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IMPACT OF INCOME ON PRICE AND INCOME RESPONSES IN THE DIFFERENTIAL DEMAND SYSTEM

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Abstract

An extension of the differential demand system model is developed that allows the demand system's income and price responses to vary with income level. The model's income flexibility and marginal propensities to consume (MPCs) out of income are made functions of real income measured by the Divisia volume index. The income flexibility is a factor of proportionality underlying all price effects and a change in this term impacts the sensitivity of all demands to prices. Price effects are also made a function of the MPCs using a uniform substitute specification. The model was used to analyze the conditional demands for a group of beverages. The findings indicate that changes in conditional total beverage expenditures result in various income and price elasticity changes across individual beverage products.

Key Words: demand, Rotterdam demand system, varying parameters.

JEL Classifications: D12, C51, Q11

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Introduction

The differential demand system is based on the fundamental matrix equation of consumer demand derived through differentiation of the first-order conditions of the utility maximization problem (Theil 1975; Barten, 1966). The basic differential demand system is known as the Rotterdam model and there are two parameterizations of this model— the absolute price version and relative price version.[1] The relative price version of the Rotterdam model has been useful to impose various separability and preference-structure restrictions. To allow for increased flexibility in the income and price responses, as well as for specification of non-price, non-income explanatory variables, various extensions of the differential model have been suggested including those that combine the features of the Rotterdam model and Almost Ideal Demand System (Barten, 1993) and those based on the Basmann, Tintner and Ichimura condition for the impacts of non-price, non-income variables (e.g., Theil, 1980b; Duffy; and Brown and Lee, 1997, 2002).

In this study a further extension of the relative price version of the Rotterdam model is proposed to analyze the impact of income level on the price and income responses in the differential demand system. In an empirical analysis, the demands for a group of beverage products are considered and the model specified is a conditional demand system. The focus is on how total expenditures on the product group (conditional income) impacts the price and income coefficients of the conditional demand equations for the group.

The relationship between income and the effects of prices on demands was earlier examined by Timmer in context to food policy. Timmer argued that as real income increases, the own-price elasticity of food tends to decline. Timmer, as well as Theil, Chung and Seale, noted that own-price elasticities based on the linear expenditure system or quadratic expenditure system (Pollak and Wales) actually supported the opposite conclusion that price elasticities increase with income level. This conclusion is apparently related to the restrictive nature of these demand models and exemplifies the importance of a flexible demand specification. Theil, Chung and Seale developed and estimated a more flexible cross-country demand model and found that the own-price elasticity for food did tend to decrease as real income increased. More recent analysis by Bouis supports this finding. Given the conditional income variable for the beverage group considered in the present study differs from the broader definition of income used by Timmer and the other studies mentioned, there is no reason, however, to believe the previous findings that increases in income reduce the price responses should apply to the present study.

Model

Consider the utility maximization problem confronting consumers—how to allocate income over available goods. The solution is the affordable bundle of goods that yields the greatest utility. Formally, this problem can be written as maximization of $u = u(q)$ subject to $p'q = x$, where u is utility; $p' = (p_1, \ldots, p_n)$ and $q' = (q_1, \ldots, q_n)$ are price and quantity vectors with p_i and q_i being the price and quantity of good i, respectively; and x is total expenditures or income. The first order conditions for this problem are $\partial u/\partial q = \lambda p$ and $p'q = x$, where λ is the Lagrange multiplier which is equal to $\partial u/\partial x$. The solution to the first order conditions is the set of demand equations $q = q(p, x)$, and the Lagrange multiplier equation $\lambda = \lambda(p, x)$. The Rotterdam demand model is an approximation of this set of demand equations and the model developed in this paper is an extension of this approximation. Analyses by Barnett, Byron and

Mountain show that the Rotterdam approximation is comparable to other flexible functional forms such as the Almost Ideal Demand System.

Rotterdam Model

Following Theil (1975, 1976, 1980a,b), the Rotterdam model can be written as

(1) $w_i \, d(\log q_i) = \theta_i \, d(\log Q) + \sum_j \pi_{ij} \, d(\log p_j) \quad i=1, \ldots, n,$

where $w_i = p_i q_i / x$ is the budget share for good i ; $\theta_i = p_i (\partial q_i / \partial x)$ is the MPC for good i; $d(\log Q) = \sum w_i \, d(\log q_i)$ is the Divisia volume index;[2] and $\pi_{ij} = (p_i \, p_j / x) \, s_{ij}$ is the Slutsky coefficient, with $s_{ij} = (\partial q_i / \partial p_j + q_j \, \partial q_i / \partial x)$ being the i,jth element of the substitution matrix S. The Rotterdam model can be obtained from the total differential of the first order conditions, as shown in Appendix A.

From equation (A7) in the Appendix, the Slutsky coefficient can be written as

(2) $\pi_{ij} = \varphi \, (\theta_{ij} - \theta_i \, \theta_j),$

where $\theta_{ij} = ((p_i \, p_j \, \lambda)/(x \, \varphi)) \, u^{ij}$, with u^{ij} being the i,jth element of the inverse of the Hessian matrix, $[u^{ij}] = [\partial^2 u / \partial q_i \, \partial q_j]^{-1}$. The parameter φ is a factor of proportionality, referred to as the income flexibility, and is equal to the reciprocal of the elasticity of the marginal utility of income with respect to income; φ is negative based on the assumption that U is negative definite for utility maximization (this result follows from the definition of $\partial \lambda / \partial x$ in equation (A5), given $\lambda >$ 0 and $x > 0$). The term $\varphi \, \theta_{ij}$ captures the specific substitution effect while the term $-\varphi \, \theta_i \, \theta_j$ captures the general substitution effect (Theil, 1975).

The general restrictions on demand (1) are (e.g., Theil 1975, 1976, 1980a,b)

(3a) adding up: $\sum_i \theta_i = 1$; $\sum_i \pi_{ij} = 0$;

(3b) homogeneity: $\sum_j \pi_{ij} = 0$;

(3c) symmetry: $\pi_{ij} = \pi_{ji}$.

Based on restrictions (3), the restrictions on Slutsky coefficient specification (2) are

(4a) adding up: $\sum_i \theta_{ij} = \theta_j$; $\sum_j \sum_i \theta_{ij} = 1$;

(4b) homogeneity: $\sum_j \theta_{ij} = \theta_i$;

(4c) symmetry: $\theta_{ij} = \theta_{ji}$.

The θ_{ij}s are referred to as normalized price coefficients since by restriction (4a) they add up to one.

