# Measuring the Gains to Groundwater Management with Recursive Utility

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**Summary:**

The current literature on non-cooperative and strategic groundwater extraction assumes user behavior that conforms to the highly stylized assumption of time-additive separability of the individual's objective criterion. This paper examines how the measured gains to management changes when this assumption is relaxed in favor of a recursive utility specification that takes path-dependency into account in modeling the behavior under both the non-cooperative and central management regimes. Application of this framework to the empirical case of Kern County, California shows that the difference in measured management gains is significantly larger than that which is measured under the assumption of time-additive separability. The paper also shows how the traditional method of calculating the benefits over time must be modified in order to properly account for these management gains.

# 1. Overview

## 1.1 Introduction

The Natural Resource Economics literature on Common Property externalities in groundwater extraction is both rich in theoretical and empirical treatments, beginning with the article of Gisser and Sanchez (1980) which examined the loss of efficiency that occurs when a groundwater aquifer moves from a sole-owner extraction regime to one in which there is competition in pumping. Various other authors have addressed the efficiency problems that arise under competitive in groundwater pumping (Allen and Gisser, 1984; Feinerman and Knapp, 1983; Kim *et al.*, 1989), but Negri's 1989 article placed the problem squarely in the realm of applied differential game theory. Other natural resource problems, such as that of fishing (Levhari and Mirman, 1980; Cave, 1987; Fisher and Mirman, 1992) have been characterized within the context of dynamic and differential games, however Negri's article was the first to characterize groundwater problems as such and to describe the *strategic* externality that arises from the dynamic gaming of the competing agents. Dixon (1991) studied the possibility for trigger-strategies giving rise to an equilibrium in groundwater extraction, while Provencher and Burt (1993) re-cast Negri's problem within a dynamic programming context, and went further to describe how risk might introduce yet another type of externality into the problem.

While these authors have examined the losses that arise from non-cooperative groundwater extraction in some detail, they all maintain a rather severe assumption on inter-temporal behavior that is widely maintained in economic models of dynamic behavior – namely, that of time-additive separability of the objective criterion. Following the original suggestion of Koopmans (1960), recent contributions to the literature have begun to examine the implications of relaxing this rather severe behavioral restriction on the inter-temporal felicity function, to allow for more generalized inter-temporal specifications, such as the iso-elastic intertemporal utility aggregator suggested by Epstein and Zin (1989, 1991). Among the few papers that have examined the implications of introducing recursive utility specifications in the context of natural resource management are those of Knapp and Olson (1996) and Howitt *et al.* (2005). However, neither of these papers go beyond the single agent case and deal with the issue of non-cooperative or strategic resource extraction behavior.

In this paper, we use the well-studied example of Kern County, California, to characterize the strategic interaction between two players pumping from the same aquifer. Several authors (Feinerman and Knapp, 1983; Dixon, 1991) have used this empirical example to illustrate the gains to groundwater management, and conjunctive use management (Knapp and Olson, 1996), while maintaining the assumption of time-additive separability in the objective criterion. By comparing the non-cooperative outcome to the central planner's solution, with both recursive and non-recursive preferences, we demonstrate the extent to which the standard assumption of time-additive separability can affect the measured gains to resource usage coordination and management. Further policy insight into groundwater management might also be gained, once we observe how the potential gains to management change with the relaxation of this behavioral assumption.

The rest of this paper is designed as follows. Following a brief description of the studies that have been done on groundwater management in Kern County, and a summary of their results, we present the dynamic game model that illustrates the strategic behavior between economic agents pumping from the same aquifer. The dynamic game will then be presented within in a recursive utility formulation, in order to illustrate the particular features of interest for this study. The section which follows compares the gains to groundwater management under both time-additive separability as well as under recursive preferences, using the central planner's problem, as a benchmark of efficiency, under its corresponding formulations. This section is followed by a discussion of the results and their policy implications for the management of groundwater in Kern County, and a final section concludes the paper.

## 1.2 The Literature on Groundwater Management in Kern County, California

In the economic literature on water resources management, Kern County, California, has provided fertile ground for cultivating theoretical and methodological ideas on the optimal management of surface and groundwater resources. While this paper focuses specifically on groundwater management, the literature that has examined water management in Kern County has also considered the optimal conjunctive use of groundwater resources with stochastic surface water supplies, which has broadened the range of policy options and management issues to consider.

Beginning with the paper of Feinerman and Knapp (1983), which forms the basis of this paper, Kern County was transformed into an experimental laboratory for the economic analysis of groundwater management, and for testing the magnitude of the gains to centralized optimal control of the underlying aquifer. In this paper, the authors examined the sensitivity of management gains to model specification, and also tested several alternative policy instruments to see how the welfare gains they generated compared to the gains realized under centralized management of the aquifer. In their study, they found that the realized gains of centralized management were less than ten percent, and that other types of economic instruments might achieve the same ends, albeit with varying consequences for welfare distribution. The conclusions of the paper echoed, to an extent, the earlier statements by Gisser and Sanchez

(1980), that the gains to centralized management may not be that large, especially when one takes the costs of management into account – which are rarely quantified in any economic analysis of natural resource management.

The next detailed analysis of Kern County was done by Knapp and Olson (1995), when they examined the conjunctive use of both surface and groundwater resources under uncertainty, in order to expand the palette of policy options available – such as artificial recharge. In this paper, the authors expanded on the work of Tsur and Graham-Tomasi (1991) and Provencher and Burt (1993) to examine how management options for conjunctive use change when one considers artificial recharge of the aquifer from a surface source which is stochastic in nature. In their study, they also found that the gains to groundwater management are somewhat small, and that they are affected by the degree of risk-neutrality of the decision-maker. In particular, they noted that the gains that are realized from groundwater management may be increased under risk aversion. This statement speaks directly to the impact that behavioral assumptions have on the measured gains to management, and to the importance of taking them into serious consideration, when attempting to conduct any empirical analysis of natural resource management under uncertainty.

The role of decision-maker preferences in dynamic economic was followed up further in the paper of Knapp and Olson (1996), which featured Kern County as a brief empirical example, among other examples of natural resource management. In this paper, the authors applied the recursive utility framework to a variety of natural resource management problems, to demonstrate the impact of relaxing the assumption of time-additive separability on the resulting decision-rules. While the sensitivity of the results to preference specification – both in terms of inter-temporal substitutability and risk aversion – were explored, no explicit calculations were made to show their effect on the realized gains to management. This is the issue that is addressed in this paper, within a deterministic setting, so as to isolate the effect of inter-temporal substitutability on management gains, and to show its sensitivity to this parameter more clearly.

Finally, the paper of Knapp *et al.* (2003) looked at Kern County within the context of out-of-basin water transfers, and evaluated the operation of a possible water market, and its impacts on groundwater resources within the basin. This analysis was done using a standard specification of a dynamic economic model, with time-additive separability built into the objective criterion, so as to give more focus to the policy questions, rather than to the methodological components of the analysis. While they did find centralized management helped to mitigate the negative impacts of out-of-basin transfers and surface water cutbacks, the overall effect on net annual benefits was still found to be small. Once again, the policy laboratory of Kern County offers little contradiction to the original assertion of Gisser and Sanchez (1980) that the gains to centralized management are small, especially when compared to the potential cost of implementing it.

With this background literature in mind, we now proceed to examine if the introduction of recursive preferences into the analysis of groundwater management will have a significant impact on the gains that can be realized from centralized control. In the next section, the strategic interactions between two players pumping, non-cooperatively, from the Kern County Aquifer will be described, in detail, so as to lay out the theoretical basis for our empirical investigation.

