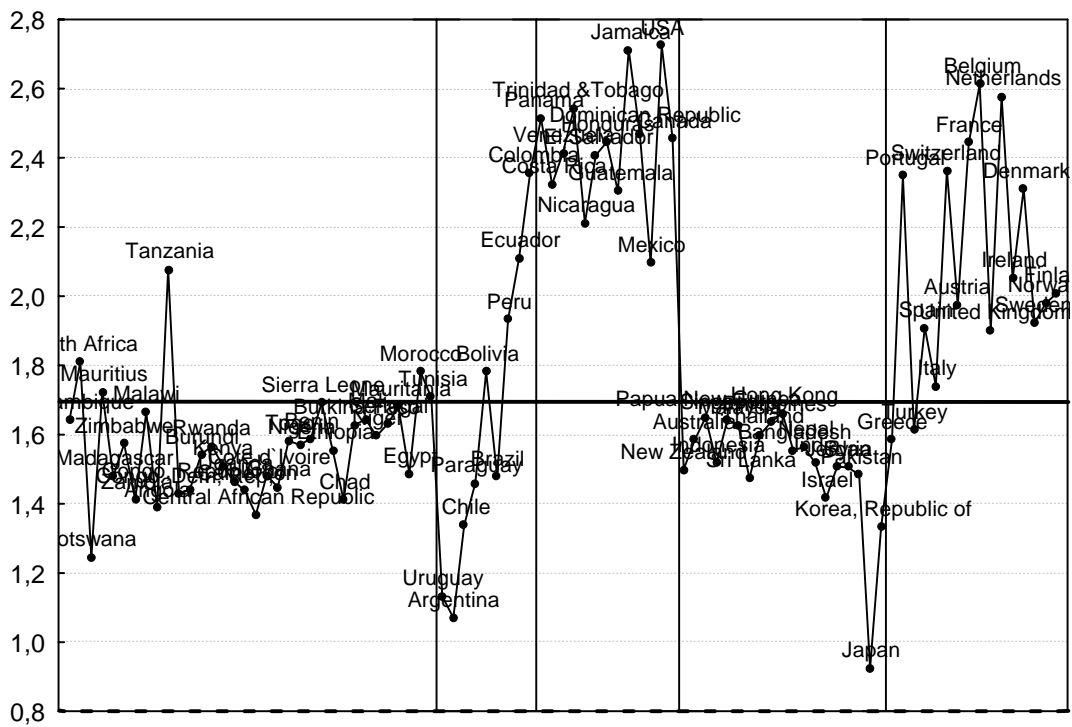
Model	Spatial aug. Solow	Spatial aug. Solow
Dep. var.	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$
Obs. / Weight matrix	91 / (W1)	91 / (W2)
<i>const.</i>	0.008 (0.858)	0.015 (0.738)
$\ln y_i(1960)$	-0.013 (0.000)	-0.012 (0.000)
$\ln s_i$	0.018 (0.000)	0.018 (0.000)
$\ln(n_i + 0.05)$	-0.035 (0.005)	-0.033 (0.008)
$W \ln y_j(1960)$	0.014 (0.000)	0.010 (0.002)
$W \ln s_j$	-0.010 (0.029)	-0.007 (0.102)
$W \ln(n_j + 0.05)$	0.032 (0.086)	0.021 (0.237)
$W \left( \frac{\ln y_j(1995) - \ln y_j(1960)}{35} \right)$	0.485 (0.000)	0.423 (0.000)
Implied $\lambda$	0.017	0.015
Half-life	40.30	46.52

