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Portfolio management implications of volatility shifts: Evidence from simulated data

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Abstract

Based on weekly data of the Dow Jones Country Titans, the CBT-municipal bond, spot and futures prices of commodities for the period 1992-2005, we analyze the implications for portfolio management of accounting for conditional heteroskedasticity and structural breaks in long-term volatility. In doing so, we first proceed to utilize the ICSS algorithm to detect volatility shifts, and incorporate that information into PGARCH models fitted to the returns series. At the next stage, we simulate returns series and compute a wavelet-based value at risk, which takes into consideration the investor's time horizon. We repeat the same procedure for artificial data generated from distribution functions fitted to the returns by a semi-parametric procedure, which accounts for fat tails. Our estimation results show that neglecting GARCH effects and volatility shifts may lead us to overestimate financial risk at different time horizons. In addition, we conclude that investors benefit from holding commodities as their low or even negative correlation with stock indices contribute to portfolio diversification.

Keywords: volatility shifts, wavelets, value at risk.

1. Introduction

To date, there is an extensive literature on the behavior of volatility of assets returns. Indeed, the GARCH model and its numerous extensions have been widely used to account for the existence of conditional heteroskedasticity in financial time series (see, for instance, the survey by Poon and Granger, 2003)[3]. However, less attention has been paid to the detection of multiple shifts in unconditional variance over time. For example, Lamoureux and Lastrapes (1990) conclude that persistence in variance may be overstated by not accounting for deterministic structural breakpoints in the variance model.

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[3] Conditional heteroskedasticity means that the variance of a return series changes over time, conditional on past information. GARCH models are designed to capture the time-series dynamics of returns, in which we observe persistence or serial correlation in volatility.

A relatively recent approach to testing for volatility shifts is Inclan and Tiao (1994)'s Iterative Cumulative Sums of Squares (ICSS) algorithm. This algorithm allows for detecting multiple breakpoints in variance in a time series. Aggarwal, Inclan and Leal (1999) present an application of this procedure to emerging markets over 1985-1995. They conclude that most events leading to volatility shifts tended to be local (e.g., the Mexican peso crisis, periods of hyperinflation in Latin America), and that the only global event over the sample period that affected several emerging markets was the October 1987 crash.

Another subject, which has received attention in recent research and that also has important implications for portfolio management, is the existence of heterogeneous investors. In a recent article, Connor and Rossiter (2005) point out that, for the specific case of commodity markets, long-horizon traders will essentially focus on price fundamentals that drive overall trends, whereas short-term traders will primarily react to incoming information within a short-term horizon. Hence, market dynamics in the aggregate will be the result of the interaction of agents with heterogeneous time horizons. In order to model the behavior of financial series at different time spans, researchers have resorted to wavelet analysis (e.g., Ramsey and Zang, 1996, 1997; Li and Stevenson, 2001; Gençay, Whitcher, and Selçuk 2003, 2005; Hong and Kao, 2004; Whitcher, 2004; Karuppiah and Los, 2005; Connor and Rossiter, 2005; Fernandez, 2005, 2006). This is a refinement of Fourier analysis that allows for decomposing a time series into its high-frequency or noisy components and its low-frequency or trend components, among many other applications.

The aim of this article is two fold. First, we analyze whether accounting for conditional heteroskedasticity and volatility shifts in asset returns really matters when comes to quantifying the potential market risk an investor faces. In doing so, we consider different time horizons by resorting a wavelet-based decomposition of Value at Risk (VaR). Second, we look at the potential diversification gains involved in investing on commodities in terms of a VaR decrease. To our knowledge, no one has conducted similar research.

This article is organized as follows. Section 2 presents the main methodological tools utilized in the empirical section of the article. Section 3 presents some descriptive statistics of the data used in the simulations carried out later on. Section 4 presents the simulation exercises involving a portfolio primarily composed of stock indices and a portfolio that also include spot and futures positions in commodities. We discuss the implications of not accounting for correlated volatility and volatility shifts for risk quantification. In addition, we focus on the benefits of holding commodities for portfolio diversification. Section 5 concludes.

