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Abstract

The use of financial ratios by analysts to compare the performance of firms from one accounting period to the next is of growing importance with continued European economic integration. Recent studies suggest that the individual component series of financial ratios exhibit nonstationarity which is not eliminated by the ratio transformation. In this paper, we derive a generalised model that incorporates stochastic and deterministic trends and allows for restricted and unrestricted proportionate growth in the ratio numerator and denominator. When the individual firm series are included in a panel structure with large N and small T, we are unable to reject convincingly a joint hypothesis of nonstationarity, whilst in about one third of the individual firm panels there is no evidence of a unit root. Although the components of financial ratios are correlated variables, our estimates show that any cointegrating effects decay rapidly.

Keywords: financial ratios, nonstationarity, proportionate growth, cointegration, panel methods

1. Introduction

Cross-border and cross-market takeover activity involving firms from different European countries is a growing business phenomenon with the continued consolidation of European economic integration. As a consequence, financial analysts are being called upon to play an ever-increasing and important role in comparing the performance of firms with their competitors on both a national and international basis. Financial ratios are commonly used by financial analysts to compare the performance of a firm with its competitors and also to assess the firm's progress from one accounting period to the next. In practice, whilst inter-firm comparison can involve financial data for large numbers of firms, trend analysis normally concerns only a short series of repeated measures. An important issue in this context is whether the simple ratio metric that is commonly used as a financial indicator provides an adequate measure on which to base both interfirm comparisons and financial trend analyses. In this paper, in addressing this issue, the context of day-to-day financial analysis for a large sample of European firms is modelled as a panel where the cross-sectional dimension N is large and the time dimension T is short.

The dynamic time series properties of financial ratios have been the subject of a number of other empirical research studies. The implications of nonstationarity and cointegration in financial ratios were first discussed by Whittington and Tippett (1995, 1999). Ioannides, Peel and Peel (2003) investigate whether nonstationarity is consistent with the well-documented mean reverting process in financial ratios (Lev, 1969; Davis and Peles, 1993; Gallizo and Salvador, 2003). Peel, Peel and Venetis (2004) reassess nonstationarity in financial ratios in the context of cross-sectional dependence.

Whittington and Tippett (1999) reach the conclusion that the components of financial ratios exhibit nonstationarity which is not eliminated by the ratio transformation. They also show that the extent of cointegration between ratio components varies considerably across different financial ratios. Ioannides, Peel and Peel (2003) find that ratios are globally stationary, but that unit root behaviour close to equilibrium results from a non-linear partial adjustment process where the rate of adjustment towards the optimal value increases with deviation from the target. Peel, Peel and Venetis (2004) demonstrate that, although the standard Dickey-Fuller tests employed in Whittington and Tippett (1999) suggest that individual financial ratio series are nonstationary, panel tests reject the null hypothesis of a joint unit root, which implies strong persistence in the ratios and places doubt on their characterization as integrated processes.

In the studies cited above, the statistical tests are carried out using a relatively small number of firm-specific ratio time series, with only Peel, Peel and Venetis (2004) employing panel estimation methods. In the present study, we use a panel with large N and small T. A suitable test for cointegration in such a panel structure is to test the hypothesis of a joint unit root in the panel by applying Pesaran's method for short panels (Pesaran, 2006). We incorporate stochastic and deterministic trends in a generalised loglinear model of repeated measures that can assume proportionate growth in accounting variables that may be restricted to firm growth, as proposed in McLeay and Trigueiros (2002). The statistical fit of the restriction is compared with alternative time series specifications, with an empirical analysis that covers a large sample of European firms over a period of eight years. The study focuses on 'pure' financial ratios (Trigueiros, 1995) that are constructed from the non-negative accounting totals which are the basic output of accounting systems, including balance sheet and income statement items.

The accounting variables of interest in this study are amongst the principal financial aggregates reported in financial statements: Shareholders' Equity (*SE*), Total Liabilities (*TL*), Total Assets (*TA*), Sales (*SA*) and Total Costs (*TC*). Total Liabilities is defined such that $TA = SE + TL$ and Total Costs includes all charges such that $SA - TC$ is equal to Earnings Available to Shareholders (EA). Consider then the following accounting construction that incorporates these variables as financial ratios, where Return on Equity = $EA/SE = SA/TA + TL/SE\ (SA/TA - TC/TL)$. The three financial ratios that explain ROE are: the Liabilities to Equity ratio (*TL/SE*), the Asset Turnover (*SA/TA*) and its counterpart that we refer to as the Liabilities Turnover (TL/TC). Each of the three drivers of return on equity in the accounting identity is a 'pure' financial ratio, having in theory the properties discussed above, and the analysis presented below is based on the properties of these three ratios.[1]

2. Accounting variables, financial ratios and nonstationary panels

Accounting variables are aggregates of like transactions that are entered into by a firm, including totals for a given period (such as sales and costs) and accumulations over the longer term (such as assets and liabilities). These variables are reported periodically as financial statement line items, and are widely available as annual time series for large numbers of firms. The initial observation for a given firm may relate to the first year of its activities or, if censored on the left, to the first fiscal year covered by the database; each series will continue until the demise of the firm, or until the censor date on the right (i.e. the last fiscal year covered by the database), or through to the present time. Financial ratios are constructed from such variables, and they too are widely available in commercial databases where, as indicated above, the cross-sectional dimension is large and the time dimension tends to be small in terms of the count of repeated observations.

In this context, a general model for an accounting variable, X, at time t is given for the j^{th} firm by:

$$\ln X_{j,t} = \alpha 1_j + \beta 1_j t + \beta 2_j \ln X_{j,t-1} + u_t \quad \text{……..……………………………… (1)}$$

This can be decomposed into lower level models by restricting the coefficients. In the first instance, consider the case of no drift and no deterministic time trend. That is, when $\alpha 1 = 0$ and $\beta 1 = 0$,

$$\ln X_{j,t} = \beta 2_j \ln X_{j,t-1} + u_t \quad \text{…………………..……..…………………………… (2)}$$

In a special case of (2), where the unit root features in the specification through the additional restriction of $\beta 2 = 1$, we have a pure random walk

$$\ln X_{j,t} = \ln X_{j,t-1} + u_t \quad \text{………………………………………………………… (2′)}$$

Now consider a financial ratio $X1/X2$ that is constructed from two variables that are each described by (2), where

$$\left\{\begin{matrix} \ln X1_{j,t} & = & \beta 21_j \ln X1_{j,t-1} + u1_t \\ \ln X2_{j,t} & = & \beta 22_j \ln X2_{j,t-1} + u2_t \end{matrix}\right\}.$$

The ratio has the following form:

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \beta21_j \ln X1_{j,t-1} - \beta22_j \ln X2_{j,t-1} + u1_t - u2_t$$

In the specific case where each variable has an autoregressive unit root, i.e. where $\beta21_j = \beta22_j = 1$, the financial ratio is described by

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \ln\frac{X1_{j,t-1}}{X2_{j,t-1}} + u1_t - u2_t$$

In this case, if the innovations $u1_t$ and $u2_t$ are uncorrelated, the ratio is characterized by a random walk over time.

Now consider an accounting variable where, in the general model given by equation (1), $\alpha1 \neq 0$, $\beta1 = 0$ and $\beta2 = 1$. This variable is described as a random walk with drift, where

$$\ln X_{j,t} = \alpha1_j + \ln X_{j,t-1} + u_t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

Consider the two accounting variables given below whose specifications are the same as that of equation (3):

$$\left\{\begin{array}{lcl} \ln X1_{j,t} & = & \alpha11_j + \ln X1_{j,t-1} + u1_t \\ \ln X2_{j,t} & = & \alpha12_j + \ln X2_{j,t-1} + u2_t \end{array}\right\}.$$

When a financial ratio is constructed from these two variables, it is defined as

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \alpha11_j - \alpha12_j + \ln\frac{X1_{j,t-1}}{X2_{j,t-1}} + u1_t - u2_t$$

This ratio follows a random walk with drift given by the constant $\alpha11_j$ - $\alpha12_j$.

