Segregation and Strategic Neighborhood Interaction*

Jason Barr
Department of Economics
Rutgers University, Newark
jmbarr@rutgers.edu

Troy Tassier
Department of Economics
Fordham University
tassier@fordham.edu

April 2007
Rutgers University Newark Working Paper #2007-001

Abstract

We introduce social interactions into the Schelling model of residential choice. These social interactions take the form of a Prisoner's Dilemma game played with neighbors. First, we study the Schelling model over a wide range of utility functions and then proceed to study a spatial Prisoner's Dilemma model. These models provide a benchmark for studying a combined model with preferences over like-typed neighbors and payoffs in the spatial Prisoner's Dilemma game. We study this combined model both analytically and using agent-based simulations. We find that the presence of these additional social interactions may increase or decrease segregation compared to the standard Schelling model. If the social interactions result in cooperation then segregation is reduced, otherwise it is increased.

JEL Classification: C63, C73, D62

Key words: Schelling Tipping Model, Spatial Prisoner's Dilemma, Cooperation, Segregation

1

1   Introduction

Racial and ethnic segregation in the United States continues to be common, despite survey results that show people to be increasingly opposed to the idea of racial segregation (Sethi and Somanathan, 2004).

One of the most compelling accounts of why segregation is still so widespread was provided by Schelling (1969, 1971). His model, seen as a forerunner of the current agent-based modeling paradigm in economics, shows how even a relatively small preference for neighbors of one's own "type" can lead to neighborhood tipping and a high level of segregation. Schelling's results may be considered quite pessimistic if one views segregation as a bad outcome.

While many agents, individually, may prefer living in majority-type neighborhoods for cultural or, sometimes, language-based reasons, segregation is considered a bad outcome because of the external effects that it can have on the society as a whole. For instance, it is commonly argued that racial segregation, especially when mixed with income inequality, may lead to unequal education and employment opportunities, the persistence of existing levels of income and wealth inequality, and poverty traps. As Cutler and Glaeser (1997) demonstrate, Blacks living in urban ghettos have reduced social and economics outcomes, such as high school graduation rates, labor market earnings and a higher prevalence of single motherhood rates. In general, severe segregation is associated with lower social well being.

Clearly people choose neighborhoods for many reasons beyond the racial and ethnic composition. In general, people weigh type-based preferences along with other location-based characteristics, such as the quality of the public schools, and the types of nearby stores. In addition, the utility derived from living in a specific neighborhood or community can be determined, in part, by the degree to which residents have positive interactions with their neighbors.

In this paper, we consider an extension of the Schelling model by also having agents play a repeated Prisoner's Dilemma game with their neighbors. In short, agents will determine their location choice by the outcome of both the Schelling and Prisoner's Dilemma games. Our interest here is in expanding the Schelling model to include other features beyond type-based preferences that can determine the residents' quality of life. People's utility derived from their residential choice is determined, in part, by the *actions* of their neighbors. In many cases, the actions of residents and their neighbors is

2

endogenous. One example is that of property maintenance. As Robert Frost has written, "good fences make good neighbors": if one neighbor does not contribute to the maintenance of a common fence, it will reduce the incentive of the other to maintain it as well.[1] Another example is parental involvement in the public schools. The more parents are involved, the more it will confer a positive benefit upon everyone in the school: increased student performance, a better sense of community, etc.

Social psychologists have documented a relationship between residents' sense of community, "neighboring" and personal well-being. Sense of community– a psychological perception of how well neighbors get along–has been found to be associated with a greater sense of personal well-being (Farrell, *et al.*, 2004). Neighboring is the exchange of goods and services among neighbors, such as the giving of information about good plumbers, the lending of power tools, or the provision of aid in an emergency. The willingness of neighbors to engage in these trades can directly influence residents, as the standard gains-from-trade models show, but can also improve the sense of community and, therefore, well-being (Farrell, *et al.*, 2004).

Demographers have investigated the degree to which peoples' perceptions about the quality of their neighborhood affect both their desire to move and their actual movement behavior. For example, Clark and Ledwith (2005) analyze a sample of households in Los Angeles, who were surveyed, among other things, about whether they perceived their neighborhoods as being close-knit or not, and whether they were dissatisfied with their current neighborhood. The authors found that holding constant other determinants of the desire to move, respondents who said they lived in communities that were not closely-knit were more likely to say that they desired to move.

Lee *et al.* (1994), found that the rate of neighborhood turnover increased peoples' desire to move. Turnover is clearly one measure of the degree to which people engage in neighborhood trade, under the assumption that the more turnover the less interaction among neighbors. Clearly the decision to move is not the same thing as the desire to move, but the desire to move is an important determinant of movement behavior, and the desire to move can be an indicator of the willingness to engage in neighboring.[2] Thus one

---

[1] Another interpretation of Frost's line is that residents prefer privacy or clear boundary demarcations over social interaction with their neighbors. However, the issue of taste for privacy in not included in the model.

[2] Lee, *et al.* (1994), for example, found in their sample that of the residents who did not *consider* moving, only 13.1% actually did so within a year; while 32.9% of those who

can envision an endogenous relationship between neighboring and mobility.

Recent research in economics and sociology has investigated the effect of social capital and trust on agent behavior (Glaeser, *et al.*, 2000). At the country level, greater degrees of trust among citizens has been found to increase economic growth and to decrease corruption. As well, research findings suggest that dense social networks can sustain trust; while interactions between different racial groups are often characterized by lower degrees of trust (Glaeser, *et al.*, 2000).[3] Marshall and Stolle (2004) found, in a sample of neighborhoods in the Detroit area, that there is a strong relationship between race and feelings of trust (holding income constant). They found, for example, that "neighborhood racial heterogeneity and neighborhood sociability significantly increase blacks' propensity to trust others" (p. 145). However, interestingly this finding did not hold for whites in the sample; heterogeneity appeared to increase feelings of distrust for whites. These findings are clearly complicated by the fact that interactions occur within the context of minority-majority relationships. However, the important point for this paper is that social interactions can foster trust, and that these social interactions can and do take place among agents of different races or ethnicities.

It is within this context that we introduce the Prisoner's Dilemma game into the Schelling model. Schelling's original model was designed to show how, even with strong preferences for integration, segregation was the only stable equilibrium. Our aim is to demonstrate that by introducing a model where cooperation (and therefore trust) among agents can develop, integrated neighborhoods can be an obtainable and stable equilibrium.

As many models have shown (discussed below), cooperation can be a sustainable outcome in a Repeated Prisoner's Dilemma framework under some conditions. We view the emergence of cooperation here as the development of neighborhood trust and, also by extension, as the gains that are available to neighbors when they engage in neighboring, such as the loaning of power tools, the provision of aid, and/or just being friendly (e.g., "I'll wave to you in the morning, if you wave to me").[4]

---

did consider moving, actually did so.

[3] Clearly education levels of both people and the neighborhoods they live in can help determine their trust levels, their attitudes toward other races, and their willingness to engage in neighboring. Here we don't explore the role of income or other class-related variables.

[4] Frank (1999) also points out that consumption "arms races" among neighbors can

The reason the Prisoner's Dilemma can be important within the Schelling game is that cooperation can potentially offset the loss of utility that neighbors receive when they live with neighbors of a different type. Our aim is to investigate under what conditions this can hold, and to what degree can we view segregation and cooperation as substitutes. That is to say, the emergence of trust among neighbors can offset or remove negative utility from living with different-type neighbors. To simplify matters, we model an equal proportion of agent types, as well as an equal initial proportion of agents who are "cooperators" and "defectors." Certainly the interaction of agents can be more complex when one group is a minority and the other is a majority. Sociological research has found that Black and Whites in the U.S. have different attitudes toward both integration and toward trust of neighbors (See Marshall and Stolle (2004) for example). We leave this complicating variation of the model for future work.

We will show that low levels of segregation can be supported in our model if high levels of cooperation can also be supported as an outcome of the Prisoner's Dilemma game. On the other hand, our model leads to even higher levels of segregation than are produced in the Schelling model when all agents defect is the outcome of the Prisoner's Dilemma game. Thus our model leads to the conclusion that increasing social interactions can be helpful in reducing segregation if the process of interaction leads to cooperation. But, social interactions should be limited if the interactions lead to non-cooperative outcomes. To the best of our knowledge no other paper has explored the effect that neighborhood cooperation can have in affecting the instability of integration in the Schelling model.

The paper proceeds as follows. The next section discusses the related literature. Then section 3 discusses the Schelling model with the inclusion of a utility function for agents. Next, in section 4 we introduce the repeated Prisoner's Dilemma game and the probability rules agents use in choosing whether to cooperate or not. Then, in section 5, we provide the model and results of the combined Schelling and Prisoner's Dilemma game. Finally, section 6 provides some concluding remarks.

---

negatively impact personal feelings of well-being.

2   Related Literature

Schelling Models   The Schelling model has become one of the central models cited in the genesis of agent-based economics (see Pancs and Vriend, 2007). It is widely cited as a pedagogical tool partly for the simplicity of the model and partly for the intrigue of the central result. Modest preferences for living near one's own type can lead to high levels of residential segregation. This central result has been subjected to various robustness checks over the years.

