Networks and Firm Location

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Abstract

This paper models the decision of vertically-linked firms to build either partitioned or connected networks of supply of an intermediate good. In each case, the locations of upstream and downstream firms are correlated. Input specificity is related both to variable costs (transport costs of the input) and fixed costs (learning costs of the use of the input). When both are low, a connected network emerges and a partitioned pattern arises in the opposite case. In the boundary region, there are multiple equilibria, either asymmetric (mixed network) or symmetric.

Keywords: Location, Vertically-linked industries, Intermediate goods, Networks, Input flexibility

JEL classification: R30, L13

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1. Introduction

The issue of the flexibility of an intermediate good concerns the choice that a supplier makes between either producing a specialized input exactly tailored to the needs of a given buyer or manufacturing a generic or standardized input that can be used by all or at least by several buyers. In spatial terms, if the sellers and buyers of the intermediate good are defined by addresses in an attribute space, the specialization strategy amounts to the input supplier competing locally in its neighborhood, while the generic strategy is equivalent to competing globally in the whole attribute space.

This issue is important on two grounds. The first is the relationship between input specificity and the incentive to vertical integration, which was established by Williamson (1981) and Joskow (1987). Under input specificity, an incentive to a long-term bilateral relationship between buyer and seller is created, which can be best governed (in the sense of minimizing transaction costs) in the context of vertical integration rather than through the open market. Although this is an important strand in the literature, the focus on the importance of input specificity in this paper will lie elsewhere.

Following Bonaccorsi and Giuri (2001), it will be stressed that the decision to specialize an input conditions the structure of the network of relations between upstream and downstream firms. Particularly, the degree of connectivity of a network depends on the decisions taken with regard to specialization. If most supplying firms decide to specialize their inputs, a "partitioned" network will emerge, in the sense that it is made up of sparse, exclusive relations. In such a network, each node (firm) has at most one connection. By contrast, if the upstream firms decide to produce generic inputs, each supplier will sell to several buyers and each buyer will procure the input from several sellers, so that a connected network will emerge, in which each node (firm) has at least two connections. The structure of the network matters because a partitioned structure implies an interdependence between the industrial dynamics at the upstream and downstream levels that is closer than the one found in the connected structure.

The choice of the degree of specialization of an input is usually regarded as the outcome of a trade-off between economies of scale (which are maximized under the standardization of the input) and adjustment costs (which are minimized if the inputs are specialized, see Lorz and Wrede, 2005). A generic input can be produced in large amounts, thus saving fixed costs, but on the other hand must be adapted to the specific needs of the users. These adaptation costs can be viewed as transport costs in relation to the distance between the seller and buyer's addresses in the attribute space.

This paper seeks to model the adjustment costs of the intermediate good, using the spatial framework. We assume that the transport cost of the intermediate good in the distance between the seller and the buyer's addresses is the variable component of the adjustment cost, as mentioned by the literature on flexible manufacturing systems (see Eaton and Schmitt, 1994; Norman and Thisse, 1999) and on the endogenous choice of the degree of input specificity (see Pontes, 2005). Furthermore, following Kranton and Minehart (2000), it is argued that input flexibility has not only a variable cost, but also a fixed cost. In order to sell the input, the upstream firm has to train the buyer to use it, and this learning cost has the nature of a fixed cost.

In section II, the model is presented. In section III, conclusions are drawn.

2. The model

2.1. Assumptions

The paper models a spatial economy that obeys the following assumptions:

1. The consumers are uniformly distributed with unit density in the space described by the interval $[0, 1]$. Each consumer has an inverse demand function $p = 1 - q$.

2. Two downstream firms, D_a and D_b, supply a consumer good in the market space. These firms have fixed locations at the end points of the market: D_a locates in 0 and D_b locates in 1. They compete in quantities at each point of the market and they transport and deliver the product to the consumers. The transport cost of one unit of the consumer good per unit of distance is given by $t \in (0, 1)$.

3. Two upstream firms U_a and U_b have variable locations s_a and s_b in $[0, 1]$. They compete in quantities in the markets represented by downstream firms D_a and D_b and they transport and deliver the intermediate good. The transport cost of the intermediate good per unit of distance is given by τ.

4. The downstream firms transform one unit of intermediate good in one unit of consumer good and the cost of the intermediate good is their only variable cost.

