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Temporal Aggregation, Causality Distortions, and a Sign Rule

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Abstract:
Temporally aggregated data is a bane for Granger causality tests. The same set of variables may lead to contradictory causality inferences at different levels of temporal aggregation. Obtaining temporally disaggregated data series is impractical in many situations. Since cointegration is invariant to temporal aggregation and implies Granger causality this paper proposes a sign rule to establish the direction of causality. Temporal aggregation leads to a distortion of the sign of the adjustment coefficients of an error correction model. The sign rule works better with highly temporally aggregated data. The practitioners, therefore, may revert to using annual data for Granger causality testing instead of looking for quarterly, monthly or weekly data. The method is illustrated through three applications.

Temporal Aggregation, Causality Distortions, and a Sign Rule

1. Introduction

A number of theoretical studies have established that temporal aggregation leads to misleading inference on Granger-causality (see Wei, 1990 and Marcellino, 1999 and references therein). Gulasekaran and Abeysinghe (2002) and Gulasekaran (2003) have derived quantitative results analytically to assess the nature of the distortions created. Overall the following conclusions emerge. Within a stationary framework, depending on the parameter magnitudes, temporal aggregation may (i) create a spurious feedback loop from a unidirectional relation, (ii) erase a feedback loop and create a unidirectional relation and (iii) erase the Granger-causal link altogether. The distortions magnify when differencing is used after temporal aggregation to induce stationarity.

These findings of distortions are not much of a comfort in practice because most available data series are either temporally aggregated or systematically sampled depending on whether the variables are flows or stocks respectively. An important finding of the Gulasekaran (2003) study is that misleading inferences are more likely at low levels of temporal aggregation. Therefore, moving towards more disaggregated data would not be of much help either unless the observation frequency coincides with the causal lag. Looking for data with a frequency that coincides with the causal lag is a far-fetched goal. Moreover, for certain variables such as GDP, there is a limit to the increase in the frequency of observations beyond which meaningful time series aggregates do not exist. (See Hoover, 2001, Chapter 6 for an eloquent discussion on this.) On the other hand, temporal aggregation creates contemporaneous correlations even when such

correlations are absent in the non-aggregate process and as temporal aggregation increases contemporaneous correlations may be all that is left between the series. As a result one may not find Granger-causality at all. Making causal inferences from contemporaneous correlations is a challenging and much needed task. For better or worse this task still remains largely in the terrain of non-sample information or beliefs of the researcher.

Hoover (2001) has proposed a very promising methodology for causal inference based on an intervention analysis. Using the logic that interventions such as strikes, wars, and policy changes do not alter a true causal relationship, Hoover suggests examining conditional and marginal distributions to see whether they are affected by interventions. The applications he has reported, however, rely on error correction (ECM) formulations which are unfortunately subject to the distortions of temporal aggregation. Hoover's approach is closely related to testing for super exogeneity (see Hendry, 1995).

Swanson and Granger (1997) have used a graph-theoretic approach advocated by philosophers and computer scientists (see Pearl, 2000) to assign a causal ordering to contemporaneous links in a structural vector autoregression (SVAR) model. Demiralp and Hoover (2003) have examined the reliability of this approach and conclude that the graph-theoretic approach combined with non-sample information may lead to a more realistic causal ordering. This approach, the SVAR approach in general, faces a number of difficulties under temporal aggregation. First, the results in Gulasekaran and Abeysinghe (2002) and Gulasekaran (2003) indicate that though unidirectional relations create contemporaneous correlations with an unambiguous sign, an ambiguity may arise if the non-aggregate process is a feedback relation with a positive and negative feedback

loop. Depending on the parameter configuration of the non-aggregate process, the contemporaneous correlation of the aggregate process may take positive, negative or zero values from which it would be difficult to infer the actual feedback relation that exists (see also Ericcson et al., 2001).[1] Second, since causal ordering is performed on the basis of residuals of an unrestricted VAR there is a possibility that the lagged variables in the unrestricted VAR acting as proxies for the omitted contemporaneous relations thereby leading to distorted residual correlations. Therefore, the final causal ordering may have to be done by estimating the full SVAR model. Third, even if the causal ordering of contemporaneous links is done successfully, temporal aggregation may distort the dynamics of the VAR model in such a way that the impulse responses may become misleading (Granger, 1988; Granger and Swanson, 1992; Marcellino 1999).