# 2. Dynamic Game of Groundwater Pumping in Kern County

## 2.1 Overview of Dynamic Game Model

In this section we present a simplified dynamic game model to represent the strategic interaction between N identical players who are pumping from the Kern County aquifer, in a non-cooperative fashion. In this model, recharge is treated as deterministic, and the decisions of each of the N players are made under certainty. The basic structure of the model follows the formulation of Feinerman and Knapp (1983).

The objective criterion of each of the N players is quadratic, with the marginal pumping costs dependent upon the depth to groundwater (h), according to the following equation

$$B^i(w_i) = a \cdot w_i - \tfrac{1}{2} b \cdot w_i - eh \cdot w_i \qquad (2.1.1)$$

where $a$ and $b$ are, respectively, the intercept and slope of the demand curve for water, and are identical for each player. The parameter '$e$' is also common to both players, and is the unit cost of energy used in pumping groundwater for every foot of lift from the groundwater table. The equation of motion for depth-to-groundwater $(h)$ is given by the expression

$$h^+ = h + \gamma(w_i + (N-1)w_{-i}) - \overline{r} \qquad (2.1.2)$$

where $h^+$ is the depth to groundwater in the next period, and which evolves from the current period according to the level of abstraction of the N players ($w$) (denoted by $i$ for the i$^{th}$ player and $-i$ for the other N-1 players) and recharge into the aquifer ($\overline{r}$). The notation in (2.1.2) is condensed, with $\gamma$ and $\overline{r}$ representing the translation of volumetric aquifer recharge and net groundwater withdrawal, into units of lift, according to the definitions

$$\gamma = \frac{(1-\theta)}{As} \quad \bar{r} = \frac{\left(1-\xi+\theta\xi\right)I+\hat{r}}{As} \tag{2.1.3}$$

In these expressions, $\theta$ represents the deep percolation into the aquifer, while A represents the areal extent of the aquifer, and $s$ is its specific yield. Recharge is given in terms of total inflow into the aquifer, $I$ , a base annual level of recharge $\hat{r}$ , and a calibrating parameter $\xi$ . The values of the model parameters are contained in Table 1, in the Appendix A of the paper.

Taken together, we can now write the structure of the dynamic game problem for player i, as follows

$$V^i\left(h\right) = \max_{w_i}\left\{a\cdot w_i - \tfrac{1}{2}b\cdot w_i - eh\cdot w_i + \beta V^i\left(h+\gamma(w_i+(N-1)w_{-i})-\bar{r}\right)\right\} \tag{2.1.4}$$

where $\beta$ is the common discount rate for all N players and $V\left(\bullet\right)$ is the maximized value of the dynamic game problem, for each player, beginning with the current level of groundwater lift ( $h$ ), and proceeding under the assumption that actions taken in subsequent periods are done optimally with respect to the groundwater lift in each period. This recursive relationship linking the implied optimality of behavior from period-to-period captures the essence of Bellman's "Principle of Optimality" (Bellman, 1957), and holds within the context of a strategic game played dynamically by two (or more) players.

The equilibrium concept used in this paper is based on the definition of a Markov strategy – also known as a closed-loop or feedback strategy – described by Dockner *et al.* (2000), in which the actions of each player depend on the past history of the 'game' only through the current value of the stock (Lockwood, 1996). As both Lockwood and Tsutsui and Mino (1990) note, the Markov equilibrium is the most interesting case to examine, and is the appropriate one to consider, when agents cannot pre-commit to a path of future actions (as in the 'open-loop' case). Clemhout and Wan (1991), also concur with the opinion that feedback strategies are more suitable for the analysis of common-property resource games, and as such, we do not consider any 'open-loop' equilibria in this paper, as other authors have done (Reinganum and Stokey, 1985; Negri, 1989; Rubio and Casino, 2002; Caputo and Lueck, 2003). The Markov equilibrium that is characterized in this paper is sub-game perfect, by construction, due to the fact that each player is solving a discrete-time dynamic programming problem which, by definition of the principle of optimality, ensures that each player's actions are optimal in each sub-period of the game, given the actions of the other player(s). Since the dynamic programming problem (and, hence, the resulting solution) of each player is independent of time, the optimal value functions and equilibrium policy functions will also be time-independent (autonomous) – which makes the resulting equilibrium strategy stationary, Markov and sub-game perfect.

The solution method that is used to obtain the stationary Markov equilibrium for this problem differs from the continuous-time approach commonly employed by authors in the literature who solve infinite-horizon differential game problems using the continuous-time optimal control approach (Mehlmann, 1988; Dockner *et al.*, 2000). We adopt a discrete-time dynamic programming approach similar to that of Levhari and Mirman (1980) and Eswaran and Lewis (1984), and adapted by Negri (1990) to the groundwater pumping problem, in order to obtain a recursive relationship that describes the evolution of the parameters of each player's carry-over value function for groundwater stock, towards its infinite-horizon value. This allow one to, numerically, obtain the exact solution to the problem, and verify that the derived infinite-horizon value function is consistent with the implied policy function that we obtain from the Euler (first-order) conditions of the dynamic problem.

For the sake of space, the full derivation of the infinite-horizon value function will not be shown, in this paper. The functional form, however is given as follows

$$V\left(h\right)=\boldsymbol{A}+\boldsymbol{B}\left(a-e\cdot h\right)+\boldsymbol{C}\left(a-e\cdot h\right)^2 \tag{2.1.5}$$

where the constants **A**, **B** and **C** are a function of all the parameters of the problem, and are found by computing a stationary value from a value-iteration process using the 'equation of motions' defined by the following system of equations

$$\begin{aligned}
\boldsymbol{A_{k+1}} &= \beta\boldsymbol{A_k}+\beta\boldsymbol{B_k}e\bar{r}-N\beta\boldsymbol{B_k}e\gamma\varphi_1+\beta\boldsymbol{C_k}\left(e\bar{r}\right)^2-\tfrac{1}{2}b\varphi_1^2-2N\beta\boldsymbol{C_k}e^2\bar{r}\gamma\varphi_1-\beta\boldsymbol{C_k}\varphi_1^2\left(e\gamma N\right)^2 \\
\boldsymbol{B_{k+1}} &= \varphi_1-b\varphi_1\varphi_2+\beta\boldsymbol{B_k}-N\beta\boldsymbol{B_k}e\gamma\varphi_2+2\beta\boldsymbol{C_k}e\bar{r}-2N\beta\boldsymbol{C_k}\varphi_1 e\gamma-2N\beta\boldsymbol{C_k}e^2\bar{r}\gamma\varphi_2 \\
&\quad +2\beta\boldsymbol{C_k}\varphi_1\varphi_2\left(e\gamma N\right)^2 \\
\boldsymbol{C_{k+1}} &= \varphi_2+\beta\boldsymbol{C_k}-\tfrac{1}{2}b\varphi_2^2-2N\beta\boldsymbol{C_k}e\gamma\varphi_2+\beta\boldsymbol{C_k}\left(e\gamma N\varphi_2\right)^2
\end{aligned} \tag{2.1.6}$$

where the constants $\varphi_1$ and $\varphi_2$ are also functions of the parameters, and defined as

$$\varphi_1=\frac{-\beta e\gamma\left[2\boldsymbol{C_k}e\bar{r}+\boldsymbol{B_k}\right]}{\left[b-2N\beta\boldsymbol{C_k}\left(e\gamma\right)^2\right]} \qquad \varphi_2=\frac{\left[1-2\beta e\gamma\boldsymbol{C_k}\right]}{\left[b-2N\beta\boldsymbol{C_k}\left(e\gamma\right)^2\right]} \tag{2.1.7}$$

at any stage, k, in the value iteration process. Numerically, this analytical expression can be approximated using orthogonal polynomial projection techniques (Judd, 1998), that are commonly employed in solving dynamic economic problems. Researchers have begun to rely more heavily on numerical methods to solving dynamic problems, as an alternative to adopting the more restrictive assumptions of the linear-quadratic formulation (Miranda and Fackler, 2002), and have applied them successfully to the solution of dynamic games, as well (Rui and Miranda, 1996).