5. Each upstream firm can choose to supply to either one or both downstream firms. The "connection" to a buyer implies a fixed cost c for the input supplier, which reflects the learning cost of using the input by the buyer.

It is assumed that this cost is borne by the seller of the intermediate good. Without loss of generality, it is assumed that, if firm U_a (respectively, U_b) sells to a single buyer, it selects firm D_a (firm D_b, respectively).

2.2. The game structure

We assume that the firms play a four stage game:

First Stage Each upstream firm decides to sell to one or to two input buyers. As a consequence, a network is formed with one of four different types: a partitioned structure, where each firm has a single connection; a connected structure, where each firm has two connections; and two mixed structures, where an upstream-downstream pair of firms has two connections, with the remaining firms having a single connection (see Figures 1, 2 and 3).

Second Stage The upstream firms simultaneously select locations s_a and s_b in $[0,1]$.

Third Stage The upstream firms select quantities of the intermediate good xaa, xab, xba, xbb, where, for instance, xab is the quantity of input sold by firm U_a to firm D_b. xab and xba can be zero, depending on the the outcome of the first stage of the game.

Fourth Stage The downstream firms D_a and D_b choose quantities $q_a(r)$ and $q_b(r)$ to be sold at each point of the market $r \in [0,1]$.

(Insert here Figures 1,2,3)

A subgame perfect equilibrium of this game is found through backward induction. In the fourth stage, the downstream firms compete in quantities at each point of the market. Their profits on sales in point $r \in [0,1]$ are:

$$\pi_{D_a}(r) = [(1-(q_a+q_b)-w_a-tr)]\,q_a \quad (1)$$

$$\pi_{D_b}(r) = [(1-(q_a+q_b)-w_b-t(1-r))]\,q_b \quad (2)$$

where q_a and q_b are the quantities and w_a and w_b are the prices of the intermediate good in the downstream firms' locations. The Nash equilibrium quantities are:

$$q_a(r) = \frac{1}{3}t - rt - \frac{2}{3}w_a + \frac{1}{3}w_b + \frac{1}{3} \quad (3)$$

$$q_b(r) = rt - \frac{2}{3}t + \frac{1}{3}w_a - \frac{2}{3}w_b + \frac{1}{3} \quad (4)$$

In order to solve the third-stage game, it is necessary to consider separately the networks that follow from the first stage and are depicted in Figures 1, 2 and 3.

The following proposition summarizes the findings (see proof in Appendix A):

Proposition 1 *In the partitioned network case (Figure 1), the Nash equilibrium quantities of the input are*

$$\mathrm{xaa} = \frac{1}{15}\tau - \frac{1}{10}t - \frac{4}{15}\tau s_a - \frac{1}{15}\tau s_b + \frac{1}{5} \quad (5)$$

$$\mathrm{xbb} = \frac{1}{15}\tau s_a - \frac{4}{15}\tau - \frac{1}{10}t + \frac{4}{15}\tau s_b + \frac{1}{5} \quad (6)$$

with $\mathrm{xab} = \mathrm{xba} = 0$ *by assumption. In the connected network case (Figure 2), the*

Nash equilibrium quantities of the input are

$$\text{xaa} = \frac{1}{9}\tau - \frac{1}{18}t - \frac{2}{3}\tau s_a + \frac{1}{3}\tau s_b + \frac{1}{9} \quad (7)$$
$$\text{xab} = \frac{2}{3}\tau s_a - \frac{2}{9}\tau - \frac{1}{18}t - \frac{1}{3}\tau s_b + \frac{1}{9} \quad (8)$$
$$\text{xba} = \frac{1}{9}\tau - \frac{1}{18}t + \frac{1}{3}\tau s_a - \frac{2}{3}\tau s_b + \frac{1}{9} \quad (9)$$
$$\text{xbb} = \frac{2}{3}\tau s_b - \frac{2}{9}\tau - \frac{1}{3}\tau s_a - \frac{1}{18}t + \frac{1}{9} \quad (10)$$

In the mixed network case (Figure 3-a), the Nash equilibrium quantities of the input are given by

$$\text{xaa} = \frac{1}{6}\tau - \frac{1}{12}t - \frac{1}{2}\tau s_a + \frac{1}{6} \quad (11)$$
$$\text{xab} = \frac{7}{12}\tau s_a - \frac{1}{4}\tau - \frac{1}{24}t - \frac{1}{6}\tau s_b + \frac{1}{12} \quad (12)$$
$$\text{xbb} = \frac{1}{3}\tau s_b - \frac{1}{6}\tau - \frac{1}{6}\tau s_a - \frac{1}{12}t + \frac{1}{6} \quad (13)$$

and $\text{xba} = 0$ *by assumption. The Nash equilibrium quantities of the input in the mixed network case of Figure 3-b are symmetric to 11, 12 and 13.*

