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Agenda Restrictions in Multi-Issue Bargaining (II): Unrestricted Agendas

Younghwan In
National University of Singapore

Roberto Serrano
Brown University

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Abstract: We study a bilateral multi-issue bargaining procedure with complete information and endogenous unrestricted agenda, in which offers can be made in any subset of outstanding issues. We find necessary and sufficient conditions for this procedure to have a unique subgame perfect equilibrium agreement.

Key words: multi-issue bargaining, complete information, agenda
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 Younghwan In, Department of Economics, National University of Singapore, 1 Arts Link, Singapore 117570, e-mail: ecsyhi@nus.edu.sg, http://courses.nus.edu.sg/course/ecsyhi, tel: +65-6874-6261, fax: +65-6775-2646. Roberto Serrano, Department of Economics, Brown University, Providence, RI 02912, U.S.A., e-mail: roberto_serrano@brown.edu, http://econ.brown.edu/faculty/serrano. In gratefully acknowledges research support from Brown University through a Stephen R. Ehrlich fellowship, and Serrano from the National Science Foundation.

1 Introduction

This note attempts to shed light on the circumstances under which an unrestricted agenda multi-issue bargaining procedure, extension of Rubinstein's (1982) alternating offers game, yields a unique subgame perfect equilibrium (SPE) agreement. In and Serrano (2000) pointed out that restricting agendas may yield multiple SPE agreements, including those with arbitrarily long delay. The procedure in In and Serrano (2000) exhibits a clear friction: bargainers are forced to negotiate one issue at a time, the one chosen by the proposer in each round.

We are thus led to examine a procedure where the issues can be bundled. A basic observation in this respect is that if one studies a procedure where all issues must be bundled in every offer, Rubinstein's uniqueness and efficiency result extends (indeed, the same proof as in Osborne and Rubinstein (1990) applies). Instead, we investigate a procedure suggested by Inderst (2000). We refer to it as unrestricted agenda bargaining. In it, each proposer can make an offer on any subset of outstanding issues. We provide necessary and sufficient conditions for this procedure to have a unique SPE, thereby generalizing Inderst's (2000) sufficiency result to a considerably larger class of utility functions (where separability across issues and concavity are not assumed). The conditions we find require the uniqueness of a stationary equilibrium payoff. In this sense, they are simple generalizations of those found for the single-issue case, and we emphasize that they are compatible with important classes of preferences ruled out in the earlier analysis of the procedure.

2 Unrestricted Agenda Bargaining

Let $L = \{1, \ldots, l\}$ be the set of issues. Bargaining takes place over potentially infinite discrete periods starting in period 0. For $i = 1, 2$ and $S \subseteq L$, the procedure $\Gamma_i^S(\delta)$ induces an infinite horizon game of perfect information and it is defined recursively.

- The first move corresponds to player i, who makes a proposal: he chooses an arbitrary subset of the pending issues $T \subseteq S, T \neq \emptyset$ and offers the split $(x_T, 1_T - x_T)$, where $0_T \leq x_T \leq 1_T$.[1]

[1] We use the following vector notation for two arbitrary vectors x and y: $x \geq y$ means

- Player j can then either accept or reject this entire proposal (he is not allowed to accept the proposal only for a strict subset of T and reject the rest).
  - If the proposal is accepted, issues T are split accordingly and the procedure $\Gamma_j^{S\setminus T}(\delta)$ is followed.
  - If the proposal is rejected, the procedure $\Gamma_j^S(\delta)$ is followed with probability δ ($0 \leq \delta < 1$), whereas negotiations break down with probability $1-\delta$. In the latter case, both players receive a zero share from those issues on which there was no agreement.

  The negotiations end either with the breakdown outcome, or when the procedure $\Gamma_i^{\emptyset}(\delta)$ must be followed for $i = 1, 2$.

We shall refer to the procedure $\Gamma_i^L(\delta)$ simply as Γ_i. Also, a subgame $\Gamma_i^S(\delta)$ will be written simply as Γ_i^S. We shall seek for the subgame perfect equilibrium (SPE) agreements of the procedure Γ_i.

Both players are von Neumann-Morgenstern expected utility maximizers. We make the following assumptions on the utility functions $u_1(x_1, \ldots, x_l)$ and $u_2(1-x_1, \ldots, 1-x_l)$:

- A0 (normalization):

$$u_i(0_L) = 0 \quad \text{for } i = 1, 2.$$

- A1 (continuity): $u_i(\cdot)$ is continuous on $[0,1]^l$.[2]

- A2 (interior strong monotonicity):

$$\begin{aligned} u_i(x) &> u_i(y) \quad \text{if} \quad x > y \text{ and } x \gg 0_L, \text{ and} \\ u_i(x) &\geq u_i(y) \quad \text{if} \quad x > y \text{ and some element of } x \text{ is zero.} \end{aligned}$$

We shall impose one more assumption that will turn out to be necessary and sufficient for uniqueness of SPE in the procedure Γ_i. This extra assumption plays the role of Lemma 3.2 in the proof of uniqueness of the

$x_k \geq y_k$ for all k; $x > y$ means $x_k \geq y_k$ for all k and $x_k > y_k$ for some k; and $x \gg y$ means $x_k > y_k$ for all k. Also, $x_T = (x_k)_{k \in T}$, and we use 0_T and 1_T to denote vectors with $|T|$ zeros and ones, respectively.

[2]Lower case letters denote cardinality of sets, i.e., $|L| = l$, $|S| = s$, and so on.

single-issue case found in Osborne and Rubinstein (1990). Before getting to it, we introduce some pieces of notation.

Consider the game Γ_i, $i = 1, 2$, and a subgame thereof $\Gamma_j^S(x_{-S})$, $j = 1, 2$, prior to which the shares $(x_k, 1 - x_k)_{k \notin S}$ have been agreed. Moreover, suppose that in this subgame both players have a non-degenerate set of feasible utilities.[3]

We define the following amounts, the maximum and minimum utilities that each player can get in this subgame:

$$\underline{u}_1(x_{-S}) = u_1(0_S, x_{-S}), \qquad \bar{u}_1(x_{-S}) = u_1(1_S, x_{-S}).$$
$$\underline{u}_2(x_{-S}) = u_2(0_S, 1_{-S} - x_{-S}), \quad \bar{u}_2(x_{-S}) = u_2(1_S, 1_{-S} - x_{-S}).$$

The *utility possibility set* in this subgame is:

$$U_S(x_{-S}) = \{(u_1(x_S, x_{-S}), u_2(1_S - x_S, 1_{-S} - x_{-S})) : x_S \in [0, 1]^s\}.$$

Given the assumptions on utility functions, the Pareto frontier of the set $U_S(x_{-S})$ is the graph of a continuous and strictly decreasing function $f_{x_{-S}}(u_1)$ defined on $[\underline{u}_1(x_{-S}), \bar{u}_1(x_{-S})]$. Let $g_{x_{-S}}(u_2)$, defined on $[\underline{u}_2(x_{-S}), \bar{u}_2(x_{-S})]$, denote the inverse function of $f_{x_{-S}}(u_1)$. Similarly, if $S = L$, the Pareto frontier of the utility possibility set U_L in the entire game is the graph of a function $f(u_1)$, whose inverse is $g(u_2)$.

We now state our last assumption. It provides a necessary and sufficient condition for uniqueness. We provide examples of specific utility functions that imply the assumption in the sequel.

- A3. For every subgame $\Gamma_i^S(x_{-S})$,
  - (1) There exists a unique solution $u_1^*(x_{-S})$ to the equation

    $$h(u_1) = u_1 - \underline{u}_1(x_{-S}) - \delta[g_{x_{-S}}((1-\delta)\underline{u}_2(x_{-S}) + \delta f_{x_{-S}}(u_1)) - \underline{u}_1(x_{-S})] = 0;$$
  - (2) If $h(z) \geq 0$, then $z \geq u_1^*(x_{-S})$; and
  - (3) If $h(z) \leq 0$, then $z \leq u_1^*(x_{-S})$.

[3]There are additional subgames of a trivial nature, where at least one of the players has only one feasible utility level (if this player has accepted a zero share in some issue and his utility is positive only when he receives positive amounts of all issues). But these subgames are irrelevant for the analysis.

Assumption A3 requires that the function $h(\cdot)$, analogue to that used in the proof of Lemma 3.2 in Osborne and Rubinstein (1990) as the basis of a stationary construction, have a unique zero. This will turn out to be weaker than additive separability and concavity, conditions used in Inderst (2000).

The next Lemma, an analogue of Lemma 3.2 in Osborne and Rubinstein (1990), will be useful and sheds additional light on Assumption A3. The simple proof is omitted.

Lemma 1 *Under Assumption A3(1), in the subgame $\Gamma_i^S(x_{-S})$ there is a unique solution $(u_1^*(x_{-S}), u_2^*(x_{-S}))$ to the system of equations*

$$\begin{aligned} u_1 &= (1-\delta)\underline{u}_1(x_{-S}) + \delta g_{x_{-S}}(u_2) \\ u_2 &= (1-\delta)\underline{u}_2(x_{-S}) + \delta f_{x_{-S}}(u_1). \end{aligned}$$

If $S = L$ and we consider the game Γ_i, the unique solution to the equations in Lemma 1 will be denoted (u_1^*, u_2^*). As in Rubinstein (1982), the equations in Lemma 1 describe the indifference of player $j = 1, 2$ when responding between accepting the proposal worth u_j and rejecting it seeking for a stationary continuation in negotiations behavior.

Proposition 1 *Consider the unrestricted agenda multi-issue bargaining procedure $\Gamma_1(\delta)$ for any $\delta \in [0, 1)$. Under Assumptions A0-A3, there exists a unique SPE payoff, which is implemented immediately and is efficient. It is the payoff pair $(g(u_2^*), u_2^*)$.*

Of course, in the game $\Gamma_2(\delta)$, the same proof also shows that the unique SPE payoff is $(u_1^*, f(u_1^*))$. With Lemma 1 in hand, the uniqueness part of the proof is by now standard, based on the ingenious argument of Shaked and Sutton (1984). Existence relies on an induction argument.

Proof of Proposition 1. First, we write down strategies that support the desired payoff. In doing this, we use induction on the number of issues. Since when $|T| = 1$, the procedure Γ_i^T reduces to Rubinstein's (1982), we make the following induction hypothesis: if $|T| < l$, the game $\Gamma_i^T(x_{-T})$ has a unique SPE payoff when the set $U_T(x_{-T})$ is non-degenerate. In it, player j receives a payoff $u_j^*(x_{-T})$ and player i its efficient projection in the subgame. Notice that this induction hypothesis holds for single-issue subgames: thanks to Assumptions A0-A3, Rubinstein's theorem applies.

Using the above induction hypothesis, SPE strategies for the game Γ_i follow. In them, let $S \subseteq L$.

- (i) In subgames $\Gamma_i^S(x_{-S})$ where the utility possibility set $U_S(x_{-S})$ is non-degenerate, player i makes a proposal on all remaining issues. He offers to player j the payoff $u_j^*(x_{-S})$ and asks for himself the efficient projection in the subgame of $u_j^*(x_{-S})$.
- (ii) In subgames $\Gamma_i^S(x_{-S})$ where the utility possibility set $U_S(x_{-S})$ is non-degenerate, player j accepts a proposal if and only if the utility u that he would get by accepting (assuming he receives the SPE payoff specified in the induction hypothesis, for the issues still pending) satisfies $u \geq u_j^*(x_{-S})$.
- (iii) In degenerate subgames where at least one of the players receives only zero utility, any SPE pair of strategies would do.

By the standard argument, which is based on the equations of Lemma 1 and involves the use of the one-time deviation property, it is easy to see that these strategies constitute a SPE of the game Γ_i.

Next, we prove that the SPE payoff is unique. Consider the games Γ_1 and Γ_2, subgames of each other. Recall that $\underline{u}_1 = \underline{u}_2 = 0$. Let M_1 and m_1 denote the supremum and the infimum of u_1, and M_2 and m_2 the supremum and the infimum of u_2 in the SPE of Γ_1. Let M_1', m_1', M_2' and m_2' denote the similar magnitudes in Γ_2.

First, $M_1 \leq g(\delta f(\delta M_1))$. This comes from combining $M_1 \leq g(\delta m_2')$ and $m_2' \geq f(\delta M_1)$. Therefore, by A3, part (3), $M_1 \leq z^*$, where z^* solves $z^* = g(\delta f(\delta z^*))$. Moreover, using the equations of Lemma 1, $z^* = g(u_2^*)$.

Second, $m_1 \geq g(\delta f(\delta m_1))$. This is obtained from combining $m_1 \geq g(\delta M_2')$ and $M_2' \leq f(\delta m_1)$. Using A3, part (2), $m_1 \geq z^*$. And therefore, $M_1 = m_1 = g(u_2^*)$.

Finally, it is clear that $M_2 \leq f(m_1)$ and $m_2 \geq f(M_1)$, but $m_1 = M_1 = g(u_2^*)$. Therefore, $M_2 = m_2 = f(g(u_2^*)) = u_2^*$. ∎

Remark 1: Under Assumptions A0-A3, there might be multiple strategy profiles supporting the unique SPE payoff of Γ_i, but this can always be supported by offers involving all issues. To get uniqueness in the equilibrium shares, additional conditions are needed (e.g., one should rule out cases like identical linear preferences, which would yield a "thick" efficient frontier in the space of issues).

Remark 2: A3 is strictly weaker than concavity of the utility functions. It is compatible with a convex frontier of the utility possibility set. For example, if $u_1(\cdot) = x_1 x_2 \cdots x_l$ and $u_2(\cdot) = (1-x_1)(1-x_2)\cdots(1-x_l)$, the unique SPE payoff of $\Gamma_1(\delta)$ is $(\frac{1}{(1+\delta^{1/l})^l}, \frac{\delta}{(1+\delta^{1/l})^l})$. This is supported by player 1 offering on all issues the same split: $(\frac{1}{1+\delta^{1/l}}, \frac{\delta^{1/l}}{1+\delta^{1/l}})$. Note how this Pareto frontier is a convex function, the more convex the higher the number of issues.

Remark 3: Additive separability across issues has not been assumed either in Lemma 1 or in Proposition 1. This, besides concavity, is the most striking difference with respect to Inderst (2000). For an example, see Remark 2 again.

Remark 4: We now show that in the absence of A3 we will have multiple SPE agreements. It suffices to develop an example to see that the multiplicity phenomenon happens as soon as the function h has multiple zeros. To emphasize the connections with the single-issue analysis, we develop an example that builds on it. For a single-issue, consider the following pair of utility functions:

$$u_1(x) = \begin{cases} \frac{b_1}{a_1} x, & x \le a_1 \\ \frac{1-b_1}{1-a_1}(x-1)+1, & x > a_1, \end{cases}$$

where $0 < b_1 < a_1 < 1$, and

$$u_2(1-x) = \begin{cases} \frac{b_2}{a_2}(1-x), & x \le a_2 \\ \frac{1-b_2}{1-a_2}(-x)+1, & x > a_2, \end{cases}$$

where $0 < b_2 \le a_2 < 1$.

The equation for the Pareto frontier is:

- If $a_1 \ge 1 - a_2$,

$$u_2 = \begin{cases} -\frac{a_1(1-b_2)}{(1-a_2)b_1} u_1 + 1, & u_1 \in [0, \frac{(1-a_2)b_1}{a_1}] \\ -\frac{a_1 b_2}{a_2 b_1} u_1 + \frac{b_2}{a_2}, & u_1 \in [\frac{(1-a_2)b_1}{a_1}, b_1] \\ -\frac{(1-a_1)b_2}{a_2(1-b_1)} u_1 + \frac{(1-a_1)b_2}{a_2(1-b_1)}, & u_1 \in [b_1, 1]. \end{cases}$$

- If $a_1 \le 1 - a_2$,

$$u_2 = \begin{cases} -\frac{a_1(1-b_2)}{(1-a_2)b_1} u_1 + 1, & u_1 \in [0, b_1] \\ -\frac{(1-a_1)(1-b_2)}{(1-a_2)(1-b_1)} u_1 + \frac{(1-a_1)(1-b_2)}{(1-a_2)(1-b_1)} - \frac{1-b_2}{1-a_2} + 1, & u_1 \in [b_1, \frac{-a_2(1-b_1)}{1-a_1} + 1] \\ -\frac{(1-a_1)b_2}{a_2(1-b_1)} u_1 + \frac{(1-a_1)b_2}{a_2(1-b_1)}, & u_1 \in [\frac{-a_2(1-b_1)}{1-a_1} + 1, 1]. \end{cases}$$

Let f denote the function in the right-hand side of the above, and g denote the inverse function of f. The Pareto frontier is a piecewise linear curve with two kinks if $a_1 \neq 1 - a_2$, and with one kink if $a_1 = 1 - a_2$.

For simplicity, assume $a_1 = 1 - a_2$ and $a_2 = b_2$. Then,

$$g(u_2) = \begin{cases} -\frac{1-b_1}{1-a_1} u_2 + 1, & u_2 \in [0, 1 - a_1] \\ -\frac{b_1}{a_1} u_2 + \frac{b_1}{a_1}, & u_2 \in [1 - a_1, 1] \end{cases}$$

and

$$f(u_1) = \begin{cases} -\frac{a_1}{b_1} u_1 + 1, & u_1 \in [0, b_1] \\ -\frac{1-a_1}{1-b_1} u_1 + \frac{1-a_1}{1-b_1}, & u_1 \in [b_1, 1]. \end{cases}$$

The two curves $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ have three distinct intersections if $a_1 > \frac{1}{2}$ and $b_1 < \frac{1}{2}$, and if $\frac{\delta}{1+\delta} > \max(1 - a_1, b_1)$. They are

$$\begin{array}{l} \left(\frac{b_1\delta}{a_1(1+\delta)}, \frac{\delta}{1+\delta}\right) \\ \left(\frac{b_1(1-b_1)\delta-(1-a_1)b_1\delta^2}{a_1(1-b_1)-(1-a_1)b_1\delta^2}, \frac{a_1(1-a_1)\delta-(1-a_1)b_1\delta^2}{a_1(1-b_1)-(1-a_1)b_1\delta^2}\right) \\ \left(\frac{\delta}{1+\delta}, \frac{(1-a_1)\delta}{(1-b_1)(1+\delta)}\right). \end{array}$$

For problems involving more than one issue, we can get a similar result, if we replace x with $x_1 x_2 \cdots x_l$ for player 1's utility, and $(1-x)$ with $(1-x_1)(1-x_2)\cdots(1-x_l)$ for player 2's utility, or if we replace x with $x_1 + x_2 + \cdots + x_l$ for player 1's utility, and $(1-x)$ with $(1-x_1) + (1-x_2) + \cdots + (1-x_l)$ for player 2's utility. These are simple examples where the equation $h(u_1) = 0$ in our assumption A3 has three solutions when $S = L$. Given the multiple zeros of the function h, the construction of stationary SPE strategies around each of them is standard and left to the reader.

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