Most recently Pancs and Vriend (2007) subject the Schelling model to a series of computational specifications (one dimension versus two dimensions, line versus ring, checkerboard versus torus) and find that the central segregation result does not depend strongly on any of these specifications. Further they expand the range of preferences incorporated in the Schelling model. In the original model, the preferences of Schelling's agents were asymmetric; agents were opposed to being a member of a small minority group in their neighborhood but not to being a member of a large majority group. In short, agents did not prefer segregation but they did not oppose it either as long as they were in the majority.

A main contribution of Pancs and Vriend (2007) is in their expansion of the utility functions considered in the Schelling model. Specifically, they remove the above-described asymmetry by considering a utility function where agents prefer integration to both being in a neighborhood where they are a small minority and being in a neighborhood where they are a large majority. Agents have single peaked, tent shaped preferences where a 50-50 integrated neighborhood is strictly preferred to all other neighborhood compositions (even one with an agent's neighborhood being composed exclusively of their own type.) Somewhat amazingly, they find that the Schelling model still produces large amounts of residential segregation. In our model, we also will investigate a large range of utility functions including a tent shaped function where all agents prefer more integration to less.

Recently the Schelling model also has been subjected to more rigorous analytical analysis from the perspective of evolutionary game theory (Zhang, 2004 and Dokumaci, 2006). This work attempts to understand the creation of segregation in the Schelling model from a formal analytical perspective.

The Prisoner's Dilemma   Studies of the maintenance of cooperation in the Prisoner's Dilemma are vast. Economists are long familiar with the

"folk theorem" (Fudenberg and Tirole, 1996), which says that agents can maintain cooperation in an infinitely repeated prisoner's dilemma as long as the future is not discounted too heavily.

More recently, other means of maintaining cooperation in the Prisoner's Dilemma have been studied. Examples range to include reputation (Nowak and Sigmund, 1998), reciprocity (Axelrod, 1984), the use of tags and signals to recognize opponent types (Riolo, 1997; Riolo *et al,* 2001; Hales, 2001; Janssen, 2007), and withdrawal from play (Aktipis, 2004; Janssen, 2007).

Most closely related to the implementation of the Prisoner's Dilemma in our paper is the work on the maintenance of cooperation in spatial models (Nowak and May, 1992; Nowak, Bonhoeffer and May, 1994; Schweitzer, Behera and Muhlenbein, 2002). In these models it is shown that repeated interaction with local neighbors may lead to the evolution of cooperation in a Prisoner's Dilemma.[5]

These spatial models are especially pertinent for our purposes because the Schelling model is clearly spatial as well. Therefore we have chosen to follow this line of research to implement a model of the Prisoner's Dilemma into the Schelling model. Specifically, we have chosen the framework of Nowak, Bonhoeffer and May (2004) for our model. Agents in the traditional Schelling model choose to move or stay based on agent types in a specified local neighborhood. In our model agents also will consider a second element in their utility function: the results of the outcomes in a Prisoner's Dilemma game with those agents located in the same specified neighborhood. We describe our model in more detail in the sections below.

3   The Schelling Model with Utility

Our paper will consist of three models. The first two will be stand-alone versions of the Schelling Segregation model and an implementation of a neighborhood-based Prisoner's Dilemma model, similar to Nowak, Bonhoeffer and May (1994). We present the results of these two models independently in order to have a benchmark for comparison to our last model which combines elements of both of the individual models.

In our first experiment we implement the standard Schelling model with a utility function. We have two types of agents. Each type of agent is different

---

[5]The introduction to Schweitzer, Behera and Mühlenbein (2002) provides a nice overview of the literature on cooperation in the Prisoner's Dilemma.

in some recognizable way from the other type, and all else equal, each agent has a utility function over the proportion of agents of the same type. We assume here that all agents have the same utility function.

In our implementation we use a 12×12 grid with 14 spots (roughly 10%) left empty. 65 of the agents are one type, and 65 are the other. Each agent interacts with the agents within her "Moore" neighborhood; that is, an agent's neighborhood consists of the eight surrounding agents. If an agent is in a corner or on an edge, then the agent will have fewer neighbors. The lattice is not a "wrapped" around torus. We make this assumption because we believe this is a more accurate model of the geographic/spatial patterns observed in physical residential neighborhoods.[6]

An agent's utility is determined based on the proportion of neighbors that are of the same type. The function is given by

$$u(s) = \left\{ \begin{array}{c} \alpha + 2(\beta - \alpha)s \text{ if } s < 0.5 \\ [\beta + \gamma(\beta - \alpha)] - \gamma 2(\beta - \alpha)s, \end{array} \right\},$$

where $s \in [0, 1]$ is the proportion of neighbors that are the same as the agent. $\gamma \in [0, 1]$ is our measure of the agents' preference for integration. This utility function is rising from 0 to 0.5 and peaks at 0.5 before decreasing. γ determines the degree of "steepness" of the negatively-sloped portion of the utility function.

Figure 1 presents a graph of the utility functions for different values of γ. When $\gamma = 0$, agent's utilities are rising from $s = 0$ to $s = 0.5$, after that point, agents are indifferent between being barely in the majority and having all neighbors of the same type. If $\gamma \in (0, 1]$, then agents globally prefer to be in a neighborhood in which exactly half of their neighbors are like them. For $\gamma \in (0, 1]$ agent utility strictly decreases as the agent moves away from a $50 - 50$ neighborhood.

We use these utility functions as benchmarks, to compare with our results presented in section 5, when we combine the Schelling game with the repeated Prisoner's Dilemma game.

Initially, agents are randomly distributed on the lattice. Then an agent is selected in turn, and the agent's utility is calculated.[7] Next a randomly open spot is chosen. The agent then compares her utility from the new spot (as

---

[6]In addition, wrapping or not wrapping edges has been shown not to be crucial to the emergence of segregation in the Schelling model (Pancs and Vriend, 2007).

[7]Since agents are randomly placed on the board, when we say that agents are selected "in turn," we mean that we begin with "agent 1" who could be located anywhere, then
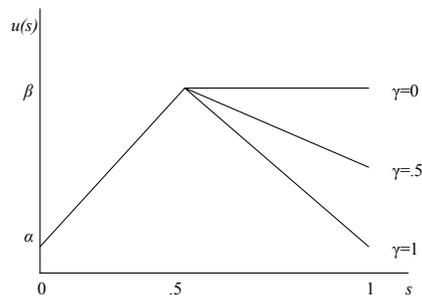
Figure 1: Different utility functions for an agent's prefence for integration.

if she was living there). If the utility of the new spot is greater than the old spot, she moves, otherwise she stays. We repeat this process for each agent in the model. Each "run" of game goes until 100,000 iterations or when agents stop moving, whichever comes first.[8]

In Schelling's (1971) model, agents are initially placed in a perfectly integrated neighborhood. Some agents are then randomly "removed." The remaining agents determine if they are satisfied by looking at the proportion of neighbors who are like them; if the proportion of like-typed agents is below a certain threshold (e.g., 50%) they are put on a list of dissatisfied agents (here we call this threshold the "Schelling threshold"). When the agent's "turn" comes up he moves to the nearest available satisfactory position. The process then continues till no agent wants to move; in the end, the board is typically highly segregated. As is now well-known, the ability of agents to leave neighborhoods when they are not satisfied with the proportion of agents like them causes a "tipping" dynamic that generates a highly segregated outcome.

For this section, we use two measures of segregation: *Similarity* and

we go to "agent 2," who could be anywhere, then on to "agent 3," etc. Then we repeat this process starting with "agent 1."

[8]A run going for 100,000 iterations means that agents have up to 790 opportunities to move. A run going for this many observations is quite rare. Furthermore, if no agents move in a round, that does not mean they would not move in future rounds, but we end the game at that point because it is sufficiently close to being an equilibrium.

Homogeneity.[9] We define *Similarity* as the average proportion of same-type neighbors for each agent across all neighborhoods. We define *Homogeneity* as the average fraction of agents that live in neighborhoods that are all of the same type.[10] Denote the agent type as t. Let the agent of one type be $t = 0$; and an agent of the other type be $t = 1$, then we have the following definition for measure of Similarity:

$$Similarity = \frac{1}{N} \sum_{i=1}^{N} s_i,$$

$$s_i = \left[\frac{1}{n_i} \sum_{j=1}^{n_i} |1 - t_i - t_{ij}| \right],$$

where t_i is agent i's type and t_{ij}, $j = 1, ..., n_i$, are the types of agent i's neighbors; N is the total number of agents. Thus s_i is the proportion of neighbors that are the same as agent i. For Homogeneity we have

$$Homogeneity = \frac{1}{N} \sum_{i=1}^{N} \chi_i,$$

$$\chi_i = \left\{ \begin{array}{ll} 1, & \text{if } s_i = 1 \\ 0, & \text{otherwise} \end{array} \right\}$$

We repeat this version of the Schelling model experiment 200 times and take averages over the runs. Figures below give examples of typical runs for four values of γ to demonstrate how the utility functions can affect segregation. We can see that γ directly affects the equilibrium level of segregation. The white squares are type 0 agents, the black squares are type 1 agents, and the gray squares are empty.