2. Methodology

2.1 The ICSS algorithm

Under Inclan and Tiao (1994)'s ICSS algorithm set-up, a time series of interest has a stationary unconditional variance over an initial time period until a sudden break takes place. The unconditional variance is then stationary until the next sudden change occurs. This process repeats through time, giving a time series of observations with a number of M breakpoints in the unconditional variance along the sample:

$$\sigma_t^2 = \begin{cases} \tau_0^2 & 1 < t < \iota_1 \\ \tau_1^2 & \iota_1 < t < \iota_2 \\ & .... \\ \tau_M^2 & \iota_M < t < n \end{cases} \quad (1)$$

In order to estimate the number of variance shifts and the point in time at which they occur, a cumulative sum of square residuals is computed, $C_k = \sum_{t=1}^{k} \varepsilon_t^2$, k=1, 2, .., n, where $\{\varepsilon_t\}$ is a series of uncorrelated random variables with zero mean and unconditional variance σ_t^2, as in (1). Inclan and Tiao define the statistic:

$$Đ_k = \frac{C_k}{C_n} - \frac{k}{n} \qquad k=1, 2,.., n, \qquad Đ_0 = Đ_n = 0. \quad (2)$$

If there are not variance shifts over the whole sample period, $Đ_k$ will oscillate around zero. Otherwise, if there is one or more variance shifts, $Đ_k$ will departure from zero. The ICSS algorithm systematically looks for breakpoints along the sample. A full description of the algorithm is given in Inclan and Tiao's article.

2.2 Wavelet-based betas

Wavelet-variance analysis consists of partitioning the variance of a time series into pieces that are associated to different time scales. It tells us what scales are important contributors to the overall variability of a series (see Percival and Walden 2000). In particular, let $x_1, x_2,..., x_n$ be a time series of interest, which is assumed to be a realization of a stationary process with variance σ_X^2. If $\upsilon_X^2(\tau_j)$ denotes the wavelet variance for scale $\tau_j \equiv 2^{j-1}$, then the following relationship holds:

$$\sigma_X^2 = \sum_{j=1}^{\infty} \upsilon_x^2(\tau_j)\,. \quad (3)$$

where the square root of the wavelet variance is expressed in the same units as the original data.

Let $n'_j = \lfloor n/2^j \rfloor$ be the number of discrete-wavelet transform (DWT) coefficients at level j, where n is the sample size, and let $L'_j \equiv \left\lceil (L-2)(1-\frac{1}{2^j}) \right\rceil$ be the number of DWT boundary coefficients[4] at level j (provided that $n'_j > L'_j$), where L is the width of the wavelet filter. An unbiased estimator of the wavelet variance is defined as

$$\tilde{\upsilon}_X^2(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j-1}^{n'_j-1} d_{j,t}^2 \,. \tag{4}$$

Given that the DWT de-correlates the data, the non-boundary wavelet coefficients at a given level ($\mathbf{d}_j$) are zero-mean Gaussian white-noise processes.

Similarly, the unbiased wavelet covariance between time series X and Y, at scale j, can be defined as

$$\tilde{\upsilon}_{XY}^2(\tau_j) \equiv \frac{1}{(n'_j - L'_j)2^j} \sum_{t=L'_j}^{n'_j-1} d_{j,t}^{(X)} d_{j,t}^{(Y)} \quad , \tag{5}$$

provided that $n'_j > L'_j$.

However, as pointed out by Percival and Walden (2000), the sample properties of the DWT variance and covariance estimators are inferior to those of non-decimated discrete wavelet transforms, also known as stationary wavelet transforms. The non-decimated DWT is a non-orthogonal variant of the DWT, which is time-invariant. That is, unlike the classical DWT, the output is not affected by the date at which we start recording a time series. In addition, the number of coefficients at each scale equals the number of observations in the original time series. A non-decimated form of the DWT is known as the maximal overlap DWT (MODWT).[5] The unbiased MODWT estimator of the wavelet variance is given by

$$\hat{\upsilon}_X^2(\tau_j) \equiv \frac{1}{M_j} \sum_{t=L_j-1}^{n-1} \tilde{d}_{j,t}^2 \tag{6}$$

where $\tilde{d}_{j,t}^2$ is the MODWT-wavelet coefficient at level j and time t, $M_j \equiv n - L_j + 1$, $L_j \equiv (2^j - 1)(L - 1) + 1$ is the width of the MODWT filter for level j, and n is the number of observations in the original time series. While there are n MODWT-wavelet coefficients at each level j, the first ($L_j - 1$)-boundary coefficients are discarded. (Retaining such boundary coefficients leads to a biased estimate).

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4 The $\lfloor x \rfloor$ and $\lceil x \rceil$ terms represent the greatest integer ≤x and the smallest integer ≥x, respectively. Boundary coefficients are those that are formed by combining together some values from the beginning and the end of the time series.