It is also simple to show the following proposition (see proof in Appendix B):

Proposition 2 *In each network structure, the Nash equilibrium locations of the upstream firms entail the location of an input supplier alongside an input buyer (i.e.* $s_a = 0, s_b = 1$*) in order to save the transport costs of the intermediate good.*

Substituting the locations of proposition 2 into 8, 9 and 12, it is easily concluded that the feasibility of connections between spatially separated firms (i.e. $\text{xab} > 0$ and $\text{xba} > 0$) implies that the transport cost of the input is bounded from above

$$\tau < \frac{1}{5} - \frac{1}{10}t \quad (14)$$

From 3, 4 and proposition 1, it can be concluded that the condition

$$t < \frac{2}{7}(1+\tau) \tag{15}$$

together with condition 14 ensures that each downstream firm located at an extreme point of the interval $[0,1]$ sells a positive amount of the consumer good at each point of the market for any network structure.

The co-location of the suppliers and buyers of the input is not sensitive to the network structure. However, it should be noticed that:

Remark 3 *Although co-location of input supplier and buyer holds in each network structure, the robustness of the equilibrium decreases with the degree of connectivity of the network. With a partitioned network, co-location is a dominating strategy equilibrium. With a mixed network, it is a unique Nash equilibrium. With a fully connected network, there are two co-location Nash equilibria.*

Hence, there is a close relationship between the location of the upstream and downstream firms. However, the robustness of this correlation decreases with the degree of connectedness of the network, confirming the findings of Bonaccorsi and Giuri (2001).

Input flexibility is expressed inversely by the unit transport cost of the input and by the fixed cost entailed by establishing a connection between a supplier and a buyer of the intermediate good.

In the first-stage game, each upstream firm decides to establish a single connection with a buyer (action "1") or to establish connections with both buyers (action "2") . The profit functions of the firms in the different outcomes are:

$$\pi_{U_a}(1,1) = \pi_{U_b}(1,1) = \frac{1}{50}t^2 - \frac{2}{25}t - c + \frac{2}{25} \tag{16}$$

$$\pi_{U_a}(2,1) = \pi_{U_b}(1,2) = \frac{5}{72}t\tau - \frac{7}{72}t - \frac{5}{36}\tau - 2c + \frac{7}{288}t^2 + \frac{19}{72}\tau^2 + \frac{7}{72} \tag{17}$$

$$\pi_{U_a}(1,2) = \pi_{U_b}(2,1) = \frac{1}{9}\tau - \frac{1}{18}t - c - \frac{1}{18}t\tau + \frac{1}{72}t^2 + \frac{1}{18}\tau^2 + \frac{1}{18} \tag{18}$$

$$\pi_{U_a}(2,2) = \pi_{U_b}(2,2) = \frac{1}{27}t\tau - \frac{2}{27}t - \frac{2}{27}\tau - 2c + \frac{1}{54}t^2 + \frac{14}{27}\tau^2 + \frac{2}{27} \tag{19}$$

The game expressed by these payoff functions is a symmetric two-person game. Let us define

$$A_1 = \pi_{U_a}(1,1) - \pi_{U_a}(2,1)$$
$$A_2 = \pi_{U_a}(2,2) - \pi_{U_a}(1,2)$$

Then, the signs of A_1 and A_2 completely determine the equilibrium of the game. It is easy to see that

$$A_1 > 0 \Leftrightarrow c > \frac{5}{72}t\tau - \frac{5}{36}\tau - \frac{31}{1800}t + \frac{31}{7200}t^2 + \frac{19}{72}\tau^2 + \frac{31}{1800} \equiv F(t,\tau) \tag{20}$$

$$A_2 > 0 \Leftrightarrow c < \frac{5}{54}t\tau - \frac{5}{27}\tau - \frac{1}{54}t + \frac{1}{216}t^2 + \frac{25}{54}\tau^2 + \frac{1}{54} \equiv G(t,\tau) \tag{21}$$