---

[9] Our measures of segregation produce qualitatively similar results as other measures commonly used in empirical studies such as the Dissimilarity Index (Duncan and Duncan 1955). Also note that most of these commonly used measures are not amenable to our population. For instance the dissimilarity index is sensitive to neighborhood size. If one is working with a city population in the thousands or hundred-thousands with hundreds or thousands of neighborhoods (as is common in many large cities) the sensitivity is reduced. However the dissimilarity index would be sensitive to the choice of neighborhood size for the population size used here.

[10] Schelling (1971) refers to these two measures as the "Share," and the "Getto rate," respectively.

$\gamma = 0$ $\gamma = .3$

$\gamma = 0.7$ $\gamma = 1$
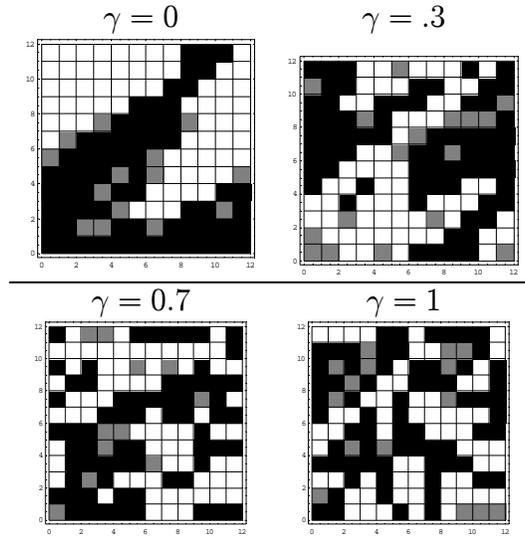
Even when $\gamma = 1$, we can still get neighborhood clusters of like type (as indicated by the patch of blocks of the same type), but globally speaking, on average each agent has half of her neighbors of a different type; and, few, in any, agents live in a completely segregated neighborhood. Table 1 presents the results; figure 2 presents the results graphically.

γ	% **Similarity**	% **Homogeneity**
0.0	85.5	54.1
0.1	84.0	51.4
0.2	80.9	42.4
0.3	74.6	27.0
0.4	73.8	26.4
0.5	66.8	15.3
0.6	62.5	10.1
0.7	59.6	6.9
0.8	55.1	3.3
0.9	52.3	1.7
1.0	49.6	0.8

Table 1: Measures of segregation for different utility functions. Averages over 200 replications.
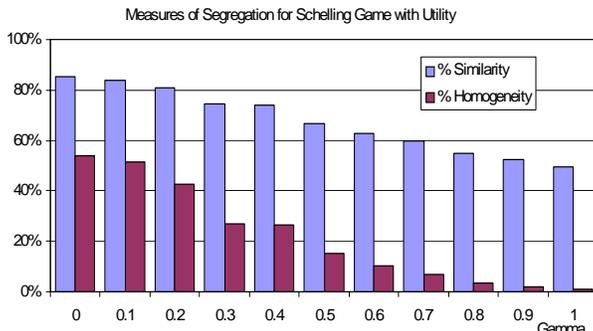
Figure 2: % Similarity and % Homogeneity for different utility functions.

The table illustrates a few facts. Segregation is monotonically decreasing in γ, as would be expected. Notice that when $\gamma = 0$, over one-half of the agents have all of their neighbors being of the same type, on average. Further, on average, 85.5% percent of neighbors are of the same type.

Note that the $\gamma = 1$ case produces results very similar to perfect integration, a $50 - 50$ neighborhood with no agents living in a purely homogeneous neighborhood. As we decrease γ the model produces increasing levels of segregation along both of the measures that we consider.

In addition, we have run a series of comparison tests. First, we ran the model with no utility function and having agents move to a randomly chosen open spot, if they had less than $1/3$ like neighbors; we then repeated this, but with the threshold equal to $1/2$. In addition, we ran the model where agents compare the utility of their current location to that of a new location, where the utility function is given by

$$u\left(s\right) = \left\{ \begin{array}{c} 0 \text{ if } s < \bar{s}, \ \bar{s} \in \left\{\frac{1}{3}, \frac{1}{2}\right\} \\ 1, \text{ otherwise} \end{array} \right\}. \tag{1}$$

This is the similar to the standard Schelling rule, but with a utility comparison added. Finally, we also included the average value of the Similarity and Homogeneity measures for 200 randomly generated initial boards. The results of these exercises are shown in Table 2.

The table illustrates that our baseline utility function ($\gamma = 0$) produces about the same segregation as the two cases where the threshold is $1/2$. Changing the threshold from $1/2$ to $1/3$ generally reduces segregation since

12

there are more neighborhoods that will be acceptable to agents. Having the threshold at 1/3 is about the same as having γ equal to 0.3 or 0.4. Interestingly, having agents make the utility comparison appears to reduce segregation by a small amount.

Lastly, the Random Board outcome is roughly the same as the $\gamma = 1$ outcome. Also note that in the $\gamma = 1$ case, agent movement will generally stop before $100,000$ iterations; meaning that the results of $\gamma = 1$ are not due simply to the fact that agents are constantly on the move.[11]

Version	% Similarity	% Homogeneity
Move if $s < \frac{1}{3}$	75.7	33.0
Move if $s < \frac{1}{2}$	87.4	59.2
Utility eq. (1), $s < \frac{1}{3}$	71.5	25.4
Utility eq. (1), $s < \frac{1}{2}$	84.5	51.4
Random Board	49.6	2.1

Table 2: Comparison tests for the Schelling model.

4   Repeated Prisoner's Dilemma

Putting aside the Schelling framework for a moment, we now turn to a repeated Prisoner's Dilemma (RPD) game. As is standard for the PD game, agents choose to play one of two actions: Cooperate or Defect. The general payoffs of the game are given in table 3.

	Rival Cooperate	Rival Defect
Agent Cooperate	A	B
Agent Defect	C	D

Table 3: Prisoner Dilemma payoffs for an agent, when $C > A > D > B$.

---

[11]For all utility functions, movement stops, on average, after 16,203 iterations. We have not found a strong relationship between the value of γ and the average number of iterations until no movement occurs.

As is well known, a one-shot Prisoner's Dilemma has only one Nash equilibrium: {Defect, Defect}. However, as is also well known, in a RPD game when the game will not end with certainty, the folk theorem says that any outcome in a RPD is a sustainable equilibrium given appropriate discount values. As mentioned in the introduction there is now a large literature beyond the folk theorem on the maintenance of cooperation in the Prisoner's Dilemma. Here we use the basic framework of Nowack, Bonhoeffer and May (1994) to install a RPD into the Schelling model. We note that this is just one possible choice of many for incorporating a Prisoner's Dilemma game that has the possibility of resulting in cooperative outcomes. Although we will report results on the outcome of the RPD game our primary focus is on the segregation outcomes when this game is combined with the standard Schelling model. Thus we restrict attention in this paper to our chosen implementation of the RPD and save additional comparisons to other implementations for future work.

We begin with a stand alone version of the RPD in order to have a benchmark for later comparison. In this paper we explore a game where each round each agent plays a one-shot prisoner's dilemma game with each of their neighbors. To simplify matters, we will set $C = A + \varepsilon$ and $D = B + \mu$, where, $\mu, \varepsilon > 0$, and we fix A, B, and μ (μ is fixed at a small value so as to preserve the PD payoff structure), and we change ε over different experiments. By changing ε we change the relative incentive to "cheat" in the RPD game. As ε increases, agents have a greater incentive to defect against a cooperating opponent; thus sustaining cooperation becomes less likely.

4.1   The RPD on the Lattice: The Model

In this paper, agents play with their neighbors, who as discussed above, are within each agent's "Moore" neighborhood. Each agent has a probability, $p_i \in [0, 1]$, of playing cooperate (and $1 - p_i$ is the agent's probability of playing defect). In this section, agents do not move.[12] When an agent is selected, she chooses an action $\{C, D\}$ according to her probability distribution function. Once an action is selected, she plays that same action with all her neighbors

---

[12]Note that agent movement can create different outcomes if agents can compare PD payoffs across locations. We do not address this type of movement here since it is not directly relevant for the comparison with the combined game section below. Furthermore, we also find that if agents move to new locations at random, there is no qualitative difference with the results presented here.

for that round. She plays the PD with each neighbor, who selects an action based on their own probability distribution function.