Recalling the univariate case given by equation (1), if $\alpha 1 \neq 0$, $\beta 1 \neq 0$ and $\beta 2 = 1$, we now have an accounting variable that is modelled as a random walk with constant drift plus a time trend. That is,

$$\ln X_{j,t} \;=\; \alpha 1_j + \beta 1_j t + \ln X_{j,t-1} + u_t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

In these circumstances, where a deterministic time trend is included and the denominator and numerator of the financial ratio are described by

$$\left\{\begin{matrix} \ln X1_{j,t} & = & \alpha 11_j + \beta 11_j t + \ln X1_{j,t-1} + u1_t \\ \ln X2_{j,t} & = & \alpha 12_j + \beta 12_j t + \ln X2_{j,t-1} + u2_t \end{matrix}\right\},$$

then the ratio yields

$$\ln \frac{X1_{j,t}}{X2_{j,t}} \;=\; \alpha 11_j - \alpha 12_j + \left(\beta 11_j - \beta 12_j\right) t + \ln \frac{X1_{j,t-1}}{X2_{j,t-1}} + u1_t - u2_t$$

Cases 2′, 3 and 4 above are all examples of a stochastic trend in accounting variables resulting in nonstationary time series. Moreover, a financial ratio constructed from such processes also exhibits nonstationarity, which is not removed by the ratio transformation.

Recall equation (1) where the natural logarithm of X_{jt} is generalized as $\alpha 1_j + \beta 1_j t + \beta 2_j \ln X_{j,t-1} + u_t$. Now applying the restriction $\alpha 1 \neq 0$, $\beta 1 \neq 0$ and $\beta 2 < 1$, the stochastic trend is no longer persistent, and the variable is stationary around a deterministic trend. For the ratio formed from two such accounting variables, where

$$\left\{\begin{matrix} \ln X1_{j,t} & = & \alpha 11_j + \beta 11_j t + \beta 21_j \ln X1_{j,t-1} + u1_t \\ \ln X2_{j,t} & = & \alpha 12_j + \beta 12_j t + \beta 22_j \ln X2_{j,t-1} + u2_t \end{matrix}\right\},$$

we now have

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \alpha 11_j - \alpha 12_j + \left(\beta 11_j - \beta 12_j\right)t + \beta 21_j \ln X1_{j,t-1} - \beta 22_j \ln X2_{j,t-1} + u1_t - u2_t$$

The above equation may be rewritten as:

$$\ln X1_{j,t} - \beta 21_j \ln X1_{j,t-1} - \left(\ln X2_{j,t} - \beta 22_j \ln X2_{j,t-1}\right) = \alpha 11_j - \alpha 12_j + \left(\beta 11_j - \beta 12_j\right)t + u1_t - u2_t$$

Now, it can be demonstrated that, in the limit, where $\beta 21_j = \beta 22_j = 0$,

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \alpha 11_j - \alpha 12_j + \left(\beta 11_j - \beta 12_j\right)t + u1_t - u2_t \quad , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$ [2]

and the ratio is equivalent to a proportionate growth model, where

$$\frac{X1_{j,t}}{X2_{j,t}} = e^{\left(\alpha 11_j - \alpha 12_j\right)} e^{\left(\beta 11_j - \beta 12_j\right)t} e^{\left(u1_t - u2_t\right)}$$

This implies that, in the absence of a stochastic trend, both the accounting variables and their corresponding financial ratios are characterized by a deterministic trend that is loglinear. Implicit in this representation of the accounting variables and resultant financial ratios, is that they are stationary processes. In such processes, deviations from the trend line are random and will die out quickly.

In the case of (5), the bivariate specification is

$$\left\{\begin{aligned} \ln X1_{j,t} &= \alpha 11_j + \beta 11_{j,t} + u1_t \\ \ln X2_{j,t} &= \alpha 12_j + \beta 12_{j,t} + u2_t \end{aligned}\right\}.$$

By adding a further restriction that $\beta 11 = \beta 12$, or in other words that each variable grows at the same rate, the ratio is modelled by

$$\ln\frac{X1_{j,t}}{X2_{j,t}} = \alpha 11_j - \alpha 12_j + u1_t - u2_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

Assuming that the innovations $u1_t$ and $u2_t$ are uncorrelated, we arrive at the conclusion that the financial ratio $X1_{jt}/X2_{jt}$ varies lognormally around a constant level as represented below:

$$\frac{X1_{j,t}}{X2_{j,t}} = e^{\left(\alpha 11_j - \alpha 12_j\right)} e^{\left(u1_t - u2_t\right)}$$

Figure 1 shows the effect of the above restriction on the ratios analysed here, for the first firm in our sample, A&C Black plc. Panel A illustrates how the restricted proportionate growth model fits a constant ratio, and it shows how this will apply to all of the financial ratios involved as we work through the return on equity identity. In contrast, Panel B shows how the unrestricted model that allows for variable-specific trends results in ratio estimates that reflect the changing structure of the firm over the period that is investigated, giving the appearance of ratio drift.

3. Ratios of cointegrated variables

In the case of stationary variables, the stochastic processes that are involved do not accumulate past errors. Such processes are described as ‘integrated of order zero’, or I(0). For nonstationary variables, the integration will be of a higher order - for example, I(1) and I(2) processes require first and second differencing respectively in order to generate a stationary series. With regard to linear combinations of nonstationary variables, it is possible in certain circumstances that the integration may cancel between series and produce an I(0) outcome (Hendry, 1995). These are cointegrated processes.

In the context of financial ratios, it is this conjecture - that a ratio transformation may have a cointegrating relationship if both of the components are nonstationary - that is tested by Whittington and Tippett (1999).[3] We provide a formal specification below of the cointegrating relationship, and demonstrate how cointegration is consistent with a proportionate growth model of financial ratios.

Consider first a constant ratio, which can be written as $X1_{j,t}/X2_{j,t} = k_j$ for all *t*. Taking logarithms of both sides

$$\ln X1_{j,t} = \ln X2_{j,t} + \delta_j$$

where $\delta_j = \ln k_j$.

In its empirical form,

$$\ln X1_{j,t} = \delta_j + \gamma_j \ln X2_{j,t} + \varepsilon_t \quad \text{.......} \quad (7)$$

For the constant ratio, $\gamma_j = 1$ and $\mathrm{E}\left[\ln\left(X1/X2\right)\right] = \delta_j$ with $\mathrm{E}\left[\varepsilon_t\right] = 0$. In other words, the logarithm of the ratio takes on random values around a constant level of δ_j. The parameter γ_j measures the long run linear growth rate that exists between the two log ratio components. If γ_j is not equal to one, then one component will be growing at a different rate than the other.

Now, if $\ln X1_{j,t}$ and $\ln X2_{j,t}$ are both unit root processes and ε_t is covariance stationary (i.e., an I(0) process), the linear model of the two log ratio components given by (7) describes the cointegrating relation between them.

More generally, for the bivariate representation of the two accounting variables that form a financial ratio, i.e.,

$$\begin{Bmatrix} \ln X1_{j,t} & = & \alpha 11_j + \beta 11_j t + \beta 21_j \ln X1_{j,t-1} + u1_t \\ \ln X2_{j,t} & = & \alpha 12_j + \beta 12_j t + \beta 22_j \ln X2_{j,t-1} + u2_t \end{Bmatrix},$$

substituting the above into (7) gives

$$\alpha 11_j + \beta 11_j t + \beta 21_j \ln X1_{j,t-1} + u1_t \quad = \quad \delta_j + \gamma_j \left(\alpha 12_j + \beta 12_j t + \beta 22_j \ln X2_{j,t-1} + u2_t \right) + \varepsilon_t$$

It follows that

$$\varepsilon_t = \left(\alpha 11_j - \gamma_j \alpha 12_j \right) + \left(\beta 11_j - \gamma_j \beta 12_j \right) t + \beta 21_j \ln X1_{j,t-1} - \gamma_j \beta 22_j \ln X2_{j,t-1} + \left(u1_t - \gamma_j u2_t \right) - \delta_j \qquad \text{.......... (8)}$$

Consider in this context that ε_t follows a first order autoregressive process:

$$\varepsilon_t \quad = \quad a + b\varepsilon_{t-1} + \eta_t \quad \text{..........(9)}$$

Also, without loss of generality, let $a = 0$. Now, if $b = 1$, then ε_t is a unit root process and, when $\ln X1_{j,t}$ and $\ln X2_{j,t}$ are also unit root processes, equation (7) is not a cointegrating relation. For $|b| < 1$, however, the ratio components are cointegrated.