5 The scaling $(\tilde{l}_k)$ and wavelet $(\tilde{h}_k)$ filter coefficients for the MODWT are rescaled versions of those of the DWT. Specifically, $\tilde{l}_k \equiv l_k / \sqrt{2}$ and $\tilde{h}_k \equiv h_k / \sqrt{2}$.

Likewise, the unbiased MODWT estimator of the wavelet covariance can be obtained as

$$\hat{\upsilon}_{XY}^2(\tau_j) \equiv \frac{1}{M_j}\sum_{t=L_j'}^{n-1}\widetilde{d}_{j,t}^{(X)}\widetilde{d}_{j,t}^{(Y)} . \tag{7}$$

In the CAPM model, as proposed by Gençay, Whitcher and Selçuk (2003), the wavelet-beta estimator for asset i, at scale j, is defined as

$$\hat{\beta}_i(\tau_j) = \frac{\hat{\upsilon}_{R_i R_m}^2(\tau_j)}{\hat{\upsilon}_{R_m}^2(\tau_j)} , \tag{8}$$

where $\hat{\upsilon}_{R_i R_m}^2(\tau_j)$ is the wavelet covariance of asset i and the market portfolio at scale j, and $\hat{\upsilon}_{R_m}^2(\tau_j)$ is the wavelet variance of the market portfolio at scale j.

An R^2 for each scale can be computed as follows

$$R_i^2(\tau_j) = \hat{\beta}_i(\tau_j)^2 \frac{\hat{\upsilon}_{R_m}^2(\tau_j)}{\hat{\upsilon}_{R_i}^2(\tau_j)} . \tag{9}$$

2.3 Wavelet-based value at risk

From the empirical representation of the CAPM, we have

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \varepsilon_i . \qquad k=1, 2,...,k. \tag{10}$$

From equation (10), the variance of excess return i and the covariance of excess returns i and j are given, respectively, by

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_\varepsilon^2 , \qquad i=1, 2,.., k,$$
$$\sigma_{ij} = \beta_i\beta_j\sigma_m^2 , \qquad i, j=1, 2,.., k, \quad i\neq j$$

where $E(\varepsilon_i^2) = \sigma_{\varepsilon_i}^2$ and $E(\varepsilon_i\varepsilon_j)=0$, $\forall i\neq j$.

Consequently, the variance-covariance matrix of the k excess returns is given by

$$\mathbf{\Omega} = \boldsymbol{\beta}\boldsymbol{\beta}'\sigma_m^2 + \mathbf{E} , \tag{11}$$

where $\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}$ and $\mathbf{E} = \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_{\varepsilon_k}^2 \end{pmatrix}$.

The (1–α) %-Value at Risk (VaR) of a portfolio of k assets is then

$$\mathrm{VaR}(\alpha) = V_0 l(\alpha)\sqrt{\boldsymbol{\omega}'(\boldsymbol{\beta\beta}'\sigma_m^2 + \mathbf{E})\boldsymbol{\omega}} \tag{12}$$

where ω is a k x 1 vector of portfolio weights, V_0 is the initial value of the portfolio, and $l(\alpha) \equiv \Phi^{-1}(1-\alpha)$, where Φ(.) is the cumulative distribution function of the standard normal.

For an equally-weighted portfolio, such that $\omega_i = 1/k\ \forall\ i$, the VaR boils down to

$$\mathrm{VaR}(\alpha) = V_0 l(\alpha)\sqrt{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i / k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2}\ . \tag{13}$$

As k becomes large, $\mathrm{VaR}(\alpha) \approx V_0 l(\alpha)\sqrt{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i / k\right)^2}$. That is, for a well-diversified portfolio, all that matters is systematic risk.

We use equation (13) to compute the value at risk at different time-scales. In particular, the VaR at scale j can be obtained by evaluating equation (13) at the j-scale components of the variance of the market portfolio return, the betas of the k stocks, and of the variances of the error terms that capture non-systematic risk:

$$\mathrm{VaR}_{\tau_j}(\alpha) = V_0 l(\alpha)\sqrt{\sigma_m^2(\tau_j)\left(\sum_{i=1}^{k}\beta_i(\tau_j) / k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)}\ . \tag{14}$$

In order to obtain $\sigma_{\varepsilon_i}^2(\tau_j)$, we use the relation $\sigma_i^2(\tau_j) = \beta_i^2(\tau_j)\sigma_m^2(\tau_j) + \sigma_\varepsilon^2(\tau_j)$. That is,

$$\sigma_\varepsilon^2(\tau_j) = \sigma_i^2(\tau_j) - \beta_i^2(\tau_j)\sigma_m^2(\tau_j)\ . \tag{15}$$

The variance of stock i at scale j, $\sigma_i^2(\tau_j)$, the beta of stock i return at scale j, $\beta_i(\tau_j)$, and the variance of the market portfolio at scale j, $\sigma_m^2(\tau_j)$, can be computed using equations (6) and (8).