Extension

Consider making the income flexibility a function of income as suggested by Theil, and Theil, Chung and Seale. Since the income flexibility is a factor of proportionality for all price effects, a change in this term results in a general change across all goods with respect to the sensitivity of demands to prices. Adding subscript t for time, the income flexibility is specified as

(5a) $\varphi_t = \varphi_{t-1} + \alpha\, d(\log Q_t)$,

where α is a coefficient to be estimated.

For the first observation (t=1), the parameter φ_t is

(5b) $\varphi_1 = \varphi_0 + \alpha\, d(\log Q_1)$.

For the second observation, $\varphi_2 = \varphi_1 + \alpha\, d(\log Q_2)$, or substituting the right-hand side of result (5b) for φ_1

(5c) $\varphi_2 = \varphi_0 + \alpha\, (d(\log Q_1) + d(\log Q_2))$.

Successively substituting in this manner, the income flexibility at time t can be written as

(5d) $\varphi_t = \varphi_0 + \alpha\, dz_t$

where $dz_t = \sum_{h=1 \text{ to } t} d(\log Q_h)$.

The Divisia volume index indicates the period-to-period percentage change in real income (the weighted average percentage change in quantity demanded). Thus the term dz_t can be viewed as a measure of the percentage difference between real income in period t and period 0.

Substituting equation (5d) into equation (2) and then equation (2) into equation (1) yields the first extended demand specification considered here, i.e.,

(6a) $w_i\, d(\log q_i) = \theta_i\, d(\log Q) + (\varphi_0 + \alpha\, dz_t) \sum_j (\theta_{ij} - \theta_i\, \theta_j)\, d(\log p_j)$

or

(6b) $w_i\, d(\log q_i) = \theta_i\, d(\log Q) + (\varphi_0 + \alpha\, dz_t) \sum_j \theta_{ij}\, [d(\log p_j) - \sum_j \theta_j\, d(\log p_j)]$,

where the restriction $\sum_j \theta_{ij} = \theta_i$ has been used to eliminate θ_i in the price term. The term $\sum \theta_j$ $d(\log p_j)$ is known as the Frisch price index (Theil, 1980a).

The other parameters of the model, the MPCs (θ_i) and normalized price coefficients (θ_{ij}), might similarly be made functions of income. An extended specification of the MPC is

(7a) $\theta_{i\,t} = \theta_{i\,t-1} + \beta_i\, d(\log Q_t)$,

where β_i is a slope coefficient specific to good i and the subscript t here is for time (not to be confused with the second subscript used in the normalized price coefficients).

Following the income-flexibility progression from (5a) through (5d), the MPC can be written as

(7b) $\theta_{it} = \theta_{i0} + \beta_i\, dz_t$,

where $\sum_i \theta_{i0} = 1$ and $\sum_i \beta_i = 0$, based on restriction (3a).

A specification where the normalized price coefficients are functions of the varying MPCs (7b) is considered subsequently. Thus, changes in the MPCs will result in both changes in the income and price effects along with the above effect of the income flexibility on the general sensitivity of demand to prices.

Relationships with the Almost Ideal Demand System

The basic relationship between the Rotterdam model and the Almost Ideal Demand System (AIDS) was noted by Deaton and Muellbauer (1980a) in their original paper on the AIDS. Except for the dependent variable, the first difference form of the AIDS is approximately the same as the Rotterdam model—the two models have the same explanatory variables but the dependent variable for the differential AIDS is the change in the budget share (dw_i) while that for the Rotterdam model is $w_i d(\log q_i)$. This similarity has given rise to several extensions of the Rotterdam model. These extensions combine the features of the basic Rotterdam and AIDS models making the MPC and Slutsky coefficients functions of the budget shares (Barten, 1993).

The AIDS can be considered a price extension of an Engel curve model proposed by Working and Leser. This latter model can be written as

(8) $$w_i = \alpha_i + \beta_i \log(x).$$

The AIDS adds to equation (8) the price term $\sum_j \gamma_{ij} \log p_j$ and replaces the income term with $\log(x/p)$ where p is a price index, i.e., $w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(x/p)$.

The income elasticities for the Working-Leser model and AIDS are[3]

(9) $$e_i = 1 + \beta_i / x.$$

Given the MPC is equal to the budget share times the income elasticity—$(p_i q_i/x)(\partial q_i/\partial x)/(x/q_i) = p_i \partial q_i/\partial x$—the MPCs corresponding to the Working-Leser or AIDS models are found by multiplying equation (9) by w_i, i.e.,

(10) $$\theta_i = w_i + \beta_i.$$

Substitution of equation (10) into equation (1) results in the CBS model (Keller and van Driel; Barten, 1993). This model has the income responses of the Working-Leser and AIDS models and the price responses of the Rotterdam model.

Consider the differential of equation (10) with respect to the log of income, i.e.,

(11a) $d\theta_i = (\partial w_i/\partial \log x)\, d\log(x)$

or, based on equation(8),

(11b) $d\theta_i = \beta_i\, d\log(x)$.

When the AIDS specification of w_i as a function of income and prices is used in the above derivations, we obtain

(11c) $d\theta_i = \beta_i\, d\log(x/p)$,

or

(11d) $d\theta_i = \beta_i\, d\log(Q)$,

where dlog(x/p) is approximated by the Rotterdam income variable dlog(Q) as suggested by Deaton and Muellbauer (1980a).

When the CBS-based equation (11d) is expressed in discrete differences with the lagged value of the MPC moved to the left-hand side of the equation, it is the same as the present paper's MPC extension, equation (7a). Thus, equation (7a) can be considered a reduced form specification of the MPC of the CBS model with respect to the income variable (the present MPC extension, however, is not a complete reduced form specification of the MPC of the CBS model, since the budget shares in the CBS specification of the MPC depend not only on income but prices and other variables). This reduced form specification avoids an endogeneity problem inherent in differential demand models that use budget shares as explanatory variables. When the CBS specification (10) is directly substituted into (1), the endogenous variable w_i appears on the right-hand side of the resulting equation. In the CBS model, this problem is handled by moving the term w_i dlog(Q) to the left-hand side of the model. However, for some similar extensions, this problem can not be handled so simply. For example, below we consider a model (uniform substitute model) which makes the normalized price coefficients a function of the MPCs. If the CBS specifications of the MPCs are used in this model, the endogeneity problem can not be removed by rearranging the equations. Brown and Lee (2000) handled this particular problem by using the lagged budget share in the MPC specification. Equation (7a) is an alternative, direct specification that does not involve endogeneity per se.

Restrictions on the Relative Price Version of the Rotterdam Model

The relative price verison of the Rotterdam model including the present extension (6b) can not be estimated unless some restriction is place on the θ_{ij}s (Theil, 1971). In the absolute price version, the MPC (θ_i) can be identified from the income variable or Divisia volume index, and the Slutsky coefficients (π_{ij})can be identified from the price variables, provided the data used to estimate the model are rich enough. Defining the matrices $\theta = [\theta_i]$, $\pi = [\pi_{ij}]$, and $\Theta = [\theta_{ij}]$, equation (2) can be written as $\pi = \varphi(\Theta - \theta\theta')$. The problem is "can φ and Θ be determined given π and θ are known?" The answer, in general, is no. The solution for Θ, given π, θ and φ is $\Theta = \pi/\varphi + \theta\theta'$; when π and θ are known but φ is unknown different values of φ can be used to generate different values of Θ, but each set of estimates of φ and Θ would be consistent with the known π and θ. However, when one constraint is put on Θ, in addition to those for homogeneity and symmetry, the parameter φ can be estimated (Theil, 1971). In this study, the restrictions underlying the uniform substitute model are placed on Θ. To examine alternative restrictions such as those resulting from separability, a reformation of the Rotterdam model is provided in Appendix B.

Uniform Substitute Model

Consider the Rotterdam model specific substitution term θ_{ij} specified in equation (2). This term equals the factor of proportionality, $\lambda/(x\varphi)$, times $p_i p_j u^{ij}$. Given u^{ij} is the i,j th

element of the inverse of the Hessian matrix, the term $p_i\, p_j\, u^{ij}$ is the i,j[th] element of the matrix $[\partial^2 u/\partial(p_iq_i)\partial(p_jq_j)]^{-1}$. Thus, the inverse of $p_i\, p_j\, u^{ij}$ is $\partial^2 u/\partial(p_iq_i)\partial(p_jq_j)$, which indicates how the marginal utility of a dollar spent on good i changes in response to another dollar spent on good j.

Let G denote a group of goods—the different types of beverage products in this study. If the goods in this group were identical, the above marginal-utility changes for these goods would be the same, say k_0; i.e., $\partial^2u/\partial(p_iq_i)\partial(p_jq_j) = k_0$, for $i,j \in G$. Instead of being exactly identical goods, assume the goods are nearly identical with respect to key attributes but unique with respect to some. The nearly identical nature of goods i and j is assumed to result in generic type changes in the marginal utilities, as indicated by k_0 (the more one beverage is consumed and thirst is satiated, the lower the marginal utility of all beverages), while the unique nature of the goods are assumed to result in product specific changes (k_i) in the marginal utilities. These two concepts can be expressed by $\partial^2u/\partial(p_iq_i)\partial(p_jq_j) = k_0 + \Delta_{ij}\, k_i$, where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if i=j, otherwise $\Delta_{ij}=0$), and both k_0 and k_i are negative. This specification of changes in marginal utilities underlies the uniform substitute model.

As shown in Appendix C, under the assumption that group G is block independent of other goods, the Slutsky coefficients for the uniform substitutes can be written as

(12) $\theta_{ij} = (1/(1- k\theta_G))\theta_i (\Delta_{ij} - k\theta_j)$, $\quad i, j \in G$,

where k is a positive parameter reflecting the commonality of the uniform substitutes in effecting utility; and θ_G is the MPC for group G. For further discussion of this derivation, see Theil (1980a); and Brown and Lee (1993).

Substituting (12) into (2), the Slutsky coefficients for uniform substitutes can be written as

(13) $\pi_{ij} = \varphi_1\, \theta_i\, (\Delta_{ij} - \varphi_2\theta_j)$, $\quad i, j \in G$,

where $\varphi_1 = \varphi/(1\text{-}k\ \theta_G)$ and $\varphi_2 = \varphi\ (1+ k /(1\text{-}k\ \theta_G)/\varphi_1$.

Conditional Rotterdam Model

Restrictions on consumer demand models, useful for empirical analysis, can be motivated through two- or multi-stage budgeting processes. In a two-stage budgeting process, a consumer first decides the amounts of income to allocate to broad groups of commodities (first stage), and then the amount allocated to each group is further allocated to individual goods in the group. The second-stage demand equations for individual goods in a group are called conditional demands, being functions of the amount of income allocated to the group and the prices of the goods in the group. In the Rotterdam model, two stage budgeting is consistent with the imposition of separability restrictions on Slutsky coefficients (Theil, 1976). The conditional Rotterdam demand equations have the same general structure as the unconditional demands specified above, equations (1) and (6), except the real income variable or the Divisia volume index is based on income allocated to the group, the prices are those for the goods in the group, and the coefficients are conditional, being functions of the unconditional coefficients (e.g., Theil ,1976; Brown and Lee, 2000).

Conditional Uniform Substitute Model

To obtain a conditional demand system for goods in group G (beverage products), an expression for the aggregate demand for group G is first obtained by summing the unconditional demand equations (1) over the goods in G, i.e.,

(14) $\quad d(\log Q_G) = \theta_G\, d(\log Q) + \sum_j \pi_{Gj}\, d(\log p_j),$

where $d(\log Q_G) = \sum_{i \in G} w_i\, d(\log q_i)$; $\theta_G = \sum_{i \in G} \theta_i$; and $\pi_{Gj} = \sum_{i \in G} \pi_{ij}$.

Rearranging (14), we find $d(\log Q) = [d(\log Q_G) - \sum_j \pi_{Gj}\, d(\log p_j)\,] / \theta_G$; and substituting this result into (1) we find

(15) $w_i\, d(\log q_i) = \theta_i^*\, d(\log Q_G) + \sum_j \pi_{ij}^*\, d(\log p_j)$,

where $\theta_i^* = \theta_i / \theta_G$; and $\pi_{ij}^* = \pi_{ij} - \theta_i^*\, \pi_{Gj}$.

At this point, the j subscript in equation (15) runs across all goods (j=1,…,n). However, under appropriate conditions, this equation becomes a conditional demand system for group G, i.e., the j subscript only runs across goods in group G. For block independence, the assumption underlying the uniform substitute specification, the Hessian matrix is group or block independent and $\partial^2 u / \partial q_i\, \partial q_j = 0$ for i and j belonging to different groups. Thus, for i and j in different groups, the inverse element $u^{ij} = 0$ and hence $\theta_{ij} = 0$. This result means that $\pi_{i,j} = -\varphi\, \theta_i\, \theta_j$, $i \in G, j \notin G$. As a result $\pi_{ij}^* = 0$, $i \in G, j \notin G$; i.e., $\pi_{ij}^* = -\varphi\, \theta_i\, \theta_j - (\theta_i / \theta_G)\, [-\varphi \sum_{i \in G} \theta_i\, \theta_j\,] = -\varphi\, \theta_i\, \theta_j + (\theta_i / \theta_G)\, \varphi\theta_G\, \theta_j = 0$. Hence, under block independence, equation (15) is the conditional demand for a good in group G.

For uniform substitute specification (13), the conditional Slutsky coefficient is

(16) $\pi_{ij}^* = \varphi_1\, \theta_i\, (\Delta_{ij} - \varphi_2 \theta_j) - \theta_i^* \sum_{i \in G} \varphi_1\, \theta_i\, (\Delta_{ij} - \varphi_2 \theta_j)$

$= \varphi^* \theta_i^*\, (\Delta_{ij} - \theta_j^*)$,

where $\varphi^* = (\varphi\, \theta_G)/(1 - k\, \theta_G)$. The parameter φ^* is negative given φ is negative and $0 < \theta_G < 1$.

Hence, for uniform substitutes, conditional demand equation (15) can be written as

(17) $w_i d(\log q_i) = \theta_i^* d(\log Q_G) + \sum_{j \in G} \varphi^* \theta_i^* (\Delta_{ij} - \theta_j^*)(d(\log p_j)$,

$= \theta_i^* d(\log Q_G) + \varphi^* \theta_i^* (d(\log p_i) - \sum_{j \in G} \theta_j^* d(\log p_j))$.

The term $\sum_{i \in G} \theta_j^* d(\log p_j)$ is known as the Frisch price index for group G (Theil, 1975, 1980a).

By dividing (17) by $w_G = \sum_{i \in G} w_i$, we obtain an alternative conditional demand specification,

(18) $w_i^* d(\log q_i) = \theta_i^* d(\log Q_G^*) + \varphi^{**} \theta_i^* (d(\log p_i) - \sum_{j \in G} \theta_j^* d(\log p_j))$,

where $w_i^* = w_i/w_G$; $d(\log Q_G^*) = \sum_{i \in G} w_i^* d(\log q_i)$, a conditional Divisia volume index; and $\varphi^{**} = \varphi^*/w_G = (\varphi\ \theta_G)/(1\text{-}k\ \theta_G)/w_G$.

In equation (18), we make the factor of proportionality φ^{**} and the conditional MPCs functions of the conditional income variable, $d(\log Q_G^*)$; i.e., following equations (5d) and (7b), $\varphi_t^{**} = \varphi_0^{**} + \alpha^{**} dz_t^*$, where $dz_t^* = \sum_{h=1 \text{ to } t} d(\log Q_h^*)$; and $\theta_{i\,t}^* = \theta_{i\,0}^* + \beta_i^* dz_t^*$. Note that the term k embedded in φ_t^{**} provides an additional motivation for the varying income-flexibility specification. Namely, $d\varphi^{**} = (\varphi^{**\,2} w_G/\varphi)dk$, so that letting $dk = \alpha d(\log Q_{G\ t}^*)\varphi/(\varphi^{**\,2} w_G)$, $d\varphi^{**} = \alpha d(\log Q_{G\ t}^*)$ or $\varphi^{**}_t = \varphi^{**}_{t-1} + \alpha\ d(\log Q^*_t)$. That is, the underlying substitution between uniform products indicated by k may depend on the level of income spent on the group.

Application

Conditional demands for beverages were studied using ACNielsen data based on retail scanner sales for grocery stores, drug stores, mass merchandisers along with an estimate of Wal-Mart sales based on a consumer panel.[5] Twelve beverages were included in the model: 1) 100% orange juice (OJ), 2) 100% grapefruit juice (GJ), 3) 100% apple juice (AJ), 4) 100% grape juice (GRJ), 5) remaining 100% juice (RJ), 6) vegetable juice (VJ), 7) less-than -100% juice drinks (JD), 8) carbonated water (CW), 9) water (W),10) diet soda (DS), 11) regular soda (RS), and 12) tea (T). Data for dairy and non-liquid beverage products were not provided.

The data are weekly running from week ending July 27, 2002 through week ending August 13, 2005 (160 weekly observations). The raw data were comprised of gallon and dollar sales. In our application, quantity demanded was measured by per capita gallon sales which was obtained by dividing raw gallon sales by the U.S. population; prices were obtained by dividing dollar sales by gallon sales. Sample mean per capita gallon sales, prices and budget shares are shown in Table 1. The infinitely small changes in quantities and prices in the differential models were measured by discrete first differences (Theil, 1975, 1976). To account for seasonality, first differences of sine and cosine variables were included—sine(2πt/52) and cosine(2πt/52) where π = 3.14..., observation t = 1, ..., 160 and 52 is the number of weeks in a year . Average budget share values underlying the differencing were used in constructing the model variables---$w^*_{i,t}$ was replaced by $(w^*_{i,t} + w^*_{i,t-1})/2$.

The demand specifications studied are conditional on expenditure or income allocated to the 12 beverage categories. Income allocated to the beverage group is measured by the conditional Divisia volume index which was treated as independent of the error term added to each beverage demand equation for estimation, based on the theory of rational random behavior (Theil, 1980a; Brown, Behr and Lee). As the data add up by construction---the left-hand-side variables in model (18) sum over i to the conditional Divisia volume index---the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten, 1969). The parameters of the excluded equation can be obtained from the adding-up conditions or by re-estimating the model omitting a different equation. The equation error terms were assumed to be contemporaneously correlated and the full information maximum likelihood procedure (TSP) was used to estimate the system of equations.

The estimates of uniform substitute model (18) with varying income flexibility and MPCs are shown in Table 2. The individual equation r-squares ranged from .47 (grapefruit juice and water) to .96 (regular soda). These measures, however, are not generally good indicators of goodness of fit, given the equation-system estimation method used (Bewley). An alternative measure is the system r-square (Buse; Bewley) which was .994. Both the constant and slope coefficient estimates for the income flexibility and 18 out of the 24 (constant and slope) coefficients estimates for the MPCs were statistically significant at the $\alpha = .10$ or smaller level. Of the 24 seasonality coefficients, 13 were statistically significant.

The constant and slope coefficients for the income flexibility are both negative, implying that the conditional beverage demands are more sensitive to price as income increases. The MPC constants are all positive while the MPC slopes are negative, expect for juice drinks, water and tea, indicating that, except for these latter three beverages, the conditional MPCs decrease as income increases; for juice drinks, water and tea, the opposite occurs, as income increases, their MPCs increase. Based on the MPC constant and slope estimates, the estimated value of the MPC for each beverage across the sample was in the zero-one interval. The income flexibility is also negative over the sample observations which, along with the MPC estimates, indicate that the estimated demand system satisfies the negativity condition of demand.

Conditional income (e_i) and price elasticity estimates (e_{ij}), calculated at sample mean budget share values, are shown in Table 3 ($e_i = (\theta_{i0}^* + \beta_i^* \, dz_t^*)/w^*_{it}$ and $e_{ij} = (\varphi_0^{**} + \alpha^{**} \, dz_t^*)(\theta_{i0}^* + \beta_i^* \, dz_t^*)(\Delta_{ij} - (\theta_{j0}^* + \beta_j^* \, dz_t^*))/w^*_{it} - w^*_{jt} e_i$). Corresponding standard error estimates are shown in Appendix D. Regular and diet soda have the highest income elasticity at 1.28 and 1.16, respectively; the income elasticities for the remaining beverages range from .64 for orange juice to .97 for apple juice. The own-price elasticities ranged from -1.37 and -1.47 for orange juice and carbonated water, respectively, to -2.25, -2.09 and -2.07 for diet soda, apple juice and regular soda, respectively. All the cross-price elasticity estimates are positive, reflecting substitution, although some are relatively small.

The impacts of income on the demand elasticities are illustrated in Table 4. The income and own-price elasticities, calculated at the minimum, mean and maximum values of the income variable dz_t^*, are shown. The largest changes in the income elasticities are for water, apple juice, vegetable juice and grape juice, while the smallest changes are for diet soda, regular soda, orange juice and grapefruit juice. For water and diet soda, the income elasticities at the maximum income level are 154% greater and 10% less than the corresponding values at the minimum income level, respectively. All income elasticities decrease with income except those for juice drinks, water and tea which increase, following the directional changes mentioned above with respect to the individual beverage MPC changes.

The largest changes in the own-prices elasticities are for water, tea and apple juice, while the smallest changes are for grapefruit juice, remaining juice and orange juice. The water and tea own-price elasticities at the maximum income level are 169% and 133% greater in absolute value, respectively, than the corresponding elasticities at the minimum income level. The grapefruit juice own-price elasticity at the maximum income level is only 1% larger than its value at the minimum income level. The own-price elasticities for orange juice, grapefruit juice, juice drinks, water, diet soda, regular soda and tea increase with income, while those for apple juice, grape juice, remaining fruit juice, vegetable juice and carbonated water decrease with income. The various impacts of income on the demand elasticities may be of interest to analysts, marketers and planners in the beverage industry, monitoring and trying to understand the underlying causes for volume changes in the market.

Given the conditional income variable for the beverage group differs from the broader definition of income used by Timmer and the other studies mentioned earlier, as well as the array of beverages considered, it may not be surprising that the present conditional demand findings for some of the beverages differ from the previous unconditional findings that increases in income reduce the price responses. Conditional demands only partially describe consumer behavior; determination of conditional income is required for a complete description. Changes in conditional income for the beverage group may be related to a number of variables, including beverage prices, prices of goods outside the beverage category and total consumer expenditures across all goods, as well as various preference variables such as consumer demographics and advertising. The impact of the conditional income variable on the price and income coefficients may thus indirectly reflect the impacts of such other factors through their impacts on conditional income. Regardless the underlying cause for changes in conditional income, it may still be useful to know how this variable impacts the beverage price and income responses. Data available to analyze and monitor sales may be limited to that for a product group, as in this study, and knowledge of whether the impacts of prices and total group expenditures on the product demands are weaker or stronger as total group expenditures change may be important to product category decision makers.

Conclusions

This paper extends the Rotterdam model to analyze the impact of income level on the price and income responses of demand. Previous extensions proposed by Barten (1993), including the synthetic model which combines features of the Rotterdam and AIDS, have made the Rotterdam model income and price coefficients functions of the budget shares of the goods. The budget shares, however, are endogenous, and their use as explanatory variables, in general results in an endogeneity problem, although for certain specifications (CBS) the problem can be handled by rearrangement of model terms. The present extension here is related to those suggested by Barten (1993) but avoids the latter endogeneity problems in that the income and price coefficients are specified as functions of changes in income level, reflecting the changes in the budgets shares due to this factor. Additional model flexibility is provided by also specifying the income flexibility underlying the price coefficients as a function of the change in income. To estimate the income flexibility, however, requires some restriction(s) on the normalized price coefficients of the Rotterdam model. In the present study, uniform-substitute-model restrictions are imposed.

The empirical analysis focuses on the conditional demands for beverages. The results indicate that income level does impact the MPCs, income flexibility and Slutsky coefficients. The range of conditional income and price elasticities, based on the income level extremes of the sample, is relatively large. Such findings may be of interest for understanding changes over time in demands for products. The increased flexibility of the varying-coefficient specification of the uniform substitute model may also be of interest for analyzing other product groups dominated by substitution, and when the uniform substitute model is not applicable, the varying MPC and income flexibility specifications can still be applied provided appropriate restrictions on the normalized price coefficients can be made for identification.

Footnotes

[1] The absolute price verion can also be derived from the difference version of the double log model by imposing the basic properties of demand— adding up, homogeneity of zero in prices and income, and symmetry (e.g., Deaton and Muellbauer, 1980b).

[2] The Divisia volume index is a close approximation of $d(\log x)-\sum w_i\, d(\log q_i)$ in (A6) in Appendix A, as shown by Theil, 1971; $d(\log Q)$ is used instead of $d(\log x)-\sum w_i\, d(\log q_i)$ to insure adding-up.

[3] Given $w_i = p_i q_i / x$, $\log(w_i) = \log(p_i) + \log(q_i) - \log(x)$, and the $\partial\log(w_i)/\partial\log(x) = \partial\log(q_i)/\partial\log(x) - 1$. Based on equation (8), $\partial\log(w_i)/\partial\log(x) = \beta_i / w_i$; and hence $e_i = \partial\log(q_i)/\partial\log(x) = 1 + \beta_i / w_i$.

[4] Additional stages in allocating income can be added resulting in a multi-stage budgeting process.

[5] Data are for U.S. grocery stores doing $2 million and greater annual sales, Wal-Mart stores excluding Sam's Clubs, mass-merchandisers, and drug stores doing $1 million and greater annual sales.

Appendix A

To obtain the Rotterdam model, totally differentiate the first order conditions of the utility maximization problem to find

(A1) $Udq = p\, d\lambda + \lambda\, dp$

(A2) $p'dq = dx - q'dp$,

where $U = [\, \partial^2 u/\partial q_i \partial q_j]$, the Hessian matrix. This differential is known as the fundamental matrix equation of consumer demand theory (Barten, 1977; Phlips).

Next, multiply (A1) by U^{-1} to obtain

(A3) $dq = U^{-1} p\, d\lambda + \lambda\, U^{-1} dp$.

Result (A3) can be viewed as a partial demand system with the second term on the right-hand side (λU^{-1}) being a matrix whose elements are known as specific price effects that show the effects of prices, given income compensations to hold both real income and the marginal utility of income (λ) constant (e.g., Theil, 1975, 1976). The uniform substitute model is based on the structure λU^{-1} .

To obtain a total demand relationship, solve for $d\lambda$ by multiplying (A3) by p', substituting the right-hand side of (A2) for p' dq, and rearranging terms to find

(A4) $d\lambda = [(dx - q'dp) - \lambda\, p'\, U^{-1} dp]/p'\, U^{-1} p$.

Substituting (A4) into (A3), we obtain the total effects of prices and income on demand---$\partial q/\partial p'$, $\partial q/\partial x$. We express these results below as Hicksian or income-compensated demand equations, i.e.,

(A5) $dq = U^{-1} p\, [\, [(dx - q'dp) - \lambda\, p'\, U^{-1} dp\,]/p'\, U^{-1} p\,] + \lambda\, U^{-1} dp$,

$= \partial q/\partial x\, (dx - q'dp) + S\, dp$, where $\partial q/\partial x = U^{-1} p / p' U^{-1} p$, $\partial\lambda/\partial x = 1 / p' U^{-1} p$, and $S = \lambda U^{-1} - (\partial q/\partial x)(\partial q/\partial x)' (\lambda/\partial\lambda/\partial x)$. The term S is the price substitution matrix---$S = \partial q/\partial p' + (\partial q/\partial x) q'$. The term $(dx - q'dp)$ is real income.

Finally, multiply both sides of equation (A5) by $\hat{p}$ (symbol ^ over a vector indicates a diagonal matrix; off diagonal elements equal zero; diagonal elements equal the vector in question) and 1/x, pre-multiply dq by the identity matrix in the form of $\hat{q}\,\hat{q}^{-1}$, post-multiply q' and S by $\hat{p}\,\hat{p}^{-1}$ to obtain the Rotterdam model:

(A6) $\hat{p}\,\hat{q}/x\,\hat{q}^{-1}\,dq = \hat{p}\,\partial q/\partial x\,(dx/x - q'\hat{p}/x\,\hat{p}^{-1}\,dp) + (\hat{p}\,S\,\hat{p}/x)(\hat{p}^{-1}\,dp)$.

The term $\hat{p}\,S\,\hat{p}/x$ is known as Slutsky matrix, denoted by π. Given the definition of S in (A5), the Slutsky matrix can be written as

(A7) $\pi = \varphi\,[(\lambda/\varphi\,x)\,\hat{p}\,U^{-1}\,\hat{p}] - \hat{p}\,(\partial q/\partial x)(\partial q/\partial x)'\hat{p}$,

where $\varphi = (\partial \log \lambda / \partial \log x)^{-1}$.

In equation (1), Model (A6) is expressed more conveniently in terms of log changes, using the relationship $dz/z = d \log (z)$ for variable z.

Appendix B

Consider the price term without the income flexibility in equation (6b), i.e., $\sum_j \theta_{ij}$ [d(log p_j) - $\sum_j \theta_j$ d(log p_j)]. Breaking out on the own-price component, this term can be written as

(B1) θ_{ii} [d(log p_i) - $\sum_j \theta_j$ d(log p_j)] + $\sum_{j \neq i} \theta_{ij}$ [d(log p_j) - $\sum_j \theta_j$ d(log p_j)].

Based on restriction (4b), $\theta_{ii} = \theta_i - \sum_{j \neq i} \theta_{ij}$. Substituting the right-hand side of this result for the first parameter θ_{ii} of equation (B1) yields

(B2) [$\theta_i - \sum_{j \neq i} \theta_{ij}$][d(log p_i) - $\sum_j \theta_j$ d(log p_j)] + $\sum_{j \neq i}$ (θ_{ij} [d(log p_j) - $\sum_j \theta_j$ d(log p_j)],

or, simplifying,

(B3) θ_i[d(log p_i) - $\sum_j \theta_j$ d(log p_j)] + $\sum_{j \neq i} \theta_{ij}$ [d(log p_j) - d(log p_i)].

Substituting result (B3) for $\sum_j \theta_{ij}$ [d(log p_j) - $\sum_j \theta_j$ d(log p_j)] in equation (6b) yields

(B4) w_i d(log q_i) = θ_i d(log Q) + ($\varphi_0 + \alpha$ dz_t) θ_i[d(log p_i) - $\sum_j \theta_j$ d(log p_j)]

+ ($\varphi_0 + \alpha$ dz_t) $\sum_{j \neq i} \theta_{ij}$ [d(log p_j) - d(log p_i)].

Equation (B4) is in a convenient form to impose separability restrictions on the cross-price parameters θ_{ij}. For example, if good i is strongly separable from the other goods $\theta_{ij} = 0$ for $j \neq i$ (Theil, 1971, 1976). Likewise, if goods i and j belong to different weakly separable groups, say groups A and B, then $\theta_{ij} = \varphi_{AB}\, \theta_i\, \theta_j$ (Theil, 1976), where φ_{AB} is another factor of proportionality.

Appendix C

Consider the partition of goods into groups 1, ..., G , ... , M . Under block independence, the utility function can be written as $u = f_1 (Q_1) + ... + f_J (Q_G) + ... f_M (Q_M)$, where f_G is a subgroup utility function for group G; and Q_G is a vector of quantities for goods in group G. In this case, the Hessian matrix U can be written

(C1) $U= diag(U_1 , ... , U_G , ... ,U_M)$,

where diag is the bock diagonal operator---diagonal matrices are U_N , N=1, ... , M; off diagonal elements are zero. The matrix $U_N = [u_{ij}] = [\partial^2 u /\partial q_i \partial q_j] = [\partial^2 f_N /\partial q_i \partial q_j]$, $i, j \in N$.

From (2) and (A7), the specific substitution matrix of the Rotterdam model is the factor of proportionality φ times the matrix $[\theta_{ij}] = (\lambda/\varphi x) \hat{p} U^{-1} \hat{p}$, which under (C1), can be written as

(C2) $[\theta_{ij}] = (\lambda/\varphi x) [\hat{p} [diag(U_1^{-1} , ... , U_J^{-1}, ... U_M^{-1})] \hat{p}]$,

where $p = (P_1 , ... P_G , ... , P_M)$, with P_N being the price vector for goods in group N.

Let the goods in group G be uniform substitutes. From (C2), note that the specific substitution terms are zero for goods from different groups. For goods from the same group, the specific substitution terms are φ times

(C3) $[\theta_{ij}] = (\lambda/\varphi x) [\hat{p}_G U_G^{-1} \hat{p}_G]$.

The inverse of (C3) is

(C4) $[\theta^{ij}] = (\varphi x/\lambda) \hat{p}_G^{-1} U_G \hat{p}_G^{-1}$

or, focusing on individual matrix elements

(C5) $\theta^{ij} = (\varphi x/\lambda) [\partial^2 u / \partial (p_i q_i) \partial (p_j q_j)]$, $i, j \in G$,

where the superscripts indicate inverse elements. Result (C5) shows the effect of another dollar spent on good j on the marginal utility of a dollar spent on good i, multiplied times a factor of proportionality $(\varphi x/\lambda)$.

Theil's uniform substitute model assumes that all cross effects in (C5) are the same given similarity of goods, while own effects are unique. Formally, this assumption can be written as

(C6) $[\theta^{ij}] = \hat{a}^{-1} + k\, \iota\, \iota\, `$, $\quad i,j \in G$,

where a is a vector of positive elements, k is a positive number and, ι is unit vector (column) and ' is the transpose operator. Since φ is negative, (C6) indicates that marginal utilities decrease with increased consumption. The i^{th} element of the vector a equals k_i (φ x/λ), and $k = k_0$ (φ x/λ).

The inverse of (C6) is

(C7a) $[\theta_{ij}] = \hat{a} - (k/(1+k\, \iota\, `\hat{a}\, \iota\,))\, \hat{a}\, \iota\, \iota\, `\hat{a}$,

or

(C7b) $[\theta_{ij}] = \hat{a} - (k/(1+k\, \iota\, `a\,))\, a\, a`$ $\quad i,j \in G$.

Given restrictions (4) and the assumption of blockwise independence, $\sum_{j \in G} \theta_{ij} = \theta_i$ for $i \in G$; and $\sum_{i \in G} \theta_{ij} = \theta_j$ for $j \in G$,. Hence, post multiplying (C7) by ι (summing columns) yields

(C8a) $\theta = a\, (1 - (k\, \iota\, `\, a \,/\, (1+k\, \iota\, `a\,))$,

or

(C8b) $a = \theta\, (1+k\, \iota\, `a\,)$,

where $\theta = [\theta_i]$, $i \in G$.

Also, pre multiplying (C8b) by ι' yields

(C9a) $\iota\, `a = \theta_G\, (1+k\, \iota\, `a\,)$,

or, after solving (C9a) for ι 'a, multiplying the result through by k, and adding one, we find

(C9b) $1 + k\, \iota\, `a = 1/(1 - k\, \theta_G\,)$,

where $\theta_G = \iota`\theta$ is the marginal propensity for group G.

Hence, result (C8b) can be written as

(C10) $a = \theta \,/\, (1 - k\, \theta_G\,)$, Substituting (C10) into (C7) gives

(C11) $[\theta_{ij}] = /(1 - k\ \theta_G) - k\ /(1 - k\ \theta_G)\ \theta\ \theta'$ $\quad i,j \in G$.

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Table 1. Descriptive Statistics of Beverage Sample, 07/27/02 Through 08/13/05

Beverage	Gallons/Week		Price: $/Gallon		Budget Share	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Orange[a]	0.052	0.004	4.35	0.06	10.2%	1.2%
Grapefruit[a]	0.002	0.000	5.04	0.44	0.5%	0.1%
Apple[a]	0.017	0.002	3.50	0.10	2.6%	0.5%
Grape[a]	0.005	0.001	5.58	0.10	1.3%	0.2%
Remaining Fruit Juice[a]	0.014	0.001	5.61	0.22	3.5%	0.3%
Vegetable[b]	0.007	0.001	6.27	0.28	1.9%	0.2%
Juice Drinks[b]	0.108	0.015	3.55	0.11	16.9%	1.0%
Carbonated Water	0.016	0.002	2.59	0.05	1.8%	0.1%
Water	0.157	0.030	1.58	0.04	10.9%	1.6%
Diet Soda	0.145	0.013	2.51	0.11	16.2%	1.0%
Regular Soda	0.289	0.032	2.48	0.11	31.8%	1.8%
Tea	0.016	0.004	3.60	0.11	2.5%	0.4%
	Mean	Std Dev	Minimum	Maximum		
Sum of Divisia Volume Index[c]	-0.0242	0.0677	-0.1732	0.2028		

[a] 100% juice.
[b] Less than 100% juice.
[c] Sum of the weekly values of the Divisia volume index over the sample.

Table 2. Full Information Maximum Likelihood Estimates of the Uniform Substitute Model with Varying Income Flexibility and MPCs

Beverage	MPC Constant		MPC Slope		Sine		Cosine	
	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error
Orange[a]	0.0650	0.0044	-0.0199	0.0527	0.0109	0.0018	-0.0026	0.0018
Grapefruit[a]	0.0037	0.0002	-0.0016	0.0025	0.0002	0.0001	-0.0003	0.0001
Apple[a]	0.0230	0.0012	-0.0943	0.0151	0.0053	0.0008	0.0005	0.0008
Grape[a]	0.0107	0.0007	-0.0232	0.0098	0.0014	0.0005	-0.0015	0.0005
Remaining Fruit Juice[a]	0.0290	0.0013	-0.0145	0.0154	0.0025	0.0008	-0.0015	0.0008
Vegetable[b]	0.0156	0.0009	-0.0389	0.0130	0.0015	0.0007	-0.0004	0.0007
Juice Drinks[b]	0.1254	0.0062	0.1122	0.0712	-0.0111	0.0042	0.0017	0.0041
Carbonated Water	0.0119	0.0006	-0.0072	0.0082	-0.0012	0.0003	-0.0004	0.0003
Water	0.1064	0.0074	0.2548	0.0788	-0.0168	0.0043	0.0041	0.0043
Diet Soda	0.1868	0.0036	-0.0513	0.0382	0.0044	0.0032	-0.0035	0.0032
Regular Soda	0.4043	0.0069	-0.1492	0.0722	0.0075	0.0062	0.0021	0.0062
Tea	0.0182	0.0015	0.0331	0.0161	-0.0047	0.0013	0.0017	0.0013
	Constant		Slope					
	Estimate	Standard Error	Estimate	Standard Error				
Income Flexibility	-2.20764	0.048156	-0.996687	0.547078				

[a] 100% juice.
[b] Less than 100% juice.

Table 3. Uniform-Substitute Model Elasticity Estimates at Sample Means[a]

Beverage	Income	Price											
		Orange	Grape-fruit	Apple	Grape	Rem. Fruit Juice	Vegetable	Juice Drinks	Carb. Water	Water	Diet Soda	Regular Soda	Tea
Orange[b]	0.641	-1.373	0.002	0.019	0.008	0.018	0.011	0.063	0.005	0.070	0.159	0.052	0.004
Grapefruit[b]	0.780	0.032	-1.700	0.023	0.009	0.022	0.014	0.077	0.007	0.085	0.194	0.447	0.010
Apple[b]	0.971	0.040	0.003	-2.092	0.012	0.028	0.017	0.096	0.008	0.106	0.242	0.556	0.013
Grape[b]	0.894	0.036	0.003	0.026	-1.940	0.026	0.016	0.089	0.008	0.098	0.222	0.512	0.012
Rem. Fruit Juice[b]	0.828	0.034	0.003	0.024	0.010	-1.784	0.015	0.082	0.007	0.091	0.206	0.474	0.011
Vegetable[c]	0.888	0.036	0.003	0.026	0.011	0.025	-1.924	0.088	0.008	0.097	0.221	0.509	0.012
Juice Drinks[c]	0.727	0.030	0.002	0.021	0.009	0.021	0.013	-1.515	0.006	0.080	0.181	0.416	0.010
Carb. Water	0.675	0.028	0.002	0.020	0.008	0.019	0.012	0.067	-1.469	0.074	0.168	0.387	0.009
Water	0.918	0.037	0.003	0.027	0.011	0.026	0.016	0.091	0.008	-1.903	0.228	0.526	0.012
Diet Soda	1.161	0.047	0.004	0.034	0.014	0.033	0.020	0.115	0.010	0.127	-2.247	0.666	0.015
Regular Soda	1.284	0.052	0.004	0.038	0.015	0.037	0.023	0.127	0.011	0.141	0.319	-2.068	0.017
Tea	0.701	0.029	0.002	0.020	0.008	0.020	0.012	0.069	0.006	0.077	0.174	0.402	-1.521

[a] Price elasticities are uncompensated and conditional.
[b] 100% juice.
[c] Less than 100% juice.

Table 4. Conditional Income and Uncompensated Own-Price Elasticity Estimates At Selected Divisia Volume Indexes

Beverages	Income Elasticity			Own-Price Elasticity		
	Min[a]	Mean[b]	Max[c]	Min[a]	Mean[b]	Max[c]
Orange[d]	0.670	0.641	0.597	-1.338	-1.373	-1.411
Grapefruit[d]	0.828	0.780	0.707	-1.682	-1.700	-1.701
Apple[d]	1.510	0.971	0.150	-2.992	-2.092	-0.363
Grape[d]	1.166	0.894	0.478	-2.353	-1.940	-1.151
Rem. Fruit Juice[d]	0.889	0.828	0.735	-1.783	-1.784	-1.751
Vegetable[5]	1.199	0.888	0.415	-2.408	-1.924	-1.000
Juice Drinks[5]	0.628	0.727	0.878	-1.248	-1.515	-1.950
Carb. Water	0.735	0.675	0.584	-1.489	-1.469	-1.403
Water	0.570	0.918	1.447	-1.150	-1.903	-3.094
Diet Soda	1.209	1.161	1.089	-2.174	-2.247	-2.339
Regular Soda	1.354	1.284	1.178	-2.001	-2.068	-2.151
Tea	0.502	0.701	1.004	-1.021	-1.521	-2.384

[a] Calculated at the minimum value of the Divisia volume index sum variable.
[b] Calculated at the mean value of the Divisia volume index sum variable.
[c] Calculated at the maximum value of the Divisia volume index sum variable.
[d] 100% juice.
[e] Less than 100% juice.

Table Appendix D. Uniform-Substitute Model Standard Errors for Elasticity Estimates at Sample Means for Table 3[a]

Beverage	Income	Price											
		Orange	Grape-fruit	Apple	Grape	Rem. Fruit Juice	Vegetable	Juice Drinks	Carb. Water	Water	Diet Soda	Regular Soda	Tea
Orange[b]	0.0366	0.0769	0.0003	0.0017	0.0010	0.0020	0.0013	0.0100	0.0009	0.0109	0.0100	0.0109	0.0005
Grapefruit[b]	0.0387	0.0068	0.0818	0.0020	0.0012	0.0024	0.0016	0.0118	0.0011	0.0138	0.0109	0.0234	0.0028
Apple[b]	0.0408	0.0082	0.0004	0.0800	0.0015	0.0026	0.0018	0.0140	0.0013	0.0163	0.0117	0.0268	0.0035
Grape[b]	0.0538	0.0079	0.0004	0.0025	0.1138	0.0028	0.0019	0.0137	0.0013	0.0158	0.0146	0.0325	0.0032
Rem. Fruit Juice[b]	0.0325	0.0072	0.0004	0.0020	0.0013	0.0668	0.0016	0.0120	0.0011	0.0140	0.0101	0.0222	0.0030
Vegetable[c]	0.0451	0.0077	0.0004	0.0023	0.0014	0.0025	0.0924	0.0129	0.0012	0.0155	0.0125	0.0277	0.0032
Juice Drinks[c]	0.0351	0.0064	0.0003	0.0018	0.0011	0.0020	0.0014	0.0742	0.0010	0.0132	0.0096	0.0232	0.0027
Carb. Water	0.0338	0.0059	0.0003	0.0018	0.0011	0.0019	0.0013	0.0104	0.0733	0.0121	0.0095	0.0212	0.0025
Water	0.0649	0.0079	0.0004	0.0027	0.0016	0.0029	0.0020	0.0156	0.0014	0.1358	0.0178	0.0395	0.0034
Diet Soda	0.0208	0.0098	0.0005	0.0026	0.0017	0.0029	0.0021	0.0162	0.0015	0.0192	0.0430	0.0278	0.0042
Regular Soda	0.0213	0.0109	0.0005	0.0028	0.0019	0.0032	0.0023	0.0183	0.0017	0.0210	0.0138	0.0345	0.0046
Tea	0.0649	0.0064	0.0003	0.0024	0.0013	0.0025	0.0016	0.0121	0.0011	0.0137	0.0177	0.0384	0.1411

[a] Price elasticities are uncompensated and conditional.
[b] 100% juice.
[c] Less than 100% juice.