For given values of t, $F(t,\tau)$ and $G(t,\tau)$ define two convex parabolas in τ. $F(t,\tau)$ has two roots, namely $\tau = \frac{1}{5} - \frac{1}{10}t$ and $\tau = \frac{31}{95} - \frac{31}{190}t$, the former root being smaller than the latter for $t \in (0,1)$. $G(t,\tau)$ has a single root $\tau = \frac{1}{5} - \frac{1}{10}t$. Given boundary condition 14, only values of (t,τ), such that $\tau < \frac{1}{5} - \frac{1}{10}t$, have any economic meaning. In this region, $F(t,\tau)$ and $G(t,\tau)$ intersect twice.

$$F(t,\tau) = G(t,\tau) \Leftrightarrow \tau = \frac{7}{215} - \frac{7}{430}t \vee \tau = \frac{1}{5} - \frac{1}{10}t \tag{22}$$

It is clear that in 22, the first root is positive and strictly smaller than the second root for $t \in (0,1)$. Summing up, it is possible to plot condition 14 and 20 and 21 in the space (τ, c) in Figure 4 (a value $t = 0.5$ is implicitly assumed in this figure, but the figure is not sensitive to the specific value of t, provided that this value is feasible according to conditions 14 and 15).

(Insert here Figure 4)

Figure 4 shows that, for high values of the transport cost of the intermediate good and for high connection costs, to compete locally, i.e. to supply only the nearby buyer, is a dominating strategy for both upstream firms. By contrast, if τ and c are low, it is a dominating strategy for both input suppliers to compete globally, i.e. to supply both downstream firms.

These dominating strategy regions are separated by two regions where Nash equilibria are multiple. If the transport cost of the intermediate good is low and the connection cost is high, we have a Coordination Game, so that there are two symmetric Nash equilibria in pure strategies: for each upstream firm, the best reply is to replicate the strategy of the competitor. By contrast, if τ is high and c is low, we have a Chicken Game, with two asymmetric equilibria. If the competitor decides to compete globally, the best reply is to compete locally and vice-versa. A mixed network as depicted in Figure 3 emerges. It should be noticed that:

Remark 4 *For any value of t, the region where the connection game has multiple asymmetric equilibria is much larger than the region with multiple symmetric equilibria. Whenever there is no dominating strategy, the connection game is more likely to be a Chicken Game than a Coordination Game. Refraining from competing globally, one firm supplies a public good that benefits the competitor (as in Choi and Yi, 2000).*

3. Conclusions

It is possible to conclude that the degree of input flexibility is inversely expressed by the transport cost of the input (a variable cost) and by the learning cost resulting from a trade connection in the input market (a fixed cost). This confirms the intuition of the literature on flexible manufacturing systems (Eaton and Schmitt, 1994, Norman and Thisse, 1999) and on the endogenous determination of input specificity (Pontes, 2005). It also confirms the idea that inputs tend to be traded through networks, where the establishment of a connection entails a fixed cost (as in Kranton and Minehart, 2000).

The locations of upstream and downstream firms always tend to be closely related, as was acknowledged by Belleflamme and Toulemonde (2003). However, the robustness of this relationship is greater in a partitioned network than in a connected network.

Two causes determine the degree of input flexibility, namely the transport cost of the intermediate good and the fixed cost of establishing a trade connection. If both are low, the input is flexible, and to compete globally is a dominating strategy, so that a connected network emerges. If both are high, the input is specific and it is always better for each upstream firm to compete locally, so that a partitioned pattern arises. The boundary cases where one of the variables is high and the other is low entail multiple equilibria, either symmetric or asymmetric. The case with multiple asymmetric equilibria occurs in a larger region of the parameter space.

The case where the downstream firms have variable locations in $[0, 1]$ is left for further research. In this case, inter-firm distance would be a cause of input specificity together with the unit transport cost of the input.

Appendix A: Derivation of the equilibrium quantities of the intermediate good

In the partitioned network case (Figure 1), the derived demand of the input in location 0 is

$$\text{xaa} = \int_0^1 q_a(r)\,dr = \frac{1}{3}w_b - \frac{2}{3}w_a - \frac{1}{6}t + \frac{1}{3} \tag{A.1}$$

where $q_a(r)$ is given by 3.

The derived demand of the input in this case in location 1 is

$$\text{xbb} = \int_0^1 q_b(r)\,dr = \frac{1}{3}w_a - \frac{1}{6}t - \frac{2}{3}w_b + \frac{1}{3} \tag{A.2}$$

where $q_b(r)$ is given by 4.