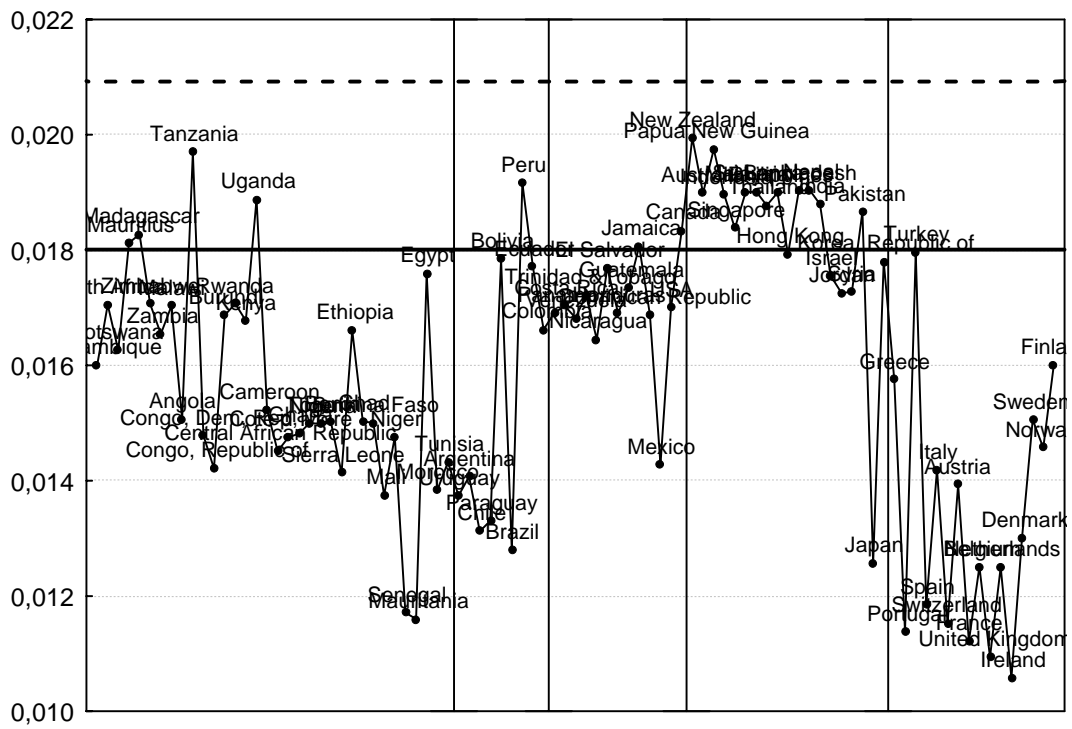
Table 5: Conditional Convergence with spatially autocorrelated errors and non linear restrictions tests

Model	Spatial aug. Solow	Spatial aug. Solow
Dependent variable	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$	$\frac{\ln y_i(1995) - \ln y_i(1960)}{35}$
Obs. / Weight matrix	91 / (W1)	91 / (W2)
<i>constant</i>	0.033 (0.349)	0.027 (0.437)
$\ln y_i(1960)$	-0.010 (0.000)	-0.008 (0.000)
$\ln s_i$	0.020 (0.000)	0.019 (0.000)
$\ln(n_i + 0.05)$	-0.041 (0.001)	-0.038 (0.002)
$\gamma$	0.531 (0.000)	0.444 (0.000)
Common factor test	10.943 (0.012)	6.432 (0.011)
Implied $\lambda$	0.012	0.092
Half-life	59.162	70.874