2.4 Long-memory processes

Connor and Rossiter (2005) discuss how to obtain the long-memory parameter of a time series from wavelet analysis. Specifically, a time series y_t is said to be a long-memory process if its autocovariance sequence decays at a slower rate than that of an ARMA process. Mathematically, if $\lambda_s = \mathrm{cov}(y_t, y_{t+s})$, s=–1,0,1, and there exist constants C and β, such that $\lim_{s\to\infty}\frac{\lambda_s}{Cs^\beta} = 1$, then y_t is long memory process. Furthermore, $\lim_{s\to\infty}\frac{\lambda_s}{Cs^\beta} = 1$ if and only if $\lim_{f \to 0} \frac{S(f)}{K|f|^{\alpha}} = 1$, where $\alpha+\beta=-1$, K is a constant, $|f|<1/2$, and S(f) is the spectral density function of the process.

The exponent α is called the spectral exponent, and it has been shown to equal $-2d$, where d represents the long-memory parameter, as usually referred to in time series analysis. Connor and Rossiter point out that d can be estimated from a regression of the logarithm of the wavelet variance on the logarithm of the scale. If $0<d<1$, y_t is a long memory process. In particular, if $0<d<0.5$, y_t is stationary but shocks decay at a hyperbolic rate, while if $0.5\leq d<1$, y_t is non-stationary. On the other hand, if $-0.5<y_t<0$ is stationary and has short memory.

3. The data

Our sample consists of weakly returns on the Dow Jones Country Titans (Australia, Canada, Germany, Hong Kong, Italy, Japan, The Netherlands, Spain, Sweden, Switzerland, and The United Kingdom), the Dow Jones Global 50,[6] the Dow Jones Industrial, Moody's commodities index[7], CBT-municipal bond, CBT-10 year US T-note, LME-spots prices of copper, nickel and zinc, and futures prices of corn and wheat. All indices and prices are expressed in US dollars and span the period 1992-2005. The data sources are Datastream and Ecowin.

Table 1 presents some descriptive statistics of the data. The returns on the nickel spot price and the wheat futures price stand out for their high volatility, measured by the interquartile range, followed by the DJ Hong Kong Titan. The least volatile return series are those on the Moody's commodity index, the CBT-municipal bond and the CBT-10 year US note. As usual, all return series strongly reject the assumption of normality, according to the Shapiro-Wilk and Jarque-Bera tests.

Given that we are ultimately interested in quantifying systematic risk, we compute the beta of each return series for different time horizons (scales). The proxies for the market portfolio and the risk-free asset are the DJ Titans Global and the CBT-10 year US note, respectively. As Table 2 shows, returns on metals and grains futures display little market risk as compared with those on the DJ Country Titans (e.g., Australia and the UK). This is particularly so for grains futures, whose betas are close to zero, and sometimes even negative, at different time horizon. In general, we observe that for the DJ Country Titans, beta tends to increase as the time horizon increases. In other words, the CAPM has greater predictive power in the long than in the short run, as Gençay, Whitcher and Selçuk (2003) conclude.

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[6] The Dow Jones Global Titans is made up by fifty internationally based and globally oriented companies, such as Microsoft, Nestle, Toyota Motor Corp., Time Warner Inc., and Coca-Cola. The Dow Jones Country Titans in turn generally represent the biggest and most liquid stocks traded in individual countries.

[7] Moody´s commodity index is an average of eighteen leading commodities, including corn, soybeans, wheat, coffee, hogs, steers, sugar, cotton, wool, aluminum, copper scrap, lead, steel scrap, zinc, rubber, hides and silver. The index is based on daily closing spot prices.

Recent literature has shown that volatility may exhibit long memory. In order to asses whether that is the case for our returns series, we follow Connor and Rossiter's procedure to compute the long-memory parameter. Table 2 presents our results. There is some evidence of long memory, particularly in the squared returns on the DJ Titans Netherlands, DJ Titans Switzerland and the wheat futures. In addition, for all the absolute and squared return series, the estimate of d is less than 1, which suggests that all volatility series are stationary.