Substituting (8) into (9) results in

$$\left(\alpha 11_j - \gamma_j \alpha 12_j\right) + \left(\beta 11_j - \gamma_j \beta 12_j\right)t + \beta 21_j \ln X1_{j,t-1} - \gamma_j \beta 22_j \ln X2_{j,t-1} + \left(u1_t - \gamma_j u2_t\right) - \delta_j$$
$$= b\left[\left(\alpha 11_j - \gamma_j \alpha 12_j\right) + \left(\beta 11_j - \gamma_j \beta 12_j\right)(t-1) + \beta 21_j \ln X1_{j,t-2} - \gamma_j \beta 22_j \ln X2_{j,t-2} + \left(u1_{t-1} - \gamma_j u2_{t-1}\right) - \delta_j\right] + \eta_t$$

$$\text{i.e., } \ln\left(\frac{X1^{\beta 21_j}}{X2^{\gamma_j \beta 22_j}}\right)_{j,t-1} = \ln\left(\frac{X1^{\beta 21_j}}{X2^{\gamma_j \beta 22_j}}\right)^b_{j,t-2} + (b-1)\left[\left(\alpha 11_j - \gamma_j \alpha 12_j\right) + \left(\beta 11_j - \gamma_j \beta 12_j\right)t - \delta_j\right]$$
$$- b\left(\beta 11_j - \gamma_j \beta 12_j\right) + \left[\left(u1_t - \gamma_j u2_t\right) - b\left(u1_{t-1} - \gamma_j u2_{t-1}\right)\right] + \eta_t$$

When the ratio components are nonstationary (i.e., $\beta 21 = \beta 22 = 1$) and are not cointegrated (i.e., b=1),

$$\ln\frac{X1_{t-1}}{X2^{\gamma_j}_{t-1}} = -\left(\beta 11_j - \gamma_j \beta 12_j\right) + \ln\frac{X1_{t-2}}{X2^{\gamma_j}_{t-2}} + \xi_t$$

where $\xi_t = \left(u1_t - u1_{t-1}\right) - \gamma_j\left(u2_t - u2_{t-1}\right) + \eta_t$.

In other words, nonstationary variables that are not cointegrated will lead to a random walk with drift in $\ln\left(X1_{j,t} \big/ X2^{\gamma_j}_{j,t}\right)$, the adjusted ratio that allows for the differential growth relationship between $X1$ and $X2$, and in the unadjusted ratio when $\gamma_j = 1$. Moreover, if $\gamma_j = \beta 11_j / \beta 12_j$, there will be no drift.

In contrast, when nonstationary ratio components are cointegrated, such that b<1 and $b \to 0$, then in the limit

$$\ln\frac{X1_{t-1}}{X2^{\gamma_j}_{t-1}} = \delta_j - \left(\alpha 11_j - \gamma_j \alpha 12_j\right) - \left(\beta 11_j - \gamma_j \beta 12_j\right)t + \phi_t$$

where $\phi_t = \left(u1_t - \gamma_j u2_t\right) + \eta_t$.

Thus, nonstationary variables that are cointegrated, with diminished autoregression in the error, form a financial ratio that tends in the limit towards the proportionate growth model, and to its restricted form of a constant when $\gamma_j = \beta 11_j / \beta 12_j$.

4. Analysis

As set out above, in this paper we evaluate a generalised model that incorporates stochastic and deterministic trends in the ratio, allowing also for restricted and unrestricted proportionate growth. The sample consists of European firms that are included in the Worldscope database. The period examined in the study is from 1992 to 1999, and the sample is restricted to firms that report for a calendar year in every period. Furthermore, we also require that all necessary financial information is provided for all eight years, and that the balance sheet and income statement information extracted from the database articulate. Firms with negative equity, liabilities, assets, sales or costs were excluded. The final sample on which the results are based comprised 609 firms over eight years, i.e. 4872 firm years.

The median values of the logarithm of equity, liabilities, assets, sales and costs are given in Table 1 for each year from 1992 to 1999. It is evident that the general trend is upwards, in all cases without exception, and Figure 2 demonstrates this pervasive effect of firm growth on each of the variables of interest. The similarity in their gradients is the key feature of these plots.

Table 1 also provides an understanding of the cross-sectional distributions of the ratios under investigation. The log distributions are particularly stable across the years. We find that the log of Sales/Assets (SA/TA) is consistently logistic, the log of Liabilities/Equity (TL/SE) is consistently log-logistic, and the log of Costs/Liabilities

(TC/TL) is consistently Weibull, as illustrated in Figure 3 using observed values for the first year, 1992.[4] The logistic, log-logistic and Weibull distributions are closely related extreme value distributions, and are special cases of the generalized Gamma function.[5] However, this function requires analytical integration as there is no closed-form equivalent. The paper proceeds on the basis of the reasonable simplifying assumption of loglinearity between the components of each of the three pure financial ratios that are examined. This is supported by Figure 4, which presents bivariate plots on a log scale, for Sales v. Assets, Liabilities v. Equity and Costs v. Liabilities.

The drawback of focussing on a small sample of long-lived firms, as analysed by Whittington and Tippett (1999), Ioannides, Peel and Peel (2003) and Peel, Peel and Venetis (2004), is that they are not representative of the population of firms. In contrast, we consider a larger number of shorter series (eight observations), which is more in keeping with the timespan over which a financial analyst may look backwards in attempting to understand how a firm's financial structure and performance arrived at their present position and how the firm compares with others. For each accounting variable, initial estimates are obtained from a vector autoregression at the firm level. Figure 5 provides an indication of the distribution of the autoregressive coefficient β across the firms for each of the five variables of interest. The mean varies between 0.6388 (Liabilities) and 0.7207 (Assets), as shown in the fourth column of Panel A in Table 2. In each case, the density rises towards 1, and the mode always lies below 1, but there is a small but significant proportion of series in each case where β is equal to or greater than 1. Indeed, the top 5% of observed estimates is always above 1. Thus, univariate analysis suggests that nonstationarity is plausible in ratio numerators and denominators, although the vast majority of series are stationary.

The potential for nonstationarity is mitigated however by the fact that any influence from prior errors is expected to decay relatively quickly. Panel B of Table 2 shows the results when the current error for the j^{th} firm is estimated from the lagged estimate as $u_{j,t} = a + bu_{j,t-1} + \eta_{j,t}$. After pooling the regression, b is shown to be between 0.5 and 0.6.[6]

To resolve the issue, a test of the joint null hypothesis of nonstationarity is required. Testing for unit roots in heterogeneous panels has already received a great deal of attention in the econometrics literature, and proposals include the cross-sectional de-meaning of series (Im, Pesaran and Shin, 1995), the incorporation of integrable functions of lagged dependent variables (Chang, 2002) and the adjustment of observed values to remove common factors (Moon and Perron, 2004). Each of these cross-sectionally augmented tests has the appropriate asymptotic properties, but they are for $T \rightarrow \infty$, and generally for $T > N$. Panel C of Table 2 reports the results of an alternative unit root test which offers the finite sample properties that are necessary (Pesaran, 2006).[7] This test also relaxes the assumption of cross-sectional independence implicit in the standard univariate Dickey-Fuller approach, and is consistent with the model structure described earlier, i.e. it is asymptotically convergent for short T and large N, and it is robust in the presence of a deterministic trend. In order to avoid the influence of nuisance parameters, the test is applied to the deviation between the dependent variable and its initial cross-section mean in year 0, where the time series is indexed $0 \ldots T$. The standard Dickey-Fuller regression of the first difference in the dependent variable on the lagged dependant variable is augmented cross-sectionally by the addition of the first difference in the yearly mean and the lagged mean. The cross-sectionally augmented Dickey-Fuller (CADF) test is based on the t-ratio of the OLS estimate of the coefficient on the lagged value of the dependent variable, and for significance we rely here on critical values for the shortest T (10) and the largest N (200) as tabulated in Pesaran (2006).[8] The results in Panel C show that, for all five variables, the null hypothesis of a joint unit root cannot be rejected convincingly, although there is some (weak) evidence in support of stationarity in Shareholders'Equity and Total Liabilities, and, for individual firms, the unit root hypothesis is strongly rejected at the 1% level in about one third of cases for each variable.

