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A MATHEMATICAL MODEL FOR DEVELOPING ETHNO-BIOLOGICALLY DIVERSE TROPICAL FORESTS

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ABSTRACT

This paper presents a dynamic optimal control model describing the benefits and costs associated with the development of tropical forests rich in plant and animal species and folk knowledge. The model is a framework to assess how various market and institutional incentives might influence both deforestation and the collection of "ethno-biological information."

CONTENTS

INTRODUCTION

The clearing of tree-covered land near the equator arouses widespread debate and concern. Linkages between vegetation and global climate remain the subject of controversy (Detwiler and Hall 1988). By contrast, there is little doubt that deforestation impacts "ethno-biological information." Although tropical forests cover less than 10% of the Earth's land surface, they contain half the world's plant and animal species (Myers 1984; Wilson 1988). In addition to species extinction, land clearing eliminates forest dwellers' environmental knowledge.

Rarely have economists addressed the management of ethno-biological information directly, concentrating instead on the study of policies that accelerate deforestation. For example, Repetto and Gillis (1988) find that, in many countries, royalties paid for access to public forests are far below the commercial value or stumpage value of the timber. This creates strong incentives to log these areas rapidly, and then move quickly to new sites. In addition, Mahar (1988) contends that tax breaks and other subsidies have stimulated cattle ranching in some parts of the Brazilian Amazon.

Although economists have not addressed the management of ethno-biological information directly, both "in situ" and collected ethno-biological information have value to humans. The genotypes of naturally- occurring plants have often been used to increase the yield of agricultural crops. Many medicines are based on wild plants. However, before wild genotypes are useful, they must be identified, collected, and analyzed to determine useful properties. In addition to this analysis, we need to scrutinize the trade-offs associated with using tropical forests as a natural storehouse of ethno-biological information. Also no one has determined the likely impacts of various policies on information stocks.

These issues are the focus of this paper. By using a control model, we identify the costs and benefits of tree clearing near the equator. We also discuss the necessary conditions for socially efficient deforestation and information collection. These conditions serve as a framework for assessing how market forces, technology, and public policy influence private decisions about the management of tropical forests. In particular, we discuss the results of establishing "ecosystem reserves," easing developing countries' debt burdens, altering tenure arrangements, and improving technology to collect and use that information.

THE MODEL

Mining models characterize an efficient schedule for depleting an exhaustible resource (Hotelling 1931; Dasgupta and Heal 1979; Devarajan and Fisher 1981). We can use such a model to analyze loss of tropical forests. Once cleared, those forests recover their original ethno-biological diversity slowly or never (Gomez-Pompa "et al." 1972). Accordingly, deforestation is irreversible.

To describe socially efficient use and management of tropical forests, we must change the basic mining model and consider agricultural rents captured by the owners of deforested land (Brazee and Mendelsohn 1990). We must also address income from forestry activities that do not require deforestation and the "in-situ" values of tree-covered land (Vousden 1973).

People usually clear forests to plant crops and to raise livestock. It is equally true of agricultural colonists in the developing world today. Ehui and Hertel (1989) say that the opening of "virgin" lands for crop and livestock production can provide a considerable stimulus for an agricultural economy. Conversely, the value of recently cleared land can be low because of poor soils and isolation from markets. In general, though, we can treat agricultural rents captured at time t, A(t), as an increasing function of cleared land. That is, agricultural rents increase but at a decreasing rate as cleared land increases. Cleared land equals land originally covered with primary forests F0, less current tree-covered area, F(t):

(1) $A(t) = A(F0-F(t))$.

Tropical forests yield myriad "in situ" values. The value of primary concern in this analysis (and in much of the scientific literature) is ethno-biological information. Distinct from other benefits of standing forests, anthropologists, botanists, and others whose time is scarce, must collect that information before using it. In addition, "archived" species and folk knowledge make up a stock, rather than a flow that collects over time.

Existing economic literature does not address these characteristics. In our model, E(t) expresses the value of archived ethno-biological information at time t as an increasing and concave function of the stock of archived information, I(t):

(2) $E(t) = E(I(t))$.

The opportunity cost, C(t), of information collection at date t is an increasing and convex function of inputs to the activity, N(t):

(3) $C(t) = C(N(t))$.