If we invert this system of derived demand functions, we obtain

$$w_a = 1 - 2\,\text{xaa} - \text{xbb} - \frac{1}{2}t \tag{A.3}$$

$$w_b = 1 - \text{xaa} - 2\,\text{xbb} - \frac{1}{2}t \tag{A.4}$$

The profit functions of the upstream firms are

$$\pi_{U_a} = (w_a - \tau s_a)\,\mathrm{xaa} \tag{A.5}$$

$$\pi_{U_b} = (w_b - \tau(1 - s_b))\,\mathrm{xbb} \tag{A.6}$$

Calculating the Cournot-Nash equilibrium yields the outputs 5 and 6.

We now deal with the case of the connected network. The derived demand function of the input in location 0 is given by

$$\mathrm{xaa} + \mathrm{xba} = \int_0^1 q_a(r)\,dr \tag{A.7}$$

where $q_a(r)$ is again given by 3. The derived demand function of the input in location 1 is given by

$$\mathrm{xab} + \mathrm{xbb} = \int_0^1 q_b(r)\,dr \tag{A.8}$$

where $q_b(r)$ is again given by 4.

Inverting the derived demand functions, we obtain

$$w_a = 1 - 2\,\mathrm{xaa} - \mathrm{xab} - 2\,\mathrm{xba} - \mathrm{xbb} - \frac{1}{2}t \tag{A.9}$$

$$w_b = 1 - \mathrm{xaa} - 2\,\mathrm{xab} - \mathrm{xba} - 2\,\mathrm{xbb} - \frac{1}{2}t \tag{A.10}$$

The profit functions of the upstream firms are

$$\pi_{Ua} = (w_a - \tau s_a)\,\mathrm{xaa} + (w_b - \tau(1 - s_a))\,\mathrm{xab} \tag{A.11}$$

$$\pi_{U_b} = (w_a - \tau s_b)\,\mathrm{xba} + (w_b - \tau(1 - s_b))\,\mathrm{xbb} \tag{A.12}$$

Calculating the Cournot-Nash equilibrium quantities gives 7 to 10.

In the case of the mixed network of Figure 3-a, the derived demand of the input in location 0 is

$$\text{xaa} = \int_0^1 q_a(r)\, dr \tag{A.13}$$

where $q_a(r)$ is again given by 3. The derived demand of the input in location 1 is given by

$$\text{xab} + \text{xbb} = \int_0^1 q_b(r)\, dr \tag{A.14}$$

where $q_b(r)$ is again given by 4. Inverting the derived demand functions, we obtain

$$w_a = 1 - 2\,\text{xaa} - \text{xab} - \text{xbb} - \frac{1}{2}t \tag{A.15}$$

$$w_b = 1 - \text{xaa} - 2\,\text{xab} - 2\,\text{xbb} - \frac{1}{2}t \tag{A.16}$$

The upstream profit functions are

$$\pi_{U_a} = (w_a - \tau s_a)\,\text{xaa} + (w_b - \tau(1 - s_a))\,\text{xab} \tag{A.17}$$

$$\pi_{U_b} = (w_b - \tau(1 - s_b))\,\text{xbb} \tag{A.18}$$

The Cournot-Nash equilibrium is given by 11 to 13.

Appendix B: The equilibrium of locations.

For each network structure, it is easy to plug the Nash equilibrium quantities of input defined in proposition 1 into the profit functions of the upstream firms, and obtain the profits as a function of the locations s_a and s_b. It can easily be concluded that the profit is a strictly convex function of the firm's own location, so that the maximum profit is attained at a boundary point of $[0, 1]$. Hence, the location game can be solved as a finite game where each upstream firm has a strategy set $\{0, 1\}$.