For any two players, define the payoff to agent i when playing a neighbor agent j as

$$\pi_i\left(x_i; x_{ij}\right) = Ax_i x_{ij} + Bx_i\left(1 - x_{ij}\right) + C\left(1 - x_i\right)x_{ij} + D\left(1 - x_i\right)\left(1 - x_{ij}\right), \quad (2)$$

where $x_i, x_{ij} = 1$ if a player cooperates; 0 if the player defects. x_{ij} represents the action of agent $i's$ neighbor agent $j \in \{1, ..., 8\}$. If we set $C = A + \varepsilon$ and $D = B + \mu$, and rearrange terms, equation (2) can be written as

$$\pi_i\left(x_i; x_{ij}\right) = Ax_{ij} + B\left(1 - x_{ij}\right) + \left[\varepsilon x_{ij} + \mu\left(1 - x_{ij}\right)\right]\left(1 - x_i\right).$$

If agent i plays against n_i neighbors, then we have the average payoff as:

$$\begin{aligned}
\bar{\pi}_i\left(x_i; x_{ij}\right) &= \frac{1}{n_i}\sum_{j=1}^{n_i}\pi_i\left(x_i; x_{ij}\right) \qquad\qquad\qquad\qquad (3)\\
&= A\rho_{ij} + B\left(1 - \rho_{ij}\right) + \left[\varepsilon\rho_{ij} + \mu\left(1 - \rho_{ij}\right)\right]\left(1 - x_i\right),
\end{aligned}$$

where $\rho_{ij} = \frac{1}{n_i}\sum_{j=1}^{n_i}x_{ij}$. Thus $\bar{\pi}_i\left(x_i; x_j\right) = \pi_i\left(x_i; \rho_{ij}\right)$. That is, agent $i's$ average payoff is determined by the proportion of neighbors playing cooperate. Notice that the agent will receive a weighted sum of $A\rho_{ij} + B\left(1 - \rho_{ij}\right)$ regardless of the agent's action. However, if the agent chooses to defect, she also will receive a "defectors bonus" given by $\varepsilon\rho_{ij} + \mu\left(1 - \rho_{ij}\right)$, a weighted sum of the defection payoffs.

Each time an agent finishes playing with her neighbors we update her p_i. To update p_i we use the rule from Nowack, Bonhoeffer and May (1994), which is given by the following formula (we do not include time subscripts to simplify notation):

$$p_i = \frac{\sum_{j=1}^{n_i}\pi_{ij}\left(x_{ij}; x_i\right)x_{ij}}{\sum_{j=1}^{n_i}\pi_{ij}\left(x_{ij}; x_i\right)}$$

where $x_i, x_{ij} = 1$, if an agent cooperates, 0 otherwise. The rule is an imitation rule where agents change their probability based on the strategies of their neighbors. Note how this rule allows for the development of social capital in that it implicitly allows for reciprocity: if a high proportion of neighbors cooperate, then the agent will be more likely to cooperate the next time the agent and her neighbors interact. Reciprocity has been found to be a common behavior in economic activities (Fehr and Gächter, 2000).

15

Thus, as "cooperate" ("defect") becomes a more lucrative action for the neighbors of agent i, p_i increases (decreases). Also note that if all of an agent's neighbors play cooperate (defect) then the agent will play cooperate (defect) in the next round with probability 1. Thus the system reaches an absorbing state whenever all agents play the same action in a given round.

This formula can be rewritten according to equation (3) as

$$p_i = \frac{[Ax_i + B(1-x_i)]\,\rho_{ij}}{[Ax_i + B(1-x_i)] + [\varepsilon x_i + \mu(1-x_i)]\left(1 - \rho_{ij}\right)}. \tag{4}$$

Notice that, *cet. par*, $\partial p_i / \partial \rho_{ij} > 0$, which means that an increasing level of cooperation in the neighborhood of agent i increases the probability that agent i will cooperate in the next round. This can generate a type of "cooperative tipping." As demonstrated below, if ε is not too large, all agents having $p_i = 1$ can become an equilibrium (absorbing state).

4.1.1 Equilibria

For the probability updating rule given by equation (4), there are two pure strategy equilibria (everyone plays cooperate or everyone plays defect) and no mixed strategy equilibrium.

Claim 1 *There are two pure strategy absorbing states of the model, $p_i = 1$ for all i and $p_i = 0$ for all i, and no mixed strategy absorbing states.*

Proof. To prove this, we begin with identifying the value of ρ_{ij} where player i will be indifferent between playing $x_i = 1$ or $x_i = 0$: Thus we set $\left(p_i | x_i = 1; \rho_{ij}\right) = \left(p_i | x_i = 0; \rho_{ij}\right)$. This gives the condition

$$\frac{B\rho_{ij}}{B + \left(1 - \rho_{ij}\right)\mu} = \frac{A\rho_{ij}}{A + \left(1 - \rho_{ij}\right)\varepsilon}.$$

Rearranging terms gives

$$\rho_{ij}\left(1 - \rho_{ij}\right)\mu A - \rho_{ij}\left(1 - \rho_{ij}\right)\varepsilon B = 0. \tag{5}$$

Assuming $A/B \neq \varepsilon/\mu$, then the only solutions to equation (5) are $\rho_{ij} = 1$ and $\rho_{ij} = 0$, $\forall i$. ∎

Next we turn to the selection of these equilibria over repeated plays.

16

4.2　The RPD on the Lattice: Experiments

Here, we fix the payoffs (given in Table 4) and vary $\varepsilon \in \{0, 0.01, 0.02, ..., 0.10\}$. Given the probability updating rule, there is a range of ε and μ where everyone cooperating is an attainable absorbing state (where "attainable" here means that it occurs with non-zero probability over repeated runs of the RPD game). Notice, there are an infinite number of payoff values that can be used, but, ε and μ must be relatively small. We leave for further research a more detailed exploration of the relationship between cooperation and the payoff space.

	Rival Cooperate	Rival Defect
Agent Cooperate	5	3
Agent Defect	$5 + \varepsilon$	3.01

Table 4: Payoffs for RPD experiment.

To perform the experiment, we assign half the agents an initial probability of 1 and the other half a probability of 0.[13] As discussed above, each agent is chosen in turn, and he randomly picks an action according to his probability function. He then plays all his neighbors, who also randomly pick an action. Note the agent selects an action once and plays the same action for that round. After he plays the game with all his neighbors, he observes his rivals' payoffs and updates his probability, p_i. We run the system for 100,000 iterations or until the system reaches an absorbing state, which ever comes first.[14] We then take averages of 200 runs to smooth out fluctuations. The figures below give illustrations of two runs where $C = 5.05$. The figures present, for each iteration, the average of all agents' probability of cooperation.

$C = 5.05$, Avg. Probabilities

---
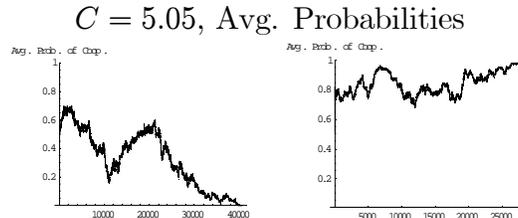
[13]We have checked other initial conditions such as $p_i = 1/2$ for all i, and other similar initial distributions, and have found that the results to be qualitatively similar.

[14]Note that the system running for 100,000 iterations is statistically rare, occurring roughly 5% or less across C payoffs.

Table 5 presents the results; Figure 3 presents the results graphically. The results presented are the proportion of runs (out of 200) that end in the game hitting an absorbing state of all agents playing defect with probability one. As the results show, the smaller the value of ε (the smaller the value of the "cheat" payoff, C), the smaller the proportion of runs that end in all agents defecting; the remaining runs end with all agents cooperating. As shown in the table, cooperation can be attained in a large percentage of runs as long as the incentives to cheat are sufficiently small.

C Payoff	% Defect
5.00	30.5
5.01	46.6
5.02	55.4
5.03	58.3
5.04	72.1
5.05	81.2
5.06	82.6
5.07	89.8
5.08	91.5
5.09	93.5
5.10	97.9

Table 5: Percentage of RPD games that end in all agents defecting with probability 1.

5   Combined Schelling and PD games

After having demonstrated the results in the stand-alone Schelling model and our implementation of the RPD we now move to analyzing the combined game. Recall that the purpose of this section of the paper is to analyze the effect of non-type based social interactions in a Schelling model. Here we are modeling the additional social interactions as a RPD game with neighbors as defined in the Schelling model.

In the combined game, we have total per-period utility given by:
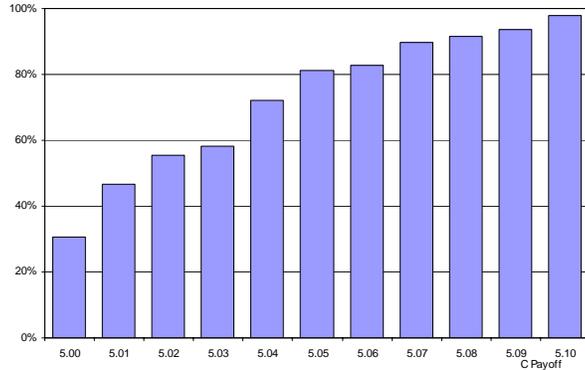
$$U(s, x) = \phi u(s) + \pi(x; \rho),$$

(6)

Figure 3: Percentage of RPD games that end in all-defect (over 200 runs).

where $x = 1$ if an agent cooperates, and $x = 0$ if an agent defects; ρ is the proportion of neighbors who cooperate. ϕ scales the relative importance of the Schelling portion of utility to the RPD portion of utility. As ϕ increases the importance of the Schelling game increases. Notice a few simplifying assumptions that we make. First the utility function is simply the sum of two game payoffs; there is no interaction effect between the two parts. Certainly we could imagine a situation where the payoff to the Prisoner's Dilemma would be determined, in part, by the number of like-type agents; and perhaps cooperation could be more likely if an agent is playing with mostly her own type as in the signalling examples of cooperation in the RPD mentioned in the introduction. In addition there is no asymmetry in utility across types. We leave these complicating variations for future work.