Unlike dynamic relations, cointegrating relations remain invariant to temporal aggregation (Granger, 1990; Pierse and Snell, 1995; Granger and Siklos, 1995; Franses and Boswijk; 1996; Marcellino, 1999). They are also not affected by linear seasonal filters (see Hendry, 1995, Section 15.6 and the references therein). Cointegration also implies G-causality (Granger, 1986) though the direction is uncertain. Establishing the direction of causality from a cointegrating relation is an important research agenda and some apparatus under weak exogeneity already exists for this purpose (Engle et al., 1983, Hendry 1995, Johansen 1995). As the level of temporal aggregation increases, a stationary VAR(p) process may tend towards VAR(0) by absorbing all causal information into contemporaneous links. However, a cointegrated VAR(p) process

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[1] For example, since early 1980s (until the onset of the Asian financial crisis) the Monetary Authority of Singapore has used exchange rate management as a means of controlling inflation. Higher imported inflation prompts for an appreciating exchange rate policy that in turn lowers the inflation rate. Therefore, the two variables form a positive and negative feedback loop. However, the observed data hardly show this relationship.

cannot shrink below   VAR(1) because of the presence of unit roots.[2] As a result some adjustment coefficients of the error correction model have to remain non-zero regardless the level of temporal aggregation. Therefore, weak exogeneity under cointegration helps not only in contemporaneous conditioning but it also helps in Granger causality inference.

Since it is of critical importance to make causal inference from relations that are invariant to temporal aggregation and given the impracticality of searching for the non-aggregate forms of the data series we propose to base causal inference on highly aggregated data series such as the annual series that are cointegrated and best modeled as a VAR(1) process though higher order VAR models may also be entertained[3] (see footnote 6 and Section 5). The objective of our study is to investigate the feasibility of this proposal. It is worth noting that Pagan (1989) came up with a scathing criticism of Granger causality inference because of the inconceivably contradictory results found in the applied literature. The fragility of causal inference emanates mainly from two sources, (i) the information set used and (ii) temporal aggregation.[4] The former is a problem that all econometric models have to deal with and the latter requires devising suitable statistical techniques that produce robust inferences regardless the level of temporal aggregation.

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[2] If $l_1, l_2, ..., l_n$ are the roots of the non-aggregate autoregressive process then $l_1^m, l_2^m, ..., l_n^m$ are the roots of the aggregated process, where *m* is the order of aggregation (Marcellino, 1999).

[3] The problem with unrestricted higher order VAR models is that they tend to pick noise as signal leading to fragile inference.

[4] Other sources include the incorrect functional forms, in particular not accounting for the multiplicative interaction between the variables that economic theory some times suggests.

2. Analytical Tools

Consider the ECM formulation (Johansen, 1995):

$$\Delta y_t = \boldsymbol{a}\boldsymbol{b}'y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + ... + \Gamma_{p-1} \Delta y_{t-(p-1)} + \boldsymbol{f}D_t + e_t \tag{1}$$

where y_t ($t = 1, 2, ..., T$) is an $(n \times 1)$ vector of I(1) variables, D_t contains deterministic terms such as the constant and time trend and $u_t = \boldsymbol{b}'y_t$ is an $(r \times 1)$ vector of cointegrating relations. Note that $Var(e_t) = \Sigma$ is not a diagonal matrix in general. If we can impose a meaningful causal ordering on the contemporaneous relations such that $B_0 e_t = \boldsymbol{e}_t$ and $Var(\boldsymbol{e}_t) = \boldsymbol{W}$ is diagonal, where $\boldsymbol{e}_t$ are the fundamental innovations, then we can formulate a structural ECM by pre-multiplying (1) by B_0. This will alter $\boldsymbol{a}$, but not $\boldsymbol{b}$.[5] The problem is that temporal aggregation alters $\boldsymbol{a}$ and $\boldsymbol{G}$ s and if this happens in a distortionary way meaningful inference may not result even if the restrictions on B_0 turn out to be correct. Since our focus in this paper is on $\boldsymbol{a}$ we assume that the data series are sufficiently temporally aggregated such that $\boldsymbol{G}$ s in the aggregated process are practically zero and we work with the model:[6]

$$\boldsymbol{D}y_t = \boldsymbol{a}\boldsymbol{b}'y_{t-1} + e_t. \tag{2}$$

For the convenience of subsequent derivations we have dropped the deterministic term from (2).

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[5] Usually structural VAR modelers are estimated by imposing diagonality on $\boldsymbol{W}$. Ideally $\boldsymbol{W}$ should be diagonal empirically because $\boldsymbol{W}$ may truly be non-diagonal due to omitted variables and misspecified contemporaneous links.