If input costs rise with use, then C(t) is an increasing and convex function of input use. That is, the cost of a unit of effort devoted to information collection increases, at an increasing rate, as that effort increases. This model assumes that maintaining the stock of archived information ("i.e.", managing "ex situ" species and folk knowledge) is costless.

We can model other "in situ" values of standing forests in a simpler fashion. The flow of watershed management and climatic benefits at time t is generally an increasing and concave function of F(t). So is the income earned from sustainable commercial forestry, which is distinct from timber income during the early stages of land clearing (Sedjo 1987). Aggregating "in situ" benefits unrelated to biological diversity and the returns to "environmentally benign logging" in a single function, B(t), one obtains:

(4) $B(t) = B(F(t))$.

Finally, stumpage values are an important component of the income earned by agents of deforestation, agriculturalists and loggers alike. We can express values captured at time t, R(t), as an increasing and concave function of the land clearing rate, D(t):

(5) $R(t) = R(D(t))$.

Combining agricultural rents, "in situ" values, income from sustainable commercial forestry, and stumpage values collected by agents of deforestation, functions (1) through (5), yields an objective function -- the net discounted social value of the area originally covered with trees:

(6) Int[0, infinity] exp[-rt]{R(D(t)) + A(F0-F(t)) + B(F(t)) + E(I(t)) - C(N(t))}dt,

where r is the real discount rate.

With the control variables D(t) and N(t), the objective function contains two state variables. We can describe by transition equations tree-covered area, F(t), and archived ethno-biological information, I(t):

(7) dF(t)/dt = - D(t) and
(8) dI(t)/dt = G[F(t),N(t)],

with both first partial derivatives of G[F(t),N/t)] positive,

where G[F(t),N(t)] relates information collection at time t to forested area and collection effort at time t. We assume that this collection (production) function is increasing in both arguments and concave. In transition equation (8), F(t) serves as a proxy for the stock of unarchived ethno-biological information that exists in tropical forests.

RESULTS

To determine socially efficient use and management of tropical forests, net present value function (6) maximizes the two control variables, D(t) and N(t), subject to transition equations (7) and (8), the initial levels of the state variables, F0 and I0, and non-negativity constraints on the control and state variables. Application of the maximum principle (Kamien and Schwartz 1981; Pontryagin "et al." 1962) provides the necessary conditions for the optimal paths of D(t) and N(t). Assume functions R(), A(), B(), and E() are concave and C( ) is convex, then the objective function (6) is concave in the control and state variables. This is sufficient to satisfy both the second-order and transversality conditions.

In addition to transition equations (7) and (8) and the non-negativity constraints, the necessary conditions for an internal maximum are:

(9) exp[-rt]dR(D(t))/dD(t) - L1(t) = 0,

(10) exp[-rt]dC(N(t))/dN(t) - L2(t) partial G[F(t),N(t)]/partial N = 0,

(11) exp[-rt]{dB(F(t))/dF(t) - dA(F0-F(t))/dF(t)} + L2(t) partial G[F(t),N(t)]/partial F + dL1(t)/dt = 0 , and

(12) exp[-rt]dE(I(t))/dI(t) + dL2(t)/dt = 0 ,

where L1(t) and L2(t) are the adjoint variables for F(t) and I(t), respectively. Equations (9) and (10) are the necessary conditions for a maximum with respect to each of the two control variables, D(t) and N(t). Conditions (11) and (12) are adjoint equations.

Equation (9) is a standard mining model result. In mining models, the net present value of benefits minus costs from extraction at every time t must equal the value of another unit of stock at time t. In the context of our model at every time t, the discounted value of net revenues from logging a hectare should equal the value of another hectare of standing forest. The value of a hectare of standing forest includes "in situ" uses and returns from sustainable forestry minus the agricultural value lost from not clearing.

Equation (10) requires that the discounted marginal cost of information collection effort at time t equals the marginal value product of that effort at time t. The marginal cost of information collection effort is the additional cost from devoting more effort to information collection. The marginal value product of effort equals the marginal value of information multiplied by the marginal product of collection effort. The marginal value of information is the value from an additional unit of information collected. The marginal product is the quantity of information collected from an additional unit of effort.

Equation (11) describes how the value of an additional unit of forest changes over time. In a standard mining model, the value of a unit of stock is constant over time. Here the value of an additional unit changes over time as the benefits of the standing forest unrelated to ethno-biological information, the value of agricultural land, and the value of ethno-biological information change overtime.

For equation (12) to hold, the discounted value of ethnobiological information collected should equal the change over time in the value of collected information. If we add to the stock of collected ethno-biological information through additional collection, then the value of this addition should equal the change in value to the total stock.

To analyze the qualitative impacts of public policy, market forces, and technology, we eliminate time derivatives from the necessary conditions and derive comparative statics for the resulting system. For this analysis to yield clear insights, we need to simplify the model. Specifically, simplify three components of the objective function (6):

(3') $E(t) = V(t) \times I(t)$,
(4') $C(t) = W \times N(t)$, and
(5') $R(t) = P(t) \times D(t)$,

where V(t) is the marginal value of information at date t, W is an externally determined wage, and P(t) is the marginal stumpage value of commercial timber extracted from a unit of deforested land at date t.

These assumptions describe the decision-making environment facing a region or country with few ethno-biologically diverse forests and change three of the necessary conditions. We then drop the adjoint variables, all time derivatives from (7), (8) and (11), and the three changed necessary conditions (9'), (10') and (12').

The result is four equations with the two control and two state variables implicit functions of the parameters. See the Appendix for details of the procedure.

The previous results describe behavior during the deforestation era but do not determine the duration of the deforestation era. To complete the model, a condition that determines the duration of the deforestation era is needed. For simplicity, we will assume that all forests not placed in "ecosystem reserves" will eventually be developed:

(13) $F_{min} - F_0 + \int_0^T D(s)\,ds = 0$,

where Fmin is the stock of tropical forests on which timber harvesting is prohibited ("e.g." land placed in parks), and T is the length of the deforestation period. Fmin is a parameter, while T is an endogenous variable. The assumption that all forests outside of ecosystem reserves will be cleared is a simplifying technical assumption. Conditions under which this assumption is valid are presented in Levhari and Liviatan (1977).

The five equations closely approximate a simultaneous system of equations. Comparative statics analysis allows us to determine the impact of shifting an exogenous parameter on endogenous variables. We can derive the comparative statics of that system by applying standard methodology (Samuelson 1948; Silberberg 1978). Table 1 summarizes the results of this analysis of comparative statics. Plus and minus signs in that table refer to direct and inverse relationships, respectively, between exogenous parameters and endogenous variables. A zero shows the absence of a relationship.

THE RETURNS TO DEFORESTATION

Throughout Africa, Asia, and Latin America, agents of deforestation collect two kinds of income. One is stumpage values, represented by P(t) in the model. The other is agricultural rents, represented by A(F0-F(t)). As noted in the introduction, public policy influences both returns. Repetto and Gillis (1988) argue that excessive logging is a classic example of rent capture brought on by under-pricing of access to publicly-owned resources. In addition, some governments have subsidized agricultural land clearing (Mahar 1988).

Reducing either P(t) or A(F0-F(t)) helps to arrest deforestation, D(t). In 1989, for example, the Brazilian government stopped offering tax breaks to cattle ranchers in the Amazon Basin. Pearce (1991) contends that this policy change helps to explain recent declines in agricultural land clearing.

As shown in the first and sixth rows of Table 1, lessening the income of agents of deforestation also has a positive impact on the gathering of ethno-biological information. This linkage arises because the cross-partial derivative of the information collection function is positive. Reduced deforestation, which by definition increases F(t), causes collection effort, N(t), as well as stock of archived species and folk knowledge, I(t), to increase.

It is important to distinguish stumpage values collected when land is cleared, P(t), from benefits associated with sustainable commercial forestry. Our model includes the latter in B(t). Note that an increase in B(t) will slow deforestation, and increase the collection of ethno-biological information. A permanent rise in stumpage price will increase both the revenues received when land is cleared and revenues received from sustainable commercial forestry. These are opposing impacts. An increase in the revenues from clearing provided incentives for faster deforestation and less gathering of ethno-biological information. An increase in the revenues from future sustainable forestry provides incentives for slower deforestation and more gathering of ethno-biological information. Which impact dominates is an empirical question.

