Optimal Timber Rotation on Multiple Stands with an Asymmetric Externality

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Abstract

Motivated by the logging ban in China and its future deregulation strategy, this paper theoretically examines the dynamic problem of forest management with spatial externality. I construct a theoretical, spatially-explicit model of a forest planner who maximizes timber profits from infinite timber rotation on all stands minus the costs of water runoff, and more importantly, asymmetric depending on the relative location of the stand. The model examines a spatial model of two stands, where the age of one stand affects the cost of the other stand, but asymmetrically. Using specific functional forms, I examine the properties of spatial and temporal substitutability between the two stands and the marginal value of staggering harvesting on one stand. The simulation results illustrate the efficiency gains of a spatial, subadditive model versus an aspatial, additive model.

Keywords: forest economics, multiple stands, non-timber goods, flood risk, spatial externality, additivity properties
JEL: Q23, Q57

1 Introduction

Forests provide valuable environmental services. In a watershed context, flood mitigation, which forests provide by reducing sedimentation and water runoff, is one important service. In China, excessive commercial logging and forest clearing for cultivation on steep sloped land in the upper and middle reaches of China's major river basins have had severe consequences in downstream areas (Asian Development Bank, 2002). Some researchers believe that increased water runoff and sedimentation from uppper and middle reaches have silted streams, reduced hydraulic capacity, destabilized channel widths causing bank erosion and caused higher flood frequencies (MacKinnon, John and Xie, Yan, 2001). Increased water runoff and sedimentation caused by deforestation is believed to be the primary cause of devastating floods in the Yangtze River Basin and northeast China during the summer of 1998, resulting in damages of 20 billion US dollars (Ministry of Water Resources, 1999).

In 1998 China's government responded to these destructive floods by a dramatic change in its forest policy: a nationwide logging restriction. The logging restriction, scheduled to be in place until 2010, includes a complete logging ban in the forests of the upper region of the Yangtze River, the upper and middle regions of the Yellow River, as well as logging reductions in northeastern China and Inner Mongolia. The logging ban has been effective in halting timber harvest; some proponents claim that rivers have already started to clear up after only four years into the ban.

The logging ban, however, has come at a high cost to rural economies. The government no longer collects tax revenue from logging profits as it previously did from the state-owned forestry bureaus. Between 1998 and 1999, more than 1 million forest workers lost their jobs (CCICED, 2002). Although there are still five more years to go, policymakers are planning to deregulate the restriction after 2010. Clearly, the method China chooses to deregulate the logging restrictions will have direct consequences on the forest sector and on downstream externalities.

The central economic question in devising the deregulation strategy is how to balance the complex tradeoffs between the profitability of the forest sector and the downstream damage cost due to timber harvest. In deciding from which forest to deregulate at what timing, policymakers need to consider the tradeoffs between timber harvesting and flood damage risk; between harvesting forest stands with less harvesting cost versus protecting stands in the watershed that are important for flood mitigation; between profitability from timber and adopting forest management technology (e.g., selective logging) that mitigate flooding.

When balancing these tradeoffs, there could be efficiency gains from taking into account the spatial interdependence between forest stands and hydrological process that relates timber production behavior to downstream damages. Unlike some other non-timber goods produced by forests, spatial interdependence between forest stands matter for water runoff and sedimentation problems. To sequester carbon, for example, where trees grow within a watershed or along a hillslope is not a primary concern. However, in order to reduce water runoff or sedimentation, *where* trees grow and *at what timing* they are harvested within a watershed matters significantly. The importance may be maginified when forest managers and policymakers need to make policy decisions at the watershed scale. This paper will compare forest systems with and without spatial interdependence to investigate what the efficiency gains are by incorporating spatial interdepence between forest stands.

Despite the magnitude of the problem in China and elsewhere, economics literature currently provides regulators with little guidance into understanding how the complex economic tradeoffs should be balanced when deregulating a logging restriction. Forest rotation models developed since the 19th century provide the theoretical basis for determining the optimal timings of harvests when maximizing profits from timber (Samuelson, 1976). In his seminal paper, Hartman extended Faustmann's model by incorporating non-timber benefits of trees as a jointly produced good along with timber. In papers that followed Hartman, several types of non-timber benefits have been examined, such as carbon sequestration and recreational values (Strang, 1983; Van Kooten et al., 1995). However, damage costs arising from negative externalities have not yet been considered in forest rotation models.

There are several analytical analyses of the economic problem behind stand interdependence. The concept of forest stand interdependence in joint production of timber and non-timber benefits was originally addressed in Bowes and Krutilla (1985). Swallow and Wear (1993) and Swallow et al. (1997) were the first to formulate explicit spatial interactions for nontimber amenity benefits between two adjacent stands, but they relied mainly on numerical approximations. Koskela and Ollikainen (2001) and Amacher et al. (2004) extended the work by Swallow and Wear (1993) by using concepts from game theory to examine the role of landowner behavior of adjacent stands. None of the previous work, however, analytically considers a model with asymmetric spatial externality. Although the spatial aspect of an externality problem has long been recognized in the economics literature, much of the theoretical literature assume that one more unit of effluent will contribute the same marginal damage regardless of the source or ambient conditions (Helfand et al., 2003). This assumption needs to be relaxed in the case of water runoff and problems associated with logging.

The objective of this paper is to theoretically model the dynamic problem of forest management with spatial externality. First, I will define the additive properties of a forest-hydrology system and discuss under what circumstances we need a spatial model. I then build a analytical, spatially-explicit model of a forest planner who maximizes timber profits in a finite time horizon on two stands minus the damage costs of water runoff. The damage cost is a function of the stand age, and more importantly, asymmetric depending on the relative location of the stand on a hillslope. Lastly, I apply the theoretical results to a two-stand example using parameters from the literature.[1]

2 Additive Properties in Forest-Hydrology System

Forests have the function of absorbing and storing water, regulate water flow, and thus reduce the risk of downstream floods (Chang, 2003). Forests reduce the overland runoff smaller, runoff timing longer, and the water yield lower through three processes. The amount of precipitation that reaches the soil is reduced by canopy interception. Some of the soil moisture is transpired to the air through the roots-stem-leaf system. Evapotranspiration, which is these two processes combined, is generally recognized as the most pronounced direct way by which watershed hydrology is changed (e.g., Croft and Hoover (1951)). Furthermore, the roots systems, organic matter, and litter floor increase the infiltration rate and soil moisture holding capacity (Chang, 2003). These three processes combined make streamflow from forested watersheds have less water runoff (Chang, 2003).

One of the factors that affect the magnitude of these processes is the age of the forest stand. In general, as trees get older, they possess wider canopies, more leaf area, a larger system of roots and stems, and a thicker litter floor. All of these characteristics are associated with more absorption of rainfall and consequently a reduction in runoff. In addition to the age of the stand, other factors such as soil type, slope, and the aspect also determines the magnitude of runoff reduction. These

[1] To simplify the discussion, in the rest of the paper I restrict the externality problem to water runoff and ignore the sedimentation problem that also is related with flood risk but is affected by timber harvesting differently.

other factors, are exogenous to the forest manager's decision whereas age of the stand is precisely the decision variable in timber production.

Given that the degree of water absorption capacity is spatially heterogeneous, there may be two ways for a forest manager to choose from which forests to harvest and at what timing. One way is to consider each forest stand as an independent system regardless of the location in the forest, collect characteristics that determine on-site water absorption such as rainfall, soil, and slope, and reserve those stands that has the most absorption potential. Another way is to consider multiple forest stands as a linked, interdependent system, take into account the hydrological system (i.e., where water goes), and reserve stands that have the highest potential of absorbing water, including the water from upslope.

Before introducing a spatial model of forest stands, we need to ask the question: when do we need a spatial model? In other words, under what conditions would modeling forest stands as a linked system yield better results? To understand the conditions, it helps to consider two extreme cases. If all forest stands in a watershed receives heavy rainfall such that the rainfall on each stand always exceeds its water absorption capacity, then the decision of which forest to harvest first will not differ whether the forest manager views the forest stands as a spatially-linked system or each as an independent system, *ceteris paribus*. Likewise, if rainfall on each stand is so low that rainfall is less than each stand's absorption capacity, the two ways to view forest stands would not make a difference either, *ceteris paribus*. In both cases the total damage cost is simply the sum of the damage cost from each stand. In most cases a watershed lies somewhere between these two extremes. The total damage cost may not be the sum of damage cost from each stand, i.e., the damage cost could be lower than the simple summation. Therefore, in such cases, the two ways to view forest stands (spatially-linked vs. independent system) may make a difference on which stand to harvest at what timing.

We define the additive properties of water runoff of a forest system in a watershed.[2]

Definition 1 *The forest system is an* **additive system** *with respect to its water runoff if the runoff at the outlet of a watershed is equal to the sum of runoff generated from each stand. The forest system is a* **subadditive system** *with respect to water runoff if*

[2] Additive properties of a resource system is examined by Sanchirico (2005) and others. The definition we use is analogous to definitions used by Sanchirico (2005), although different in the sense that in a hydrological system there could not be a case of a supraadditive system.

runoff at the outlet of a watershed is strictly less than the sum of runoff generated from each stand.

The relationship between rainfall and water absorption capacity on each stand, and the relative location of each stand within a watershed determines when we should view forest stands as a spatially-linked system. For example, suppose there are two adjacent stands along a single hillslope, one downslope (stand 1) and the other upslope (stand 2). Suppose each stand receives rainfall R_1 and R_2 and each stand has its own water absorption capacity to absorb some of the rainfall that falls on its *own* stand, denoted by A_{11} and A_{22}, respectively. Stand 1 also has the potential to absorb some of the rainfall that falls on stand 2 and gets carried over to stand 1, denoted by A_{12}. However, whether or not stand 1 has such potential depends on the situation on both stand 1 and stand 2. We can classify the relationships between rainfall, water absorption capacity, and the total runoff at the watershed outlet into five different cases (Table 1). Cases 1 through 4 exhibit an ***additive*** system, where the runoff at the watershed outlet does not differ whether or not we consider the two stands as a spatially-linked system or not. Only Case 5 exhibits a ***subadditive*** system, where the runoff at the watershed outlet is less when we consider the two stands as a spatially-linked system. In this case, stand 1 is absorbing a part or all of water runoff from stand 2 that comes through either surface or subsurface flow, preventing and/or delaying the time it takes for the runoff to reach the watershed outlet.

The additive property of a system has an important implication for a forest manager's problem. For an additive system, the social planner can solve the maximization problem separately for each stand. For a subadditive system, however, the social planner needs to solve the problem for the forest stands jointly. In the next section I develop a spatial model of two forest stands where the forest manager solves the problem jointly for two stands.

3 A Two Stand Model with Damage Cost

Suppose there are two forest stands that are on a single hillslope, where stand 1 is adjacent to a waterway and stand 2 is the upper stand.[3] If the rainfall on stand 2 exceeds its absorption capacity, then stand 2 creates runoff to stand 1. If stand 1 has some extra absorption capacity, then a part of runoff from stand 2 can be absorbed in stand 1 and the remainder will enter the waterway. How

[3]See Appendix A for a one stand model that extends Hartman's model to a negative externality problem.

much stand 1 actually absorbs depends on how much runoff comes from stand 2 (endogenous to harvesting decision on stand 2), its own forest cover (endogenous to harvesting decision on stand 1), and other factors that determine the absorption capacity on stand 1. Therefore, runoff from stand 2 that gets absorbed by stand 1 is a function of both T_1 and T_2, where T_1 refers to the rotation age for stand 1 and T_2 refers to the rotation age of stand 2. We can express water runoff that originates from stand 1 and 2 that reaches the waterway, W_1 and W_2, as follows:

$$W_1 = R_1 - A_{11}(T_1) \tag{1}$$

$$W_2 = R_2 - A_{22}(T_2) - A_{12}(T_1, T_2) \tag{2}$$

where A_{ij} represents water that comes from stand j that is absorbed by stand i.

Therefore, the total runoff from the two stands can be expressed as:

$$W_{total} = R_1 + R_2 - A_{11}(T_1) - A_{22}(T_2) - A_{12}(T_1, T_2) \tag{3}$$

where A_{12} stands for runoff from stand 2 that is absorbed by stand 1. If the two stands are *not* spatially linked, the total runoff from the two stands can be expressed as:

$$W_{total} = R_1 + R_2 - A_{11}(T_1) - A_{22}(T_2) \tag{4}$$

Note that even if the two stands are spatially linked on a hillslope, the total water runoff can be expressed by (4) if $R_2 \leq A_{22}$ or $R_1 \geq A_{11}$ because in either case there is no potential for stand 1 to absorb runoff from stand 2.

We can say that (4) exhibits an *additive* system (i.e., the total runoff is the same as if the two forest stands were spatially independent), whereas (3) exhibits a *subadditive* system (i.e., the total runoff could be less if we consider the system spatially linked than when we consider the system spatially independent.)

Let us extend the water runoff function to a damage cost function. We assume that there is a non-linear damage cost associated with water runoff due to flood risks downstream. The damage cost for each time period is denoted as D_1 for damage cost arising from stand 1 and D_2 for the damage cost from stand 2 as a function of its own age and the adjacent stand 1's age. Then, for each time period,

$$D_1 = D_1 (W_1) = D_1 (R_1 - A_{11}(T_1)) \tag{5}$$

$$D_2 = D_2 (W_2) = D_2 (R_2 - A_{22}(T_2) - A_{12}(T_1, T_2)) \tag{6}$$

The asymmetric damage function reflect the unidirectional externality; since water flows only from upslope to downslope, stand 1's absorption capacity can be a substite for stand 2 but not vice versa. The total damage function can be expressed as:

$$D_{Total} = D (W_1 + W_2) = D (R_1 - A_{11}(T_1) + R_2 - A_{22}(T_2) - A_{12}(T_1, T_2)) \tag{7}$$

A forest planner's model of two rotations and two stands with timber profits and externality

We describe a basic framework to determine the rotation ages for two adjacent stands, stand 1 and stand 2, that are spatially linked. We assume that the forest planner values net harvest revenue and also internalizes the downstream externality cost of flood. Following Swallow and Wear (1993); Koskela and Ollikainen (2001); Amacher et al. (2004) we assume that the stands are dependent in terms of water absorption but independent with regard to timber production.

Timber volume at harvest is denoted by $f(T_1)$ and $g(T_2)$. Timber price, p, and real interest rate, r, are assumed to be common to both stands and constant over time. Regeneration costs c_1 and c_2 are allowed to differ between the stands. These assumptions reflect the typical situation where timber production costs differ across stands due to site characteristics, such as slope, tree species, or accessibility.

Assuming that the two stands are initially bare land, the present value of timber production for two rotations for each stand are, respectively,

$$V_1 = pf(T_1)e^{-rT_1} - c_1 + pf(T_1)e^{-r2T_1} - c_1 e^{-rT_1} \tag{8}$$

$$V_2 = pg(T_2)e^{-rT_2} - c_2 + pg(T_2)e^{-r2T_2} - c_2e^{-rT_2} \tag{9}$$

We now introduce damage cost as a negetive externality of timber harvesting which reflect asymmetric stand interdependence. Let E_1 describe the total damage cost from stand 1 for the whole time horizon and E_2 describe the damage cost from stand 2 as a function of its own age and the adjacent stand 1's age. For two periods, the damage cost functions can be expressed as:

$$E_1 = \int_0^{T_1} \left(R_{1,t} - A_{11,t}(s_1)\right) e^{-rs_1} ds_1 + \int_{T_1}^{2T_1} \left(R_{1,t} - A_{11,t}(s_1)\right) e^{-rs_1} ds_1 \tag{10}$$

$$E_2 = \int_0^{T_2} \left(R_{2,t} - A_{22,t}(s_2) - A_{12}(s_1, s_2)\right) e^{-rs_2} ds_2 + \int_{T_2}^{2T_2} \left(R_{2,t} - A_{22,t}(s_2) - A_{12}(s_1, s_2)\right) e^{-rs_2} ds_2 \tag{11}$$

Note that rainfall and absorption capacity for both stands should have a time subscript, but they are abbreviated in these equations.

The forest planner, as the sole owner of both stands, is assumed to choose the rotation ages of both stands to maximize:

$$\max_{T_1, T_2} \ \Omega = V_1(T_1) + V_2(T_2) - E_1(s_1) - E_2(s_1, s_2) \tag{12}$$

The first-order necessary conditions are:

$$\Omega_{T_1} = V_{1,T_1}(T_1) - E_{1,T_1}(T_1) - E_{2,T_1}(T_1, s_2) \overset{set}{=} 0, \tag{13}$$

$$\Omega_{T_2} = V_{1,T_2}(T_2) - E_{2,T_2}(s_1, T_2) \overset{set}{=} 0 \tag{14}$$

The second-order necessary conditions are:

$$\Omega_{T_1T_1} = V_{1,T_1T_1}(T_1) - E_{1,T_1T_1}(T_1) - E_{2,T_1T_1}(T_1, s_2) \leq 0, \tag{15}$$

$$\Omega_{T_2T_2} = V_{1,T_2T_2}(T_2) - E_{2,T_2T_2}(s_1, T_2) \leq 0 \tag{16}$$

which can be argued to hold when the damage cost is monotonically declining with respect to stand age.[4]

Specific functional forms for timber growth and absorption capacity

We assume that the growth of stands 1 and 2 is symmetric and is a quadratic function of rotation age:[5]

$$f(T_1) = -aT_1^2 + bT_1 + d \tag{17}$$

$$g(T_2) = -aT_2^2 + bT_2 + d \tag{18}$$

where $a > 0$ and $b > 0$.