To test for cointegration, firm level estimates of δ and γ are obtained from firm-specific fits of Equation (7), the empirical form of the loglinear ratio. First, it should be noted that Panel A of Table 3 gives mean values of γ as 0.8582 in the case of Sales to Assets, 0.5280 for Liabilities to Equity and 0.6782 for Costs to Liabilities. In each case, the range of the estimates includes 1, the value at which an assumption of equal growth rates in the two variables would hold. In effect, the proportion of cases where $\gamma \geq 1$ is 38% for Sales to Assets, 27% for Liabilities to Equity and 20% for Costs to Liabilities. To test for cointegration between the ratio components, the error term is subject to a pooled Dickey-Fuller test where $\varepsilon_{j,t}$ = a + b $\varepsilon_{j,t\text{-}1}$ + $\eta_{j,t}$. As with the univariate stationarity tests of ratio components, by construction the constant, a, in the cointegrating regression is equal to zero, as the estimates reported in Panel B of Table 3 confirm. The cross-sectionally augmented unit root test reported in Panel C finds no support for the joint hypothesis that b=1 for each of the three pairings, and, for individual firms, the test rejects b=1 at the 10% level in 57% of cases for Sales to Assets, 68% for Liabilities to Equity and 79% for Costs to Liabilities. Indeed, the average estimates of b are 0.097 for Sales to Assets, 0.125 for Liabilities to Equity and 0.106 for Costs to Liabilities, suggesting again a rapid decay in the effects carried forward to future accounting periods. Furthermore, given that cointegration between the financial ratio components seems to tend to its limit with $b \rightarrow 0$, these results imply that, in cases where there is nonstationarity in the variables examined here, proportionate growth may be a plausible model for the financial ratios involved.[9]

Table 4 gives the results from the proportionate growth regression in Panel A. When we introduce the restriction that $\beta 1_j$ is equal for each variable involved, the joint estimate across the five accounting variables provides a measure of the firm's growth during the period. The mean estimate of the continuous growth rate $\beta 1_j$ across all firms is 8.59% p.a. and the 5th and 95th percentiles of the distribution range from -6.03% to 29.3%. When the proportionate growth estimation is unrestricted, such that $\beta 1_j$ varies not only across firms but also across accounting variables, it can be seen that the mean estimates of growth vary little from one variable to another, the lowest being 8.40% (Total Costs) and the highest 8.81% (Total Liabilities).

In order to assess the proportionate growth model, we compare the fit of each of the processes that have been used to describe the component variables of the financial ratios. Equation (1) is the full model defined as $\ln X_{jt} = \alpha 1_j + \beta 1_j t + \beta 2_j \ln X_{j,t-1} + u_t$, with the restriction $\beta 1$=0 leading to the stochastic trend model $\ln X_{jt} = \alpha 1_j + \beta 2_j \ln X_{j,t-1} + u_t$. If each of these processes are nonstationary time series, then, as demonstrated earlier, financial ratios constructed from such variables are also nonstationary. The restriction $\beta 2$=0 leads to the deterministic trend model of proportionate growth, $\ln X_{jt} = \alpha 1_j + \beta 1_j t + u_t$. This model may be further restricted by constraining $\beta 1_j t$ to be equal across all variables, in which case the ratio of two such variables would tend towards a constant as cointegration between the variables tends towards its limit. Finally, as shown in Panel B of Table 4, the null is the firm mean where $\ln X_{jt} = \alpha 1_j + u_t$. It can be seen that the adjusted mean squared error falls for the average firm from the null of 0.1505 to

0.0508 (F=12.781, prob<0.0001) when a deterministic trend is fitted to each variable for each firm, and to 0.0682 (F=8.245, prob<0.0001) when a stochastic trend is fitted to each variable for each firm.[10] The deterministic trend is a better fit to all variables than the stochastic trend. On average, across all five variables, the R^2 is 62.2% for the stochastic trend, 72.9% for the deterministic trend and 76.3% for the full model. Although the latter will always provide the best fit overall, the gain in explanatory power requires an additional parameter for each variable, and this seems to have little statistical support. That is to say, the indicative F-ratios for the average firm are 0.735 (prob=0.6040) when a stochastic trend is added to the unrestricted proportionate growth model. On the other hand, the explanatory power that is lost when the growth estimates are constrained to be equal across all variables is not of great statistical significance, the F-ratio between the restricted and unrestricted models being 2.120 (prob=0.1081). Furthermore, given the degrees of freedom that are gained, the model of restricted proportionate growth (average mean squared error 0.0586) provides a more plausible and parsimonious model than the stochastic trend (average mean squared error 0.0682).

5. Conclusions

This paper sets out to provide a comprehensive model of deterministic and stochastic trends in accounting variables and the financial ratios which they form, building on innovative research by Whittington and Tippett (1999), Ioannides, Peel and Peel (2003) and Peel, Peel and Venetis (2004). The focus of the present paper differs from the above however, as the main concern here is not primarily with the potential for spurious regression when potentially nonstationary variables are employed as regressors, which is well documented in econometrics[11], but instead with the statistical validity of the simple ratio metric that is commonly used in business and finance as a measure on which to base both interfirm comparisons and financial trend analyses.

When account is taken of cross-sectional dependence between companies, and also of the relatively short length of accounting time series, we are unable to reject a joint hypothesis of nonstationarity in accounting variables, although there is no significant evidence of a unit root in about one third of the individual firm series. By their very nature, however, the components of financial ratios are correlated variables, and our estimates show that any cointegrating effects will decay rapidly.

Furthermore, it is important to recognise that line items in income statements and balance sheets, such as sales or total assets, will tend to grow as the firm grows. Over a given period, accounting variables may grow at a rate that is higher or lower than the firm as a whole, especially if it is in the process of altering its financial or operating structure. This transitory divergence in deterministic trends within the same firm may give the appearance of drift in the ratio of two variables as the level changes. However, it is shown here that ‘pure’ financial ratios can be defined parsimoniously by their lognormal variation around an expected value, and that such ratios can be represented succinctly by a statistically valid model of proportionate growth in the firm.

Appendix

Although the analytical result in this paper relies on the restrictive assumption that there is no stochastic trend in either ratio component, *i.e.* that $\beta 21_j = \beta 22_j = 0$ in (5), a more general ratio model that allows us to relax this assumption may be derived with recursive substitution.

Consider the accounting variable, X, such that

$\ln X_t = \alpha 1 + \beta 1 t + \beta 2 \ln X_{t-1} + u_t$, as in Equation (1) in the text.

Then, by substitution

$$\ln X_t = \alpha 1 + \beta 1 t + \beta 2(\alpha 1 + \beta 1(t-1) + \beta 2 \ln X_{t-2} + u_{t-1}) + u_t .$$

This may be rearranged as

$$\ln X_t = \alpha 1(1+\beta 2) + \beta 1 t + \beta 1 \beta 2(t-1) + \beta 2^2 \ln X_{t-2} + u_t + \beta 2 u_{t-1}.$$

By substituting for $\ln X_{t-2}$, it follows that

$$\ln X_t = \alpha 1(1+\beta 2) + \beta 1 t + \beta 1 \beta 2(t-1) + \beta 2^2(\alpha 1 + \beta 1(t-2) + \beta 2 \ln X_{t-3} + u_{t-2}) + u_t + \beta 2 u_{t-1}$$

which may be rearranged in turn as

$$\ln X_t = \alpha 1(1+\beta 2+\beta 2^2) + \beta 1 t + \beta 1 \beta 2(t-1) + \beta 1 \beta 2^2(t-2) + \beta 2^2 \ln X_{t-3} + u_t + \beta 2 u_{t-1} + \beta 2^2 u_{t-2}.$$

With recursive substitution through t=0, and given an initial value $X_{j,0}$ on the accounting variable X reported by firm j, the specification in (1) may be rewritten as:

$$\ln X_{j,t} = \alpha 1_j \left(1 + \beta 2_j + \beta 2_j^2 + \ldots + \beta 2_j^{t-1}\right) + \beta 1_j t + \beta 2_j \beta 1_j (t-1) + \beta 2_j^2 \beta 1_j (t-2) + \ldots + \beta 2_j^{t-1} \beta 1_j + \beta 2_j^t \ln X_{j,0} + u_t + \beta 2_j u_{t-1} + \ldots + \beta 2_j^{t-1} u_1$$

It follows that the ratio of two such variables *X*1 and *X*2 is equal to

$$\begin{aligned}\ln \frac{X1_{j,t}}{X2_{j,t}} = {} & \alpha 11_j \left(1 + \beta 21_j + \beta 21_j^2 + \ldots + \beta 21_j^{t-1}\right) - \alpha 12_j \left(1 + \beta 22_j + \beta 22_j^2 + \ldots + \beta 22_j^{t-1}\right) + \\ & \left(\beta 11_j - \beta 12_j\right) t + \\ & \left(\beta 21_j \beta 11_j - \beta 22_j \beta 12_j\right)(t-1) + \left(\beta 21_j^2 \beta 11_j - \beta 22_j^2 \beta 12_j\right)(t-2) + \ldots + \left(\beta 21_j^{t-1} \beta 11_j - \beta 22_j^{t-1} \beta 12_j\right) + \\ & \beta 21_j^t \ln X1_{j,0} - \beta 22_j^t \ln X2_{j,0} + \\ & u1_t - u2_t + \beta 21_j u1_{t-1} - \beta 22_j u2_{t-1} + \ldots + \beta 21_j^{t-1} u1_1 - \beta 22_j^{t-1} u2_1\end{aligned}$$

which, as in (6), simplifies to $\alpha 11_j - \alpha 12_j + u1_t - u2_t$ when

$\beta 11_j = \beta 12_j$ and $\beta 21_j = \beta 22_j = 0$.

If we now relax the restriction by holding both trends, deterministic and stochastic, equal across the ratio components (*i.e.* $\beta 11_j = \beta 12_j$ and $\beta 21_j = \beta 22_j = \beta 2_j$), it follows that

$$\begin{aligned}\ln \frac{X1_{j,t}}{X2_{j,t}} = {} & \left(\alpha 11_j - \alpha 12_j\right)\left(1 + \beta 2_j + \beta 2_j^2 + \ldots + \beta 2_j^{t-1}\right) + \beta 2_j^t \left(\ln \frac{X1_{j,0}}{X2_{j,0}}\right) \\ & + u1_t - u2_t + \beta 2\left(u1_{t-1} - u2_{t-1}\right) + \ldots + \beta 2_j^{t-1}\left(u1_1 - u2_1\right)\end{aligned}$$

Footnotes

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[1] It is not the aim of this paper to derive the distributional form of return on equity, a ratio that has non-convergent moments (i.e. if book equity reaches its lower bound of zero, the ratio is infinity, and if earnings are also at break-even, the ratio is undefined).

[2] Note that a model that allows us to relax the assumption $\beta 21_j = \beta 22_j = 0$ may be derived with recursive substitution. The derivation of this more general model is given in the Appendix.

[3] Whittington and Tippett (1999) provide an excellent overview of cointegration and unit roots. Their evidence is based on Dickey-Fuller tests at the firm level, and suggests that the ratio does not always remove the effects of nonstationarity in the ratio components, even when drift in the ratio is accounted for with an additional trend term.

[4] The stability of the distributions of the three pure ratios is in contrast to Return on Equity, where the best fit varies between normal, Student t, logistic, beta and log-logistic, as is also shown in Table 1. However, this overfitting is attributable to small numbers of extreme values - if the extreme negative values that arise when Shareholders' Equity approaches its lower bound of zero are removed, this results in a consistently best fit by the logistic distribution.

[5] Extreme value distributions are commonly applied in survival analysis - see George and Devidas (1992).

[6] By construction, the mean residual is zero, and the estimate of a is not significantly different to zero (the 0.05 and 0.95 confidence limits are below and above zero respectively for each of the five variables of interest).

[7] For fixed and random effects models, where the time dimension is small and the cross-sectional dimension is large, maximum likelihood estimators have also been obtained and their finite sample properties documented (Binder, Hsiao and Pesaran, 2005).

[8] The critical values of the limit distribution of the test statistic are tabulated in Pesaran (2006) for $N = 10 \ldots 200$ and $T = 10 \ldots 200$. When the model has an intercept but no trend, the critical values for the largest $N = 200$ and the shortest $T = 10$ are as follows:

[9] As the main interest is not in the contemporaneous correlation between ratio components but in any systematic effects in the error term, firm level estimates of β are not required, and a panel data analysis may be undertaken to allow for between-firm variation as fixed effects. In

this case, the fixed effects estimates of b are: Sales to Assets 0.3569 (s.e.=0.0140); Liabilities to Equity 0.4118 (s.e.=0.0129); and Costs to Liabilities 0.3436 (s.e.=0.0143), and b=1 is rejected for each of the three pairings.

[10] Although a predicted value may be fitted to year one for the null and deterministic trend models, there is no initial prediction in the case of the full and stochastic trend models, which are autoregressive. Given that the firm series are short, and the degrees of freedom are a function of series length, we compare all model fits by excluding year 1 from the mean squared error of the deterministic trend models and the null. The number of observations in years 2 to 8 is 35 per firm when all 5 variables are considered jointly in the seemingly unrelated regression that restricts the proportionate growth model. The mean squared error is calculated as the sum of squared errors divided by the adjusted degrees of freedom (i.e. 35 less the number of parameters in the model), and standard F-tests are used here to compare models and to evaluate model restrictions, given the small samples by firm.

[11] With regard to spurious regression in panel data, see Baltagi (2003, chapter 12).

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Table 1. Median values of observed variables and ratios, and their fitted probability distributions

	Years	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Accounting Variables									
Log_e Sales		12.046	12.093	12.165	12.331	12.408	12.485	12.578	12.609
Log_e Costs		12.006	12.090	12.126	12.269	12.364	12.428	12.506	12.554
Log_e Assets		12.034	12.076	12.143	12.254	12.349	12.355	12.408	12.568
Log_e Liabilities		11.392	11.450	11.591	11.645	11.718	11.743	11.888	11.997
Log_e Equity		11.122	11.098	11.218	11.385	11.437	11.483	11.627	11.698
Financial Ratios									
Log_e (Sales/Assets)		0.128	0.132	0.131	0.158	0.172	0.165	0.141	0.131
	Best fit	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*
Log_e (Liabilities/Equity)		0.300	0.258	0.256	0.248	0.285	0.279	0.300	0.287
	Best fit	*LL*	*LL*	*LL*	*LL*	*LL*	*LL*	*LL*	*LL*
Log_e (Costs/Liabilities)		0.730	0.750	0.736	0.761	0.787	0.753	0.713	0.699
	Best fit	*W*	*W*	*W*	*W*	*W*	*W*	*W*	*W*
Return on Equity									
- Observed		0.090	0.100	0.106	0.111	0.113	0.123	0.118	0.118
	Best fit	*B*	*ST*	*N*	*ST*	*LL*	*LL*	*LS*	*ST*
- Truncated		0.092	0.100	0.106	0.111	0.113	0.123	0.119	0.120
	Best fit	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*	*LS*

Best fitting distributions: B Beta General. LL Log-logistic. LS Logistic. N Normal. ST Student. W Weibul. The *BEST FIT* statistical software provides estimates of the best fitting distribution from a wide range of potential models.

Table 2. Cross-sectional tests of stationarity

A. First order autoregression by firm

$\ln X_{j,t} = \alpha 1_j + \beta 2_j \ln X_{j,t-1} + u_{j,t}$	$\overline{\alpha}$	5^{th} %	95^{th} %	$\overline{\beta}2$	5^{th} %	95^{th} %	$\overline{MSE}$	5^{th} %	95^{th} %
Shareholders' Equity	3.7299	-3.9520	13.1232	0.6804	-0.1972	1.3773	0.1007	0.0016	0.3303
Total Liabilities	4.2641	-3.7009	13.6857	0.6388	-0.1786	1.3430	0.0847	0.0029	0.2975
Total Assets	3.4798	-5.0692	12.5607	0.7207	-0.0310	1.4118	0.0450	0.0021	0.1579
Sales	3.7156	-2.7705	12.3210	0.7064	0.0026	1.2141	0.0544	0.0016	0.1600
Total Costs	3.9033	-2.8701	13.5490	0.6957	-0.0778	1.2480	0.0561	0.0017	0.1660

B. Error decay (pooled)

$u_{j,t} = \mathbf{a} + \mathbf{b} u_{j,t-1} + \eta_{j,t}$	$\hat{a}$	s.e.	0.05	0.95	$\hat{b}$	s.e.	0.05	0.95
Shareholders' Equity	0.0047	0.0048	-0.0047	0.0141	0.4752	0.0133	0.4491	0.5012
Total Liabilities	0.0058	0.0045	-0.0030	0.0146	0.5579	0.0109	0.5365	0.5794
Total Assets	0.0049	0.0033	-0.0017	0.0114	0.6539	0.0087	0.6368	0.6710
Sales	0.0051	0.0037	-0.0021	0.0123	0.6049	0.0099	0.5856	0.6242
Total Costs	0.0038	0.0037	-0.0035	0.0111	0.6055	0.0099	0.5861	0.6249