Table 1. Comparative Statics Results
(Pr means partial below)

		Variables [note 1]				
Parameters	Description	D(s)	F(t)	I(s)	N(t)	T
A(F0-F(t))	agricultural rents	+	-	-	-	-
B(F(t))	forest values unrelated to ethno-biological information	-	+	+	+	+
Fmin	forest land on which timber harvesting is prohibited	0	0	0	0	-
Pr G F(F(t),N(t)) / Pr F	marginal product of standing forest in information collection	-	+	+	+	+
Pr G N(F(t),N(t) / Pr N	marginal product of collection effort in information collection	+	-	-	-	-
(t)	current stumpage price	+	-	-	-	-
dP(t)/dt	temporal change in stumpage prices	-	+	+	+	+
r	real interest rate	+	-	-	-	-
V(t)	marginal value of archived information	-	+	+	+	+
W	marginal cost of	-	+	+	+	+

effort devoted to
information collection

1. Assume Pr [Pr G]/Pr F Pr N to be positive throughout this analysis.

FOREST MANAGEMENT, DEBT RELIEF, AND ECOSYSTEM RESERVES

International efforts to save tropical forests now have a dual focus. First they help establish ecosystem reserves by prohibiting logging and agricultural colonization. Also they help ease developing countries' debt burdens. The thinking is that as real interest rates drop, the present value of future benefits from forest conservation increases.

We can use comparative statics results to assess both initiatives, often tied together in "debt-for-nature swaps." In this paper's model, Fmin. represents the total area placed in ecosystem reserves. As reported in the third row of Table 1, increasing Fmin brings forward the date, T, when the "post-deforestation era" begins. Significantly, however, this affects neither deforestation nor collection of ethno-biological information before T. This is not to deny, of course, that we may recover more species and folk knowledge after T because of a larger standing forest.

An important caveat for this result arises from the model's aggregative structure. We could place easily accessible forests in a reserve. This might cause timber harvesting and agricultural colonization to lessen for awhile because loggers, farmers, and ranchers would have to relocate to more remote areas. Also, the delayed deforestation would allow information collection to increase.

The results of dealing with tropical deforestation by easing developing countries' debt burdens are separate from the total impacts of establishing ecosystem reserves. Although several factors determine the real interest rate, debt relief by decreasing the demand for financial capital could reduce the real interest rate. Reducing the real interest rate slows deforestation while leading to an increase in collection effort, the size of the standing forest, and the stock of archived ethno-biological information (see eighth row of Table 1). Figures 1A and 1B depict the differences between the impacts of debt relief and the results of creating ecosystem reserves.

Comparative statics analysis of interest rate changes is also subject to a caveat. In addition to reflecting society's time preferences, real interest rates guide the allocation of scarce financial capital. As those rates fall, capital becomes more available, which causes the economy to grow. This can, in turn, stimulate demand for timber, agricultural commodities, and other products extracted from tree-covered land.

TREES, TENURE, AND TECHNOLOGY

Property arrangements, rights in ethno-biological information and its substitutes, and technology used to exploit those commodities, also influence development of tropical forests.

Land Tenure

Agricultural use rights are the obvious feature of most frontier tenure regimes in the developing world. Raising crops or livestock where trees used to grow is necessary for new land claims along agriculture's extensive margin. Similarly, continuous agricultural land use is essential to keep others from violating one's formal or informal property rights (Southgate 1990).

This type of tenure regime makes the capital gains associated with delayed deforestation insecure. One capital gain arises from an increase over time in stumpage prices, $dP(t)/dt$. The seventh row of Table 1 shows that strengthening the property rights of forest dwellers to enhance their ability to capture this capital gain actually increases forested area and the stock of archived ethno-biological information in all time periods.

Weakened property rights discourage loggers from managing tree-covered land for the capital gains generated by rising stumpage values. In the same way, tenurial conditions keep agricultural colonists from following guidelines like those described by Ehui and Hertel (1989). Fearing that anyone willing to clear "unimproved" land can claim it, colonists will always deforest immediately if they can capture any agricultural rent. Colonists are better off capturing a small rent than losing the land entirely because they decided to delay clearing.