We assume that the absorption capacity is an increasing function of stand age. At each time period the absorption of rainfall on its respective stand, A_{11} and A_{22}, cannot exceed rainfall volume during that period. A simple functional form that satisfies these properties is:

$$A_{11} = \frac{R_1 s_1}{\varphi_1 + s_1} \tag{19}$$

$$A_{22} = \frac{R_2 s_2}{\varphi_2 + s_2} \tag{20}$$

The water runoff from stand 2 that is absorbed by stand 1, A_{12}, is an increasing function of age of stand 1, but it is bounded from above by the volume of runoff from stand 2. Therefore:

$$\begin{aligned} A_{12} &= \frac{\left(R_2 - A_{22}(s_2)\right) s_1}{\varphi_1 + s_1} \\ &= \frac{\left(R_2 - \frac{R_2 s_2}{\varphi_2 + s_2}\right) s_1}{\varphi_1 + s_1} \end{aligned} \tag{21}$$

Let us examine the properties of the damage functions. For now, I assume that the damage

[4]Shown in appendix of the full version of the paper.

[5]In the next version I need to change this to the functional form used in the simulation.

cost is linear with respect to water runoff and that per unit cost of water runoff equals 1. Inserting (19), (20), and (21) into (5) and (6), we have:

$$D_1 = R_1 - \frac{R_1 s_1}{\varphi_1 + s_1} \tag{22}$$

$$D_2 = R_2 - \frac{R_2 s_2}{\varphi_2 + s_2} - \frac{\left(R_2 - \frac{R_2 s_2}{\varphi_2 + s_2}\right) s_1}{\varphi_1 + s_1} \tag{23}$$

When examining the properties of these damage costs, we find the following properties:

Lemma 1 *Using functional forms in equations (22) and (23) stand 2 is spatial independent of stand 1, but stand 1 is a spatial substitute of stand 2.*

Taking the first derivatives with respect to rotation age of the other stand, we can show that:

$$\frac{\partial D_1}{\partial T_2} = 0 \tag{24}$$

$$\frac{\partial D_2}{\partial T_1} = -A_{12,T_1} = -\frac{\left(R_2 - \frac{R_2}{\varphi_2 + T_2}\right) \varphi_1}{(\varphi_1 + T_1)^2} < 0 \tag{25}$$

Using the definition of *spatial dependence* as defined in Koskela and Ollikainen (2001) and also used in Amacher et al. (2004), (24) shows that stand 2 is a *spatial independent* of stand 1; the marginal damage cost of stand 1 is independent of rotation age of stand 2. To the contrary, equation (25) shows that the stand 1 is *spatial substitute* of stand 2; the marginal damage cost of stand 2 decreases with the rotation age of stand 1. Therefore, we have a setting where the spatial relationship of two stands are asymmetric.

In addition to spatial dependence, the literature also highlights the importance of *temporal dependence*, i.e., how spatial dependence between stands is affected by rotation age choices on its own stand. Using the specific functional form, and using the definition from Koskela and Ollikainen (2001) we find the following:

Lemma 2 *Using functional forms defined in equations 22 and refeq:damagett2 the spatial substitutability between stand 1 and stand 2 decreases with the rotation age on stand 2.*

This is obtained by differentiating (25) by T_2:

$$\frac{\partial^2 D_2}{\partial T_1 \partial T_2} = -A_{12,T_1T_2} = \frac{R_2}{(\varphi_2 + T_2)^2 (\varphi_1 + T_1)^2} > 0 \tag{26}$$

A two rotation model with specific functional forms

Using the specific functional forms, the timber profits for two periods for each stand are:

$$V_1 = p\left(-aT_1^2 + bT_1 + d\right)e^{-rT_1} - c_1 + p\left(-aT_1^2 + bT_1 + d\right)e^{-r2T_1} - c_1 e^{-rT_1} \tag{27}$$

$$V_2 = p\left(-aT_2^2 + bT_2 + d\right)e^{-rT_2} - c_2 + p\left(-aT_2^2 + bT_2 + d\right)e^{-r2T_2} - c_2 e^{-rT_2} \tag{28}$$

Damage cost for each stand for two periods is obtained by inserting (19), (20), and (21) into (10) and (11):

$$E_1 = \int_0^{T_1} \left(R_{1,t} - \frac{R_1 s_1}{\varphi_1 + s_1}\right) e^{-rs_1} ds_1 + \int_{T_1}^{2T_1} \left(R_{1,t} - \frac{R_1 s_1}{\varphi_1 + s_1}\right) e^{-rs_1} ds_1 \tag{29}$$

$$\begin{aligned} E_2 = &\int_0^{T_2} \left(R_{2,t} - \frac{R_2 s_2}{\varphi_2 + s_2} - \frac{\left(R_2 - \frac{R_2 s_2}{\varphi_2 + s_2}\right) s_1}{\varphi_1 + s_1}\right) e^{-rs_2} ds_2 \\ &+ \int_{T_2}^{2T_2} \left(R_{2,t} - \frac{R_2 s_2}{\varphi_2 + s_2} - \frac{\left(R_2 - \frac{R_2 s_2}{\varphi_2 + s_2}\right) s_1}{\varphi_1 + s_1}\right) e^{-rs_2} ds_2 \end{aligned} \tag{30}$$

Importantly, note that stand age is zero in the beginning of each rotation and T_j when harvesting.

The first derivaties of the timber profits for each stand $i = 1, 2$ with respect to T_1 and T_2 are:

$$
\begin{aligned}
V_{i,T_i} &= pe^{-rT_i}\left((-2aT_i+b) - r(-aT_i^2+bT_i+d)\right) + rc_ie^{-rT_i} \\
&\quad + pe^{-r2T_i}\left((-2aT_i+b) - 2r(-aT_i^2+bT_i+d)\right)
\end{aligned} \tag{31}
$$

If we maximize only timber profits, the first order necessary condition will be:

$$
p(-2aT_i+b)(1+e^{-rT_i}) = r\left(p\left(-aT_i^2+bT_i+d\right)-c_i\right) + 2rp\left(-aT_i^2+bT_i+d\right)e^{-rT_i} \tag{32}
$$

The intuition for (32) is that at the optimal rotation, the forest manager balances the marginal benefit of waiting to harvest for another year with marginal cost of waiting another year, both for two rotations.

The first order necessary conditions with respect to T_1 and T_2 for damage costs from harvesting each stand are[6]

$$
\begin{aligned}
E_{1,T_1} &= \left(R_{1,t=T_1} - \frac{R_{1,t=T_1}T_1}{\varphi_1+T_1}\right)e^{-rT_1} + \left(R_{1,t=2T_1} - \frac{R_{1,t=2T_1}T_1}{\varphi_1+T_1}\right)e^{-r2T_1} - R_{1,t=T_1}e^{-rT_1} \\
&= \left(-\frac{R_{1,t=T_1}T_1}{\varphi_1+T_1}\right)e^{-rT_1} + \left(R_{1,t=2T_1} - \frac{R_{1,t=2T_1}T_1}{\varphi_1+T_1}\right)e^{-r2T_1}
\end{aligned} \tag{33}
$$

$$
E_{1,T_2} = 0, \tag{34}
$$

[6]Note that in the beginning of each rotation the age of the stand is zero, i.e., $s_j = 0$ in the starting point of the integration.

$$
\begin{aligned}
E_{2,T_2} =& \left(R_{2,t=T_2} - \frac{R_{2,t=T_2}T_2}{\varphi_2 + T_2} - \frac{\left(R_{2,t=T_2} - \frac{R_{2,t=T_2}T_2}{\varphi_2+T_2} \right) s_1}{\varphi_1 + s_1} \right) e^{-rT_2} \\
&+ \left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2 + T_2} - \frac{\left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2+T_2} \right) s_1}{\varphi_1 + s_1} \right) e^{-r2T_2} - \left(R_{2,t=T_2} - \frac{R_{2,t=T_2}s_1}{\varphi_1 + s_1} \right) e^{-rT_2} \\
=& \left(-\frac{R_{2,t=T_2}T_2}{\varphi_2 + T_2} - \frac{\left(R_{2,t=T_2} - \frac{R_{2,t=T_2}T_2}{\varphi_2+T_2} \right) s_1}{\varphi_1 + s_1} \right) e^{-rT_2} \\
&+ \left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2 + T_2} - \frac{\left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2+T_2} \right) s_1}{\varphi_1 + s_1} \right) e^{-r2T_2} - \left(-\frac{R_{2,t=2T_2}s_1}{\varphi_1 + s_1} \right) e^{-rT_2}
\end{aligned}
\tag{35}
$$

$$
E_{2,T_1} = -\int_0^{T_2} \frac{\left(R_{2,t} - \frac{R_{2,t}s_2}{\varphi_2+s_2} \right) \varphi_1}{(\varphi_1 + T_1)^2} e^{-rs_2} ds_2 - \int_{T_2}^{2T_2} \frac{\left(R_{2,t} - \frac{R_{2,t}s_2}{\varphi_2+s_2} \right) \varphi_1}{(\varphi_1 + T_1)^2} e^{-rs_2} ds_2 \tag{36}
$$

Equations (33) and (35) show the temporal effect of waiting another year to harvest each stand on the damage cost from the respective stand. The last term in each equation is the *increase* in damage cost by waiting another year (although some of it can be mitigated by having a mature stand), and the rest shows the *decrease* in damage cost by postponing the harvesting year's damage cost for another year. Importantly, equation (35) shows that the marginal damage on stand 2 can be reduced by stand 1, although the magnitude depends on the age of stand 1 at the time of stand 2's harvest. Age of stand 1 when stand 2 is harvested can be different each time stand 2 is harvested (see numerical example).