As in the original Schelling model, we will define a threshold utility above which agents are satisfied and remain at their current location; agents whose utility is below the threshold are not satisfied and subsequently move. In this implementation we set the Schelling portion of utility to be negative and the RPD portion of utility to be positive and set the threshold at 0. Thus agents need to have sufficiently positive RPD utility in order to be satisfied. Further, we can use ϕ to scale the importance of the two components of the utility function. One also can think of changes to ϕ as an indirect adjustment of the move threshold. As we increase ϕ we increase the weight on the negative portion of utility and thereby increase the movement threshold indirectly. As ϕ increases, either agents need to achieve larger RPD payoffs or live in a

19

neighborhood with more like typed agents in order to be satisfied.

This general framework means that agents gain utility by moving to neighborhoods with higher concentrations of their type (here we fix $\gamma = 0$), and/or by learning to cooperate or defect depending on the actions of their neighbors. We show here that tradeoffs exist between the two portions of utility: a high cooperating neighborhood can offset living in a neighborhood with few of an agent's same type. Recall that for the Schelling game we use the following utility function (assume $\gamma = 0$):

$$
\begin{aligned}
u(s) &= \alpha + 2(\beta - \alpha)s, \text{ if } s \in [0, .5) \\
&= \beta, \qquad \text{if } s \in [.5, 1].
\end{aligned}
$$

To make the utility from the two games directly comparable, we set α and β equal to minus the RPD payoffs, $-A$ and $-B$, respectively, and scale the Schelling utility according to the parameter ϕ:

$$
\begin{aligned}
u(s) &= -\phi[A + 2(B - A)s] \text{ if } s \in [0, .5) \qquad (7) \\
&= -\phi B, \qquad \text{if } s \in [.5, 1] \text{ (assume } \gamma = 0),
\end{aligned}
$$

where $\phi > 0$ is the relative weight given to $u(s)$ in the combined utility function.

5.1   Combined Utility

If we substitute the Schelling utility and the RPD payoff function into equation (6), and rearrange terms, we can write the combined utility function as (with subscripts dropped)

$$
\begin{aligned}
U(s, x; \phi) &= (A - B)\rho + A(1 - \phi) + (A - B)(\phi 2s - 1) + [\varepsilon\rho + \mu(1 - \rho)](1 - x) \text{ if } s \in [0, .5) \\
&= (A - B)\rho + B(1 - \phi) + [\varepsilon\rho + \mu(1 - \rho)](1 - x), \qquad \text{if } s \in [.5, 1].
\end{aligned}
$$

Here combined (total) utility may be positive or negative depending on the values of ρ, s, and ϕ. First, consider the case where $s \in [0, .5)$. The first and last terms are clearly positive, while the middle two terms can be positive or negative depending on the size of ϕ. Below we trace through the various elements of the utility function more carefully in order to illuminate the computational results that follow.

5.1.1 The Case of $\phi = 1$

For a moment, suppose that each of the two components of utility have equal weight, this is the case when $\phi = 1$. Therefore we have

$$
\begin{aligned}
U(s, x; \phi) &= (A - B)\rho + (A - B)(2s - 1) + [\varepsilon\rho + \mu(1 - \rho)](1 - x) \ \text{if } s \in [0, .5) \\
&= (A - B)\rho + [\varepsilon\rho + \mu(1 - \rho)](1 - x), \qquad\qquad \text{if } s \in [.5, 1].
\end{aligned}
$$

Recall that if an agent cooperates then $x = 1$ and if an agent defects then $x = 0$. Consider the case when $s > 0.5$. In this case the Schelling utility is a constant and only the level of ρ and the agent's action (cooperate or defect) determine utility. In the case where $\phi = 1$ and $s > 1/2$ the agent always has positive utility and will not move regardless of the of outcome of the Prisoner's Dilemma game with her neighbors.

Now, consider the case where an agent cooperates ($x = 1$) and we have $s < 0.5$. In this case, the utility function is given by $(A - B)\rho + (A - B)(2s - 1)$. Therefore utility will be positive and an agent will not move if $(A - B)(\rho + 2s - 1) > 0$. This implies that the agent needs $\rho + 2s > 1$ in order to be satisfied. Note that there is a substitution effect between the amount of cooperation in the neighborhood and the fraction of like-typed neighbors. High levels of cooperation can induce an agent to remain in a neighborhood in which she is a minority member. Further, if $\rho = 1$ then any level of s will yield utility greater than or equal to 0. In this case the agent will not move. In sum, when $\phi = 1$, the all-cooperate absorbing state makes an agent satisfied even if he is a very small minority in his neighborhood.

Now, if the agent defects ($x = 0$) and $s < 0.5$, then the agent receives an added utility bonus of $\varepsilon\rho + \mu(1 - \rho)$. Thus a defecting agent is more likely to have positive utility and stay than a cooperating agent, but the tradeoff between cooperation and like-typed agents still exists. Agents will stay in a non-cooperating neighborhood if they have sufficient like-typed neighbors and agents will stay in a neighborhood that has few like typed-neighbors if there is sufficient cooperation.

Notice however that the dynamics of this process are complex. Even though there exists a tradeoff between cooperation and having sufficient like typed neighbors, a neighborhood with high levels of cooperation is attractive to defecting agents. A defecting agent in a highly cooperative neighborhood will receive large payoffs and thus lead to other agents becoming more likely to defect themselves. This would lead to agents wanting to move to neigh-

borhoods with more like-typed neighbors. In the remainder of this paper we will investigate the equilibrium conditions and the dynamics of this process.

5.1.2 Equilibria when $\phi = 1$

We proceed to discuss an equilibrium of the combined game when $\phi = 1$. An equilibrium requires that (1) the RPD component of the game has reached an absorbing state, and (2) all agents in the population have utility greater than or equal to 0 so that no agent changes location. Above, we sketched the requirements for agents not to move. We can now state these requirements a bit more formally as functions of s and ρ for a cooperating and a defecting agent in a combined game equilibrium.

In an equilibrium where all agents cooperate, $x = \rho = 1$ for each agent. Recall that when $x = 1$, for no movement to occur we need $\rho + 2s > 1$ when $\phi = 1$. Since $\rho = 1$ in equilibrium, this inequality always holds in an all cooperate equilibrium. This means that an agent will accept any level of integration or segregation at this equilibrium. But the level of segregation that emerges will be a result of the complex, stochastic dynamics that unfold.

On the other hand, if we have the all-defect equilibrium then $x = \rho = 0$. In this equilibrium, an agent will not move if $s \geq \frac{1}{2}\left(\frac{A-D}{A-B}\right)$, since this is the value of s for which utility is greater than or equal to zero. Since $D > B$, this threshold will be achieved when s is close to, but less than, $s = 1/2$. That is to say, even in the case where everyone defects, less segregation than in the "pure" Schelling case is possible. Notice that if D were to be increased, it would allow an even lower level of segregation to exist. Interestingly, as will be discussed below, we find in the simulation results, then when all agents are in the all-defect absorbing state, the average level of segregation is higher than in the "pure" Schelling game, without the RPD game; this is due do what occurs "out of equilibrium."

If we compare this to the equilibrium with all agents cooperating we see that an equilibrium where all agents defect requires a sufficient level of like-typed neighbors. The all-cooperate equilibrium can be sustained for any level of s. If we think about these equilibrium outcomes in the spirit of Schelling's original model we see that an equilibrium with all-defect being the RPD outcome implies that there will be a non-zero threshold for like-typed neighbors for each agent. Thus we are in a situation very similar to the original Schelling model and should expect high levels of segregation as in the original model. On the other hand, if we reach an all-cooperate absorbing

22

state of the RPD then this threshold for like-typed neighbors is removed thereby creating the possibility of integration in the Schelling model.

This leads to several questions: (1) Under what conditions can we reach the all-cooperate absorbing state in the combined game; just because an equilibrium exists, does not mean that we can attain it? (2) If we reach the all-cooperate absorbing state, what level of segregation will be attained? Recall that the all-cooperate absorbing state can co-exist with any level of segregation when $\phi = 1$. Integration is possible but segregation is possible as well.

We next describe the equilibrium conditions for the more general model (we allow for $\phi \neq 1$) and then proceed to explore the parameter space of our model using simulations.