In the case of the partitioned network of Figure 1, the profits of the upstream firms in locations 0 and 1 are respectively:

$$\pi_{U_a}(0,1) = \pi_{U_b}(0,1) = \frac{1}{50}t^2 - \frac{2}{25}t + \frac{2}{25} \tag{B.1}$$
$$\pi_{U_a}(0,0) = \pi_{U_b}(1,1) = \frac{4}{75}\tau - \frac{2}{25}t - \frac{2}{75}t\tau + \frac{1}{50}t^2 + \frac{2}{225}\tau^2 + \frac{2}{25} \tag{B.2}$$
$$\pi_{U_b}(0,0) = \pi_{U_a}(1,1) = \frac{8}{75}t\tau - \frac{16}{75}\tau - \frac{2}{25}t + \frac{1}{50}t^2 + \frac{32}{225}\tau^2 + \frac{2}{25} \tag{B.3}$$
$$\pi_{Ua}(1,0) = \pi_{U_b}(1,0) = \frac{2}{25}t\tau - \frac{4}{25}\tau - \frac{2}{25}t + \frac{1}{50}t^2 + \frac{2}{25}\tau^2 + \frac{2}{25} \tag{B.4}$$

It is easy to conclude that $s_a = 0$ is a dominating strategy for firm U_a and $s_b = 1$ is a dominating strategy for firm U_b.

In the case of the connected network of Figure 2, the upstream firms' payoffs as a function of their locations are

$$\pi_{Ua}(0,1) = \pi_{U_b}(0,1) = \frac{1}{27}t\tau - \frac{2}{27}\tau - \frac{2}{27}t + \frac{1}{54}t^2 + \frac{14}{27}\tau^2 + \frac{2}{27} \tag{B.5}$$
$$\pi_{U_a}(0,0) = \pi_{U_b}(0,0) = \frac{1}{27}t\tau - \frac{2}{27}\tau - \frac{2}{27}t + \frac{1}{54}t^2 + \frac{2}{27}\tau^2 + \frac{2}{27} \tag{B.6}$$
$$\pi_{U_a}(1,1) = \pi_{U_b}(1,1) = \pi_{U_a}(0,0) = \pi_{U_b}(0,0) \tag{B.7}$$
$$\pi_{Ua}(1,0) = \pi_{U_b}(1,0) = \pi_{Ua}(0,1) = \pi_{U_b}(0,1) \tag{B.8}$$

It is easily seen that this game has two Nash equilibria $(0, 1)$ and $(1, 0)$.

In the case of the mixed structure depicted in Figure 3-a, the payoff functions are

$$\pi_{U_a}(0,0) = \frac{1}{72}t\tau - \frac{1}{36}\tau - \frac{7}{72}t + \frac{7}{288}t^2 + \frac{7}{72}\tau^2 + \frac{7}{72} \quad \text{(B.9)}$$

$$\pi_{U_b}(0,0) = \frac{1}{18}t\tau - \frac{1}{9}\tau - \frac{1}{18}t + \frac{1}{72}t^2 + \frac{1}{18}\tau^2 + \frac{1}{18} \quad \text{(B.10)}$$

$$\pi_{U_a}(1,1) = \frac{1}{12}t\tau - \frac{1}{6}\tau - \frac{7}{72}t + \frac{7}{288}t^2 + \frac{1}{6}\tau^2 + \frac{7}{72} \quad \text{(B.11)}$$

$$\pi_{U_b}(1,1) = \frac{1}{72}t^2 - \frac{1}{18}t + \frac{1}{18} \quad \text{(B.12)}$$

$$\pi_{U_a}(0,1) = \frac{5}{72}t\tau - \frac{5}{36}\tau - \frac{7}{72}t + \frac{7}{288}t^2 + \frac{19}{72}\tau^2 + \frac{7}{72} \quad \text{(B.13)}$$

$$\pi_{U_b}(0,1) = \frac{1}{9}\tau - \frac{1}{18}t - \frac{1}{18}t\tau + \frac{1}{72}t^2 + \frac{1}{18}\tau^2 + \frac{1}{18} \quad \text{(B.14)}$$

$$\pi_{U_a}(1,0) = \frac{1}{36}t\tau - \frac{1}{18}\tau - \frac{7}{72}t + \frac{7}{288}t^2 + \frac{2}{9}\tau^2 + \frac{7}{72} \quad \text{(B.15)}$$

$$\pi_{U_b}(1,0) = \frac{1}{9}t\tau - \frac{2}{9}\tau - \frac{1}{18}t + \frac{1}{72}t^2 + \frac{2}{9}\tau^2 + \frac{1}{18} \quad \text{(B.16)}$$

From B.9 to B.16, it can easily be concluded that this game has a unique Nash equilibrium $s_a = 0$ and $s_b = 1$, provided that condition 14 is met.