Equations (34) and (36) show the the spatial effect of waiting another year to harvest each stand. Equation (34) merely reflects the setting that stand 2 is spatial independent of stand 1; since stand 2 is upslope of stand 1, rotation on stand 2 cannot affect the amount of water runoff originating from stand 1. To the contrary, equation (36) reflects the setting that stand 1 is spatial substitute of stand 1: damage cost from stand 2 can be decreased by postponing harvesting for another year on stand 1. Furthermore, the degree of this decrease in damage cost depends on the particular age of stand 2 when stand 1 is harvested.

Using equations (33) through (36), (13) and (14), the first-order necessary conditions for T_1 and

T_2 are:

$$\begin{aligned}
\Omega_{T_1} =& p\left(-2aT_1+b\right)\left(e^{-rT_1}+e^{-r2T_1}\right) - rp(-aT_1^2+bT_1+d)e^{-rT_1} + rc_1e^{-rT_1} - 2rp(-aT_1^2+bT_1+d)e^{-r2T_1} \\
& - \left(-\frac{R_{1,t=T_1}T_1}{\varphi_1+T_1}\right)e^{-rT_1} - \left(R_{1,t=2T_1} - \frac{R_{1,t=2T_1}T_1}{\varphi_1+T_1}\right)e^{-r2T_1} \\
& + \int_0^{T_2} \frac{\left(R_{2,t} - \frac{R_{2,t}s_2}{\varphi_2+s_2}\right)\varphi_1}{\left(\varphi_1+T_1\right)^2} e^{-rs_2}ds_2 + \int_{T_2}^{2T_2} \frac{\left(R_{2,t} - \frac{R_{2,t}(s_2-T_2)}{\varphi_2+(s_2-T_2)}\right)\varphi_1}{\left(\varphi_1+T_1\right)^2} e^{-rs_2}ds_2 \\
& = 0 \\
& \Leftrightarrow \frac{p\left(-2aT_1+b\right) + \frac{R_{1,t=T_1}T_1}{\varphi_1+T_1} + \frac{E_{2,T_1}}{e^{-rT_1}+e^{-r2T_1}} - \frac{R_{1,t=2T_1}}{e^{rT_1}+1}}{p(-aT_1^2+bT_1+d)(1+2e^{-rT_1})-c_1} = \frac{r}{1+e^{-rT_1}}
\end{aligned} \tag{37}$$

where E_{2,T_1} is given by equation (36).

$$\begin{aligned}
\Omega_{T_2} =& p\left(-2aT_2+b\right)\left(e^{-rT_2}+e^{-r2T_2}\right) - rp(-aT_2^2+bT_2+d)e^{-rT_2} + rc_1e^{-rT_2} - 2rp(-aT_2^2+bT_2+d)e^{-r2T_2} \\
& - \left(-\frac{R_{2,t=T_2}T_2}{\varphi_2+T_2} - \frac{\left(R_{2,t=T_2} - \frac{R_{2,t=T_2}T_2}{\varphi_2+T_2}\right)s_1}{\varphi_1+s_1}\right)e^{-rT_2} \\
& - \left[\left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2+T_2} - \frac{\left(R_{2,t=2T_2} - \frac{R_{2,t=2T_2}T_2}{\varphi_2+T_2}\right)s_1}{\varphi_1+s_1}\right)e^{-r2T_2} - \left(-\frac{R_{2,t=2T_2}s_1}{\varphi_1+s_1}\right)e^{-rT_2}\right] \\
& = 0 \\
& \Leftrightarrow \frac{p(-2a+b) - \frac{E_{2,T_2}}{e^{-rT_2+-r2T_2}}}{p(-aT_2^2+bT_2+d)(1+2e^{-rT_2})-c_1} = \frac{r}{1+e^{-rT_2}}
\end{aligned} \tag{38}$$

where E_{2,T_2} is given by equation (35).

Two points are worth noting here. First, the asymmetry in the first order condition again reflects the setting that since stand 1 is downhill of stand 2, water runoff from stand 2 reaching the water way can be reduced by extending the rotation on stand 1 but not vice versa. Second, despite the unidirectional relationship, the stand age of both stands are in the first order conditions for both T_1 and T_2.

Comparative Statics when Rotation Age of the Other Stand is Exogenous

In this section I examine the comparative statics to examine the marginal value of staggering. To do so, suppose the forest planner only owns one of the stands and the decision of the rotation of the other stand is exogenous. When the forest manager owns only stand 2, the upslope stand, then the rotation of stand 1 is exogenous. We examine how stand 1's age affects stand 2's optimal rotation.

Proposition 1 *When the downslope stand's rotation is longer, the optimal rotation on the upslope stand is shorter.*

From Implicit Function Theorem,

$$\frac{\partial T_2^*}{\partial T_1} = -\frac{\Omega_{T_2T_1}}{\Omega_{T_2T_2}} \tag{39}$$

$\Omega_{T_2T_2}$ is negative by the second-order condition. We can show that

$$\begin{aligned}
\Omega_{T_2T_1} &= -E_{2,T_2T_1}(T_1, T_2) \\
&= -\left(-A_{12,T_1}(e^{-rT_2} + e^{-2rT_2}) + A_{11,T_1}(e^{-rT_2})\right) \\
&= -\left(\left(-\frac{\left(R_2 - \frac{R_2}{\varphi_2+T_2}\right)\varphi_1}{(\varphi_1 + T_1)^2}\right)(e^{-rT_2} + e^{-2rT_2}) + \frac{R_2\varphi_1}{(\varphi_1 + T_1)^2}(e^{-rT_2})\right) < 0
\end{aligned} \tag{40}$$

iff $\left(R_2 - \frac{R_2}{\varphi_2+T_2}\right)\varphi_1(e^{-rT_2} + e^{-2rT_2}) < R_2\varphi_1(e^{-rT_2})$ holds. We can show that this is equivalent to:

$$e^{-rT_2} + e^{-2rT_2} > e^{-2rT_2}(\varphi_2 + T_2) \tag{41}$$

To show that this holds, we increase the number of rotations to infinity. Then equation (41) becomes

$$\frac{e^{-rT_2}}{1 - e^{-rT_2}} > e^{-\infty rT_2}(\varphi_2 + T_2) = 0 \tag{42}$$

Therefore, $\frac{\partial T_2^*}{\partial T_1} < 0$ as the number of rotations reach infinity.

Conversely, suppose the forest manager owns only stand 1, the downslope stand, and the rotation of stand 2 iss exogenous.

Proposition 2 *When the upslope stand's rotation is longer, the optimal rotation on the downslope stand is shorter.*

From Implicit Function Theorem,

$$\frac{\partial T_1^*}{\partial T_2} = -\frac{\Omega_{T_1T_2}}{\Omega_{T_1T_1}} \tag{43}$$

$\Omega_{T_1T_1}$ is negative by the second-order condition. We can show that

$$\begin{aligned}\Omega_{T_1T_2} &= -E_{2,T_1T_2}(T_1, s_2) \\ &= -\left(-\int_0^{T_2} \frac{\partial A_{12}}{\partial T_1 \partial T_2} e^{-rs_2} ds_2 - \int_{T_2}^{2T_2} \frac{\partial A_{12}}{\partial T_1 \partial T_2} e^{-rs_2} ds_2\right) \\ &= -\int_0^{T_2} \frac{R_2}{(\varphi_2 + T_2)^2 (\varphi_1 + T_1)^2} e^{-rs_2} ds_2 - \int_{T_2}^{2T_2} \frac{R_2}{(\varphi_2 + T_2)^2 (\varphi_1 + T_1)^2} e^{-rs_2} ds_2 < 0 \end{aligned} \tag{44}$$

from equation (26). Therefore, $\frac{\partial T_1^*}{\partial T_2} < 0$.

In sum, in this section I extended the single-stand model to a spatial model of two stands, where the age of one stand affects the cost of the other stand, but asymmetrically. Harvesting behavior on one stand affects the cost on its own stand and also directly but asymmetrically affects its neighboring stands depending on their relative location on a hillslope. The two stands are assumed to be located on a single hillslope, with stand 1 adjacent to a waterway and stand 2 adjacent to and above stand 1. A fraction of rainfall on stand 2 gets retained by both stand 1 and 2, whereas rainfall on stand 1 only gets retained by stand 1. In other words, stand 1 provides external retention service for rainfall that falls on stand 2, but not vice versa. Because of this asymmetry, the costs of harvesting on each stand is asymmetric for a given stand age. I also illustrate a case where the forest planner only needs to consider the timber benefits of a stand 2 (i.e., ignore the downstream externality cost) if the water from stand 2 is fully retained by stand 1, the downslope stand.