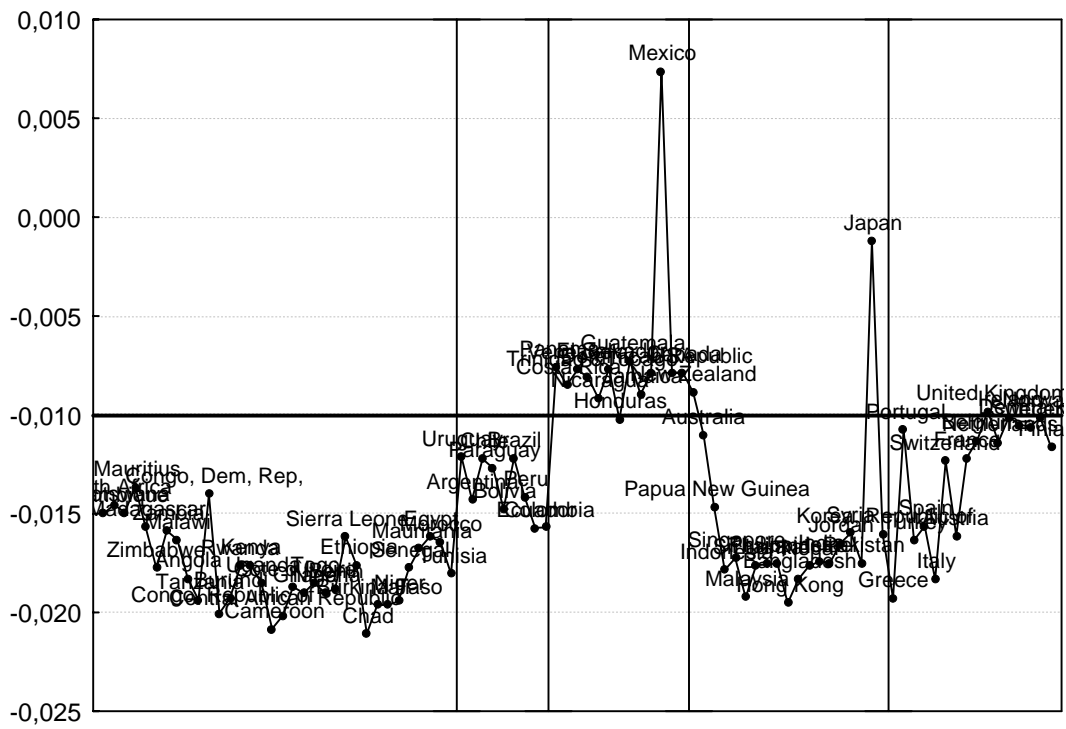


Figure 5: Distribution of the estimates of lagged saving rate





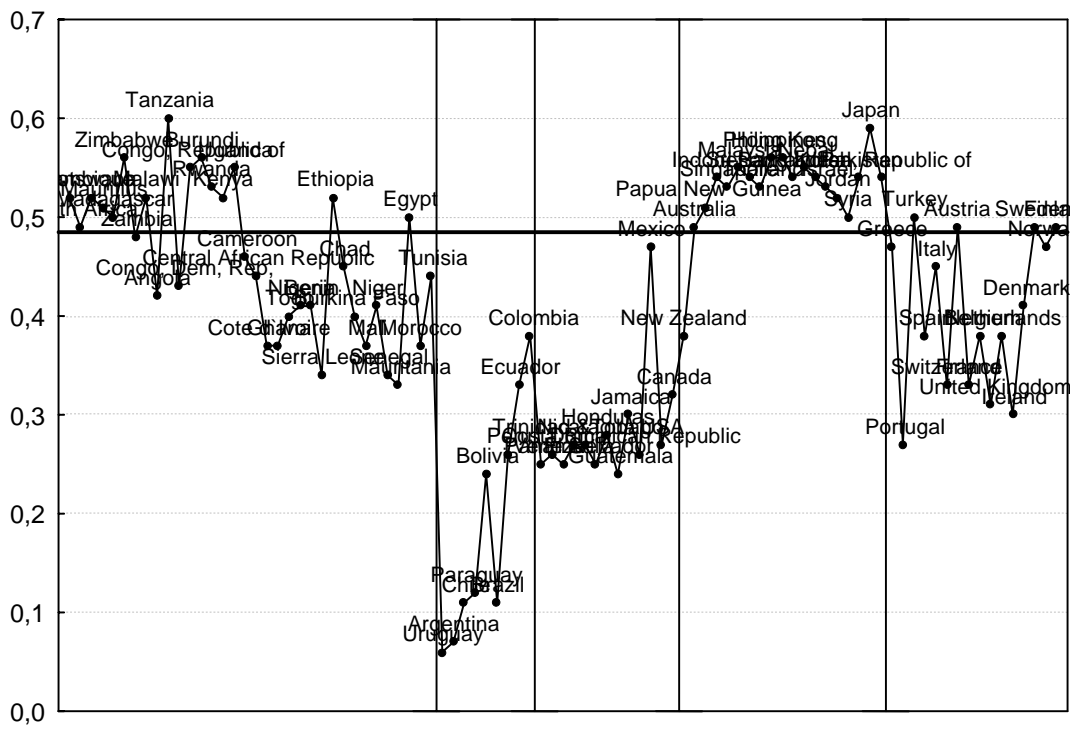


Figure 8: Distribution of the estimates of the lagged growth rate