4. Portfolio simulations

We follow two procedures to quantify the portfolio risk. One consists of fitting a generalization of a GARCH model to the individual return series, after accounting for structural breaks in volatility. In order to determine such breaks, we utilize the ICSS algorithm. The output obtained from the ICSS algorithm is used to construct dummy variables, which are incorporated into the variance equation of each return series. Table 4 reports the volatility shifts detected in the weekly returns. Most series exhibit structural breaks around the Asian crisis and at the beginning of the Iraq invasion. Three series do not present any shifts at all: the DJ Titan Australia index, the copper spot price, and the wheat futures price.

Given that we previously found some evidence of long memory in the returns volatility, a standard GARCH model may be inadequate. A possibility would be to utilize a fractionally integrated GARCH model. Alternatively, a generalization of the GARCH model, which allows for longer memory in the conditional variance than a standard GARCH model, may prove suitable. In particularly, we resort to a Power GARCH model (PGARCH):

$$r_t=\delta'\mathbf{x}_t+\varepsilon_t , \qquad \varepsilon_t=\sigma_t z_t,\ z_t\sim \text{IID } (0,1),\ t=1, 2..., T \tag{16}$$

where

$$\sigma_t^{\delta} = \alpha_0 + \sum_{t=1}^{p}\alpha_i(|\,\varepsilon_{t-i}\,| +\gamma_i\varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q}\beta_j\sigma_{t-i}^{\delta} ,$$

and $\alpha_0>0$, $\delta>0$, $\alpha_i\geq 0$, i=1,..., p, $\beta_j\geq 0$, j=1,..., q and $|\gamma_i|<1$, i=1,..., p.

Many GARCH variants can be nested in the PGARCH model. For instance, if $\delta=2$ and $\gamma_i=0\ \forall i$, we have a GARCH model; if $\delta=1$, we have the threshold GARCH model, etcetera. For some of our return series, the estimated δ is close to 2, indicating that a GARCH model seems satisfactory.[8] Given the existence of structural breaks in unconditional variance in most return series, we consider a more general function for the

[8] We fitted FIGARCH(1,1) models to the return series, but in some cases the sum of the ARCH and GARCH coefficients was greater than 1, giving rise to a non-stationary process.

conditional variance equation, $\sigma_t^\delta = \alpha_0 + \sum_{t=1}^{p} \alpha_i (|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^\delta + \sum_{k=1}^{m-1} \varpi_k d_k$, where d_k is a dummy variable that takes on the value of 1 between dates of breakpoints and zero otherwise. If there are m structural breakpoints, m–1 dummy variables are included in the conditional variance equation.

The second approach we use to model the behavior of returns consists of a semi-parametric procedure, which is discussed in Carmona, 2004. Specifically, the tails of the distribution can be modeled by means of the generalized Pareto distribution, while the empirical distribution can be used to model the center of the distribution. That is, parametric and non-parametric approaches are used to model the tails and the center of the distribution, respectively.

In order to carry out the simulation exercises, we first form an equally-weighted portfolio made up by nineteen assets—the DJ Country Titans, The Dow Jones Industrial, Moody´s commodity index, the municipal bond, the three metals (copper, nickel, zinc), and the grains futures (corn and wheat). The first simulation exercise consists of fitting PGARCH models to the returns on the nineteen portfolio assets and simulating returns data from the fitted models.[9] The simulated data is used at the next stage to compute the portfolio Value at Risk for the raw data and the five wavelet scales, as described in Section 2.3. The same procedure is repeated one hundred times. The second simulation exercise is meant to quantify the diversification loss incurred by not investing on the metals and the grains. The third and fourth simulation exercises are in the same vein, but they are based on the semi-parametric procedure referred to above. The computer code involved in the estimation process was written in S-Plus 7.0.

The simulation results are reported in Table 5. If we look at Panels (a) and (b), where the PGARCH models are reported, we see that there is a clear diversification benefit from investing on metals and grains. Indeed, for the raw data, the 95-percent weekly VaR for a USD 1000-investment on the portfolio made up by the nineteen assets (base portfolio) is USD 9.73, whereas for the portfolio excluding the metals and grains the weekly 95-percent VaR increases to USD 13.21. Now, if we look at different time horizons, we see that short-term investors are subject to greater potential losses than long-term investors. For instance, for a 8-16 week horizon (scale 3), the 95-percent weekly VaR of the base portfolio is USD 3.48, whereas this amounts to only USD 1.69 for a 32-64 week horizon (scale 5).

On the other hand, our simulations based on the semi-parametric procedure show that neglecting conditional heteroskedasticity and volatility shifts can lead us to overestimate market risk substantially. Indeed, as Panels (c) and (d) of Table 5 show, the semi-parametric method yields VaR estimates that are twice as large as those reported in Panels (a) and (b), respectively.

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[9] We also fit PGARCH models to our proxies of the market portfolio and the risk-free rate in order to simulate returns series for the two of them.

5. Conclusions

In this study, we quantify the extent to which modeling conditional heteroskedasticity and structural breaks in long-term volatility matters to determine systematic risk. In doing so, we compute a wavelet-based measure of value at risk, which makes it possible to take account of investors´ heterogeneous time horizons.

Our simulation results, based on weekly data of the Dow Jones Country Titans and spot and futures prices of commodities for the period 1992-2005, show that neglecting GARCH effects and volatility shifts may lead us to overestimate financial risk considerably, at various investment horizons. In addition, we conclude that investors benefit from holding commodities—particularly futures—as their low or even negative correlation with stock indices contribute to portfolio diversification.

A potential extension of this research would be to simulate returns from a multivariate distribution rather than from marginal distributions, as assets returns will generally exhibit some correlation. Most likely, a smaller number of assets should be considered in order to make the estimation process computationally tractable.

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Appendix: Data description

Abbreviation	Description
DJTIAU	DOW JONES AUSTRALIA TITANS 30, USD
DJTICA	DOW JONES CANADA TITANS 40 ,USD
DJTIBD	DOW JONES GERMANY TITANS 30, USD
DJTIHK	DOW JONES HONG KONG TITANS 30, USD
DJTIIT	DOW JONES ITALY TITANS 30, USD
DJTIJP	DOW JONES JAPAN TITANS 100, USD
DJTINL	DOW JONES NETHERLAND TITANS 30, USD
DJTISP	DOW JONES SPAIN TITANS 30, USD
DJTISW	DOW JONES SWEDEN TITANS 30, USD
DJTICH	DOW JONES SWISS TITANS 30, USD
DJTIUK	DOW JONES UK TITANS 50, USD
DJINDUS	DOW JONES INDUSTRIALS
DJTITAN	DOW JONES GLOBAL TITANS 50, USD
CMDTY	MOODY'S COMMODITIES INDEX
CMB	CBT-MUNICIPAL BOND
T-BILL	CBT-10 YEAR US T-NOTE
COPPER	COPPER, SPOT, LME, USD
NICKEL	NICKEL, SPOT, LME, ASK, SETTLEMENT, USD
ZINC	ZINC, SPOT, LME, ASK, SETTLEMENT, USD
CORN	CORN, FUTURES 1-POS, CBT, CLOSE, USD
WHEAT	WHEAT, FUTURES 1-POS, CBT, CLOSE, USD

Table 1 Some descriptive statistics of the return series

	DJTIAU	DJTICA	DJTIBD	DJTIHK	DJTIIT	DJTIJP	DJTINL	DJTISP	DJTISW	DJTICH	DJTIUK
Min	-0.102	-0.131	-0.133	-0.149	-0.131	-0.100	-0.181	-0.119	-0.199	-0.180	-0.107
1st. Qu.	-0.014	-0.011	-0.013	-0.018	-0.017	-0.020	-0.013	-0.013	-0.017	-0.012	-0.012
Median	0.003	0.003	0.002	0.003	0.003	-0.001	0.002	0.002	0.003	0.002	0.001
Mean	0.002	0.002	0.001	0.002	0.001	0.000	0.002	0.002	0.002	0.002	0.001
3rd. Qu.	0.017	0.017	0.019	0.024	0.020	0.018	0.018	0.019	0.022	0.017	0.015
Max	0.073	0.098	0.138	0.134	0.117	0.150	0.156	0.080	0.142	0.128	0.117
Interq. range	0.032	0.028	0.033	0.042	0.037	0.038	0.030	0.031	0.039	0.029	0.027

	DJINDUS	CMDTY	CMB	T-BILL	DJTITAN	COPPER	NICKEL	ZINC	CORN	WHEAT
Min	-0.092	-0.054	-0.050	-0.029	-0.122	-0.123	-0.192	-0.223	-0.312	-0.245
1st. Qu.	-0.011	-0.006	-0.006	-0.005	-0.011	-0.015	-0.022	-0.013	-0.018	-0.023
Median	0.002	0.001	0.001	0.000	0.002	0.001	-0.001	0.000	0.000	-0.001
Mean	0.002	0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.000	0.000
3rd. Qu.	0.014	0.009	0.007	0.006	0.013	0.017	0.025	0.016	0.016	0.024
Max	0.098	0.077	0.175	0.029	0.127	0.135	0.254	0.118	0.138	0.216
Interq. range	0.025	0.015	0.013	0.011	0.024	0.032	0.047	0.030	0.034	0.047

Table 2 Wavelet-based betas of the return series

	Betas						R^2					
	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Raw	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
DJTIAU	0.540	0.518	0.522	0.638	0.612	0.767	0.312	0.263	0.314	0.470	0.528	0.628
DJTICA	0.533	0.536	0.518	0.659	0.525	0.521	0.352	0.311	0.388	0.450	0.470	0.638
DJTIBD	0.509	0.493	0.528	0.582	0.482	0.583	0.458	0.424	0.491	0.556	0.485	0.666
DJTIHK	0.330	0.345	0.350	0.340	0.259	0.264	0.245	0.230	0.299	0.284	0.220	0.286
DJTIIT	0.366	0.386	0.382	0.263	0.321	0.446	0.274	0.287	0.285	0.166	0.290	0.517
DJTIJP	0.386	0.368	0.373	0.517	0.406	0.365	0.291	0.244	0.300	0.465	0.318	0.296
DJTINL	0.559	0.527	0.582	0.645	0.606	0.654	0.508	0.471	0.504	0.609	0.649	0.790
DJTISP	0.481	0.474	0.469	0.502	0.552	0.552	0.326	0.305	0.312	0.354	0.530	0.561
DJTISW	0.446	0.437	0.445	0.494	0.551	0.520	0.443	0.421	0.429	0.504	0.646	0.662
DJTICH	0.574	0.566	0.555	0.660	0.557	0.597	0.472	0.449	0.438	0.572	0.653	0.588
DJTIUK	0.677	0.644	0.659	0.845	0.905	0.948	0.488	0.473	0.454	0.563	0.709	0.724
DJINDUS	0.757	0.741	0.710	0.875	0.843	0.807	0.526	0.461	0.543	0.727	0.800	0.837
CMDTY	0.402	0.408	0.426	0.406	0.436	0.673	0.072	0.067	0.091	0.069	0.067	0.274
CMB	0.192	0.198	0.282	0.080	-0.040	0.336	0.005	0.005	0.012	0.001	0.000	0.013
COPPER	0.214	0.253	0.217	0.148	0.119	0.354	0.071	0.082	0.091	0.036	0.027	0.214
NICKEL	0.143	0.164	0.122	0.158	0.074	0.195	0.064	0.072	0.058	0.078	0.020	0.153
ZINC	0.207	0.222	0.198	0.159	0.200	0.413	0.062	0.061	0.063	0.036	0.060	0.302
CORN	0.045	0.034	0.068	0.069	0.010	-0.035	0.004	0.002	0.012	0.011	0.000	0.004
WHEAT	0.050	0.058	0.082	0.014	-0.044	-0.054	0.006	0.008	0.021	0.000	0.005	0.007

Notes: (1) Scale 1: 2-4 weeks, scale 2: 4-8 weeks scale 3: 8-16 weeks, scale 4: 16-32 weeks, and scale 5: 32-64 weeks. (2) The wavelet-beta estimate for asset i, at scale j, is computed as $\hat{\beta}_i(\tau_j) = \frac{\hat{\upsilon}^2_{R_i R_m}(\tau_j)}{\hat{\upsilon}^2_{R_m}(\tau_j)}$, whereas the corresponding R^2 is calculated as $R^2_i(\tau_j) = \hat{\beta}_i(\tau_j)^2 \frac{\hat{\upsilon}^2_{R_m}(\tau_j)}{\hat{\upsilon}^2_{R_i}(\tau_j)}$.

Table 3 Long-memory in volatility

	Absolute returns		Squared returns	
Series	d	s.e.	d	s.e.
DJTIAU	–0.02	0.03	0.04	0.03
DJTICA	0.08	0.03	0.08	0.03
DJTIBD	0.08	0.03	0.15	0.03
DJTIHK	0.05	0.03	0.10	0.03
DJTIIT	0.07	0.03	0.08	0.03
DJTIJP	0.06	0.03	0.05	0.03
DJTINL	0.18	0.03	0.23	0.03
DJTISP	0.08	0.03	0.12	0.03
DJTISW	0.11	0.03	0.12	0.03
DJTICH	0.12	0.03	0.18	0.03
DJTIUK	0.17	0.03	0.22	0.03
DJINDUS	0.10	0.03	0.13	0.03
CMDTY	0.04	0.03	0.07	0.03
CMB	–0.01	0.03	–0.13	0.02
T-BILL	–0.03	0.03	0.00	0.03
DJTITAN	0.14	0.03	0.22	0.02
COPPER	0.10	0.03	0.16	0.03
NICKEL	0.03	0.03	0.07	0.03
ZINC	0.06	0.03	0.08	0.03
CORN	0.08	0.03	0.09	0.02
WHEAT	0.12	0.03	0.21	0.02

Table 4 ICSS-volatility breakpoints

DJTIAU	DJTICA	DJTIBD	DJTIHK	DJTIIT	DJTIJP	DJTINL	DJTISP
--	22-Jul-98	22-Jul-98	29-Sep-93	5-Oct-94	7-Apr-93	12-Feb-97	17-May-95
	20-Jun-01	23-Apr-03	15-Mar-95	19-Mar-03	10-Sep-97	1-Apr-98	10-Sep-97
	27-Nov-02	19-May-04	24-Sep-97	19-May-04	17-Mar-99	24-Mar-99	18-Feb-98
			21-Oct-98		10-Dec-03	10-Jul-02	19-Mar-03
			10-Oct-01			19-Mar-03	
			5-Dec-01			19-May-04	

DJTISW	DJTICH	DJTIUK	DJINDUS	CMDTY	CMB
24-Aug-94	7-Jan-98	14-Apr-93	13-Dec-95	6-Jul-94	1-Dec-93
20-Mar-96	5-Aug-98	7-May-97	26-Mar-97	20-Jul-94	31-May-95
12-Mar-97	10-Jul-02	8-Jul-98	13-Sep-00	3-Jun-98	7-Jun-95
29-Jul-98	19-Mar-03	28-Apr-99	19-Mar-03		4-Sep-96
17-Jul-02	16-Apr-03	10-Jul-02			29-Aug-01
6-Nov-02		19-Mar-03			9-Oct-02
19-May-04					4-Dec-02
					14-Apr-04

T-BILL	DJTITAN	COPPER	NICKEL	ZINC	CORN	WHEAT
29-Aug-01	9-Feb-94	--	24-May-00	21-Apr-93	27-Mar-96	--
14-Apr-04	1-Oct-97			13-Oct-93	2-Oct-96	
	29-Jul-98			5-Feb-97		

Table 5 Value at Risk (VaR) of an equally-weighted portfolio: simulation results

	Raw data	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
(a) PGARCH(1,1) model accounting for volatility breakpoints (base portfolio)						
Average 95%-VaR (USD)	9.73	7.28	4.61	3.48	2.21	1.69
Std (USD)	0.35	0.24	0.21	0.16	0.13	0.18
(b) PGARCH(1,1) model accounting for volatility breakpoints, excluding metals and grains						
	Raw data	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Average 95%-VaR (USD)	13.21	9.88	6.26	4.72	3.00	2.30
Std (USD)	0.47	0.32	0.29	0.22	0.18	0.24
(c) Semi-parametric procedure (base portfolio)						
	Raw data	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Average 95%-VaR (USD)	17.42	12.18	8.60	7.16	4.64	3.45
Std (USD)	0.18	0.14	0.16	0.19	0.17	0.19
(d) Semi-parametric procedure, excluding metals and grains						
	Raw data	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Average 95%-VaR (USD)	23.65	16.53	11.68	9.71	6.30	4.68
Std (USD)	0.25	0.19	0.21	0.26	0.22	0.26

Notes: (1) In Panels (a) and (c), the equally-weighted portfolio (base portfolio) is made up by the DJ Country Titans, The Dow Jones Industrial, Moody´s commodity index, the municipal bond, the three metals (copper, nickel, zinc), and the grains futures (corn and wheat). In Panels (b) and (d), the metals and grain are excluded. (2) The portfolio investment is USD 1,000 and the VaR is expressed on a weekly basis. (3) The number of simulation is 100 in each case. (3) Scale 1: 2-4 weeks, scale 2: 4-8 weeks scale 3: 8-16 weeks, scale 4: 16-32 weeks, and scale 5: 32-64 weeks.

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