By examining the case when a forest owner owns only one of the stands and takes the rotation age of the other stand as exogenous, we showed that the optimal rotation on each stand becomes longer (shorter) as the other stand is harvested (becomes older). Equations (40) and (44) demonstrate the *marginal value of staggering* the rotation period.

This model is distinct from previous studies in two ways. This model follows the rotation model as in Hartman instead of analyzing in a dynamic programming framework as in Swallow and Wear (1993); Swallow et al. (1997), which clarifies the ways in which rotations on each stand affects the production of non-timber good on each other's stand. Swallow and Wear (1993); Swallow et al. (1997) also numerically illustrate the case when the production of the non-timber good of two stands are asymmetric, but in their case the age of one stand does not directly affect the production of the non-timber good on the other stand. Rather, their application assumes that the price of the non-timber good is endogenous and therefore production quantity affects the prices. To the contrary, in this model the production of the non-timber good, which is the damage cost by harvesting a stand, is directly affected from a harvesting decision on another stand.

4 Numerical Simulation

I apply the theoretical model to a two-stand system using parameters from the literature.[7] The timber growth function for each stand ($n = 1, 2$)is a logistic growth function given by

$$V_n = \frac{K_n}{(1 + e^{\alpha_n - \theta_n T_n})} \tag{45}$$

where V_n is volume (thousands of board feet) per acre on stand n of age T_n. Table 2 lists base level parameter values. The timber growth function for alternative θ values are depicted in Figure 1. Using the given equations and the base parameters, the timber model reaches the harvest age that maximizes timber benefits alone (the Faustmann age) at age 33.

The water runoff damage functions for stand 1 and 2 are specfied as follows:

$$D_1 = R_1 - \frac{R_1 s_1}{\varphi_1 + s_1} \tag{46}$$

$$D_2 = R_2 - \frac{R_2 s_2}{\varphi_2 + s_2} - \frac{\left(R_2 - \frac{R_2 s_2}{\varphi_2 + s_2}\right) s_1}{\varphi_1 + s_1} \tag{47}$$

Table 2 lists the base parameter values for rainfall on each stand (R_n) and a parameter in water absorption function (φ_n). The water runoff damage function is sensitive to values of φ_n, as illustrated for stand 1 (D_1) in Figure 2. In addition, note that in equation (47) the age of stand 1 (s_1) varies in cycle of stand 2. When stand 1's age is young, it does not absorb as much water runoff from stand 2 as when it is older. Therefore, then younger the age on stand 1, the more the damage cost from stand 2 is (Figure 3).

[7] In future work, the parameters are to be replaced by parameters from Sichuan.

Simulation Scenarios

Using the specific functional forms, I conduct simulations for four scenarios. The base case simulates two stands with identical water runoff function (both stands using equation (46)). The base case implies that the two stands are in an additive system and thus is an aspatial model: the total runoff is simply the sum of water runoff generated from each stand. Next, to assess the implications of the subadditive, spatial model, we develop four case scenarios using the model of two stands with asymmetric water runoff function, i.e., using equation (46) for stand 1 and using equation (47) for stand 2. Case 1 uses the base parameters. Case 2 illustrates a case where there is more runoff from stand 2. In general, upslope stands can have higher slopes and thus harvesting them may result in more runoff than otherwise. Case 3 demonstrates the case with different harvesting costs: harvesting cost on stand 2 is higher than stand 1. To implement Case 3 we let the net price for timber of stand 1 to be twice the net price of stand 1. This case is relevant because harvesting cost is presumably lower in the downhill of a hillslope which introduces an additional tradeoff to the problem.

We perform two sets of these four scenarios. The first set examines the steady state by only allowing a unique rotation period for each stand, assuming that the system reaches the steady state from the first rotation, i.e., forest manager repeats the same rotation infinitely. The second set examines how the system reaches the steady state by allowing two choices per stand: we allow the first rotation to be different from the second rotation and on, assuming that the system reaches its steady state at the second rotation. By steady state we mean that the same rotation (or a combination of rotations) is repeated infinitely.

Next, we change the initial age of both stands to examine how sensitive the optimal rotation on each stand is to intial conditions. Here the model is set up so that if the initial age exceeds a rotation year candidate, the stand is forced to be harvested immediately and then start that particular rotation infinitely. Examining alternative initial conditions is relevant in the case of China because by 2010 the forested area under the logging ban will start from a non-bare land. This exercise is conducted under the assumption that the system reaches the steady state from the second rotation.

In all of the simulations, the timber revenue and damage cost are calculated over a time horizon of 1000 years. The optimal solutions are found through simple grid search over a set of possible combinations of rotation on the two stands. A unique solution (i.e., combination of optimal rotation on each stand) was found for all of the results.

Simulation Results

The simulations illustrate the efficiency gains of a spatial model(Table 3). We found that whether or not to model a forest system as an additive or a non-additive system makes a difference on the optimal rotation of the two stands. When the system is modeled as an additive system, the negative externality from harvesting each stand is symmetric, and thus the optimal timber rotation is the same ($T_1 = T_2 = 37$). The optimal rotations are both two years longer than the Faustmann solution due to the additional damage cost in each point in time which is a function of the stand age. When the system is modeled as a sub-additive system, where stand 1 has the capacity to absorb a part of the runoff from stand 2, the optimal timber rotation is longer for stand 1 ($T_1 = 39$) and shorter for stand 2 ($T_2 = 33$) (Case 1). Compared to the additive model, the subadditive model resulted in a slightly lower timber profit (-0.2%) but nearly 30 percent less damage cost, resulting in a three times higher total net present value.

Sensitivity analyses with respect to alternative parameter values illustrate that the optimal rotation changes consistently with the theoretical model (Tables 3). Interestingly, when stand 2 (the upslope stand) has more runoff, it is optimal to have a longer rotation on stand 1 and maintain the Faustmann rotation on stand 2, contrary to an expectation of an aspatial model. When stand 1 has a higher net price (Case 3), it is optimal to have a shorter rotation on stand 1 (37 years) and maintain the same rotation on stand 2, resulting in twice as high timber profit but only 2 percent higher damage cost.

When the model is allowed more flexibility in its rotation choices for each rotation, the optimal rotations show a different pattern (Table 4). In this table, the first number is the optimal rotation for the first rotation and the second number indicates the optimal rotation for the second rotation and all other rotations in the rest of the time horizon. The Faustmann rotation and the optimal rotation for the additive system is identical in both models, which means the system goes into a steady state from the first rotation. However, when the model assumes that the system reaches the steady state from the second cycle resulted in different results for the sub-additive system. With the base parameters, the optimal rotation for the first rotation was 39 years and for the second rotation and on was 38 years for stand 1; for stand 2, these were 32 and 34, respectively. They are slightly different compared to the results from Table 3 (39 years and 33 years for stand 1 and 2, respectively), which suggests that it is optimal not to go quickly into the steady state.

Simulation results suggest that the optimal rotations are sensitive to the initial conditions, i.e., the initial age of each stand (Table 5). A general rule that comes out of the simulations is that when stand 1 starts out with some forests but of an age less than the optimal rotation starting from bare land and stand 2 starts with a bare land, the optimal rotation on stand 1 is shorter and the optimal rotation on stand 2 is longer than the case starting with bare land (Table 5, rows 1-3). To the contrary, when stand 1 starts out with an age beyond the optimal rotation under the assumption of bare land, it is optimal to harvest the stand after a few years (rows 4-5). When stand 2 starts with some forests and stand 1 starts out as bare land, then it is optimal to harvest stand 2 earlier than the Faustmann at the expense of higher damage cost (rows 6-8). When both stands start out as a non-bare land, stand 2 should be harvested right away but maintain more or less the same rotation age as the base case on stand 1 (rows 13-15).

The simulations conducted in this paper only allows up to two rotation choices per stand and we do not yet have the solution to the full-blown problem where the number of rotation each stand takes to reach the steady state is endogenous. We suspect that the solution to the full blown problem is akin to a Tahovenen-like problem in the sense that because of the nonlinearity in the dynamic profit function the optimal rotations are sensitive to intial conditions and may take a long time to reach the steady state cycle.

5 Conclusion and Future Extensions

Motivated by the logging ban in China and its future deregulation strategy, this paper theoretically examined the dynamic problem of forest management with spatial externality. I constructed a theoretical, spatially-explicit model of a forest planner who maximizes timber profits from infinite timber rotation on all stands minus the costs of water runoff, and more importantly, asymmetric depending on the relative location of the stand. The model examined a spatial model of two stands, where the age of one stand affects the cost of the other stand, but asymmetrically. Using specific functional forms, I examined the properties of spatial and temporal substitutability between the two stands. I also examined the first order conditions to tease out the marginal value of staggering harvesting on one stand.

