Representation of Real Assets Behavior
by a Hysteresis Process

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Abstract

In modeling a behavior of real assets, such as a value of project, enterprise, and corporate brand, we must analyze how real assets differ from financial assets, and implement their distinguishable features into a model. For the field financial assets, we usually discuss under an assumption that a behavior of financial assets follows a Brownian motion; that assumption fits well and has brought many contributions in the field of derivatives. On the other hand, for real assets such an assumption is not appropriate, because a value of real assets includes the management decision makings, which lead to a behavior far from stochastic process. Out of all different features from a Brownian motion, we emphasize a hysteresis. While there are some preceding studies of hysteresis in economics or finance, we could find no analyses of hysteresis in those fields by applying quantitative model used in Physics. Therefore, in this paper we will analyze how we can apply Preisach Model in Physics to a behavior of real assets.
1. Motivation

In valuing of a project, real options analysis is superior to decision analysis or dynamic programming in that we can value real assets by using the objective risk free rate without arguing cost of capital which is hard to determine (Trigeorgis (1996, p.200)). In real options analysis, we introduce twin securities, supposed to move in parallel with the real assets, and apply a binominal lattice model or Black-Scholes formula, which is popular in the valuation of financial derivatives. Indeed, we should evaluate the significance of real options analysis in the managerial context, and it might be less important argument how much exactly the value of real assets is. However, we are motivated to discuss in which points real assets differ from financial assets, how we will implement those different features into the model of real options, and how the valuation will change from that of financial options.

Copeland and Antikarov (2001, p.110-112) have discussed the following differences between real options and financial option: 1. We cannot observe implied volatilities due to the illiquid market, 2. While both the buyer and seller of financial options cannot affect the behavior of underlying financial assets, the buyer of real options can affect real assets, and 3. While risk is given exogenously for both financial and real assets, the actions of a company owing real options can affect the uncertainty surrounding it. Other than these differences, we may point out other differences such as excludability of owing options; however, we will focus on the behavior of real assets, because, we consider, the above arguments 2. and 3. are concerned with the cause and effect of
executives in the context of their decision making processes, and results of their decision making are followed by the behavior of real assets.

Thus at the outset of model building of real assets, we need to at least modify the model where financial assets are assumed to follow a Brownian motion. Out of many modifications, we can think of the preceding studies of path dependent options. However, as Tavella (2002, p.73) have discussed, there are two notions of path dependency: the dependency of the payoff of derivatives and the dependency of the underlying processes themselves. Whereas many studies have accumulated concerning the former, we have rarely found the latter, because we cannot obtain a value of options without further assumptions under the situation of the latter. Another modification may be the model involving the correlation between the prices in a discreet time horizon. Such a model may work and includes practical applications, because we can simulate and obtain the value of options by generating random variables following the fixed correlation coefficient. Nevertheless, remaining issue is raised: what parameters determine the correlation coefficient. Even if we may think of time series of correlation coefficients, we will face the complexity of model building and lose the managerial implication of real options.

Therefore, in this paper, we attempt to discuss how the model of financial derivatives changes, if we implement hysteresis into the behavior of real assets. Also, our attempt may be the first to apply hysteresis in Magnetism in physics to the behavior of assets. However, with limited source of preceding studies, our analysis is preliminary and we are only
at the standpoint of this study.

2. The Definition of Hysteresis (Narrow Sense)

Before we analyze applications of hysteresis, we should review the definition of hysteresis in a narrow sense, because we can, from time to time, find the term hysteresis is used just as a metaphor. Thus we need to examine the meaning of hysteresis in the field of Magnetism of physics. In magnetic materials, the relation between applied field and magnetization is not simple and linear. In Figure 1, when we apply magnetic field \( H \) from a point \( O \) in the positive direction, a magnetization \( M \) will move along the curve \( a \) and arrive at the saturation point \( b \). However, even if we apply magnetic field in the reverse direction, \( M \) will not move back along the curve \( a \). It shows the trajectory along the curve \( cde \). In short, the behavior of \( M \) will not move back along the path it has come over. That is, it moves back slowly. We call this phenomenon as hysteresis, because the magnetization \( M \) depends on the path it has moved. The phenomenon hysteresis is also seen in Thermodynamics.