5.2   The General Case

We now move to the general case with $\phi > 0$. Recall that an agent's utility function is:

$$
\begin{aligned}
U(s, x; \phi) &= (A - B)\rho + (A - B)(\phi 2s - 1) + A(1 - \phi) + [\varepsilon\rho + \mu(1 - \rho)](1 - x) \text{ if } s \in [0, .5) \\
&= (A - B)\rho + B(1 - \phi) + [\varepsilon\rho + \mu(1 - \rho)](1 - x) \qquad\qquad \text{if } s \in [.5, 1].
\end{aligned}
$$

Given whether the game is in an all-cooperate or all-defect state, we can solve for the levels of s as a function of ϕ and the PD payoffs that will create positive utility and thus give agents the desire to stay. We call this level of s the *Implied Schelling Threshold* of the combined game:

Definition: *The Implied Schelling Threshold, s^*, is the minimum level of like-typed neighbors that can support a given equilibrium of the RPD game (all-cooperate or all-defect) in the combined game.*

One should think of s^* as creating the same effect as the threshold in the original Schelling model. A larger threshold should produce more segregation than a smaller threshold. As we will see below s^* will be very important in understanding the outcomes of the model.

For an all-cooperate equilibrium s^* is given by:

$$
s^* = \left\{ \begin{array}{c} \frac{A(\phi-1)}{2\phi(A-B)} \text{ if } \phi \in \left(1, \frac{A}{B}\right] \\ 0 \text{ if } \phi \in (0, 1] \end{array} \right\}. \tag{8}
$$

In short, for $\phi \in (0, 1]$, any level of liked-typed neighbors can be supported in the all-cooperate equilibrium. For $\phi \in \left(1, \frac{A}{B}\right]$, the ability to sustain

integration will depend on both the value of $\frac{A}{B}$ and how close ϕ is to it. Thus there is a non-zero Implied Schelling Threshold level of like-typed neighbors that must occur to support the all cooperate equilibrium for $\phi \in (1, \frac{A}{B}]$. But for $\phi < 1$, $s^* = 0$, i.e., there is no threshold. Note that this meets with intuition; as ϕ increases the Schelling game becomes more important and we should expect a "Schelling-style" threshold to emerge as ϕ increases. Also note that for $\phi > \frac{A}{B}$, no all-cooperate equilibrium exists since utility is always less than 0 for all levels of s.

For an all-defect equilibrium, s^* is given by:

$$s^* = \left\{ \begin{array}{c} \frac{A\phi - D}{2\phi(A-B)} \text{if } \phi \in \left(\frac{D}{A}, \frac{D}{B}\right] \\ 0 \text{ if } \phi \in \left(0, \frac{D}{A}\right] \end{array} \right\}. \tag{9}$$

Note that $s^* = 0$ at $\phi = \frac{D}{A}$. Thus at this level of ϕ and below any level of like-typed neighbors can be supported in an all defect equilibrium. The threshold increases as ϕ increases up to $\phi = \frac{D}{B}$ where $s^* = 1/2$. Also note that the range of $\phi \in \left(0, \frac{D}{B}\right]$, is just the range for ϕ which preserves the structure of the Schelling and PD games. No all-defect equilibrium exists for $\phi > \frac{D}{B}$ since utility is always less than 0 for all levels of s.

If we compare s^* in the all-cooperate equilibrium to s^* in the all-defect equilibrium, we see that a lower ϕ is required in the all-defect equilibrium to have $s^* = 0$. This implies that there must be less weight on the Schelling portion of utility in an all-defect equilibrium in order to remove the threshold. This leads one to believe that in order to achieve integration there has to be a smaller weighting on the Schelling portion of utility in an all-defect equilibrium than in an all-cooperate equilibrium. We can also directly compare s^* for the entire range of feasible levels of ϕ.

Claim 2 *(1) The Implied Schelling Threshold, s^*, is equal to 0 for $\phi \leq \frac{D}{A}$ in both the all-cooperate equilibrium and the all-defect equilibrium.*

(2) The Implied Schelling Threshold, s^*, is lower in an all-cooperate equilibrium than in an all-defect equilibrium for $\phi \in (\frac{D}{A}, \frac{A}{B})$.

Proof. (1) is shown directly in equations (8) and (9) above. For (2) we need to consider two ranges of ϕ. For $\phi \in (\frac{D}{A}, 1]$, $s^* = 0$ in an all-cooperate equilibrium and $s^* = \frac{A\phi - D}{2\phi(A-B)} > 0$ for an all defect equilibrium. For $\phi \in (1, \frac{A}{B})$ we have $s^* = \frac{A\phi - A}{2\phi(A-B)}$ for the all-cooperate equilibrium and $s^* = \frac{A\phi - D}{2\phi(A-B)}$ for the all defect equilibrium. Now, $A\phi - A < A\phi - D$ since $A > D$. Thus the

24

Schelling threshold is strictly lower in the all-cooperate equilibrium over this range of ϕ. ∎

This claim states that s^* in an all-cooperate equilibrium is less than or equal to s^* in an all-defect equilibrium for all levels of ϕ. Thus we should expect lower levels of segregation when an all-cooperate equilibrium is reached compared to when an all-defect equilibrium is reached.

5.3   Combined Game Results

We now present simulation results of the combined game. All results are averages over 1000 runs of the simulations for each value of C. To review, agents are randomly distributed over the lattice; one-half of the agents are type 0; the other one-half are type 1. Initially, one-half of the agents play cooperate with probability zero; the other one-half play cooperate with probability 1. The Prisoner's Dilemma game payoffs are fixed according to Table 4 (note ε is a parameter that we change over different simulations). We use the our base utility function for the Schelling part, with $\gamma = 0$.

Each agent is selected in turn, and the agent plays the Prisoner's Dilemma with her neighbors according to her probability distribution function. The neighbors also choose an action according to their probability distribution functions. After the agent plays with her neighbors, her average payoff is determined. Next, the agent determines her "Schelling utility," given by equation 7. Then total utility is determined. If total utility is less than zero, the agent moves to a randomly chose open spot. The game continues until an absorbing state is reached in the RPD *and* all agents have utility greater than or equal to 0 (thus no agent wishes to move.)

5.3.1   Cooperation and Similarity versus the Cheating Payoff

Figure 4 demonstrates the relationship between cooperation and segregation as we increase C, the payoff to "cheating," when $\phi = 1$. Here we see that as we increase C, there is a corresponding increase in the level of defection as occurred in the stand-alone RPD game. In addition we see that segregation also increases, on average, as C increases. Thus we see a first indication of a direct relationship between segregation and defection.

The reason that Similarity and defection move in the same direction is because as we increase C there are relatively fewer games that end in everyone cooperating. When the game does hit an all-cooperation absorbing state,
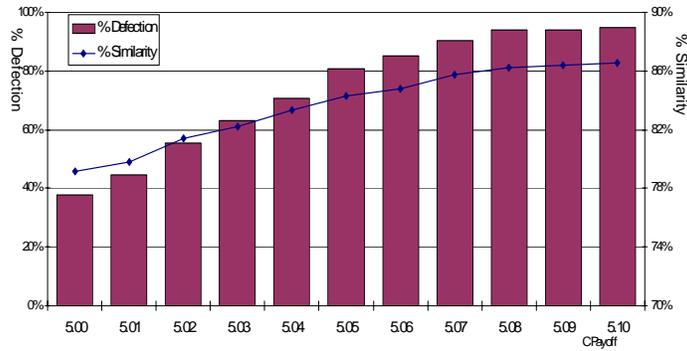
Figure 4: % Defection and % Similarity versus C Payoff in Combined Game, $\phi = 1$.

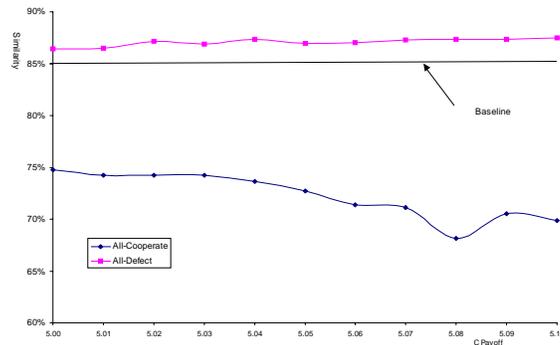then, in fact we see decreases in the similarity index, for the reasons outlined above.

Figure 5: % Similarity for All-coopering and All-defecting outcomes, $\phi = 1$.

Figure 5 shows that games that end in all-cooperation have consistently lower similarity values than when the game ends in all players defecting. Notice, however, that as we increase C, the all-cooperate case becomes increasing rare, and therefore there is more variation in the Similarity measure. In sum, as we increase the temptation to cheat, if the players do in fact learn to cooperate, then they are also willing to live with relatively more agents of the other type.