The simulation results illustrated the efficiency gains of a spatial, subadditive model versus an aspatial, additive model. When examining alternative parameter values, the simulations indicated that the optimal rotation changes consistently with the theory to differences in production costs or the parameters that determine the extent of runoff. They also suggest that when the initial stand age exceeds the optimal rotation starting with a bare land, it is optimal to harvest the existing stand immediately and start the same rotation as the ones optimal under bare land. Our simulation results, however, may be driven by the specific functional forms for timber growth and runoff-stand age relationship. Future work needs to investigate how the results change with other functional forms and parameters.

Despite these caveats, the analytical model and the simulation results have critical implications in devising a deregulation strategy of the logging ban. First, there could be efficiency gains by devising a deregulation strategy based on a spatial, subadditive model compared to an aspatial, additive model. It is important to note, however, that a forest can be located in a setting where there could be no gains from a spatial model. These include situations where the forest-hydrology system is always be in an additive system, such as areas that only have little rain or those that have heavy, short duration rainfalls such as in the tropics. In such areas the downslope forest stands provide litte benefit in terms of absorbing water runoff from upslope forest stands. Finally, it is important to note that a given watershed system can flip between an additive and a subadditive system. This implies that when we empiricize this type of a forest-hydrology bioeconomic model we may need a spatially-explicit hydrological model that can capture the relationship between forest and hydrology over time and space.

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Table 1: Illustration of addtive and non-additive properties in a two-stand forest system

	Stand 2	Stand 1	Total water runoff entering a river system
Case 1	$R_2 \leq A_2$	$R_1 \leq A_1$	None
Case 2	$R_2 \leq A_2$	$R_1 \succ A_1$	$R_1 - A_1$
Case 3	$R_2 \succ A_2$	$R_1 = A_1$	$R_2 - A_2$
Case 4	$R_2 \succ A_2$	$R_1 \succ A_1$	$R_2 - A_2 + R_1 - A_1$
Case 5	$R_2 \succ A_2$	$R_1 \prec A_1$	$R_2 - A_{22} - A_{12} - R_1 - A_{11}$

Table 2: Base Value of Parameters Used in the Simulation

Parameter	Base Value
Net Price (per thousand board feet)	p=600
Discount Rate	r=0.04
Parameters in Growth Function	
Carrying capacity (thousand board feet per acre)	K=10
Other parameters	ω=3.5; θ=0.099;
Rainfall on stand 1	R_1=10
Rainfall on stand 2	R_2=10
Parameter in stand 1's absorption capacity	$\varphi_1 = 5$
Parameter in stand 2's absorption capacity	$\varphi_2 = 5$

Table 3: Simulation Results: Steady State Cycle Starting from First Rotation

Scenario	Optimal Rotation		Timber Profit	Damage Cost	Total NPV
	Stand 1	Stand 2			
Faustmann (Timber Revenue only)	33	33	196.9	N.A.	194.46
Base case: Additive System	37	37	194.46	-169.11	25.35
Scenarios for Sub-additive system					
Case 1: Base Parameters	39	33	194.02	-119.38	74.64
Case 2: More Runoff from Stand 2 ($\varphi_2 > \varphi_1$) [a]	40	33	192.93	-152.65	40.28
Case 3: Higher Net Price for Stand 1 ($p_1 > p_2$)[b]	37	33	391.36	-121.52	269.85

[a]The parameters used for this scenario were $\varphi_2 = 50$ and $\varphi_1 = 5$.
[b]The parameters used for this scenario were $p_1 = 120$ and $p_2 = 60$.

Table 4: Simulation Results: First Rotation as Free Choice and then Steady State from Second Rotation

Scenario	Optimal Rotation[a]		Timber Profit	Damage Cost	Total NPV
	Stand 1	Stand 2			
Faustmann (Timber Revenue only)	$33/\overline{33}$	$33/\overline{33}$	196.9	N.A.	196.9
Base case: Additive System	$37/\overline{37}$	$37/\overline{37}$	194.46	-169.11	25.35
Scenarios for Sub-additive system					
Case 1: Base Parameters	$39/\overline{38}$	$32/\overline{34}$	194.11	-119.43	74.67
Case 2: More Runoff from Stand 2 ($\varphi_2 > \varphi_1$) [b]	$40/\overline{39}$	$33/\overline{34}$	193.14	-152.84	40.29
Case 3: Higher Net Price for Stand 1 ($p_1 > p_2$)[c]	$37/\overline{36}$	$32/\overline{33}$	293.08	-121.57	171.51

[a]The number with overline is the optimal rotation at the steady state cycle.

[b]The parameters used for this scenario were $\varphi_2 = 50$ and $\varphi_1 = 5$.

[c]The parameters used for this scenario were $p_1 = 120$ and $p_2 = 60$.

Table 5: Simulation Results for Alternative Initial Conditions

Initial Age of		Optimal Rotation[a]		Timber Profit	Damage Cost	Total NPV
Stand 1	Stand 2	Stand 1	Stand 2	(NPV)	(NPV)	
10	0	$36/\overline{37}$	$36/\overline{36}$	205.45	-85.50	119.95
20	0	$36/\overline{37}$	$35/\overline{35}$	222.08	-80.88	141.20
30	0	$37/\overline{38}$	$34/\overline{34}$	243.57	-91.26	152.30
40	0	$42/\overline{40}$	$33/\overline{34}$	256.95	-103.63	152.32
50	0	$63/\overline{34}$	$34/\overline{34}$	158.34	-74.09	84.25
0	10	$38/\overline{37}$	$30/\overline{35}$	207.26	-106.45	100.81
0	20	$37/\overline{36}$	$27/\overline{36}$	231.55	-108.57	122.97
0	30	$39/\overline{38}$	$30/\overline{33}$	258.57	-116.71	141.87
0	40	$36/\overline{37}$	$40/\overline{40}$	263.84	-117.53	146.31
0	50	$39/\overline{38}$	$51/\overline{33}$	184.95	-113.77	71.18
0	100	$38/\overline{37}$	$119/\overline{34}$	104.70	-94.92	9.78
0	0	$39/\overline{38}$	$32/\overline{34}$	194.11	-119.43	74.67
20	20	$36/\overline{38}$	$20/\overline{35}$	267.03	-80.15	186.88
30	30	$38/\overline{38}$	$30/\overline{34}$	305.47	-88.91	216.56
40	40	$44/\overline{40}$	$40/\overline{40}$	316.91	-197.29	219.62

[a] The number with overline is the optimal rotation at the steady state cycle.

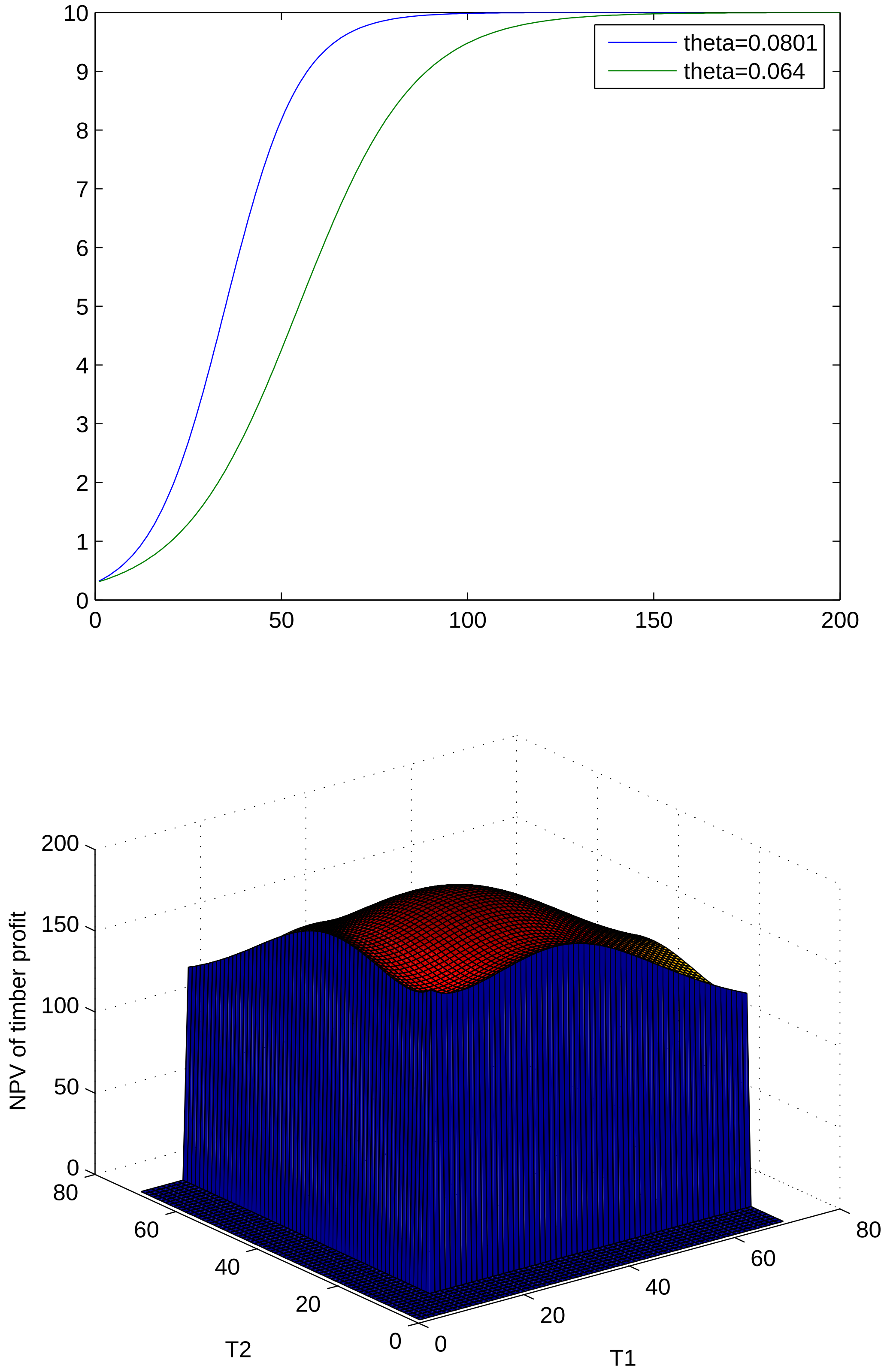

Figure 1: Timber Growth Function for Alternative Parameters and 3D Representation of Present Value of Timber Revenue from Two Stands

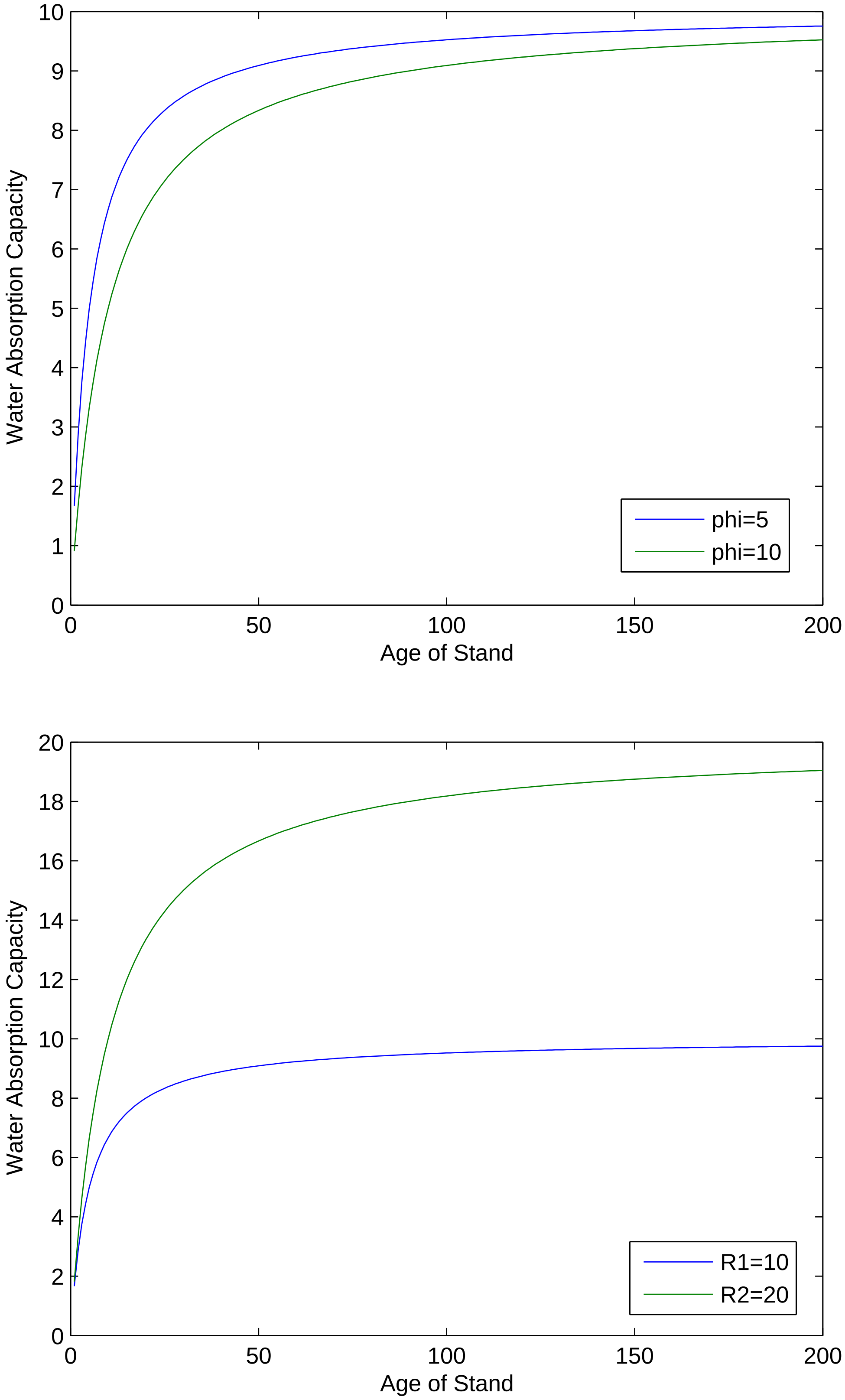

Figure 2: Water Absorption Function for Alternative Parameters

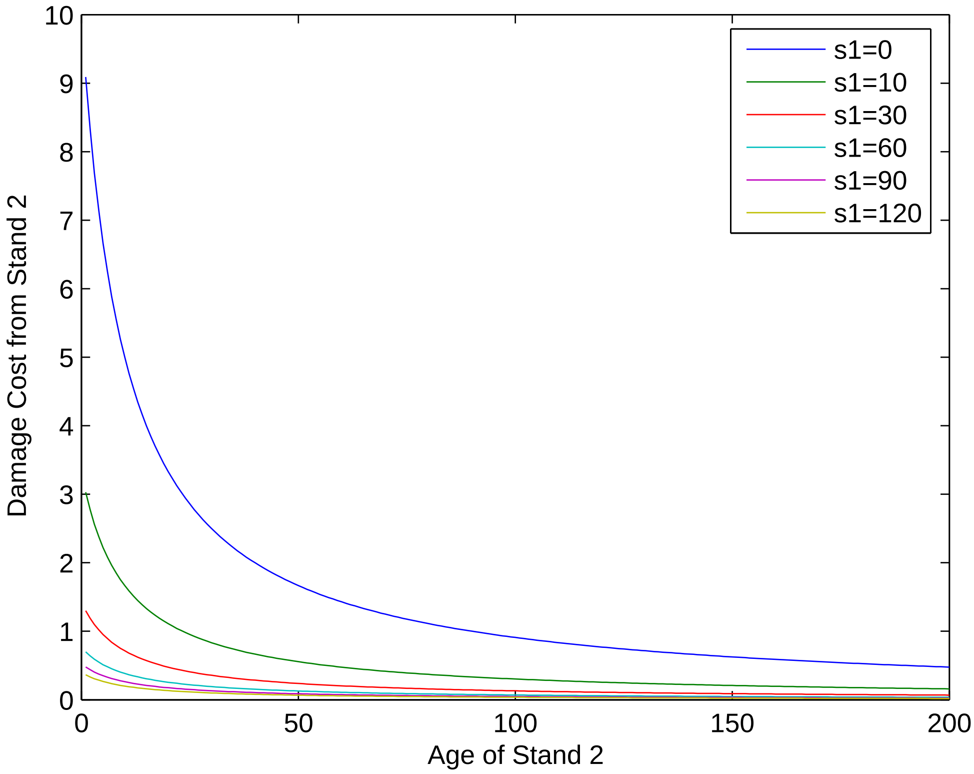

Figure 3: Damage Cost from Stand 2 for Alternative Ages on Stand 1

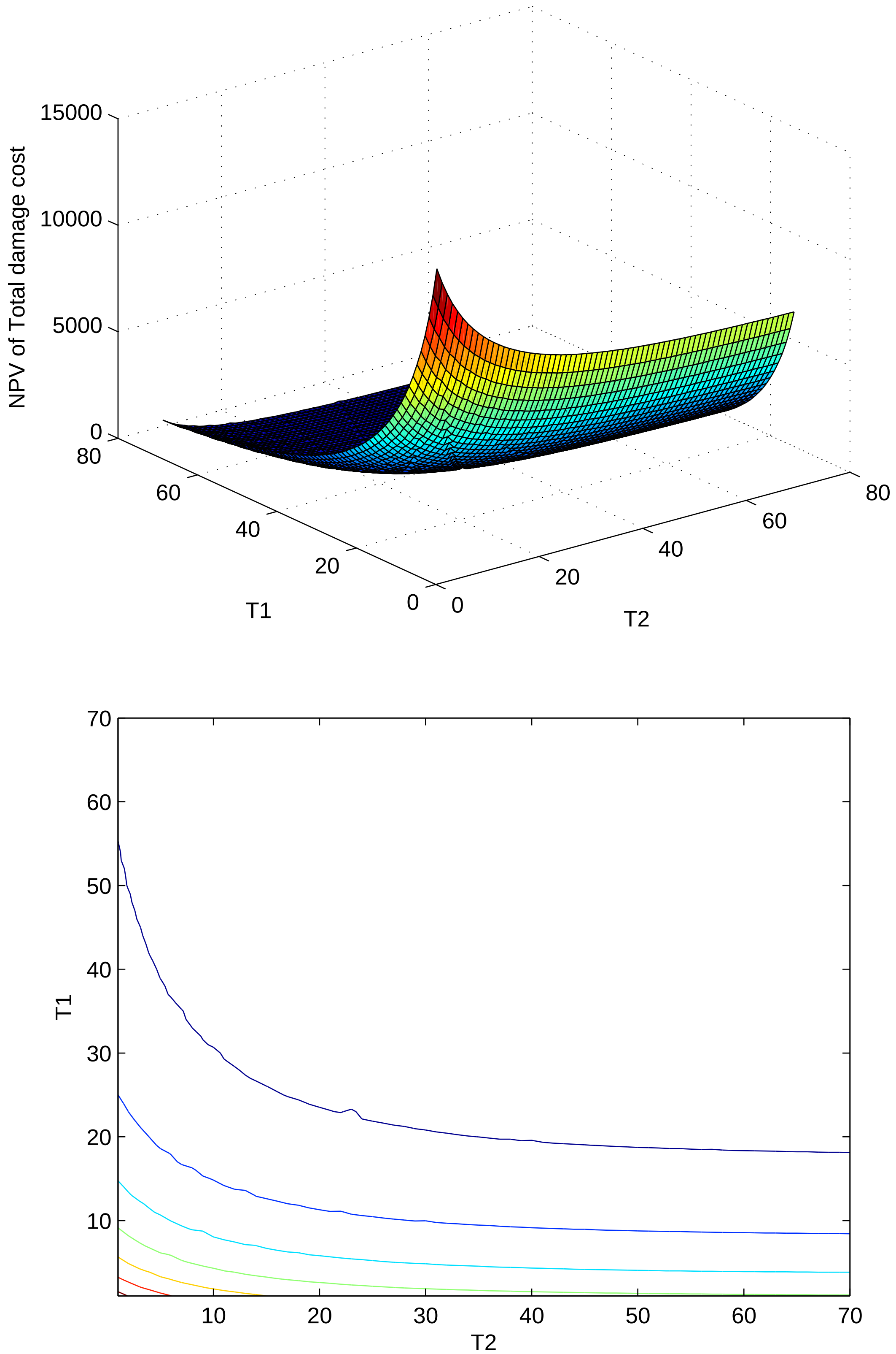

Figure 4: Present Value of Total Damage Cost From Two Stands, 3D and Contour Plots

Appendix A Optimal Timber Rotation on Single Stand with Externality

Consider a model of a public forest planner who owns a single, even-aged stand with infinite repeated rotation of the same length. I modify Hartman's timber rotation model with joint non-timber production. Instead of non-timber benefits like in models by Hartman and others, I introduce an external cost of flood damage as a function of stand age. The planner is assumed to choose the optimal harvesting time so as to maximize the present value of sum of net timber revenue and external damage cost over an infinite series of rotations:

$$\max_{T} \; PV \overset{def}{=} \frac{1}{1-e^{-rT}} \left[pQ(T)e^{-rT} - c - \int_0^T D(R-A(s))e^{-rs}ds \right] \tag{48}$$

where p is the timber price per volume that is fixed throughout the time horizon, $Q(T)$ is timber growth function where I assume $Q'(T) > 0$ and $Q''(T) < 0$ in the relevant range, and c is a fixed cost for replanting. In the damage function, $D(\cdot)$ is the damage cost associated with water runoff where R is the rainfall in each period, and $A(s)$ is the absorption of water by the stand as a function of the stand age s where I assume $A'(S) > 0$ and $A''(S) < 0$, and also $R \geq A(s)$. The first-order necessary condition, if it exists, is:

$$pQ'(T) - D(R-A(s)) \overset{set}{=} r\left(pQ(T) - c\right) + r\left[\frac{1}{1-e^{-rT}}\left(pQ(T)e^{-rT} - c - \int_0^T D(R-A(s))e^{-rs}ds\right)\right]$$

$$\Leftrightarrow pQ^{'}(T) - D(R-A(s)) \overset{set}{=} r\left(pQ(T) - c\right) + rPV \tag{49}$$

The first-order condition is analogous to Hartman's result. At the optimal rotation, the forest planner balances the net marginal benefit of postponing another year (marginal timber benefit minus the damage cost in the rotation year) with marginal cost of postponing rotation (forgone interest by extending additional year plus the site value.)

The second-order necessary conditions is:

$$pQ''(T) - D'(R-A(T))(-A'(T)) - rpQ'(T) < 0 \tag{50}$$

The first term is negative in the range of year where trees are mature enough to harvest; the second term is negative for any T from the assumption of $Q'(T) > 0$. The second term is positive because of the assumptions $A'(T) > 0$ and $\frac{\partial D}{\partial(R-A(T))} > 0$, but the term is likely to be small around $t = T^*$ if we assume a damage cost function that declines over time at a decliing rate, i.e., $D'(T) < 0$ and $D'' > 0$.

Whether or not the optimal rotation in this problem (T^*) is shorter or longer compared to the Faustmann's rotation (T^F) depends on the shape of the cost function. To compare the first order condition in equation (49)with Faustmann's formula, I simplify the equation and rewrite:

$$\frac{pQ'(T) - D(T)}{pQ(T) - c} + \frac{1}{pQ(T) - c}\frac{1}{1-e^{-rT}}\int_0^T D(R-A(s))e^{-rs}ds \overset{set}{=} \frac{r}{1-e^{-rT}} \tag{51}$$

Recall that the Faustmann's formula can be expressed as (Clark p.273):

$$\frac{pQ'(T) - D(T)}{pQ(T) - c} \overset{set}{=} \frac{r}{1-e^{-rT}} \tag{52}$$

Compared to equation (52), equation (51) has two extra terms on the left hand side. The first term is:

$$\frac{-D(T)}{pQ(T)-c} \tag{53}$$

which can be interpreted as the damage cost in the harvesting year $t = T^*$ relative to the stumpage value. The second term is:

$$\frac{1}{pQ(T)-c}\frac{1}{1-e^{-rT}}\int_0^T D(R-A(s))e^{-rs}ds \tag{54}$$

which can be interpreted as the present value of the damage cost during the infinite rotations relative to the stumpage value. Whther or not the optimal rotation in this problem is longer or shorter than the Faustmann's solution depends on the relative size of these two terms. The optimal rotation of this problem will be longer than Faustmann's solution iff:

$$\begin{aligned}&\frac{D(T)}{pQ(T)-c} < \frac{1}{pQ(T)-c}\frac{1}{1-e^{-rT}}\int_0^T D(R-A(s))e^{-rs}ds\\ &\Leftrightarrow D(T) < \frac{1}{e^{rT}-1}\int_0^T D(R-A(s))ds\end{aligned} \tag{55}$$

Note that using l'Hopital's rule we can show that as $r \rightarrow 0$,

$$\lim_{r\rightarrow 0} \frac{1}{e^{rT}-1}\int_0^T D(R-A(s))ds = \frac{1}{T}\int_0^T D(R-A(s))ds \tag{56}$$

Therefore, when r=0, the condition in equation (55) becomes

$$D(T) < \frac{1}{T}\int_0^T D(R-A(s))ds \tag{57}$$

The condition (57) means that as long as the damage cost in year $t = T^*$ is less than the average damage cost from $t = 0$ to $t = T^*$, then the optimal rotation is longer than the Faustmann's rotation. Intuitively, this means that if the damage cost is decreasing with respect to stand age, then the optimal rotation could be longer than the Faustmann's rotation. Conversely, if the damage cost is increasing with respect to stand age, then the optimal rotation could be shorter than the Faustmann's rotation. If the damage cost is fixed regardless of the stand age, the optimal rotation is the same as the Faustmann's rotation. The damage cost due to harvesting a forest stand is considered to be the highest when the stand is just harvested and decline over time as the vegetation cover recovers.

Comparative Statics

Assuming that the second order necessary condition holds, we can unequivocally show that:

$$\frac{\partial T}{\partial p} < 0, \frac{\partial T}{\partial r} < 0, \frac{\partial T}{\partial c} > 0 \tag{58}$$

which are all expected and consistent with Faustmann and Hartman models. In addition, we can show that as long as the damage cost function is monotonically decreasing over time,

$$\frac{\partial T}{\partial R} > 0. \tag{59}$$

Finally, if there are other stand-specific factors Z than timber rotation that affect absorptive capacity, i.e., $A(T;Z)$, we can show that:

$$\frac{\partial T}{\partial Z} \geq 0 \tag{60}$$

if $R \geq A$.