C. Cross-sectionally augmented Dickey-Fuller (CADF) test

	Average: full distribution $\overline{CADF}$	Average: truncated distribution $\overline{CADF}''$	By firm: % rejection of unit root hypothesis $CADF_j$		
		Signif:	0.10	0.05	0.01
Shareholders' Equity	-2.3248 ***	-2.0264 *	50.4%	41.5%	34.0%
Total Liabilities	-2.1475 **	-2.0391 *	54.2%	44.0%	34.5%
Total Assets	-2.0464 *	-1.9455	51.4%	42.7%	34.0%
Sales	-2.0385 *	-1.9713	54.4%	47.0%	36.9%
Total Costs	-1.9733	-1.9276	54.0%	44.8%	32.2%

The estimates in Panel A were obtained from vector autoregressions by firm for 609 firms, as specified by Equation (2). The coefficients and mean squared error are reported as averages across firms, together with the 5th and 95th percentiles. Panel B reports the estimates $(\hat{a}, \hat{b})$ from a pooled regression of the error terms from firm-specific autoregressions, and the range of plausible coefficient estimates is indicated in each case by the 0.05 and 0.95 confidence levels. Panel C reports on the cross-sectionally augmented Dickey-Fuller (CADF) test of the unit root hypothesis, based on the t-ratio of the coefficient b_j in the CADF regression $\Delta y_{j,t} = a_j + b_j y_{j,t-1} + c_j \overline{y}_{t-1} + d_j \Delta \overline{y}_t + e_{j,t}$, where $y_{j,t}$ represents the deviation between $\ln X_{j,t}$ and $\overline{\ln X_0}$ (i.e., the initial cross-section mean is set to zero to eliminate the effect of nuisance parameters - see Pesaran, 2006).

Table 3. Cross-sectional tests of cointegration

A. Loglinear regression of ratio components (by firm)

$\ln X1_{j,t} = \delta_j + \gamma_j \ln X2_{j,t} + \varepsilon_{j,t}$	$\overline{\delta}$	5th %	95th %	$\overline{\gamma}$	5th %	95th %	$\overline{MSE}$
Sales : Total Assets	1.9416	-8.8039	14.0120	0.8582	-0.1012	1.7430	0.0315
Total Liabilities : Shareholders' Equity	5.7067	-12.0300	21.5440	0.5280	-0.8038	2.0004	0.0769
Total Costs : Total Liabilities	4.4905	-4.4057	13.7180	0.6782	-0.1211	1.5059	0.0355

B. Cointegrating regression (by firm)

$\varepsilon_{j,t} = \mathbf{a} + \mathbf{b}\varepsilon_{j,t-1}$	$\overline{a}$	5th %	95th %	$\overline{b}$	5th %	95th %
Sales : Total Assets	-0.0032	-0.0411	0.0285	0.0975	-0.5654	0.7609
Total Liabilities : Shareholders' Equity	-0.0059	-0.0898	0.0758	0.1252	-0.4600	0.8266
Total Costs : Total Liabilities	-0.0090	-0.0592	0.0288	0.1060	-0.5504	0.8096

C. Cross-sectionally augmented Dickey-Fuller (CADF) test

	Average: full distribution $\overline{\text{CADF}}$	Average: truncated distribution $\overline{\text{CADF}}''$		By firm: % rejection of unit root hypothesis CADF_j		
			Signif:	0.10	0.05	0.01
Sales : Total Assets	-2.1194 **	-2.0097 **		57.0%	42.9%	29.6%
Total Liabilities : Shareholders' Equity	-2.3970	-2.3116 ***		68.0%	54.2%	39.4%
Total Costs : Total Liabilities	-2.7557	-2.6734 ***		79.3%	71.1%	60.6%

The estimates $(\hat{\mu}, \hat{\gamma})$ in Panel A were obtained from OLS regression by firm of the empirical form of the ratio specification in Equation (7). Panel B reports on the cointegration of ratio components, based on the autoregressive process specified in Equation (9). In both panels, the coefficients and mean squared error are reported as averages across firms, together with the 5th and 95th percentiles. Panel C reports on the cross-sectionally augmented Dickey-Fuller (CADF) test of the unit root hypothesis, based on the t-ratio of the coefficient b_j in the CADF regression $\Delta\varepsilon_{j,t} = a_j + b_j\varepsilon_{j,t-1} + c_j\overline{\varepsilon}_{t-1} + d_j\Delta\overline{\varepsilon}_t + e_{j,t}$, (see Pesaran, 2006).

Table 4. Proportionate growth

A. Estimation of deterministic trends by firm (seemingly unrelated regression)

$\ln X_{j,t} = \alpha 1_j + \beta 1_j t$	$\overline{\alpha}$	5th %	95th %	$\overline{\beta 1}$	5th %	95th %	$\overline{\text{MSE}}$	5th %	95th %
Restricted proportionate growth									
Shareholders' Equity	11.8549	8.7150	15.6367				0.0942	0.0128	2.0377
Total Liabilities	11.5319	8.3556	14.8574				0.0720	0.0200	1.2975
Total Assets	11.5690	8.6141	15.1591	0.0859	-0.0634	0.2929	0.0342	0.0080	0.6679
Sales	12.1027	8.5661	15.6953				0.0455	0.0089	0.8116
Total Costs	11.8583	8.5638	15.3888				0.0472	0.0100	0.8910
Unrestricted proportionate growth									
Shareholders' Equity	11.1431	8.1552	14.5844	0.0876	-0.0882	0.2975	0.0714	0.0011	0.2663
Total Liabilities	11.4208	7.9617	15.1324	0.0881	-0.1151	0.3333	0.0622	0.0019	0.2134
Total Assets	12.0702	8.7873	15.5261	0.0861	-0.0649	0.2963	0.0341	0.0011	0.1243
Sales	12.1568	9.0519	15.7460	0.0853	-0.0778	0.2940	0.0423	0.0010	0.1328
Total Costs	12.1185	8.9628	15.6890	0.0840	-0.0814	0.2931	0.0439	0.0010	0.1371

B. Comparison between proportionate growth and stochastic trend models

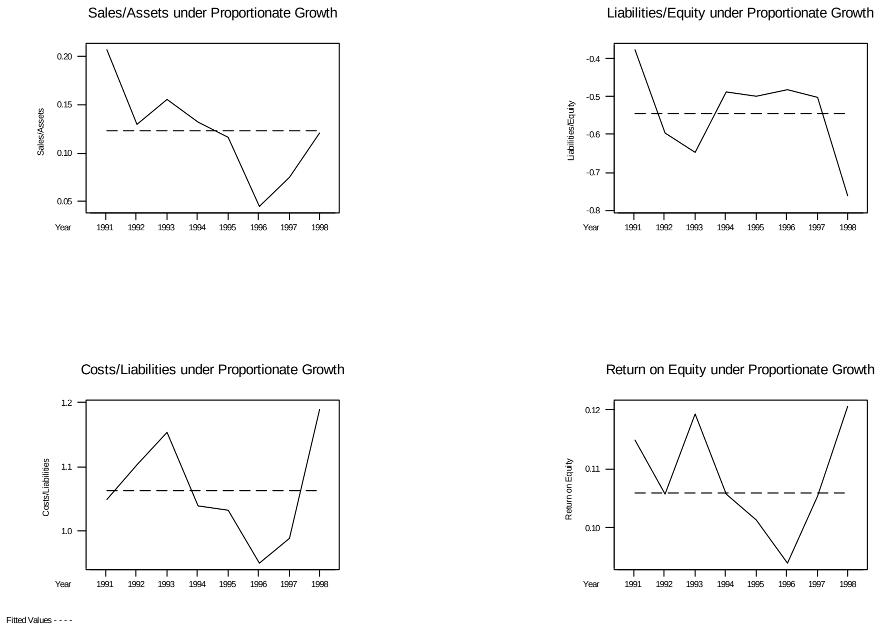
	Sums of Squared Errors and Mean Squared Errors of Fitted Models									
	Full Model		Stochastic Trend		Deterministic Trend: Unrestricted Proportionate Growth		Deterministic Trend: Restricted Proportionate Growth		Mean	
	$\ln X_{j,t} = \alpha 1_j + \beta 1_j t + \beta 2_j \ln X_{j,t-1}$		$\ln X_{j,t} = \alpha 1_j + \beta 2_j \ln X_{j,t-1}$		$\ln X_{j,t} = \alpha 1_j + \beta 1_j t$		$\ln X_{j,t} = \alpha 1_j + \beta 1_j t$ (**$\beta 1$ jointly estimated**)		$\ln X_{j,t} = \alpha 1_j$	
	$\overline{\text{SSE}}$	$\overline{\text{MSE}}$	$\overline{\text{SSE}}$	$\overline{\text{MSE}}$	$\overline{\text{SSE}}$	$\overline{\text{MSE}}$	$\overline{\text{SSE}}$	$\overline{\text{MSE}}$	$\overline{\text{SSE}}$	$\overline{\text{MSE}}$
Shareholders' Equity	0.2997	0.0749	0.5033	0.1007	0.3572	0.0714	0.5462	0.0942	1.0644	0.1774
Total Liabilities	0.2675	0.0669	0.4237	0.0847	0.3108	0.0622	0.4178	0.0720	1.0604	0.1767
Total Assets	0.1442	0.0361	0.2249	0.0450	0.1704	0.0341	0.1986	0.0342	0.7491	0.1248
Sales	0.1795	0.0449	0.2719	0.0544	0.2116	0.0423	0.2638	0.0455	0.8175	0.1363
Total Costs	0.1815	0.0454	0.2805	0.0561	0.2195	0.0439	0.2740	0.0472	0.8234	0.1372
All variables	1.0724	0.0536	1.7044	0.0682	1.2695	0.0508	1.7003	0.0586	4.5148	0.1505
R^2	76.3%		62.2%		71.9%		62.3%			