![Figure 1 Hysteresis Image](image-url)
3. Preceding Studies of Hysteresis in Economics or Finance

Whereas the term hysteresis has its own meaning in physics, it has been also used in the field of economics and finance to represent real dynamic economic phenomena. Cross (1988) has discussed, in its anthology, while Neo Classical General Equilibrium Theory did not include this uncomfortable hysteresis, considerations of hysteresis is needed for empirical studies, such as a consumption function, unemployment rates and international trades. However, we cannot find rigid or quantitative studies in the anthology, even if some of papers developed graphs or simple models. For example, in the field of foreign exchange rates, Krugman (1989) discussed how the change of foreign exchange rates affects overseas investments or international trades of firms by using the concept of hysteresis. Also, in the field of corporate finance, Dixit (1992) pointed out a phenomenon of hysteresis in the cause and effect between the change of forecasted cash flow and managerial decision making process, because of the existence of sunk cost. Nevertheless, both discussions are rather descriptive and limited in a sense they did not take quantitative approach to examine hysteresis. Therefore, there remain rooms for further research of hysteresis by using quantitative models in physics.

4. Preisach Model

Then, what are quantitative models of hysteresis in physics? There are many models for Magnetism; however, we focus on typical Preisach Model with the reference to Della Torre, Edward (1999).
In Figure 2, we suppose a magnetization $M = +1$ in case a given magnetic field $H$ is $H > X$, $M = -1$ in case $H$ is $H < Y$, and $M = \pm 1$ in case $H$ is $Y < H < X$. In addition, a probability density $P(X, Y)$, which depends on Switching Field Coordinates $(X, Y)$, is defined as

$$M = \iint P(X, Y) \, dX \, dY$$

That is, we suppose a magnetic material is composed of a collection of square-loop hysterons, which are a unique normalized magnetization taking a value of either $+1$ or $-1$; we assume hysterons are distributed all over the plane $(H, M)$. A hysteron is supposed to work as an element or device where an input $H$ produces nonlinear output $\pm M$. In Figure 2, a hysteron is depicted as a square and extended two horizontal lines. It has a value of either $+1$ or $-1$ in the horizontal lines and either $X$ or $Y$ in the vertical line. The magnetization $M$ depends on the value of the magnetic field $H$ and previous magnetization $M$. If $H$ takes a value of $H > X$, $M$ becomes $M = +1$ regardless the previous value of $M$. On the contrary, if $H$ is $H < X$, $M$ becomes either $+1$ or $-1$ dependent on the previous value of $M$. Furthermore, if $H$ is in a critical point $H = X$, the previous value $M = -1$ will abruptly move up to $M = +1$. This is called phase transition. That is, when an input is applied, the previous value of $M$ changes counterclockwise in a square portion of Figure 2.
5. Path Dependent Model in Finance

Our objective is to represent the behavior of real assets using the Preisach Model in Section 4. However, we have a difficulty in depicting it generally using continuous stochastic models and in examining the difference quantitatively from the financial models, even if we could represent it. Thus we need to proceed to discreet simulation analysis. Before we move on a simulation analysis, we overview preceding studies of path dependent options in this section.

Broadie & Glasserman (1997) have expressed Non-Recombining Tree Method; that is, we generate many random paths based on discreet state variable and time. It include the modification of a typical Recombining Tree Method known as Cox-Ross-Rubinstein (CRR from now on) Lattice Method, in a sense the number of nodes expands to a
level where simulation analysis is necessary. That is, their approach is to calculate as many paths as possible. We will take this approach; however, we will not take Non-Recombining Tree Method, because we need tough calculations and our objective is to show the difference of models between real and financial assets. Thus we will take the same Recombining Binominal Lattice (CRR Lattice) as is used in the valuation of financial assets. Hull & White (1993) have showed Forward Shooting Grid (FSG) Method for Asian options pricing. They assumed Recombining Lattice; however, they calculate all paths to obtain information about history. This method includes the concept of hysteresis in the function of contingent claim. Even if we consider the hysteresis in the paths themselves, their work is suggestive in that they attempt to calculate all paths.