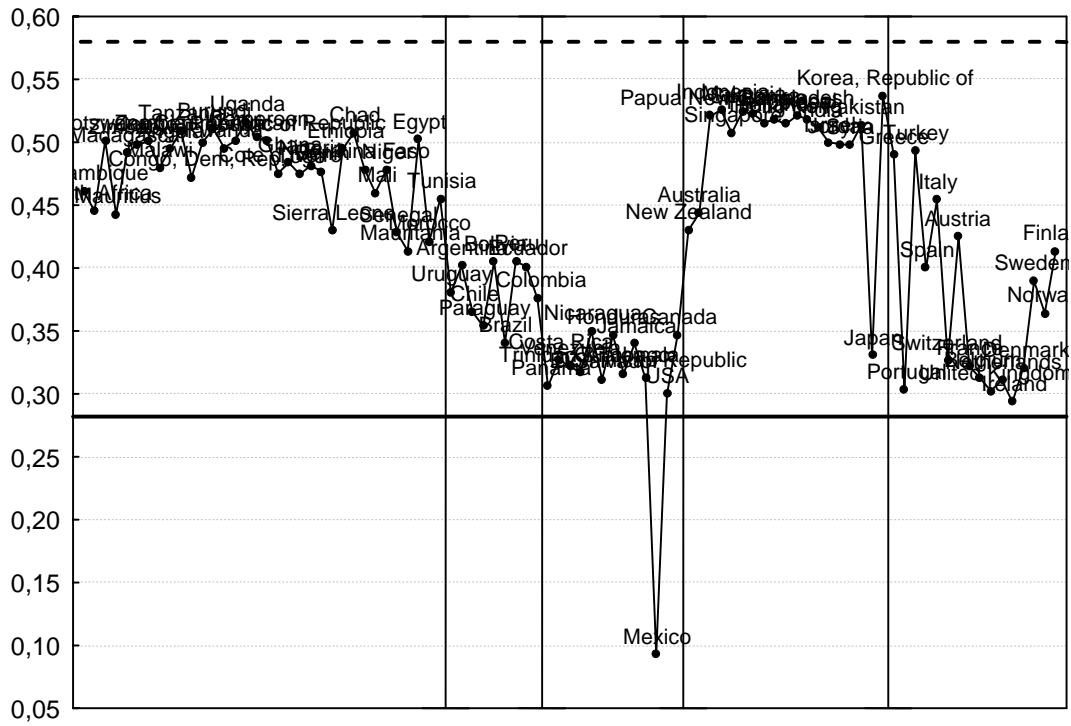


Figure 9: Distribution of the estimates of the income capital share ( $\alpha$ )



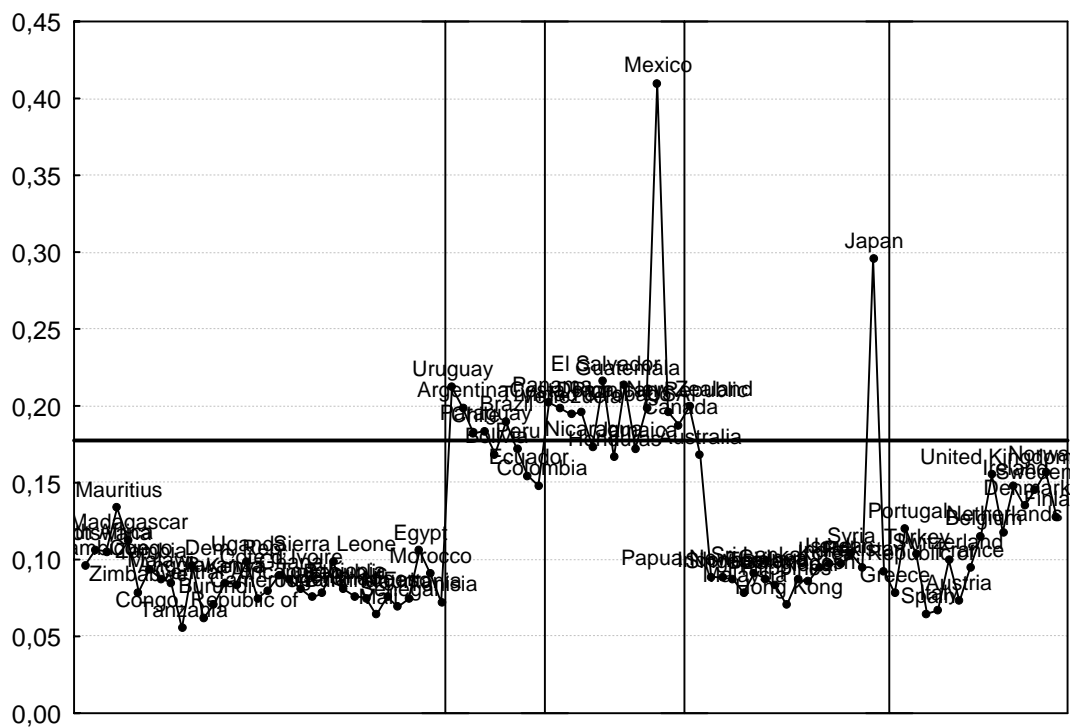


Figure 10: Distribution of the estimates of the physical capital externalities ( $\phi$ )