Parameters, P	15	10	10	6	5
Degrees of freedom, DF	20	25	25	29	30
F : against null (mean effects model)		8.245 prob<0.0001	12.781 prob<0.0001		
F : against full model		2.357 prob=0.0776	0.735 prob=0.6040		
F : proportionate growth restriction				2.120 Prob=0.1081	

The estimates in Panel A were obtained from a seemingly unrelated regression, allowing restriction on coefficients across firms. In order to compare the sum of squared errors (SSE) and the mean squared error (MSE) across models, all fits exclude year 1. The coefficients and MSEs in Panel A are reported as averages across firms, together with the 5^{th} and 95^{th} percentiles, as are the SSEs in Panel B. The MSE is calculated here as SSE/DF, where the available degrees of freedom (DF=N-P) is equal to the number of observations across all five variables (N=35) less the number of parameters (P). The indicative R^2 is unadjusted, and is computed in aggregate over all variables as 1 – SSE(model)/SSE(null).

Figure 1. Ratios that allow for proportionate growth in the numerator and denominator

(A) Restricted proportionate growth



(B) Unrestricted proportionate growth

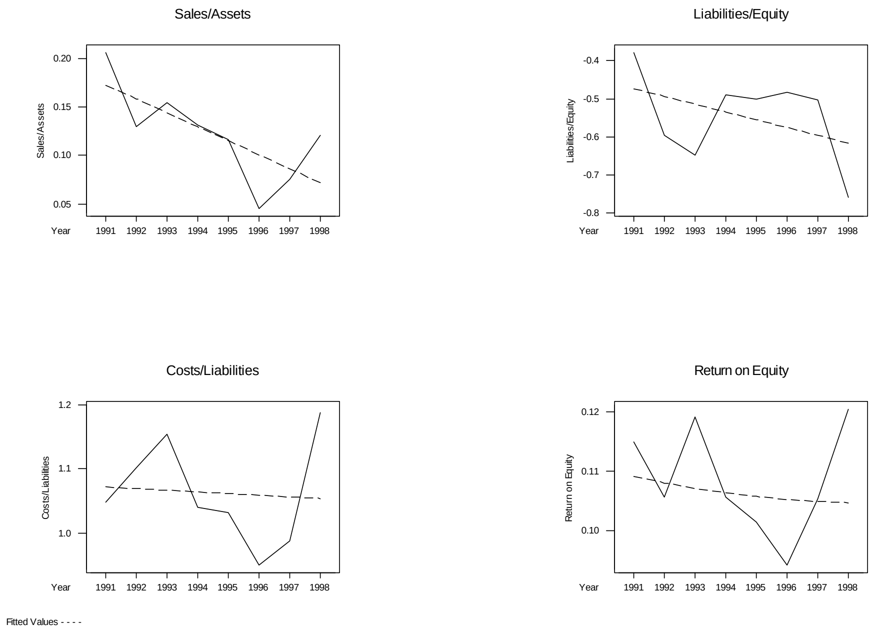


Figure 2. Ratio components grow at similar rates

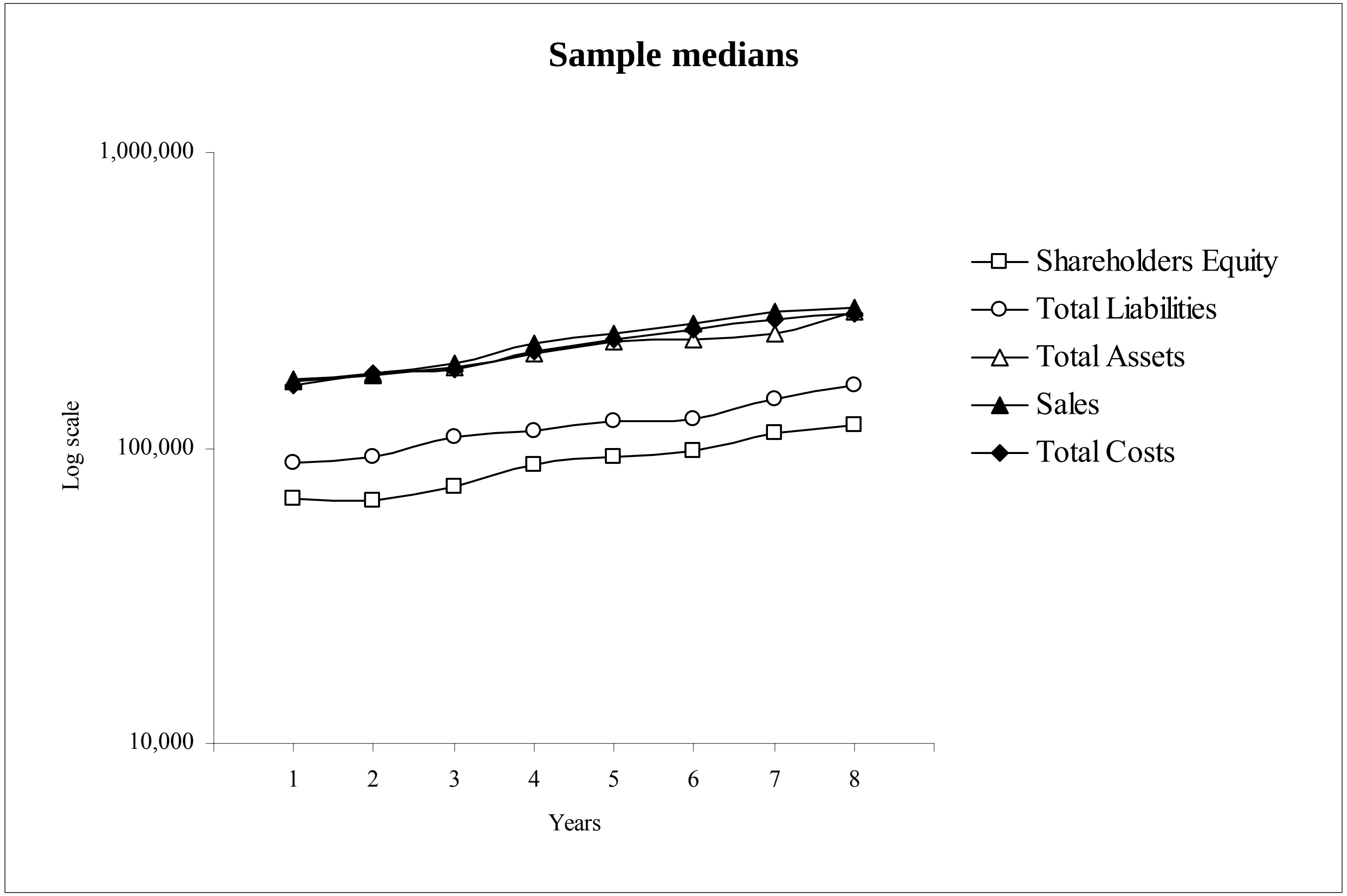


Figure 3. Best Fits to Ratio Frequency Distributions

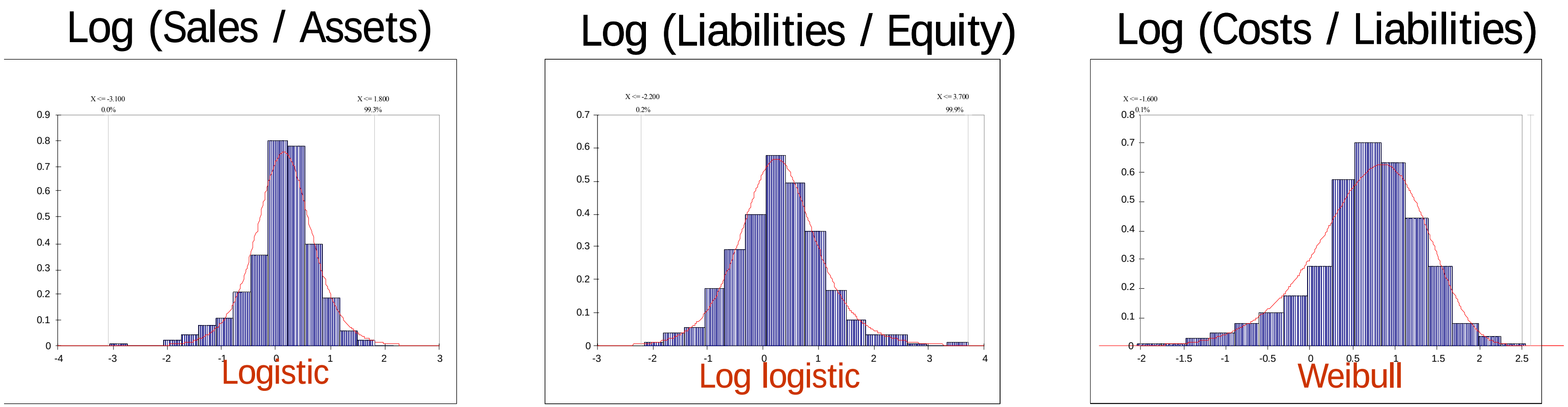


Figure 4. Ratio bivariate log plots

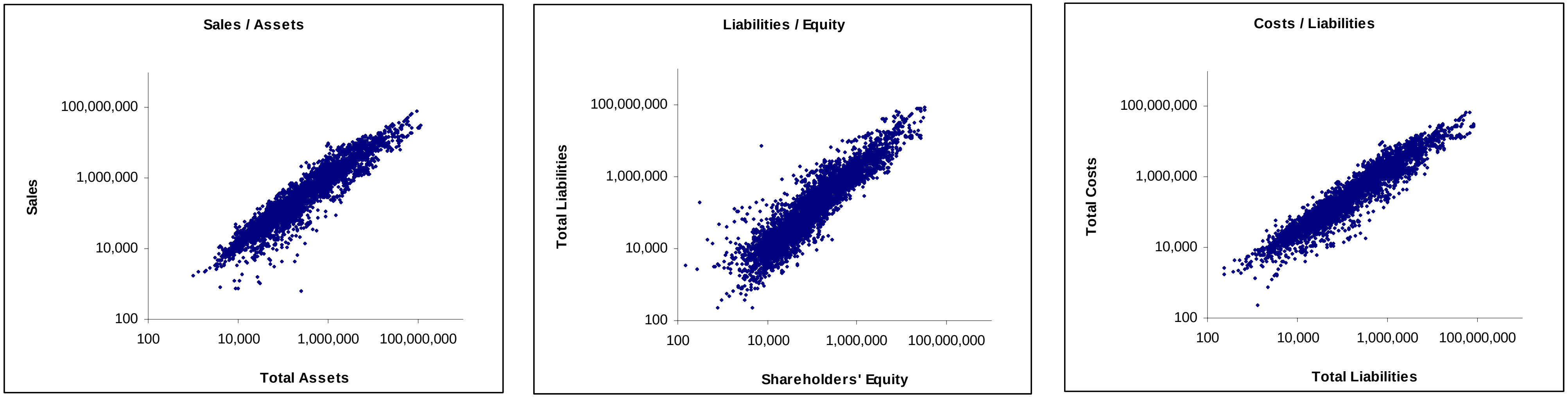
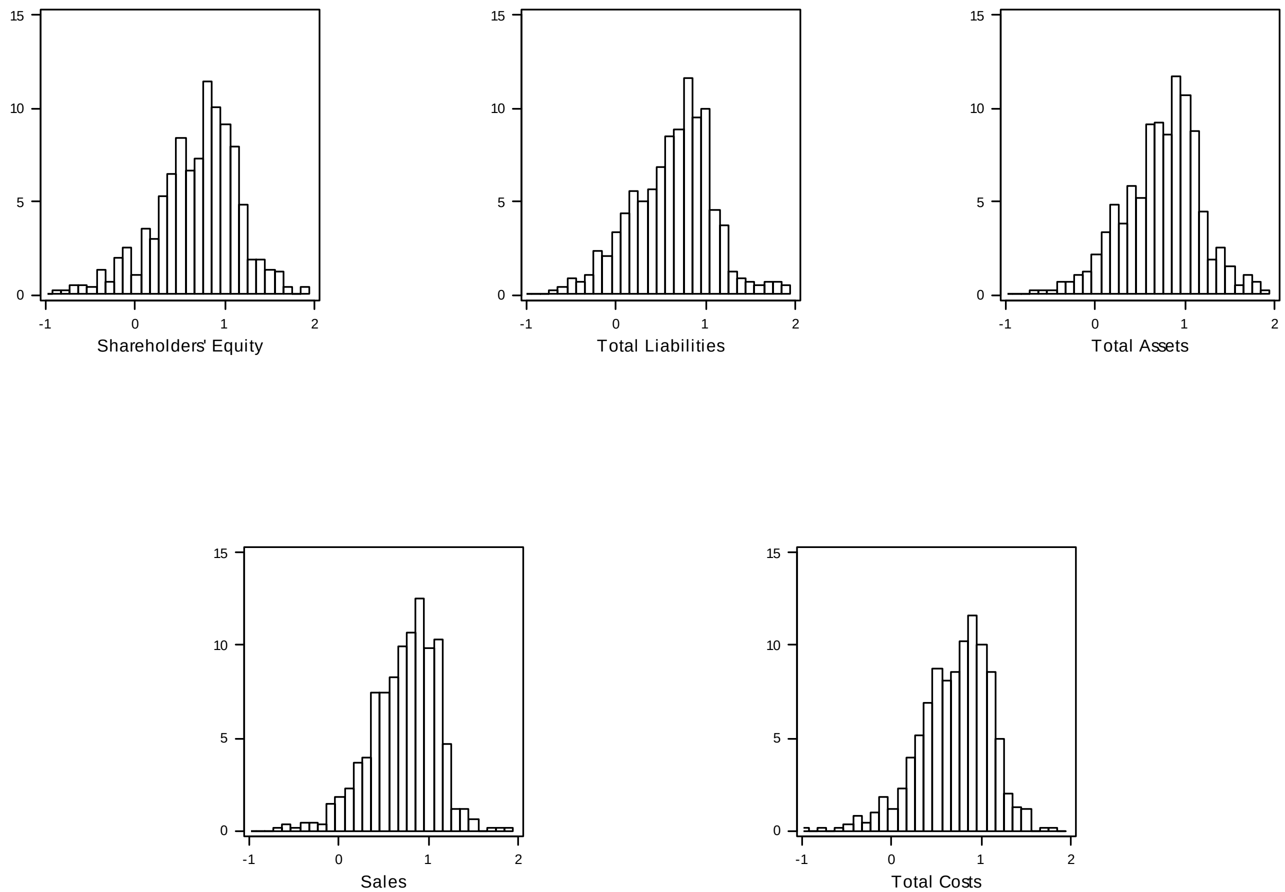


Figure 5. AR (1) betas



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