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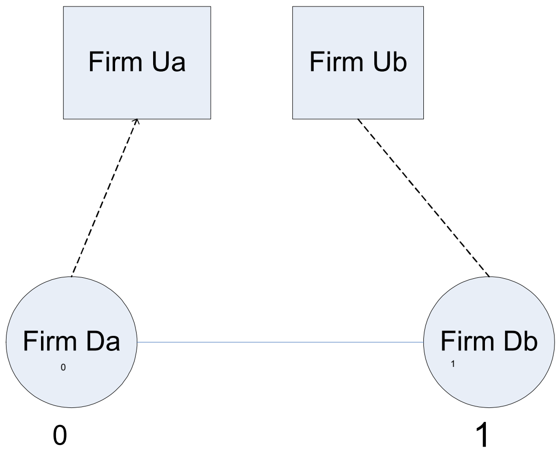
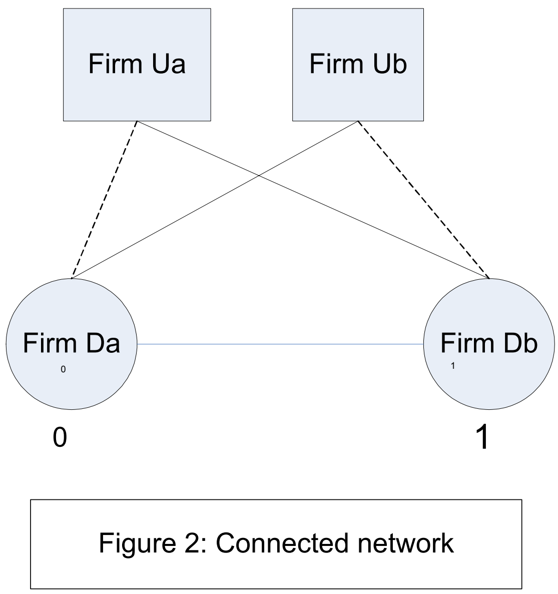


Figure 1: Partitioned network

Figure 1:


Firm Ua
Firm Ub
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0
Firm Db
1
0
1
Figure 2: Connected network

Figure 2:

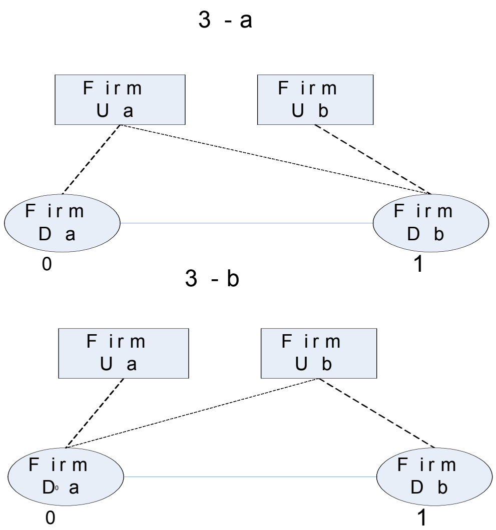

Figure 3: Mixed networks

Figure 3:

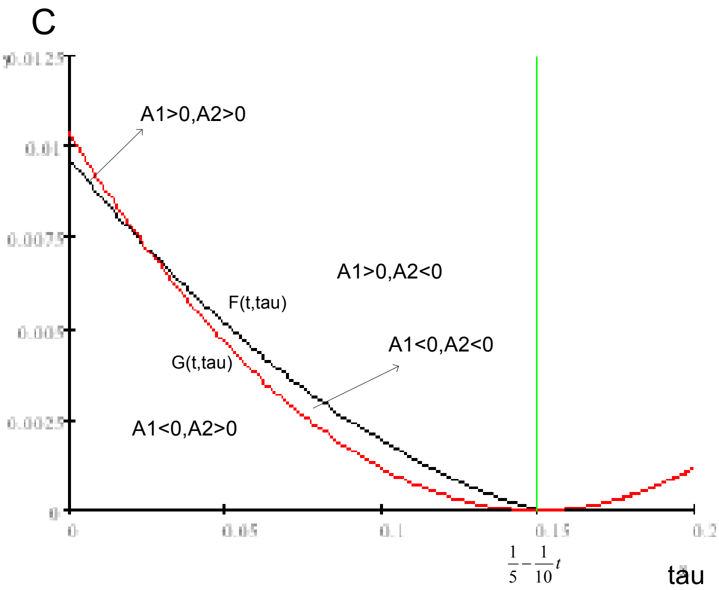

Figure 4: Network equilibria

Figure 4: