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A Fictitious-Play Model of Bargaining To Implement the Nash Solution

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Abstract: We present a fictitious-play model of bargaining, where two bargainers play the Nash demand game repeatedly. The bargainers make a deliberate decision on their demands in the initial period and then follow a fictitious play process subsequently. If the bargainers are patient, the set of epsilon-equilibria of the initial-demand game is in a neighborhood of the division corresponding to the Nash bargaining solution. As the bargainers make a more accurate comparison of payoffs and become more patient accordingly, the set of epsilon-equilibria shrinks and the only equilibrium left is the division of the Nash bargaining solution.

JEL classifications: *C71, C72, C78, D83.*
Keywords: *fictitious play, Nash demand game, epsilon-equilibrium, Nash bargaining solution, Nash program.*

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1 Introduction

Since Nash's work in the early 1950's, there have been two different approaches to analyzing bargaining problems: the strategic approach and the axiomatic approach. Nash (1953) claimed that these two approaches should be complementary, and presented a strategic model of bargaining (the perturbed Nash demand game) to implement his axiomatic solution (the Nash (1950) bargaining solution). The idea of relating axiomatic solutions to equilibria of strategic models is now known as the "Nash program." Binmore et al. (1986) showed that the subgame-perfect equilibrium of the Rubinstein's (1982) alternating-offers model approaches the Nash bargaining solution as the friction becomes smaller. For more results in the Nash program, see Osborne and Rubinstein (1990) and Serrano (2004). Serrano also explains how one can view the Nash program in the context of the implementation theory. We attempt to contribute to the research in the Nash program by presenting a strategic model based on a fictitious play of the Nash demand game to implement the Nash bargaining solution.

Our strategic model is partly related with evolutionary models. Several authors have studied evolutionary models of the Nash demand game. Young (1993, 1998) presented a model where bargainers make their demands by choosing best replies based on an adaptive play process with incomplete sampling and showed that the stochastically stable divisions converge to the Nash solution. Santamaria-Garcia (2004) showed similar convergence results using Kandori et al.'s (1993) matching framework. Binmore et al. (2003) showed that the Nash solution is the unique stochastically stable equilibrium in best-response dynamics. Skyrms (1994) showed that the equal split has a larger basin of attraction among the evolutionarily stable equilibria. Ellingsen (1997) investigated the dynamics of the "obstinate" strategy and the "sophisticated" strategy in the evolutionarily stable equilibria. Oechssler and Riedel (2001) showed that the equal split is stable with a continuous strategy space.

Although our model is partly related with evolutionary models, our model is different from the standard evolutionary models. Our model attempts to explain what a fixed pair of individual bargainers will act initially if they expect a long-term relationship with the other bargainer whereas other evolutionary models attempt to explain dynamics of the population(s). The most important difference is the equilibrium-selection mechanism. In our model the initial actions are determined endogenously by the bargainer's deliberate choices whereas in most evolutionary models they are given exogenously. The role that mutation or random experimentation plays for the equilibrium selection in other evolutionary models is played by the small error that the bargainers can make in their calculation of expected payoffs in our model.

We first show that the fictitious play resolves eventually the inefficiency due to miscoordination. If players miscoordinate in the initial period by demanding too little, they get to coordinate eventually in the way that the player who is less greedy in his initial demand gets a better share. If players miscoordinate in the initial period by demanding too much, they get to coordinate eventually in the way that the player who is greedier in his initial demand gets a worse share.

Our main result is that in the initial-demand game they play, all the ϵ-equilibria (à la Radner (1986)) are in a neighborhood of the division corresponding to the Nash bargaining solution if the bargainers are patient (the time discount is small). Furthermore, as the bargainers make a more accurate comparison of payoffs and become more patient (the time discount vanishes) accordingly, the set of ϵ-equilibria shrinks and the only equilibrium left is the division of the Nash bargaining solution.

2 The strategic model

Two players 1 and 2 are playing the Nash (1953) demand game infinitely many times, starting at time $t = 0$. In each period both players simultaneously announce their demands x and y respectively, where $0 < x, y \leq 1$. That is, the size of the pie has been normalized to 1. If $x+y \leq 1$, they receive $u(x)$ and $v(y)$ in that period respectively. Otherwise, they receive $u(0)$ and $v(0)$ respectively. The utility functions are strictly increasing, concave, and normalized so that $u(0) = v(0) = 0$.

There is a discount of payoff by δ between periods, where $0 \leq \delta < 1$. Alternatively, $(1 - \delta)$ can be interpreted as a probability of breakdown, as Binmore, et al. (1986) did. Player 1 receives $u(x_t)I_{[x_t+y_t \leq 1]}(x_t, y_t)$ in each period t, and the average payoff of the infinite sequence of payoffs is

$$\bar{u} \equiv (1 - \delta) \sum_{t=0}^{\infty} \delta^t u(x_t) I_{[x_t+y_t \leq 1]}(x_t, y_t),$$

where I is an indicator function. Similarly, player 2 receives $v(y_t)I_{[x_t+y_t \leq 1]}(x_t, y_t)$ in each period t, and the average payoff of the infinite sequence of payoffs is

$$\bar{v} \equiv (1 - \delta) \sum_{t=0}^{\infty} \delta^t v(y_t) I_{[x_t+y_t \leq 1]}(x_t, y_t).$$

From the time $t = 1$, both players use a simple learning rule and make a decision according to the fictitious play. For any $t \geq 1$, let $f_t(x)$ denote the relative frequency with which player 1 has chosen x up to time $(t-1)$. Similarly, let $g_t(y)$ denote the relative frequency with which player 2 has chosen y up to time $(t-1)$. According to the fictitious play, players choose x_t and y_t for any $t \geq 1$ as follows:

$$x_t = \arg\max_x \sum_{y:g_t(y)>0} g_t(y)u(x)I_{[x+y\leq 1]}(x,y),$$

$$y_t = \arg\max_y \sum_{x:f_t(x)>0} f_t(x)v(y)I_{[x+y\leq 1]}(x,y).$$

That is, in each period each player chooses his best response to the observed historical frequency of his opponent's choices. For simplicity, we assume that ties are broken in favor of a higher demand.

In the initial period $t=0$, however, there has been no opponent's action to refer to. In this initial period, the players exercise a higher level of rationality to choose their initial demands because they will determine unambiguously the subsequent demands by the fictitious play process. This initial-demand game is virtually a one-shot Nash demand game where the payoffs are the average payoff of the infinite sequence of payoffs that they expect in the initial and subsequent periods. We employ Radner's (1986) ϵ-equilibrium concept for this initial-demand game.

DEFINITION A strategy profile is an **ϵ-equilibrium** if no player has an alternative strategy that increases his payoff by more than ϵ.

3 The implementation result

Lemma 1 *For any (x_0, y_0), the following hold:*
(1) $x_1 = 1-y_0$ and $y_1 = 1-x_0$.
(2) For any $t \geq 2$, x_t must be either x_0 or $(1-y_0)$, and y_t must be either y_0 or $(1-x_0)$.
(3) For any $t \geq 1$, $f_t(x_0)+f_t(1-y_0)=1$ and $g_t(y_0)+g_t(1-x_0)=1$.

Proof. (1) Clearly, $x_1 = \arg\max_x u(x)I_{[x\leq 1-y_0]}(x) = 1-y_0$, and $y_1 = \arg\max_y v(y)I_{[y\leq 1-x_0]}(y) = 1-x_0$.
(2) We prove statement (2) by mathematical induction. x_2 must be either x_0 or $(1-y_0)$ because $x_1 = \arg\max_x[\frac{1}{2}u(x)I_{[x\leq x_0]}(x) + \frac{1}{2}u(x)I_{[x\leq 1-y_0]}(x)]$. Similarly, y_2 must be either y_0 or $(1-x_0)$.
Suppose that x_t is either x_0 or $(1-y_0)$, and y_t is either y_0 or $(1-x_0)$ for any $t\geq 2$ (induction hypothesis). Then x_{t+1} must be either x_0 or $(1-y_0)$ because $x_{t+1} = \arg\max_x[g_{t+1}(1-x_0)u(x)I_{[x\leq x_0]}(x)+g_{t+1}(y_0)u(x)I_{[x\leq 1-y_0]}(x)]$.

Similarly, y_{t+1} must be either y_0 or $(1-x_0)$.
(3) Statement (3) follows immediately from statements (1) and (2). ■

Let (x^N, y^N) be the division of the Nash bargaining solution, given the utility functions u and v. That is,

$$(x^N, y^N) = \arg\max_{(x,1-x)} u(x)v(1-x).$$

We define a function ϕ that assigns $y = \phi(x)$ in $[0,1]$ to each number x in $[0,1]$ as follows (see Figure 1):

- $\phi(x^N) = y^N$.
- If $x \neq x^N$, $\phi(x)$ is the solution of the following equation which is different from $(1-x)$:

$$u(x)v(1-x) = u(1-\phi(x))v(\phi(x)).$$

Since u and v are strictly increasing and concave, $\phi(x)$ is uniquely determined for each $x \in [0,1]$. The function $\phi(x)$ is strictly increasing and reflects the shape of the Pareto frontier of the feasible alternatives. For example, if u and v are linear then $\phi(x) = x$.

Lemma 2 *(1) If $x_0 + y_0 = 1$, then $\bar{u} = u(x_0)$ and $\bar{v} = v(y_0)$.*
(2) If $x_0 + y_0 < 1$ and $y_0 = \phi(x_0)$, then

$$(1-\delta)u(x_0) < \bar{u} < (1-\delta+\delta^2)u(x_0), \quad \lim_{\delta\to 1}\bar{u} = \frac{u(x_0)^2}{u(1-y_0)},$$

$$(1-\delta)v(y_0) < \bar{v} < (1-\delta+\delta^2)v(y_0), \quad \textit{and} \quad \lim_{\delta\to 1}\bar{v} = \frac{v(y_0)^2}{v(1-x_0)}.$$

(3) If $x_0 + y_0 > 1$ and $y_0 = \phi(x_0)$, then

$$(1-\delta)\delta u(1-y_0) < \bar{u} < \delta u(1-y_0), \quad \lim_{\delta\to 1}\bar{u} = \frac{u(1-y_0)^2}{u(x_0)},$$

$$(1-\delta)\delta v(1-x_0) < \bar{v} < \delta v(1-x_0), \quad \textit{and} \quad \lim_{\delta\to 1}\bar{v} = \frac{v(1-x_0)^2}{v(y_0)}.$$

(4) If $x_0 + y_0 < 1$ and $y_0 > \phi(x_0)$, then

$$(1-\delta)u(x_0) + \delta^T u(1-y_0) \leq \bar{u} \leq (1-\delta^T)u(x_0) + \delta^T u(1-y_0) \quad \textit{and}$$

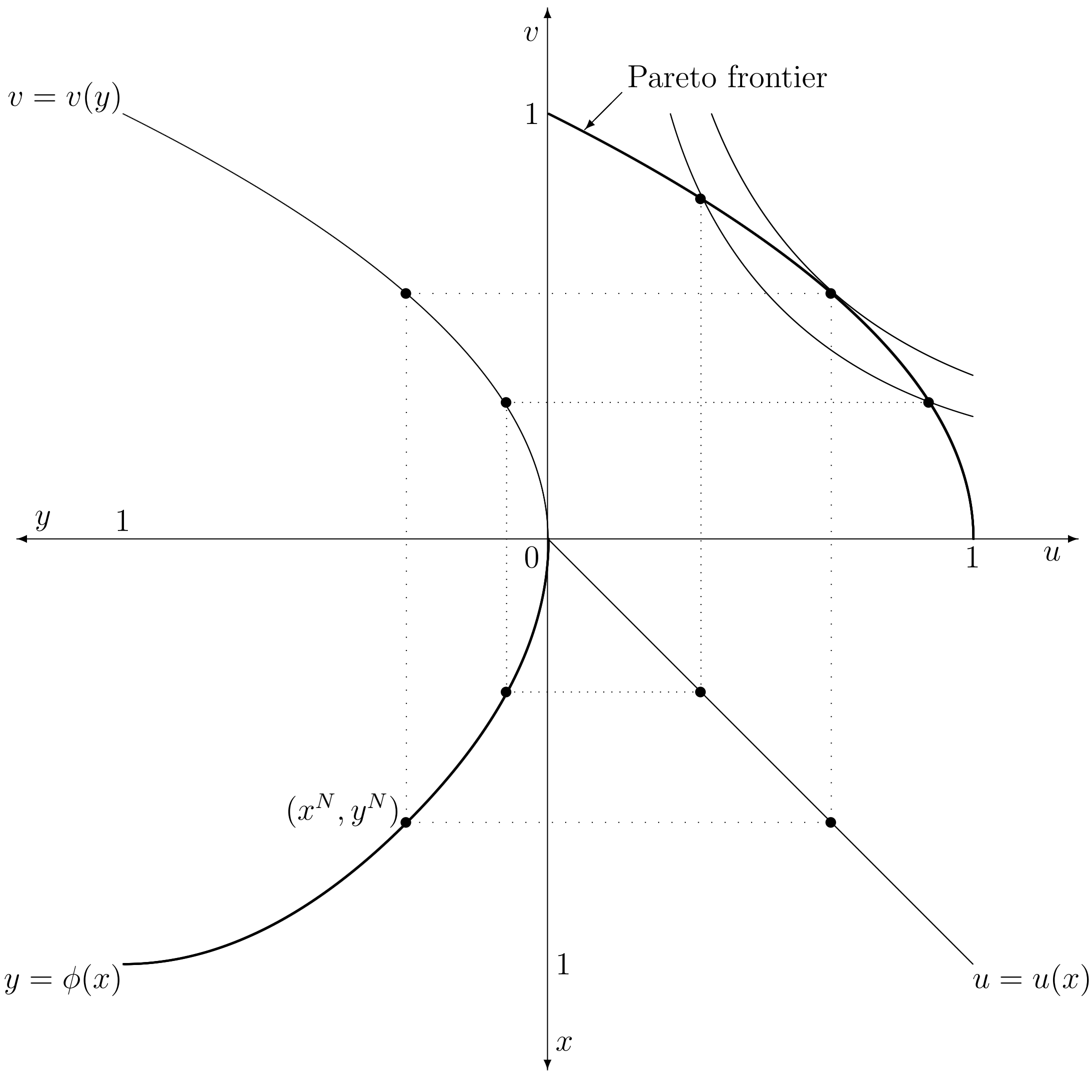

Figure 1: Construction of Function ϕ

$$(1-\delta+\delta^T)v(y_0) \le \bar{v} \le v(y_0) \quad \textit{for some positive integer } T.$$

If δ *is sufficiently large, as* $\delta \to 1$, $\bar{u}$ *monotonically increases towards*

$$\lim_{\delta\to 1} \bar{u} = u(1-y_0)$$

and $\bar{v}$ *monotonically increases towards*

$$\lim_{\delta\to 1} \bar{v} = v(y_0).$$

Similarly, if $x_0 + y_0 < 1$ *and* $y_0 < \phi(x_0)$, *then*

$$(1-\delta+\delta^T)u(x_0) \le \bar{u} \le u(x_0) \quad \textit{and}$$

$$(1-\delta)v(y_0) + \delta^T v(1-x_0) \le \bar{v} \le (1-\delta^T)v(y_0) + \delta^T v(1-x_0)$$

for some positive integer T.

If δ *is sufficiently large, as* $\delta \to 1$, $\bar{u}$ *monotonically increases towards*

$$\lim_{\delta\to 1} \bar{u} = u(x_0)$$

and $\bar{v}$ *monotonically increases towards*

$$\lim_{\delta\to 1} \bar{v} = v(1-x_0).$$

(5) If $x_0 + y_0 > 1$ *and* $y_0 > \phi(x_0)$, *then*

$$(1-\delta+\delta^T)u(x_0) \le \bar{u} \le u(x_0) \quad \textit{and}$$

$$(1-\delta)v(y_0) + \delta^T v(1-x_0) \le \bar{v} \le (1-\delta^T)v(y_0) + \delta^T v(1-x_0)$$

for some positive integer T.

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$$\lim_{\delta\to 1} \bar{v} = v(1-x_0).$$

Similarly, if $x_0 + y_0 > 1$ *and* $y_0 < \phi(x_0)$, *then*

$$(1-\delta)u(x_0) + \delta^T u(1-y_0) \le \bar{u} \le (1-\delta^T)u(x_0) + \delta^T u(1-y_0) \quad \textit{and}$$

$$(1-\delta+\delta^T)v(y_0) \leq \bar{v} \leq v(y_0) \quad \textit{for some positive integer } T.$$

If δ *is sufficiently large, as* $\delta \to 1$, $\bar{u}$ *monotonically increases towards*

$$\lim_{\delta \to 1} \bar{u} = u(1-y_0)$$

and $\bar{v}$ *monotonically increases towards*

$$\lim_{\delta \to 1} \bar{v} = v(y_0).$$

Proof. (1) Statement (1) follows from Lemma 1 because $x_0 + y_0 = 1$.
(2)-(5) We prove only statement (2) (the case of $[x_0+y_0 < 1 \text{ and } y_0 = \phi(x_0)]$) and the first part of statement (4) (the case of $[x_0 + y_0 < 1 \text{ and } y_0 > \phi(x_0)]$) omitting the tedious repetition for the other cases.

We first prove the first part of statement (4). Let $f^* \equiv \frac{u(x_0)}{u(1-y_0)}$ and $g^* \equiv \frac{v(y_0)}{v(1-x_0)}$. For any $t \geq 1$,

$$f_{t+1}(x_0) = \begin{cases} \frac{tf_t(x_0)}{t+1} & \text{if } g_t(y_0) \geq f^* \\ \frac{tf_t(x_0)+1}{t+1} & \text{if } g_t(y_0) < f^*, \end{cases}$$

and

$$g_{t+1}(y_0) = \begin{cases} \frac{tg_t(y_0)}{t+1} & \text{if } f_t(x_0) \geq g^* \\ \frac{tg_t(y_0)+1}{t+1} & \text{if } f_t(x_0) < g^*. \end{cases}$$

We define the following four states regarding the pair of relative frequencies $(f_t(x_0), g_t(y_0))$:

- state [>>]: $f_t(x_0) \geq g^*$ and $g_t(y_0) \geq f^*$,
- state [><]: $f_t(x_0) \geq g^*$ and $g_t(y_0) < f^*$,
- state [<>]: $f_t(x_0) < g^*$ and $g_t(y_0) \geq f^*$,
- state [<<]: $f_t(x_0) \geq g^*$ and $g_t(y_0) \geq f^*$.

We define state [>>]* and [<<]* as follows:

- state [>>]*: $f_t(x_0) \geq g^*$, $g_t(y_0) \geq f^*$, and $f_t(x_0) = g_t(y_0)$,
- state [<<]*: $f_t(x_0) < g^*$, $g_t(y_0) < f^*$, and $f_t(x_0) = g_t(y_0)$.

Note that $0 < f^* < g^* < 1$ because $x_0 + y_0 < 1$ and $y_0 > \phi(x_0)$. At $t = 1$, $f_1(x_0) = g_1(y_0) = 1$ and therefore the pair of relative frequencies is in the state [>>]*. At $t = 2$, $f_2(x_0) = g_2(y_0) = \frac{1}{2}$ and the state can be either [>>]*, [<>], or [<<]* depending on the values of f^* and g^*. However, it cannot be [><] because $f^* < g^*$. We can establish the following regarding the transition between states:

- If the current state is [<>], the next state is always [<>] (Borrowing a term from the Markov chain theory, the state [<>] is an absorbing state).
- If the current state is [>>]*, the next state must be either [>>]*, [<>], or [<<]*.
- If the current state is [<<]*, the next state must be either [>>]*, [<>], or [<<]*.

Therefore, the state must be either [>>]*, [<>], or [<<]* for any $t \geq 2$. Furthermore, the state becomes [<>] eventually because the change in the relative frequency between two periods becomes smaller than $(g^* - f^*)$ eventually. That is, an oscillation between the states [>>]* and [<<]* cannot last for ever.

Let T_1 be the number of periods when the state is [>>]* before the state becomes [<>] eventually, and T_2 the number of periods when the state is [<<]* before the state becomes [<>] eventually. The numbers T_1 and T_2 are nonnegative integers. By taking $T \equiv T_1 + T_2 + 1$, we have

$$(1-\delta)u(x_0) + \delta^T u(1-y_0) \leq \bar{u} \leq (1-\delta^T)u(x_0) + \delta^T u(1-y_0) \quad \text{and}$$

$$(1-\delta+\delta^T)v(y_0) \leq \bar{v} \leq v(y_0) \quad \text{for some positive integer } T.$$

If δ is sufficiently large, as $\delta \to 1$, $\bar{u}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{u} = u(1-y_0)$$

and $\bar{v}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{v} = v(y_0).$$

This ends the proof of the first part of statement (4). Now, we prove statement (2).

If $x_0 + y_0 < 1$ and $y_0 = \phi(x_0)$, then $f^* = g^*$. Therefore, the state must be either [>>]* or [<<]* for any $t \geq 1$. The state oscillates between the states [>>]* and [<<]* for ever, and the sequence $\{f_t(x_0)\}$ converges to $\frac{u(x_0)}{u(1-y_0)}$ $(= \frac{v(y_0)}{v(1-x_0)})$. Therefore, we obtain

$$(1-\delta)u(x_0) < \bar{u} < (1-\delta+\delta^2)u(x_0), \quad \lim_{\delta\to 1} \bar{u} = \frac{u(x_0)^2}{u(1-y_0)},$$

$$(1-\delta)v(y_0) < \bar{v} < (1-\delta+\delta^2)v(y_0), \quad \text{and} \quad \lim_{\delta\to 1} \bar{v} = \frac{v(y_0)^2}{v(1-x_0)}.$$

This ends the proof of the first part of statement (2). ■

Therefore, the fictitious play resolves eventually the inefficiency due to miscoordination. If players miscoordinate in the initial period by demanding too little, they get to coordinate eventually in the way that the player who is less greedy in his initial demand gets a better share. If players miscoordinate in the initial period by demanding too much, they get to coordinate eventually in the way that the player who is greedier in his initial demand gets a worse share. In this model, the perpetual miscoordination, as in Young (1993) p. 152, does not happen generically. It only happens when $y_0 = \phi(x_0)$.

The limits for the linear utility case were studied by He (2004). To see how long it takes for the demands to reach the limit in the linear utility case, we refer readers to He.

Theorem 1 *For any open neighborhood of* (x^N, y^N)*, there exist* $\epsilon > 0$ *and* $\delta^*(\epsilon) < 1$ *such that all the* ϵ*-equilibria are in the neighborhood for any* $\delta \geq \delta^*(\epsilon)$*. As* $\epsilon \to 0$ *and* $\delta^*(\epsilon) \to 1$ *accordingly, the set of* ϵ*-equilibria shrinks and the only equilibrium left is* (x^N, y^N)*.*

Proof. Using the limit average payoffs $\lim_{\delta\to 1} \bar{u}$ and $\lim_{\delta\to 1} \bar{v}$ that we have obtained in Lemma 2, we can get the best response correspondences for player 1 (illustrated in Figure 2)

$$x^*(y) = \begin{cases} [0, \phi^{-1}(y)) \cup [1-y, 1] & \text{if } y < y^N \\ [0, 1] & \text{if } y = y^N \\ \emptyset & \text{if } y > y^N, \end{cases}$$

and for player 2

$$y^*(x) = \begin{cases} [0, \phi(x)) \cup [1-x, 1] & \text{if } x < x^N \\ [0, 1] & \text{if } x = x^N \\ \emptyset & \text{if } x > x^N. \end{cases}$$

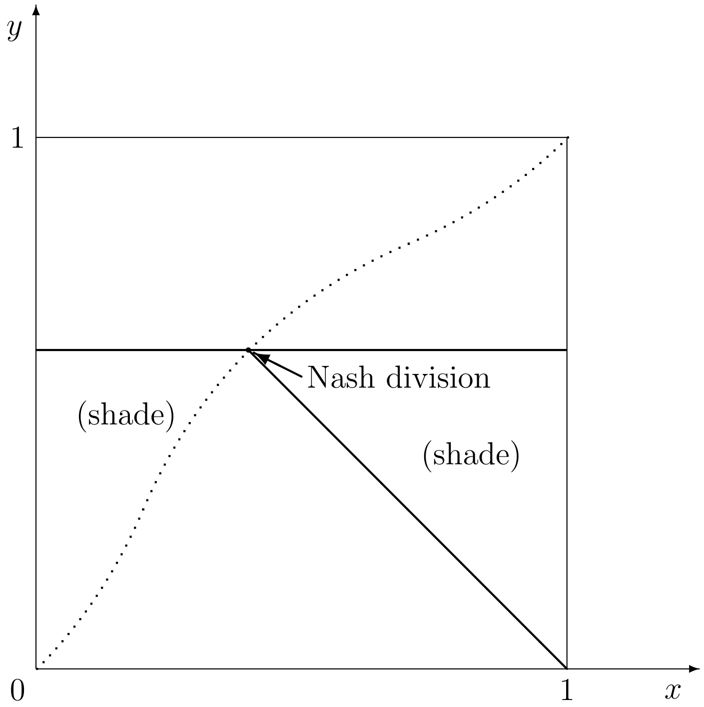

Figure 2: Limit-Average-Payoff Best Response Correspondence for Player 1

Note that the average payoff functions are continuous except at the points of $y = \phi(x)$. One can see easily that the only pure-strategy Nash equilibrium in this case is (x^N, y^N).

Using the limit average payoffs $\lim_{\delta\to 1} \bar{u}$ and $\lim_{\delta\to 1} \bar{v}$ that we have obtained in Lemma 2, we can get the ϵ-best response correspondence for player 1 (illustrated in Figure 3)

$$x^*_\epsilon(y) = \begin{cases} [0, \phi^{-1}(y)) \cup [u^{-1}(u(1-y)-\epsilon), 1] & \text{if } y < y^{(1)} \\ [0, \phi^{-1}(y)) \cup (\phi^{-1}(y), 1] & \text{if } y^{(1)} \le y < y^{(2)} \\ [0, 1] & \text{if } y^{(2)} \le y \le y^{(3)} \\ [0, \phi^{-1}(y)) \cup (\phi^{-1}(y), 1] & \text{if } y^{(3)} < y \le y^{(4)} \\ [u^{-1}(u(\phi^{-1}(y))-\epsilon), \phi^{-1}(y)) & \text{if } y > y^{(4)}, \end{cases} \quad (1)$$

where

$$y^{(1)} \text{ is the solution of } \phi^{-1}(y) = u^{-1}(u(1-y)-\epsilon),$$

$$y^{(2)} \text{ is the solution of } \frac{u(\phi^{-1}(y))^2}{u(1-y)} = u(1-y)-\epsilon,$$

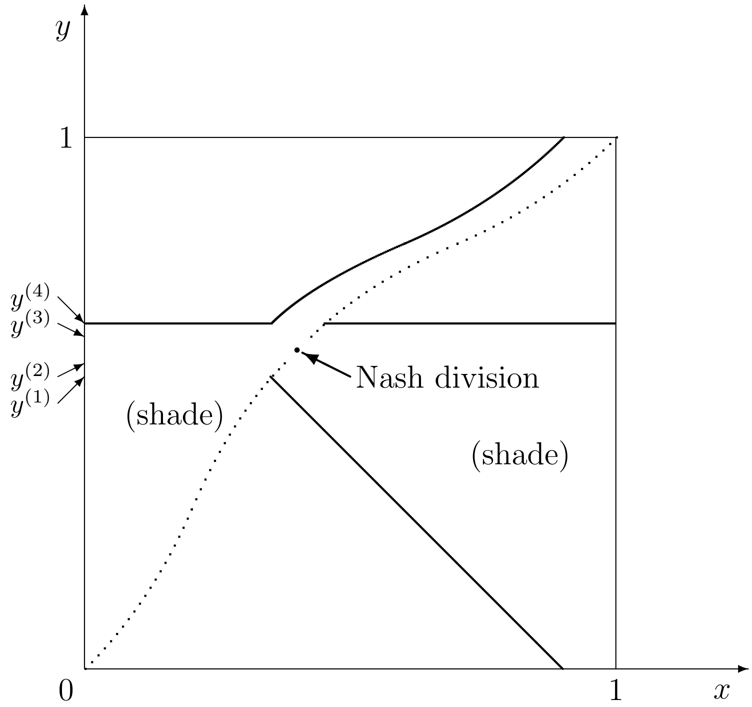

Figure 3: LIMIT-AVERAGE-PAYOFF ϵ-BEST RESPONSE CORRESPONDENCE FOR PLAYER 1

$$y^{(3)} \text{ is the solution of } \frac{u(1-y)^2}{u(\phi^{-1}(y))} = u(\phi^{-1}(y)) - \epsilon, \text{ and}$$

$$y^{(4)} \text{ is the solution of } \phi^{-1}(y) = u^{-1}(u(1-y)+\epsilon).$$

Similarly, the ϵ-best response correspondence for player 2 is

$$y^*_\epsilon(x) = \begin{cases} [0,\phi(x)) \cup [v^{-1}(v(1-x)-\epsilon), 1] & \text{if } x < x^{(1)} \\ [0,\phi(x)) \cup (\phi(x), 1] & \text{if } x^{(1)} \le x < x^{(2)} \\ [0,1] & \text{if } x^{(2)} \le x \le x^{(3)} \\ [0,\phi(x)) \cup (\phi(x), 1] & \text{if } x^{(3)} < x \le x^{(4)} \\ [v^{-1}(v(\phi(x))-\epsilon), \phi(x)) & \text{if } x > x^{(4)}, \end{cases} \tag{2}$$

where

$$x^{(1)} \text{ is the solution of } \phi(x) = v^{-1}(v(1-x)-\epsilon),$$

$$x^{(2)} \text{ is the solution of } \frac{u(x)^2}{u(1-\phi(x))} = v(1-x) - \epsilon,$$

$$x^{(3)} \text{ is the solution of } \frac{u(1-\phi(x))^2}{u(x)} = v(\phi(x)) - \epsilon, \text{ and}$$

$x^{(4)}$ is the solution of $\phi(x) = v^{-1}(v(1-x)+\epsilon)$.

The set of ϵ-equilibria based on the limit average payoffs is illustrated in Figure 4. This set is a subset of

$$\{x : 1 - v^{-1}(v(y^{(4)})+\epsilon) \leq x \leq \phi^{-1}(v^{-1}(v(y^{(4)})+\epsilon))\} \times$$
$$\{y : 1 - u^{-1}(u(x^{(4)})+\epsilon) \leq y \leq \phi(u^{-1}(u(x^{(4)})+\epsilon))\}. \quad (3)$$

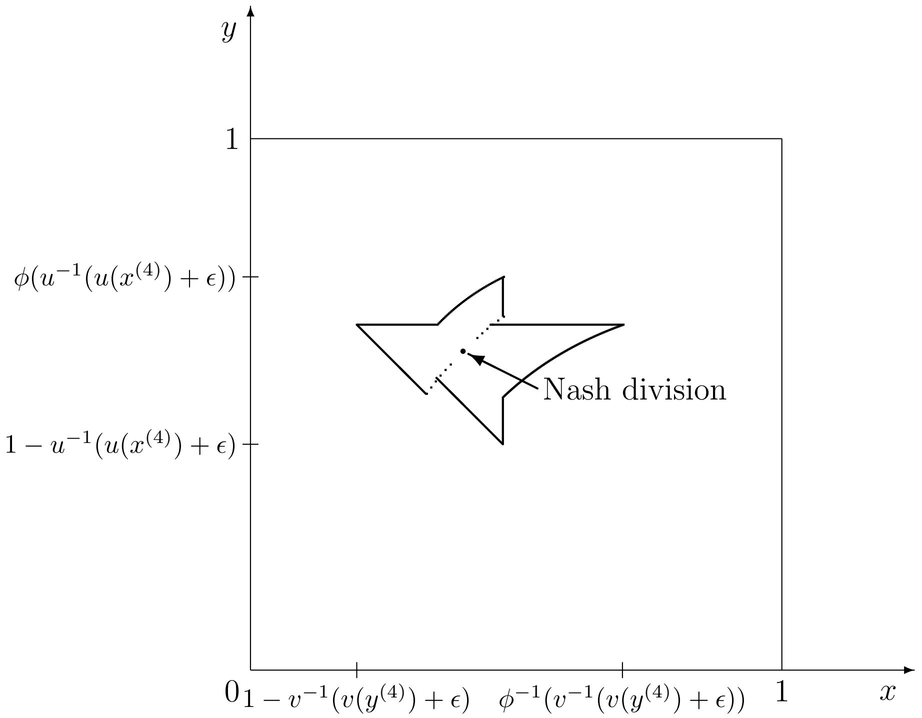

Figure 4: Limit-Average-Payoff ϵ-Equilibria

If we choose a sufficiently large $\delta^*(\epsilon) < 1$, then the set of ϵ-equilibria will be a subset of (3) above for any $\delta \geq \delta^*(\epsilon)$. As $\epsilon \to 0$,

$$1 - v^{-1}(v(y^{(4)})+\epsilon) \to x^N,$$
$$\phi^{-1}(v^{-1}(v(y^{(4)})+\epsilon)) \to x^N,$$
$$1 - u^{-1}(u(x^{(4)})+\epsilon) \to y^N, \text{ and}$$
$$\phi(u^{-1}(u(x^{(4)})+\epsilon)) \to y^N.$$

This implies that as $\epsilon \to 0$ and $\delta^*(\epsilon) \to 1$ accordingly, only (x^N, y^N) remains as the limit of ϵ-equilibria. ■

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