As mentioned above, as C increases it becomes increasingly rare for the agents to coordinate on the all-cooperate outcome. Also recall that the model begins with one-half of the agents playing cooperate and one-half of the agents playing defect; the game is starting in the middle of the state space on average. With large cheat payoffs there are large incentives to move toward the all-defect absorbing state. Thus one can imagine that in order to move to the all-cooperate absorbing state the system must get there relatively quickly because of the large incentives to move toward defection. This is what we observe in our simulations. As C increases the runs that converge to all-cooperate get there more quickly than when C is small.[15] If we combine this with the fact that agents begin in an integrated state we can see why lower levels of similarity, or more integration, emerges as C increases for the runs that end in all cooperate. If C is large and the agents converge to the all-cooperate outcome they must do so quickly and thus they have relatively little time for segregation to develop. Thus, the relationship shown in Figure 7 is largely due to the convergence properties of our probability adjustment rule combined with a lower Implied Schelling Threshold. Different probability-updating rules may not generate the same convergence properties and therefore the same outcome. Nonetheless, what the rule does demonstrate is that the faster the agents learn to cooperate the lower the over-all segregation.

In addition, we can compare the level of segregation attained here to the level of segregation attained in our stand-alone Schelling model when $\gamma = 0$. In the stand-alone model, Similarity is equal to 85.5% for $\gamma = 0$. The combined game results in lower levels of similarity (or segregation) for all levels of C when the combined game reaches an all-cooperate absorbing state compared to the stand-alone model. But, if the game reaches an all-defect absorbing state, then the combined game model has a slightly larger level of Similarity (segregation) compared to the stand-alone model. Thus social interactions can have a positive effect and reduce segregation if the social interactions result in cooperative outcomes. But social interactions can lead to even worse levels of segregation if the interactions lead to non-cooperative outcomes.

---

[15]For brevity, we do not present the results of convergence rates versus C; they are available upon request.

Cooperation and Similarity versus ϕ Next we compare changes in segregation and defection rates as we increase ϕ. As ϕ increases more weight is placed on the Schelling portion of utility. Thus one should expect more segregation as compared to the original Schelling model due to the increase in weight placed on the type-based portion of utility. As shown in Figure 6 this is exactly what we find: an increase in ϕ leads to more segregation. The figure presents the results for three different values of $\phi \in \{0.9, 1.0, 1.003\}$.[16]
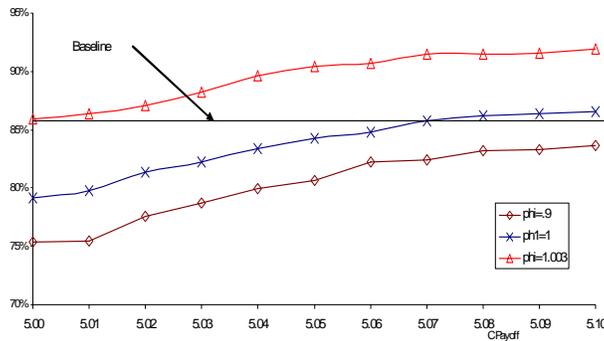
Figure 6: % Similarity versus C payoff for three different values of ϕ.

Further, as we saw in Section 5.2, the implied Schelling threshold of the combined game is increasing in ϕ. And as shown earlier in the paper, there is a direct relationship between segregation and defection. Again we see results that match expectations.[17]

Figure 7 shows the relationship between similarity and the C payoff for the three different values of ϕ when the game ends in the all-cooperate equilibrium. In general, we see that a larger C value is associated with a lower Similarity value, and that increasing ϕ increases the similarity index.

---

[16]Note we limit ϕ to a value less than but close to $D/B = 1.00333$; this is the largest value for ϕ for which both the all-cooperate and all-defect equilibria exist.

[17]Interestingly, the relationship between ϕ and defection appears to be relatively weak. For $\phi > 1$, defection rates are slightly above those for $\phi \leq 1$, for all values of C. We do not present these results here.
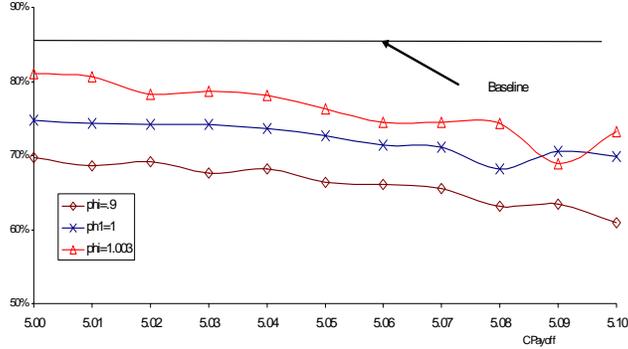
28

Figure 7: % Similarity versus C payoff for three different values of ϕ, when the game ends in all-cooperate.

5.4   Utility and Welfare

Given our definition of utility we can investigate which outcomes provide the greatest total utility or welfare for all the agents in the population, where welfare is given by

$$W = \sum_{i=1}^{N} U_i\left(s, x\right) = \sum_{i=1}^{N} \left[\pi_i\left(x; \rho\right) - \phi u_i\left(s\right)\right].$$

In equilibrium, the largest payoff an agent can receive will come if she is in the majority type in the neighborhood (she will get $-\phi B$) and if everyone is cooperating (she gets A). Thus to make comparisons across games we normalize welfare by calculating

$$W' = \sum_{i=1}^{N} \frac{U_i\left(s, x\right)}{A - \phi B}. \tag{10}$$

Note that this function only measures the welfare to agents in the game, but does note include any external costs or benefits that may accrue to society.

On average, as C increases, welfare will decline, since a larger fraction of games end in all defect. Because of the plateau in the Schelling utility function, agents will be indifferent between any neighborhoods where the fraction of like-type agents is greater than or equal to 0.5. In all cases of the combined game discussed here, all equilibria that emerge have agents, on average, living in neighborhoods that have more than half the agents like

29

themselves, thus the increasing proportion of games that end in all-defect is driving the drop in utility as C increases; this result can be seen in Figure 8.
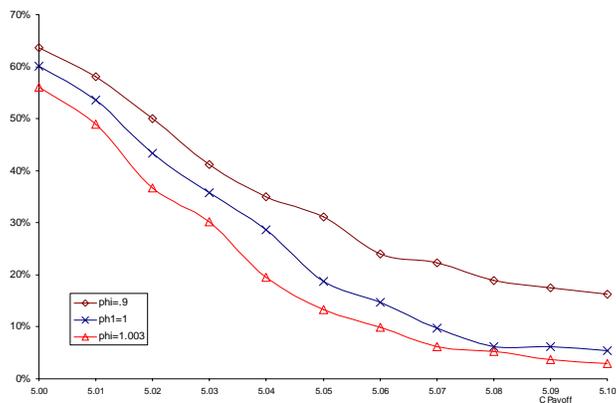

Figure 8: Welfare versus C payoff for different values of ϕ.

Furthermore, the figure shows that as we decrease ϕ, we increase the total welfare available to agents, because when the game ends in the all-defect outcome, a smaller value of ϕ increases welfare. Reducing ϕ yields both a cost and benefit to total welfare. First, it increases the denominator and thus reduces welfare. But reducing ϕ also lowers the Schelling part of total utility. However, because segregation is higher (and virtually all agents will be on the plateau part of the Schelling utility function) in the all-defect case, lowering ϕ increases the numerator more than the denominator, and thus welfare rises.

However a different story occurs in the all-cooperate outcome. When we reduce ϕ, welfare decreases. However, reducing ϕ also lowers segregation, which reduces total welfare. In the end, a lower ϕ causes a net reduction in total welfare. The relationship between ϕ and welfare is illustrated in Figure 9 where $C = 5.01$.

However, when the game ends in all-cooperate, welfare is much higher than when the game ends in all-defect. Figure 10 demonstrates how welfare evolves versus the C payoff, for the case when $\phi = 1$. We see that when the games end in all cooperation, total welfare is roughly 95% of the maximum obtainable welfare. The reason is because all the agents are cooperating and,
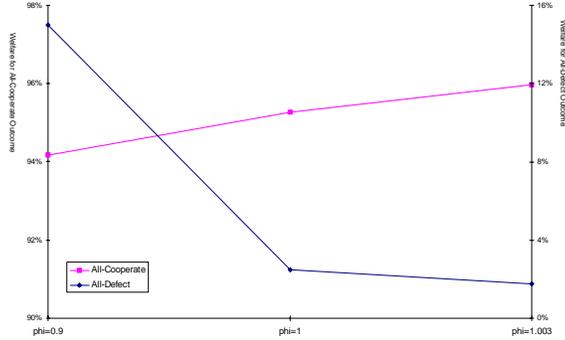
Figure 9: Welfare versus ϕ, $C = 5.01$.

on average, a vast majority of the agents are in neighborhoods where they are in the majority (as Figure 5 demonstrates).

To give a numerical illustration, take the case where $\phi = 1$, this gives a denominator for equation (10) of $5 - 3 = 2$. Furthermore, let's assume that in equilibrium, 90% of the agents are in neighborhoods with at least 50% like-type agents (which gives payoff of $-B = -3$) and 10% of the agents are in less-than-majority neighborhoods, and are getting $-B=-4$, on average. Thus in the all-cooperating outcome, the numerator of equation (10) is $5 - 0.9(3) - 0.1(4) = 2.9$. This gives total welfare of $2.9/3 = 0.95$. Now let's say, the game ends in the all-defect outcome this gives a total welfare of $(3 - 2.9)/2 = 0.05$. In short, the outcome of the Prisoner's Dilemma game is driving the total welfare.