The effects of strengthening property for the standing forest are similar to the impacts of a lower discount rate, illustrated in Figure 1. Several authors including (Ehui and Hertel, 1989, Mahar 1989, Repetto and Gillis 1988, and Southgate "et.al." forthcoming) describe these impacts in more detail for several less developed countries.

Property Rights in Ethno-Biological Information

Land rights are not the only tenurial factor influencing the development of tropical forests. Legal interests in species and folk knowledge and in substitutes for ethno-biological information are also important. Sedjo (1985) and others stress that no one has tenure in tropical forests' plant and animal species. Forest dwellers' rights in their environmental folk knowledge is also very weak. Under these circumstances, individuals, groups, and governments can capture only a small portion of the marginal social value, $V(t)$, of additions to archived ethno-biological information. Strengthening property rights in information would increase that portion, which would in turn cause deforestation to fall and information collection to rise (see ninth row of Table 1).

Technology for Using Ethno-Biological Information and Its Substitutes

Improved techniques for collecting ethno-biological information enhance the marginal productivity of the standing forest, partial G[F(t),N(t)/partial F. They also enhance marginal productivity of effort devoted to collecting species and folk knowledge, partial G[F(t),N(t)]/partial F. Table 1 shows a rise in partial G[F(t),N(t)]/partial F tends to diminish deforestation and to stimulate information collection. By contrast, an increase in partial G[(F(t),N(t)]/partial N substitutes collection effort, N(t) for forest area, F(t), both now and in all later periods. This allows current deforestation to increase. Meanwhile, current information collection declines to accommodate an efficient increase in future information collection. [A decline in the marginal cost of collection effort, W, has the same effects as an increase in partial G[(F(t),N(t)]/partial N.

How improved technology for the recovery of ethno-biological information affects use and management of tropical forests is an empirical question that depends on the nature of the technological advance. If the technological advance increases the usefulness of the standing forest relative to the effectiveness of labor devoted to collection effort, then the impact will be to slow deforestation and increase information collection. If the technological advance increases the effectiveness of labor devoted to collection effort relative to the usefulness of the forest, then the rate of deforestation will increase, and the rate of information collection will decrease.

The results of advances in biotechnology are similarly ambiguous.

If advances in biotechnology increase the demand for prior human knowledge in the form of collected genotypes of existing organisms relative to the cost of producing substitutes for collected genotypes, then deforestation should decrease, and information collection increase. However if advances in biotechnology decrease the cost of producing substitutes for collected genotypes relative to the demand for collected genotypes, then deforestation will increase and information collection decrease.

SUMMARY AND CONCLUSIONS

Any theoretical treatment of resource development trade-offs has limitations. Lacking a geographic dimension, this analysis offers no specific insights into which tropical forests deserve special protection because they feature unusually high concentrations of threatened and potentially valuable species. Sedjo (1985) addresses this issue. In addition, our analysis is deterministic and risk neutral. So, it is not possible to discuss how option values associated with tropical forests' uncollected ethno-biological information should affect land clearing and information collection.

Data needed for empirical application of this paper's model are also lacking. There is wide variation among the few available estimates of forested area in the developing world. No long-term time series on agricultural land clearing exists for any country.

Rarer still are data on tropical forests' ethno-biological information. No country keeps comprehensive reports on effort devoted to information collection or on newly-discovered species or efforts to enhance the value of previously-discovered species.

Regardless of the impediments to empirical application, this paper's analysis sheds light on factors influencing the development of ethno-biologically diverse tropical forests. Policies enhancing the rents associated with deforestation also diminish the collection of species and folk knowledge. Establishing ecosystem reserves could have a lesser impact on total deforestation and collection of ethno-biological information than debt relief, which reduces real interest rates.

Giving loggers and agricultural colonists the land rights they need to capture capital gains associated with delayed deforestation encourages conservation of tropical forests. Finally, improved techniques for gathering species and folk knowledge and advancements in biotechnology might or might not strengthen incentives to conserve tropical forests. The size of technological innovation's impacts depends on evolving property rights for ethno-biological information. These findings, we hope, will contribute to the development of effective strategies for managing and conserving tree-covered land near the equator.

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