6. An Application of Preisach Model to a Binominal Lattice

In this Section, we analyze how CRR Lattice Model changes, if we implement hysteresis using Preisach Model. We assume the followings. $H (> 0)$ means some sorts of information which affect the value of a firm. Fundamentally, $+H$ has a positive effect, while $-H$ has a negative effect. Each information is appeared with the probability $p$ or $1-p$ respectively. For the value of output, we assume, $M=u$ or $M=d$. In addition, to clearly show the characteristics of the model, we set a hysteron as $Y_i = \begin{cases} -H < Y_i < 0 & \text{for } X_i < H < X_i \end{cases}$ and the probability density as $P(X_i)$ or $P(Y_i)$ in the switching field $X_i$ or $Y_i$. That is, indeed the information given outside has potential impacts of either positive or negative; however, we will not necessarily obtain the output $u$ or $d$ directly.

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by the effect of hysteresis. Incidentally, since we need to show the initial conditions, we assume at $t=1$ the output unconditionally becomes $u$ or $d$ with the probability $P$ or $1-P$ respectively.

We start examining by the case at the end of $t=2$. We can consider four cases of history: two cases at $t=1$ and two cases at $t=2$. We express these four cases as $(H, H)$, $(H, -H)$, $(-H, H)$ and $(-H, -H)$; the first term in the parentheses means the value at $t=1$ and the second term means that at $t=2$. In Figure 3, in the case of history $(H, H)$, even if the value of input $H$ at $t=2$ is $2H < X$, the value of output becomes $u$ and the total value in these two periods becomes $u^2$. This case has resulted in the same as that of CRR Lattice model and we can obtain the same result in the case of history $(-H, -H)$ as $d^2$.

The interesting cases are the history $(H, -H)$ or $(-H, H)$, which include different directions of input in two periods. The results depend on the distribution of hysterons. In the case of history $(H, -H)$, the accumulated value of input at $t=2$ is $H = 0$. While the value of output at $t=1$ is on the upper horizontal line, the output at $t=2$ depends on hysterons. At $t=2$, we can classify two cases whether the left vertical line $Y_i$ generate a critical state and the output changes from $u$ to $d$; the hysteron with the left vertical line $Y_i (> 0)$ causes this change, while those with $Y_i$ or $Y_i (< 0)$ does not. As a result, the total value in these two periods becomes $P(Y_i)ud + (1 - P(Y_i)) d'$, which clearly differs from that of CRR Lattice Model as $ud$. In contrast, in the case of history $(-H, H)$, the right vertical line does not cause a critical state, because of the asymmetrical distributions of hysterons. In this case, the total value is
determined $d^2$ as due to the strong effect of the initial condition, which also differs from the result of CRR Lattice Model. Thus as we show in Table 1, even if a simple case of binominal lattice at $t=2$, our Hysteresis Binominal Tree Model needs modifications from CRR Model.

![Figure 3 An Application of Hysteron](image)

**Table 1 The Value of Output in Hysteresis Model ($t=2$)**

<table>
<thead>
<tr>
<th>History</th>
<th>Accumulated Input</th>
<th>Value CRR</th>
<th>Value Hysteresis Model</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H,H$</td>
<td>$2H$</td>
<td>$u'$</td>
<td>$u'$</td>
<td>$p^2$</td>
</tr>
<tr>
<td>$H,-H$</td>
<td>0</td>
<td>$ud$</td>
<td>$p(Y_i)ud+(1-P(Y_i))u'$</td>
<td>$p(1-p)$</td>
</tr>
<tr>
<td>$-H,H$</td>
<td>0</td>
<td>$ud$</td>
<td>$d^2$</td>
<td>$p(1-p)$</td>
</tr>
<tr>
<td>$-H,-H$</td>
<td>$-2H$</td>
<td>$d'$</td>
<td>$d^2$</td>
<td>$(1-p)^2$</td>
</tr>
</tbody>
</table>

At the end of $t=3$, we can obtain more interesting and complex results. In Table 2, we show the results of all eight cases of histories. We can observe the density of both edge sides, corresponding to the maximum $u'$ or minimum $d'$ in CRR Model, has increased. In the context of management, if we implement hysteresis into the behavior of real assets, we can observe the implication of learning effect, because the
results will become clearer, either success or failure, as time passes. This observation maybe a common sense; however, we consider our Hysteresis Model has significance in setting parameters of hysterons and examining quantitatively how speedy the results of output converge to both edge points. If we remember the previous analysis at \( t=2 \), the asymmetrical distribution of hysterons causes the difference of values between \( P(Y_i) u_d + (1-P(Y_i)) d' \) and \( d' \). That is, if we would like to set modeling so that the value at the end of many periods will most rapidly converges, we can set only one hysteron, which has vertical lines \( X_i \) and \( Y_i \) in the range of \(-H<X_i<0<Y_i<H\). Also, we should take the highest density for \( P(X_i) \) and \( P(Y_i) \). These probability densities are given how pessimistically executives count in the situation of given positive information; or how opportunistically executives count in the situation of given negative information.

<table>
<thead>
<tr>
<th>History</th>
<th>Accumulated Input</th>
<th>Value</th>
<th>CRR</th>
<th>Value Hysteresis Model</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H, H, H )</td>
<td>3H</td>
<td>( u' )</td>
<td>( u' )</td>
<td>( p'(1-p') )</td>
<td></td>
</tr>
<tr>
<td>( H, H, -H )</td>
<td>H</td>
<td>( u'd )</td>
<td>( u' )</td>
<td>( p'(1-p') )</td>
<td></td>
</tr>
<tr>
<td>( H, -H, H )</td>
<td>H</td>
<td>( u'd )</td>
<td>( P(Y_i) u_d (P(X_i) u+P(X_i) d) + (1-P(Y_i)) u' )</td>
<td>( p'(1-p') )</td>
<td></td>
</tr>
<tr>
<td>( H, -H, -H )</td>
<td>-H</td>
<td>( u'd )</td>
<td>( P(Y_i) u_d^+ (1-P(Y_i)) u' P(Y_i) d+P(Y_i) u )</td>
<td>( p(1-p')^2 )</td>
<td></td>
</tr>
<tr>
<td>( -H, H, H )</td>
<td>H</td>
<td>( u'd )</td>
<td>( P(Y_i) u_d^2 + P(X_i) d^3 )</td>
<td>( p'(1-p) )</td>
<td></td>
</tr>
<tr>
<td>( -H, H, -H )</td>
<td>-H</td>
<td>( u'd )</td>
<td>( d' )</td>
<td>( p'(1-p') )</td>
<td></td>
</tr>
<tr>
<td>( -H, -H, H )</td>
<td>-H</td>
<td>( u'd )</td>
<td>( d' )</td>
<td>( p(1-p')^2 )</td>
<td></td>
</tr>
<tr>
<td>( -H, -H, -H )</td>
<td>-3H</td>
<td>( d' )</td>
<td>( d' )</td>
<td>( (1-p)^3 )</td>
<td></td>
</tr>
</tbody>
</table>

7. Conclusion and Prospects

In this paper, we have attempted to represent the behavior of real assets using Preisach Model and demonstrated how models change for
CRR Binominal Lattice to our Hysteresis Binominal Lattice. We observed a trend of convergence to both edge points, as time passes. We have concluded the critical parameters are the distributions of hysterons. Although our analysis is limited, we believe our model has potentials to develop. In prospect, we will move on simulation studies to obtain the sensitivity of hysteron parameters and accumulate experimental results.

1 This paper is based on the presentation in Japanese at the 26th annual conference by Japan Finance Association at the University of Tokyo on October 4 - 6, 2002.
2 Note X and Y can take positive or negative numbers. The figure 2 is intended to show the relation X > Y.

References


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