5.4.1   Policy Implications

In summary, we can make the following conclusions about the model.

- The outcome of the Prisoner's Dilemma game can reduce segregation because it can lower the "Schelling threshold," especially in cases where the all-cooperate outcome emerges.

- As C increases, when all agents cooperate, there is a reduction in the degree of segregation due, in part, to the speed of convergence to this equilibrium.
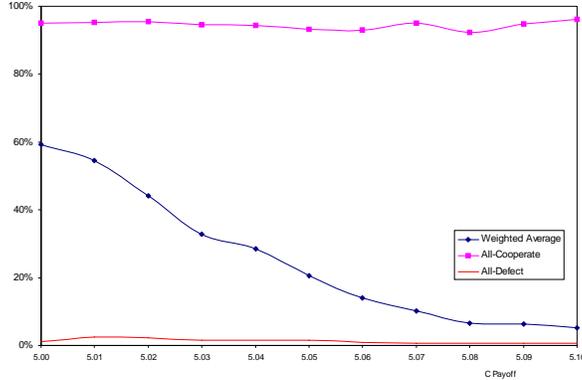
31

Figure 10: Welfare vs. C payoff, for all-cooperating and all-defecting outcomes, $\phi = 1$.

- However, given the PD game payoffs and the ϕ parameters used here, there is never an outcome that yields full integration (where full integration is all agents living with exactly 50% of like types).

- As a result of this, total welfare to "society" is, in essence, affected by the outcome of the RPD game, with the all-cooperate outcome generating roughly 95% of total possible welfare, while the all-defect outcome generates roughly 5% of total possible welfare.

- Decreasing ϕ decreases segregation, and its effect on total welfare depends on whether the game ends in all-cooperate or all-defect.

These findings suggest that if there was a "social planner" she should attempt to influence the game in the following manner. By attempting to reduce ϕ, the planner would decrease segregation. This would be a "good" outcome from a social point of view since, as described in the introduction, it can increase the external benefits to society from less segregation and it can have a positive or only slightly negative effect on agent welfare, as described above. Secondly, the planner should encourage cooperation among agents, at least in the early stages of the game to generate an all-cooperate equilibrium. The all-cooperate equilibrium reduces segregation as well as improves agent welfare.

32

The next issue that arises is how do the policy suggestions given within the context of the game translate into actual policy recommendations? ϕ would translate into the relative taste for type-based preferences to the game outcome. One can argue, as is discussed in Sethi and Somanathan (2004), for example, that ϕ has been decreasing over time simply due to the evolution of society and the global economy, as well as the evolution of educational curricula that includes discussion of ethnic and racial diversity in the U. S. In regards to moving agents to increasing neighborhood cooperation, a policy prescription would be for state and/or local governments to create or promote the creation of community-based organizations that facilitate trust and cooperation among residents (Nyden, *et al.*, 1997).

6   Conclusion

This paper has introduced non-typed based social interactions into the Schelling model of residential segregation. In general we find that social interactions can help to dampen the forces within the Schelling model that lead to segregation if the social interactions lead to a cooperative outcome. The ability to produce cooperative social outcomes can help mitigate individual preferences for like-typed neighborhoods. On the other hand, if social interactions lead to non-cooperative outcomes, then levels of segregation are increased beyond those found in the traditional Schelling model. In sum, non-cooperative social interactions lead to greater levels of segregation but cooperative social interactions lead to integration. We have also demonstrated that integration and cooperation are obtainable and stable equilibria by showing how the Prisoner's Dilemma game can reduce the "Schelling Threshold," i.e., the proportion of like-type neighbors an agents needs to have, not to move, and thus increase integration. We have also shown that as we increase the temptation for players to play defect, when the game ends in an all-cooperate outcome, we have increased levels of integration.

There are many possible extensions to this work. For example, agents may have heterogeneous utility functions both between types and more generally within types. Agents may have heterogeneous numbers of neighbors or social interactions, or the structure of social interactions may be more complex than that studied here. Social interactions also may include other games or processes than the RPD. The most fruitful extension may be to have agents in the model recognize the type of their neighbors and react to that type

in their social interactions. For instance, we could allow an agent's strategy to be a function of her opponent's type as in the signaling RPD framework. We expect the dynamics of a model of this type to be complex but of great interest.

References

[1] Aktipis, C.A. (2004). "Know When to Walk Away: Contingent Movement and the Evolution of Cooperation." Journal of Theoretical Biology, 231, 249-260.

[2] Axelrod, R. (1984). The Evolution of Cooperation. New York: Basic Books.

[3] Clark, W. A. V., and Ledwith, V. (2005). "Mobility, Housing Stress and Neighborhood Contexts: Evidence from Los Angeles." California Center for Population Research Working Paper, CCPR–005-05.

[4] Cutler, D. M. and Glaeser, E. L. (1997). "Are Ghettos Good or Bad?" Quarterly Journal of Economics, 112(3), 827-872.

[5] Dokumaci, E. and Sandholm, W.H. (2006). "Schelling Redux: An Evolutionary Dynamic Model of Segregation." University of Wisconsin Working Paper.

[6] Duncan, O. and Duncan, B. (1955). "A Methodological Analysis of Segregation Indexes." American Sociological Review, 20, 210-217.

[7] Farrell, S. J., Aubry, T. and Coulombe, D. (2004). "Neighborhoods and Neighbors: Do They Contribute to Personal Well-being?" Journal of Community Psychology, 32(1), 9-25.

[8] Fehr, E. and Gächter, S. (200). "Fairness and Retaliation: The Economics of Reciprocity." Journal of Economics Perspectives, 14(3), 159-181.

[9] Frank, R. H. (1999). Luxury Fever: Why Money Fails to Satisfy in an Era of Excess. New York: Free Press.

[10] Fudenberg, D. and Tirole, J. Game Theory. Cambridge: The MIT Press.

[11] Glaeser, E. I., Laibson, D. I., Scheinkman, J. A. and Soutter, C. L. (2000). "Measuring Trust." Quarterly Journal of Economics, 115(3), 811-846.

[12] Hales, D. (2001). Tag-based Cooperation in Artificial Societies. Ph.D. Thesis, University of Essex.

[13] Janssen, M.A. (2007). "Evolution of Cooperation in a One-shot Prisoner's Dilemma Based on Recognition of Trustworthy and Untrustworthy Agents." Journal of Economic Behavior and Organization, forthcoming.

[14] Lee, B. A., Oropesa, R. S. and Kanan, J. W. (1994). "Neighborhood Context and Residential Mobility," Demography, 31(2), 249-270.

[15] Marschall, M. J. and Stolle (2004). "Race and the City: Neighborhood Context and the Development of Generalized Trust." Political Behavior, 26(2), 125-153.

[16] Nowak, M.A., Bonhoeffer, S, and May, R.M. (1994). "Spatial Games and the Maintenance of Cooperation." Proceedings of the National Academy of Science, 91, 4877-4881.

[17] Nowak, M.A. and May, R.M. (1992). "Evolutionary Games and Spatial Chaos." Nature, 359, 826-829.

[18] Nowak, M.A. and Sigmund, K. (1998). "Evolution of Indirect Reciprocity by Image Scoring." Nature, 393, 573-577.

[19] Nyden, P., Maly, M. and Lukehart, J. (1997). "The Emergence of Stable Racially and Ethnically Diverse Urban Communities: A Case Study of Nine U. S. Cities." Housing Policy Debate, 8(2), 491-534.

[20] Pancs, R. and Vriend, N. J. (2007). "Schelling's Spatial Proximity Model of Segregation Revisited." Journal of Public Economics, 91, 1-24.

[21] Riolo, R. (1997). The Effects of Tag Mediated Selection of Partners in Evolving Populations Playing the Iterated Prisoner's Dilemma." Santa Fe Institute Working Paper # 97-02-016.

[22] Riolo, R., Cohen, M, and Axelrod, R. (2001). "Evolution of Cooperation without Reciprocity." Nature, 414, 441-443.

[23] Schelling, T. C. (1969). "Models of Segregation." American Economic Review, Papers and Proceedings, 59, 488-493.

[24] Schelling, T. C. (1971). "Dynamic Models of Segregation." Journal of Mathematical Sociology, 1(2), 143-186.

[25] Sethi, R. and Somanathan, R. (2004). "Inequality and Segregation." Journal of Political Economy, 112(6), 1296-1321.

[26] Schweitzer, F., Behera, L., and Mühlenbein, H. (2002). "Evolution of Cooperation in a Spatial Prisoners Dilemma." Advances in Complex Systems, 5, 269-299.

[27] Zhang, J. (2004). "A Dynamic Model of Residential Segregation." Journal of Mathematical Sociology, 28(3), 147-170.