This numerical approximation to the infinite-horizon carry-over value function takes the form $\hat{V}\left(x(h^+)\right)=\sum_{n=1}^{n}a_n\phi_n\left(x(h^+)\right)$, where $a_n$ is a coefficient which is fitted by iterative numerical algorithm, and $\phi_n(x)$ is a basis function for the orthogonal terms of the Chebychev polynomial we employ. Each basis function is defined over a domain x, which is restricted to the [-1,+1] interval, and onto which the state variable $h^+$ must be mapped. Further description of other types of orthogonal polynomials and of the numerical algorithm that we implement to find the polynomial coefficients are deferred to the more detailed discussion in Judd (1998), Miranda and Fackler (1999, 2002), and Howitt *et al.*(2002).

## 2.2 Dynamic Game in Groundwater with Recursive Preferences

The dynamic game model that we now present exhibits similar non-cooperative behavior between the players, but relaxes the restrictive assumption of time-additive separability in their respective objective criteria. Using the iso-elastic formulation of the recursive Kreps and Porteus preferences, following Epstein and Zin (1991), we can embed the quadratic form of the net benefit function used in (2.1.1) within a nested functional that also includes the carry-over benefits, according to the following expression

$$U^i(w_i) = \left[ (1-\beta)\left(B^i(w_i)\right)^{\alpha} + \beta\left(U^i_+(w_i)\right)^{\alpha} \right]^{\frac{1}{\alpha}} \quad (2.2.1)$$

where $U^i(\cdot)$, generically, represents the ‘felicity’ or benefit realized by the agent, as a function of the immediate net benefit in the current period, and the carry-over benefit $U^i(\cdot)$ realized in the next. By implication of the recursive nature of this formulation, the benefits in the next period also embed those realized in all subsequent periods, as well. The CES-formulation of this functional implies that there is substitution between the benefits realized in adjacent periods, and the parameter $\alpha$ determines the rate of substitution that occurs. $\beta$ remains the subjective discount factor of the agent, while $\alpha$ is defined as a constant of ‘resistance’ to inter-temporal substitution, which is less than one and non-zero. Using this parameter ($\alpha$), Epstein and Zin (1991) define the 'elasticity of inter-temporal substitution' as $\sigma = \frac{1}{1-\alpha} \in [0, +\infty)$, which gives it a more intuitive interpretation – a high value representing a reluctance to trade-off benefits between adjacent periods (i.e. a high level of 'resistance' to inter-temporal substitution).

Embedding this new formulation of the objective function within our groundwater extraction problem, we can now write the structure of the dynamic game problem for player i, as follows

$$V^i(h) = \max_{w_i} \left[ (1-\beta)\left(a \cdot w_i - \tfrac{1}{2} b \cdot w_i - eh \cdot w_i\right)^{\alpha} + \beta\left(V^i\left(h + \gamma(w_i + (N-1)w_{-i}) - \bar{r}\right)\right)^{\alpha} \right]^{\frac{1}{\alpha}} \quad (2.2.2)$$

The recursive relationship of optimal period-to-period behavior implied by Bellman's Principle of Optimality is maintained for each of the players in the strategic game, and the interpretation of the optimal carry-over value function $V^i(h)$ remains unchanged from the time-additive separable case. However, given the more complex nature of the objective functional, the analytic solution of the problem becomes even more challenging.

As before, a recursive system of equations can be derived and simulated to give the infinite-horizon value function. Omitting the details of their derivation, we present the final functional form of the infinite-horizon value function below

$$V(h) = (1-\beta)^{1/\alpha} \left[ \left(\mathbf{A} + \mathbf{B} \cdot h + \mathbf{C} \cdot h^2\right)^{\alpha} + \mathbf{D}\left(\mathbf{E} + \mathbf{F} \cdot h\right)^{2\alpha} \right]^{1/\alpha} \quad (2.2.3)$$

where the constants **A**, **B**, **C** and **D** are a function of all the parameters of the problem, and arise from a similar recursive scheme to that of (2.1.6). As before, we approximate it by a numerical scheme that has a polynomial representation similar to that used in the previous section, for the time-additive separable case.

Figure 1 shows the results that are created by simulating the central planner's model with recursive preferences (2.2.2) under a fixed discount rate of 5%. From this figure, we see that the inter-temporal substitution parameter ($\alpha$) has a pronounced effect on the extraction path, as seen from the levels of groundwater lift. In this figure, we can see that a more negative $\alpha$ value (which implies a lower elasticity of inter-temporal substitution) causes the extraction profile to 'flatten' out, such that the groundwater stock is not mined as heavily in the initial periods. The extraction path which takes place under time-additive separability (TAS) is the 'envelope' that defines the outer bound of behavior, and represents the least level of 'resistance' to inter-temporal substitution[1], and in which the decision-maker is the most willing to forgo net benefits of consumption in the next period for the sake of realizing them sooner.

Figure 2 shows the behavior of the model with strategic and non-cooperative extraction, under recursive preferences and an increasing number of players. As expected, the behavior of the model converges to that of the myopic extraction regime, as the number of players becomes large. Both Figures 3 and 4 show that the dynamic game model is affected in a similar way to the central planner's model, when the elasticity of inter-temporal substitution is changed. Even though both the 2 and 10-player examples show that the degree of groundwater mining is bounded by the TAS case, the profiles become less distinguishable, as the number of players increases, and the strategic behavior of the players converges to that of myopic extraction. Negri (1989) showed this result, analytically, as $N \to \infty$, and we can also make a similar argument, based on the derived infinite-horizon policy rule for pumping as a function of depth. A brief exposition is given in Appendix B, for the benefit of the reader.

[1] Using the definition of Epstein and Zin (1991), the elasticity of inter-temporal substitution becomes infinite in the case where $\alpha$=1, which corresponds to time-additive separability.

# 3. Measuring the Gains to Groundwater Management

In this section, we define a benchmark of allocative efficiency between the $N$ economic agents, with the groundwater extraction problem of the central planner. The central planner's solution will be derived under both time-additive separable and recursive preferences, so that the gains to management can be measured against the respective game solutions and compared. The difference between the gains to management will then be evaluated under varying degrees of inter-temporal substitutability, in order to better understand the importance of this parameter on the magnitude of management gains that are realized.

## 3.1 Central Planning under Recursive and Non-recursive Preferences

The problem facing the central planner is that of maximizing the combined welfare of all agents drawing from the aquifer, and maximizing the joint net present value of benefits that accrue over the planning horizon for all players. If the central planner were to operate under the standard assumption of time-additive separability, then his problem could be written as

$$V^{CP}\left(h\right)=\max_{\{w_i\}_{i=1}^{i=N}}\left\{\sum_{i=1}^{N}\left[a\cdot w_i-\tfrac{1}{2}b\cdot w_i-eh\cdot w_i\right]+\beta V^{CP}\left(h+\gamma\sum_{i=1}^{N}w_i-\overline{r}\right)\right\} \tag{3.1.1}$$

where the inter-temporal optimization is carried out with respect to the pumping of all players in each period.

If, however, we were to relax the assumption of time-additive separability of the objective criterion for the central planner – then his problem could be re-written as

$$V^{CP}(h)=\max_{\{w_i\}_{i=1\ldots N}}\left[\left(1-\beta\right)\left(\sum_{i=1}^{N}a\cdot w_i-\tfrac{1}{2}b\cdot w_i-eh\cdot w_i\right)^{\alpha}+\beta\left(V^{CP}\left(h+\gamma\sum_{i=1}^{N}w_i-\overline{r}\right)\right)^{\alpha}\right]^{\frac{1}{\alpha}} \tag{3.1.2}$$

which retains the same basic structure as that of the individual optimization programs in the non-cooperative dynamic game formulation. Likewise, the functional form of the optimal carry-over value functions remains the same as that for the non-cooperative case, and can be derived in the similar fashion. The numerical approximations to these functions shall be employed in this paper, in order to compare the gains to management under both time-additive separability and recursive preferences of the objective criteria. The results of these numerical simulations will be presented in the next sub-section.

## 3.2 Comparing the Gains to Management

Now we compare the computed gains to groundwater management under both the recursive and non-recursive utility specifications, to see the effect of relaxing the assumption of time-additive separability. The gains to management are normally computed by comparing the net present value of the stream of maximized benefits that accrue to the players under centralized management of the aquifer and under de-centralized, non-cooperative extraction. This has been the standard approach to evaluating any stream of net benefits that accrue over time as a result of resource extraction, and is based upon common accounting practices which calculate financial gain (or loss) of any stream benefits (or costs) on the basis of discounted values which represent foregone opportunities in alternative investments.

In order to calculate the gains to management, we add the total net benefits of each player to its discounted value for each period along both the competitive and cooperative solution paths. By calculating the difference between the cumulative net benefits that accrue over the path of each extraction regime

$$\left\{w_{i,t}^{CP}\right\}_{t=1}^{t=T},\left\{w_{i,t}^{Game}\right\}_{t=1}^{t=T},\left\{h_{t}^{CP}\right\}_{t=1}^{t=T},\left\{h_{t}^{Game}\right\}_{t=1}^{t=T}$$

we can measure the gains that would be realized under central groundwater management, by making the following computation

$$
\begin{aligned}
Gain = & \sum_{t=1}^{t=T}\left[\beta^{t-1}\sum_{i}\left(a\cdot w_{i,t}^{CP}-\tfrac{1}{2}b\cdot w_{i,t}^{CP}-eh_t\cdot w_{i,t}^{CP}\right)\right] \\
& -\sum_{t=1}^{t=T}\left[\beta^{t-1}\sum_{i}\left(a\cdot w_{i,t}^{Game}-\tfrac{1}{2}b\cdot w_{i,t}^{Game}-eh_t\cdot w_{i,t}^{Game}\right)\right]
\end{aligned}
\tag{3.2.1}
$$

The percentage gains that we see in Table 2 match the values obtained by Feinerman and Knapp (1983), in their study of Kern County, California, when taken with respect to myopic extraction behavior – which is the only case they considered. Dixon (1991) considered the strategic extraction case in Kern County, and obtained percentage gains of close to 3.7%, which corresponds closely to the two-player level reported in Table 2. Furthermore, Dixon reported that the proportion of the management gains captured by strategic behavior is 75%, which also corresponds closely to the two-player results in this paper, and which is due to the fact that the closed-loop game, although non-cooperative in nature, still embodies forward-looking, dynamic behavior which accounts for a large proportion of the marginal user costs of resource extraction. As the number of players increases, however, the extent to which they can internalize the marginal user cost decreases, and their behavior approaches that of the myopically-extracting agent, as has been shown by Negri (1989), Provencher and Burt (1993).

Many authors have reported very modest or non-existent gains to groundwater management, either under sole groundwater use (Gisser and Sanchez, 1980; Allen and Gisser, 1984; Worthington et al., 1985; Nieswiadomy, 1985; Reichard, 1987) or within a conjunctive use setting (Knapp and Olson, 1995; Knapp et al., 2003). These results are summarized in Table 3, for comparison, and represent the gains to management as measured with respect to non-cooperative and myopic extraction behavior.

By adopting the recursive-utility specification of equation (3.1.2), for the social planner, we can evaluate the effect that relaxing the assumption of time-additive separability has on the measured gains to management. By varying the inter-temporal substitution parameter, $\alpha$ , we can then re-calculate the difference shown in (3.2.1) to see how the gains to management change with an increasing degree of resistance to inter-temporal substitution, both from the perspective of each player and of the social planner. By doing so, we obtain the results shown in Table 4, which show that the gains to management decrease as the elasticity of inter-temporal substitution decreases (i.e., as $\alpha$ becomes more negative).

Given the fact that Figure 1 showed the steady-state groundwater level under decreasing levels of the inter-temporal elasticity, these results seem somewhat paradoxical, as they suggest that things are worsening, when in fact the long-run sustainability of the aquifer seems to be further enhanced when the central planner's inter-temporal preferences deviate farther from time-additive separability. The reasons for this apparent contradiction are revealed when we compare Figures 5 and 6, which show the difference between the stream of net benefits realized under central planning and myopic extraction, both in the absence and presence of discounting, respectively. From Figure 5 we see that in the early part of the extraction horizon, the net benefits that accrue to the pumpers under myopic extraction exceed those realized under central planning – making the gains to management negative. This is due to the fact that myopic pumpers mine the groundwater resource early on, ignoring the inevitable future rise in pumping costs, which eventually results in the long-run net benefits being lower than those realized under groundwater management. In Figure 5, the long-run gains realized under groundwater management are shown to be much larger when the social planner's preferences exhibit lower levels of inter-temporal elasticity, when reported in terms of their nominal value. However, when these values are discounted over time, the near-term losses begin to overwhelm the gains that are realized farther in the future under increasingly inelastic inter-temporal substitution, and the long-run benefits (in terms of lower pumping costs) are drastically under-stated, as seen in Figure 6.

Considering that the net-present value from the stream of benefits over time corresponds to the time-additive calculation, below

$$\sum_{t=1}^{t=T}\left(\beta^{t-1}\cdot NB_t\right) \tag{3.2.2}$$

its incongruity with the nature of the social planner's objective function becomes apparent, as the planner's objective criterion is now no longer strictly time-additive in nature. To date, the groundwater management literature has only considered the time-additive separable case, and has used the discount factor $(\beta)$ as the sole inter-temporal preference parameter of relevance. Given the more generalized framework of the iso-elastic recursive utility functional, the way in which we calculate gains over time may also need to be more generalized with respect to both the pure rate of time preference and the elasticity of inter-temporal substitution, so that the accounting of management gains that accrue over time remains consistent with the nature of the inter-temporal preferences embedded within the structure of the optimal program which defines the new benchmark of allocative efficiency.

## 3.3 Re-Calculation of the Gains to Management

In order to account for the gains to management in a way that is consistent with the functional form of the social planner's objective criterion, we should evaluate the stream of benefits that accrue over time with respect to both the discount factor and the elasticity of inter-temporal substitution. In order to do this, we propose the calculation scheme shown below

$$V_t=\left[(1-\beta)\left(NB_t\right)^{\alpha}+\beta\left(NB_{t+1}\right)^{\alpha}\right]^{\frac{1}{\alpha}} \tag{3.3.1}$$

which shows the benefits in period t as a function of those that accrue from time t+1 forward, such that if we were to start from the last period t = T, the calculation for the last four periods can be written as

$$V_{T-3} = \left[ (1-\beta)\left(NB_{T-3}\right)^{\alpha} + \beta\left( \left[ (1-\beta)\left(NB_{T-2}\right)^{\alpha} + \beta\left( \left[ (1-\beta)\left(NB_{T-1}\right)^{\alpha} + \beta\left(NB_{T}\right)^{\alpha} \right]^{\frac{1}{\alpha}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \quad (3.3.2)$$

This scheme can be extended, recursively, to include the entire planning horizon, starting from t=1, so that the accounting of benefits over time corresponds to the way in which the dynamically-optimizing social planner accounts for them at each point along the optimal consumption path.

By adopting this scheme for both the calculation of benefits accruing to the groundwater users under myopic and centrally-managed extraction, we obtain the results shown in Table 5, which contrast sharply with those calculated previously. While the per-acre benefits appear in a different unit-of-measure than before[2], they values increase in a manner that is consistent with our intuitive belief that higher steady-state groundwater levels (and lower levels of lift) correspond to greater social, long-run benefits. Given that the recursive scheme in (3.3.2), corresponds to the summation below

$$V_{t=1} = \left\{ (1-\beta)\left[ \sum_{t=1}^{t=T-1} \beta^{t-1}\left(NB_t\right)^{\alpha} \right] + \beta^{T}\left(NB_T\right)^{\alpha} \right\}^{\frac{1}{\alpha}} \quad (3.3.3)$$

the computed gains for the case where $\alpha = 1$ will equal those obtained from the simple present-value calculation when the sum of present-value net benefits in the first T-1 periods are divided by the factor $(1-\beta)$[3].

[2] They are now expressed in units of the utility function, rather than in monetary terms

[3] The reader can easily verify that the value of 8 utility units corresponds to the value given for the simple discounted sum, when divided by 1-β, for β = 0.952 (r = 5%).

The results in that last column of Table 5 now show much larger percentage gains to centralized management than those that were shown in Table 3, which represent the results from studies that have all imposed the assumption of time-additive separability on the inter-temporal preferences of the decision-maker. Perhaps if we were to introduce a greater degree of groundwater modeling sophistication, following the suggestion of Brozovic (2002) and Brozovic *et al.* (2003), or by addressing the risk externalities that might be present when considering stochastic surface water supplies (Provencher and Burt, 1993) – the gains to management that are reported here might be weakened, somewhat. However, given the severe restrictions that time-additive separability on inter-temporal behavior, it is likely that a more plausible set of inter-temporal preferences for the central planner would create management gains that would remain sizeable, even under attenuation by the factors mentioned above. This will remain, however, a question for further empirical research.

# 4. Conclusions

In this paper, we have extended the basic dynamic groundwater extraction model of Feinerman and Knapp (1983) to incorporate both strategic behavior and variable degrees of 'resistance' to inter-temporal substitution. By relaxing the assumption of time-additive separability in the model, we have found that the impact of changing the rate of inter-temporal substitution to have a significant effect on the path of groundwater extraction, under both central planning and non-cooperative and strategic extraction. Furthermore, when evaluating the gains to centralized groundwater management, we also found that changing the rate of inter-temporal substitution increases the realized gains to centralized groundwater management, provided that these gains are measured with a criterion which is consistent with the functional form of the objective criterion.

While many authors, beginning with Gisser and Sanchez (1980), have asserted that the gains to management are small, compared to the potential costs of its implementation, they have restricted themselves to the time-additive separable case. Using the empirical case of Kern County, we have found reason to question these long-held beliefs, and argue that the case for management might be greatly strengthened if one uses a more generalized functional form to represent inter-temporal preferences. As it seems unlikely that a groundwater manager's behavior would conform to the case of strict time-additive separability, the case for re-examination of the gains to groundwater management in the empirical literature is strengthened.

The results of this paper show the need for researchers to look more closely into the behavioral assumptions embedded in the resource modeling tools that are commonly employed to investigate policy options in natural resource management. The relaxation of time-additive separability on the inter-temporal objective criterion has dramatic effects both on the path of resource extraction as well as on the calculation of gains to groundwater management, and should be taken investigated in all analyses that rely on the result of dynamic models. Even though dynamic optimization techniques have been at the disposal of resource economists for over 40 years, the implications of relaxing such a severe restriction on inter-temporal behavior have not been fully investigated, and warrant further examination if we are to continue to rely on dynamic resource management models for insight and guidance in answering important policy questions.

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## Appendix A: Figures and Tables

**Figure 1: Comparison of Groundwater Lift Under Central Planning (TAS and Recursive Preferences, r = 5%)**

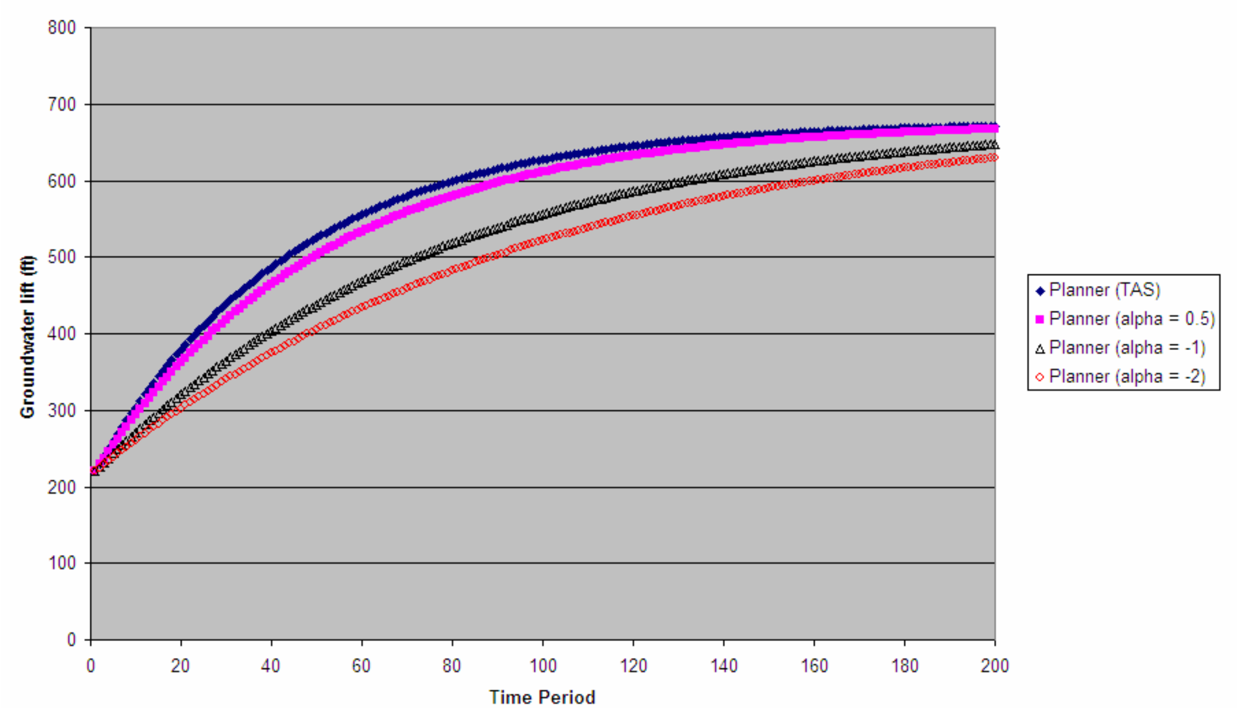


**Figure 2: Comparison of Groundwater Lift under Non-Cooperative Extraction (Recursive Preferences, $\alpha$ = 0.5, r = 5%)**

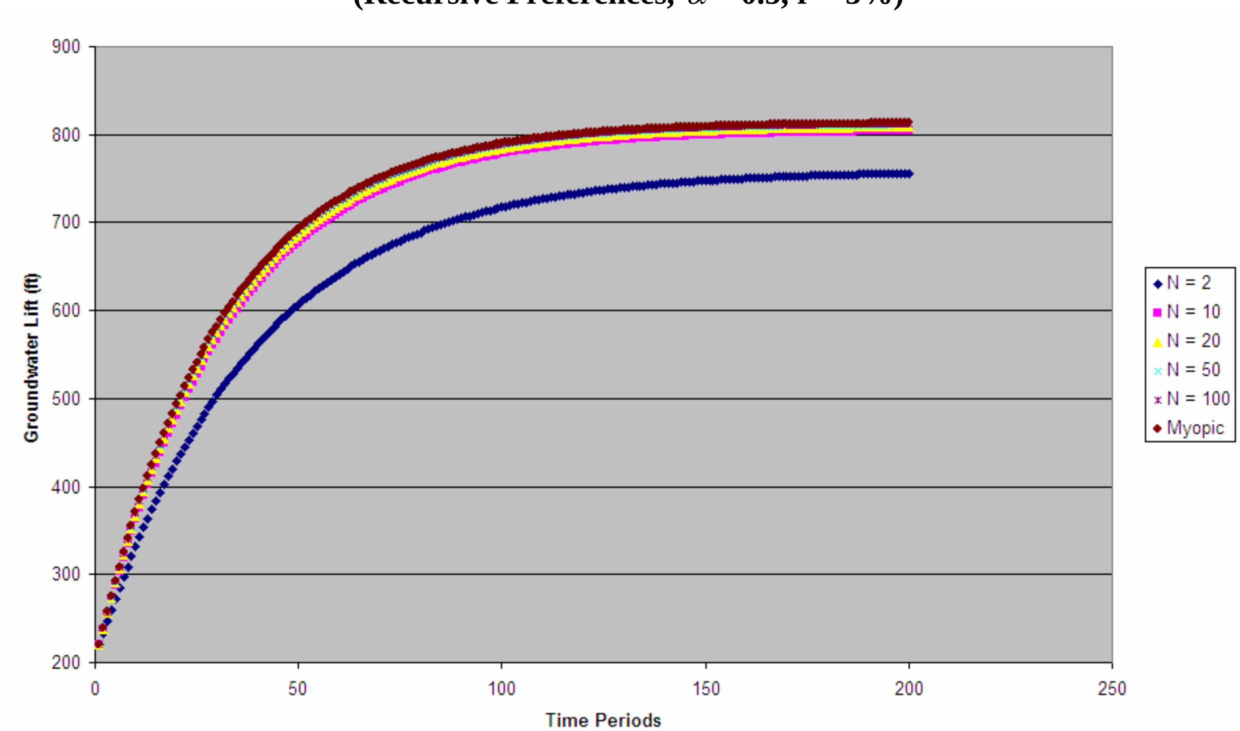

**Figure 3: Comparison of Groundwater Lift under 2-Player Non-Cooperative Extraction (TAS and Recursive Preferences, r = 5%)**

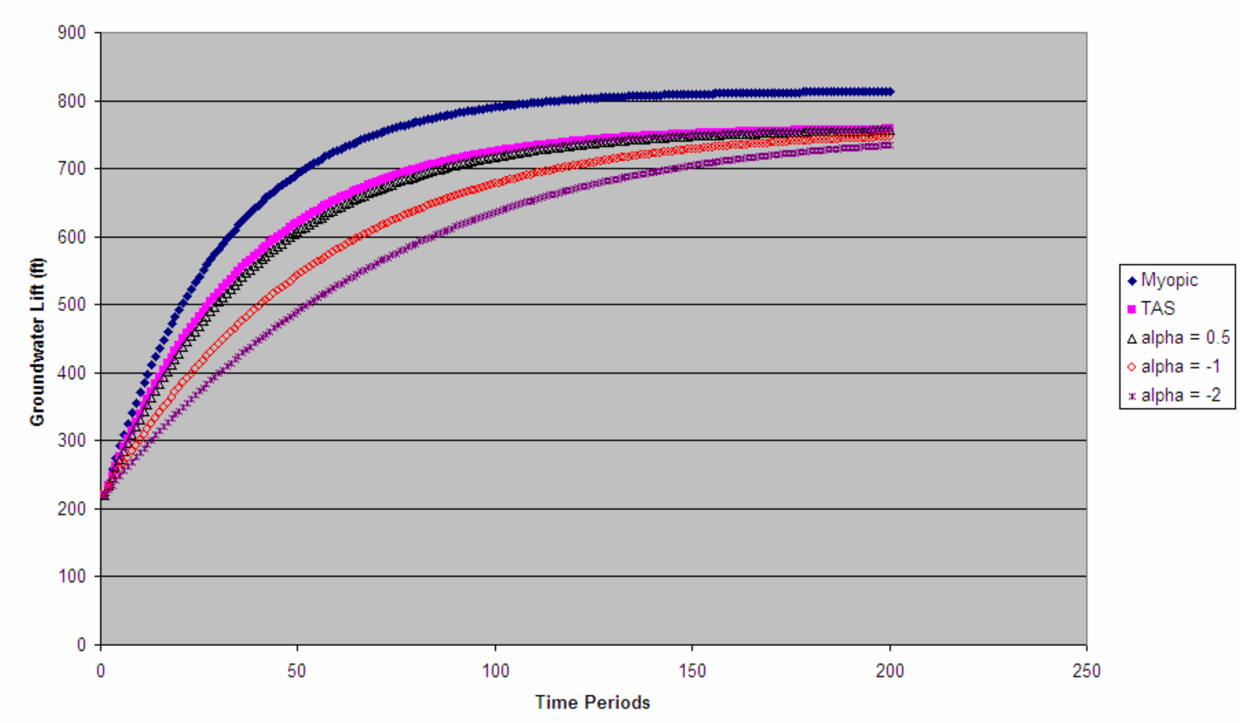


**Figure 4: Comparison of Groundwater Lift under 10-Player Non-Cooperative Extraction (TAS and Recursive Preferences, r = 5%)**

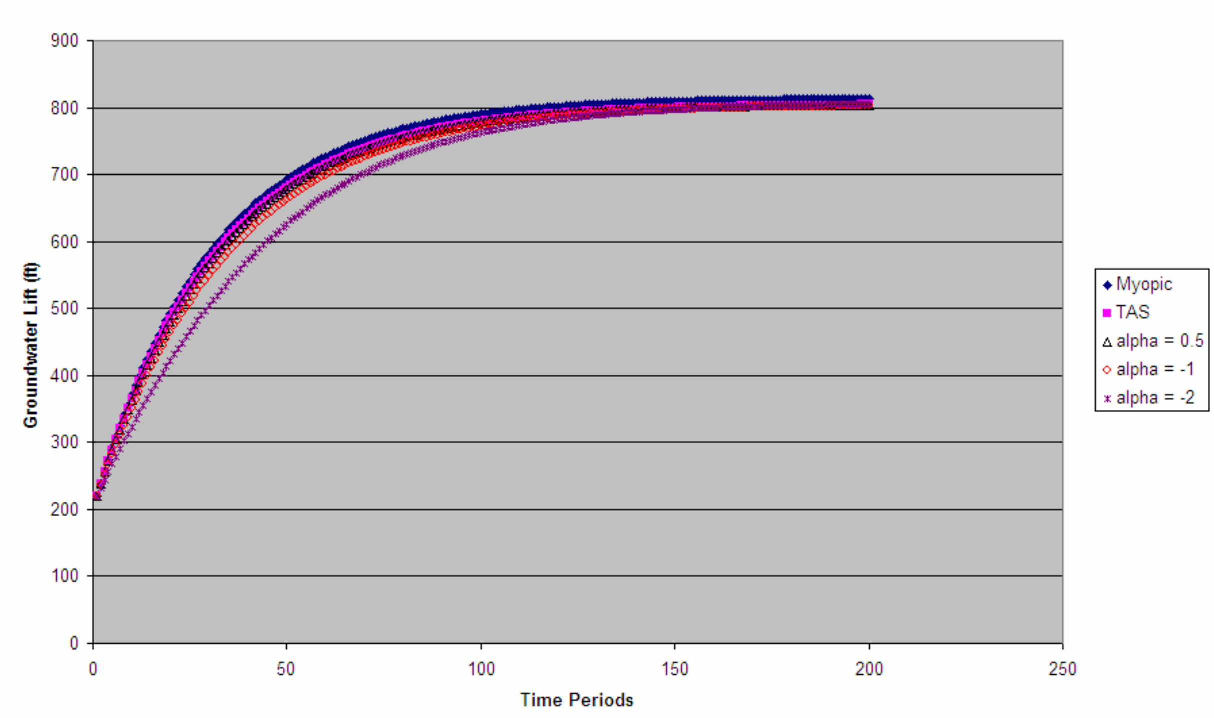

**Figure 5: Profile of Un-Discounted Management Gains over Time (TAS and Recursive Preferences, r = 5%)**

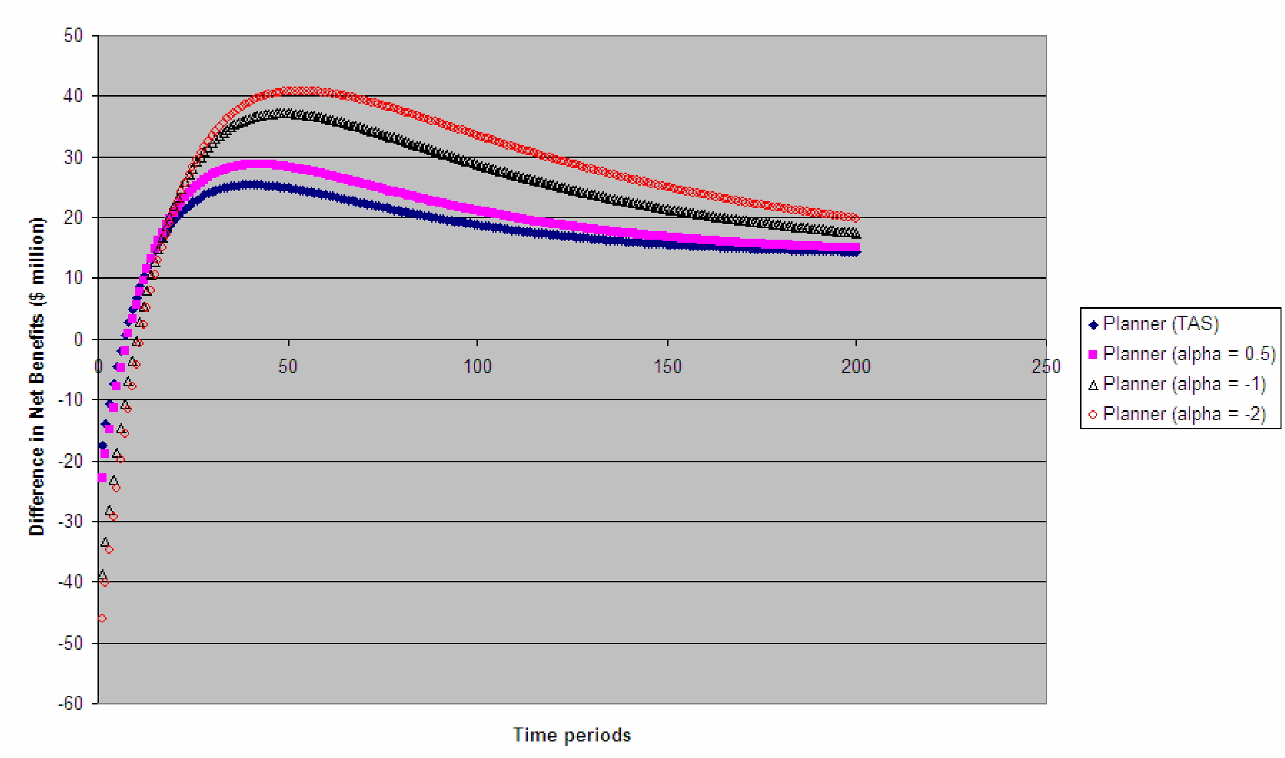


**Figure 6: Profile of Un-Discounted Management Gains over Time (TAS and Recursive Preferences, r = 5%)**

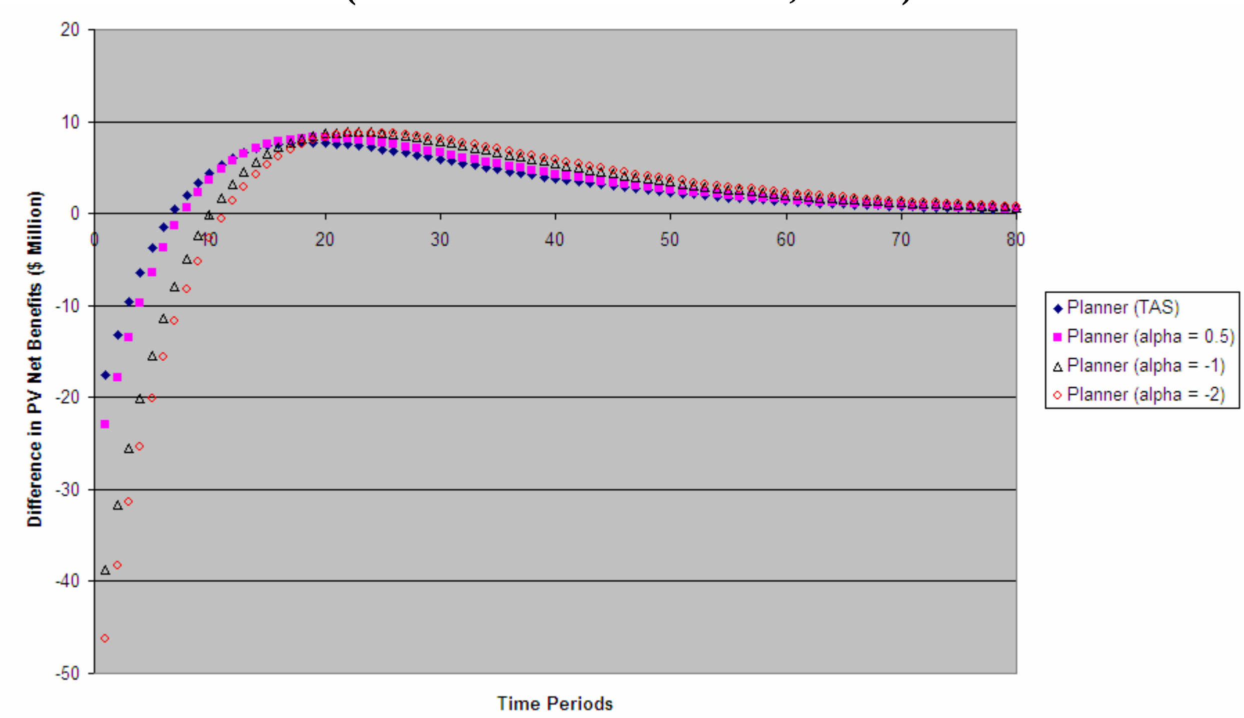

***Table 1: Parameters for Kern County Model***

| Parameter | Description | Value |
|---|---|---|
| A | Area Overlying aquifer | 1.26 (million acres) |
| s | Specific Yield of Aquifer | 0.1 |
| $\theta$ | Deep percolation coefficient | 0.2 |
| e | Energy cost per unit pumping lift | \$0.09 acre-ft/ft |
| $h_1$ | Initial lift (depth-to-water) | 220 ft |
| $\hat{r}$ | Reference level for aquifer recharge | 1410 ft |
| $\xi$ | Calibrating parameter for recharge eqn | 0.7 |
| I | Average annual surface water inflow | 1.90 (million acre-ft) |
| a | Demand curve intercept | \$92.7/acre-ft |
| b | Demand curve slope | \$0.0000175/(acre-ft)$^2$ |
| i | Real interest rate | 0.05 |
| ($\beta$) | (discount factor) | (0.952) |

***Table 2: Gains in Cumulative Net Benefits to Adopting Centralized Management of Groundwater over Non-Cooperative Allocation***

| | Total Gains from Centralized GW Management (% gains) | | |
|---|---|---|---|
| **Number of Agents** | **Strategic Behavior** | **Myopic Behavior** | **% of Management Gains Captured by Strategic Behavior** |
| 2 | $ 49/acre<br>(3.3 %) | $ 166/acre<br>(12.2 %) | 71% |
| 10 | $ 140/acre<br>(10.1 %) | $ 166/acre<br>(12.2 %) | 16 % |
| 20 | $153/acre<br>(11.2 %) | $ 166/acre<br>(12.2 %) | 8 % |
| 50 | $ 161/acre<br>(11.8 %) | $ 166/acre<br>(12.2 %) | 3 % |
| 100 | $ 163/acre<br>(12.0 %) | $ 166/acre<br>(12.2 %) | 2% |

***Table 3: Reported Gains to Groundwater Management in Literature***

| Authors | Percentage Gain in Benefits over Unregulated Base Case |
|---|---|
| GW Use Only | |
| Bredehoeft and Young (1970) | (roughly) 14-16% |
| Gisser and Sanchez (1980) | Nearly zero |
| Feinerman and Knapp (1983) | Up to 14% |
| Allen and Gisser (1984) | Nearly zero |
| Worthington et al. (1985) | Up to 34% |
| Nieswiadomy (1985) | 0.16% to 6.5% |
| Reichard (1987) | 11.5% |
| GW and SW Usage | |
| Noel, Gardner and Moore (1980) | (roughly) 20% |
| Knapp and Olson (1995) | 2.6% |
| Knapp et al. (2003) | 5% to 11% |

***Table 4: Gains to Groundwater Management under Recursive Preferences***

| | | Total Gains from Centralized GW Management (% gains) | | |
|---|---|---|---|---|
| **Inter-temporal Substitution Parameter** $(\alpha)$ | **Elasticity of Inter-temporal Substitution** | **Strategic Behavior ( N = 20)** | **Strategic Behavior ( N = 50)** | **Myopic Behavior** |
| 1 | ∞ | \$ 152/acre (11.2 %) | \$ 161/acre (11.8 %) | \$ 166/acre (12.2 %) |
| 0.5 | 2 | \$ 149/acre (10.9 %) | \$ 157/acre (11.6 %) | \$ 163/acre (12.0 %) |
| -1 | 0.5 | \$110/acre (8.0 %) | \$ 118/acre (8.9 %) | \$ 123/acre (9.1 %) |
| -2 | 0.33 | \$ 77/acre (5.6 %) | \$ 85/acre (6.3 %) | \$ 91/acre (6.7 %) |

***Table 5: Revised Calculation of Gains to Adopting Centralized Management (over Myopic Extraction of Groundwater)***

| | | **Total Gains from Centralized GW Management (% gains)** | |
|---|---|---|---|
| **Inter-temporal Substitution Parameter $(\alpha)$** | **Elasticity of Inter-temporal Substitution** | **Simple Sum of Discounted Net Benefits** | **Value Measured With Recursive Scheme[1]** |
| 1 | $\infty$ | $ 166/acre (12.2 %) | 8 units/acre (11.4 %) |
| 0.5 | 2 | $ 163/acre (12.0 %) | 10 units/acre (16.6 %) |
| -1 | 0.5 | $ 123/acre (9.1 %) | 22 units/acre (47.7 %) |
| -2 | 0.33 | $ 91/acre (6.7 %) | 30 units/acre (83.0 %) |

***1 – these numbers are expressed in terms of the units of the utility function, rather than dollar values per acre***

## Appendix B: Optimal Policy Rule

For the dynamic game, we have the optimal policy rule, which can be derived from taking the first-order conditions of the maximization problem embodied in the Bellman equation (2.1.4) for the i$^{th}$ player. The optimal policy rule takes the following form

$$w_i^* = \varphi_1 + \varphi_2 (a - eh)$$

\where the parameters $\varphi_1$ and $\varphi_2$ are defined by the relationships in (2.1.7). For the case of myopic extraction, the agent simply equates the marginal benefits of water with the marginal pumping cost of groundwater, such that we have

$$a - b \cdot w_i^* = eh$$

Which ignores the carry-over value of water, which is embodied in the ‘marginal user cost’ that comes from the derivative of (2.1.5) with respect to the pumping variable[4]. As such, the optimal pumping level, under myopic groundwater extraction is

$$w_i^* = \frac{a - eh}{b}$$

whereas the pumping rule suggests that it should be

$$w_i^* = (a - eh)\frac{[1 - 2\beta \mathbf{C} e\gamma]}{\left[b - 2N\beta \mathbf{C}(e\gamma)^2\right]} - \frac{\beta e\gamma\left[\mathbf{B} + 2\mathbf{C} e\bar{r}\right]}{\left[b - 2N\beta \mathbf{C}(e\gamma)^2\right]}$$

As N becomes larger, it is easy to see that the second term becomes small, however the first term needs to be re-arranged as follows $\frac{(a - eh)}{b} \cdot \frac{[1 - 2\beta \mathbf{C} e\gamma]}{\left[1 - 2\beta \mathbf{C} e\gamma \cdot \left(\frac{Ne\gamma}{b}\right)\right]}$

So that it can be shown that the term $\left(\frac{Ne\gamma}{b}\right)$ approaches 1 as N gets large. In so doing, the first term approaches the value of the myopic extraction rule, and the strategic, non-cooperative, dynamic game converges to a myopic extraction regime with many agents.

[4] Once the variable h in equation (2.1.5) is replaced with the ‘carry-over’ value of $h + \gamma(w_i + (N-1)w_{-i}) - \bar{r}$ , this derivative can be taken with respect to $w_i$