[6] Johansen (1995) argues, based on experience, that seasonally adjusted quarterly data are often well modeled as VAR(2). If longer lags are required he suggests to look for omitted variables and increase n instead of p. For example, consider that the n variables are well modeled as a VAR(1) process. If we throw away half of the variables then the remaining half requires a VARMA(2,1) model instead of a VAR(1). If model diagnostics do not warrant a VAR(1) because of autoregressive effects we could still proceed with a higher order VAR provided that the $\boldsymbol{G}$ s are empirically diagonal.

As stated earlier, cointegration (also unit roots) is invariant to temporal aggregation. This is in general true for static relations. Note also that if y consists of both flow and stock variables we have to apply the same transformation, temporal aggregation or averaging, to all the variables to keep $\boldsymbol{b}$ invariant. Therefore, it is advisable to use temporal averaging of all the variables when both flow and stock variables enter the same model. Since $\boldsymbol{b}$ is invariant to temporal aggregation we proceed with the assumption that $\boldsymbol{b}$ is known and concentrate on $\boldsymbol{a}$.

If the n variables are partitioned into two groups such that y_{1t} is $(n_1 \times 1)$ and y_{2t} is $(n_2 \times 1)$, $n_1 + n_2 = n$ with $\boldsymbol{a}$ and $\boldsymbol{b}$ split conformably then the ECM can be written as

$$\begin{pmatrix} \boldsymbol{D}y_{1t} \\ \boldsymbol{D}y_{2t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{b}_1' & \boldsymbol{b}_2' \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}. \tag{3}$$

Johansen (1995, Ch. 8) shows that if $\boldsymbol{a}_2 = 0$ then y_{2t} is weakly exogenous for $\boldsymbol{b}$ and $\boldsymbol{a}_1$. Writing (3) in levels it can be seen that $\boldsymbol{a}_2 = 0$ (assuming $\boldsymbol{b} \neq 0$) also implies the presence of Granger causality from y_{2t} to y_{1t} (Mosconi and Giannini, 1992) though we cannot exclude the possibility of lagged feedback effects since they may have disappeared into contemporaneous links. Following Hendry and Mizon (1998) Granger causality associated with the adjustment coefficients may be referred to as causality in levels and that associated with short-run coefficients as causality in differences. Although the former implies the latter, the converse does not necessarily hold. Results in this study as well as those in Gulasekaran (2003) indicate that causality in levels is more robust to temporal aggregation compared to causality in differences. Since causality in levels bears on weak exogeneity that forms the basis for super exogeneity which is essential for policy

evaluations the causality in levels plays an important role in practice. Since policy variables are, in general, subject to feedback effects, causality in differences does not play such a useful role in policy evaluations.

2.1 The sign of the adjustment coefficient

During our analysis we realized that the sign of the a coefficients plays a useful role in signaling how temporal aggregation has affected them. Johansen (1995) draws attention to "correct" and "wrong" signs of the adjustment coefficients in a number of places. In this section we define the "correct sign" formally for $r = 1$ and discuss the consequences of "wrong sign".

When $r = 1$, the ith equation of (2) can be written as

$$y_{it} = y_{it-1} + a_i u_{t-1} + e_{it},$$ (4)

where $u_t = \sum_{i=1}^{n} b_i y_{it}$.

The long run equilibrium implies that $u_{t-1} = 0$ which gives:

$$y_{it-1} = \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1}).$$

If the system is in disequilibrium at date $t-1$ then either $u_{t-1} > 0$ or $u_{t-1} < 0$.

Case 1: $\mathbf{b}_i > 0$

If $u_{t-1} > 0$, then $y_{it-1} > \dfrac{-1}{\mathbf{b}_i}(\mathbf{b}_1 y_{1t-1} + \mathbf{b}_2 y_{2t-1} + ... + \mathbf{b}_{i-1} y_{i-1t-1} + \mathbf{b}_{i+1} y_{i+1t-1} + ... + \mathbf{b}_n y_{nt-1})$ and

we expect $\mathbf{a}_i u_{t-1} < 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} > 0$, $\mathbf{a}_i$ has to

be negative ($\mathbf{a}_i < 0$).

If $u_{t-1} < 0$, then $y_{it-1} < \dfrac{-1}{\mathbf{b}_i}(\mathbf{b}_1 y_{1t-1} + \mathbf{b}_2 y_{2t-1} + ... + \mathbf{b}_{i-1} y_{i-1t-1} + \mathbf{b}_{i+1} y_{i+1t-1} + ... + \mathbf{b}_n y_{nt-1})$

and we expect $\mathbf{a}_i u_{t-1} > 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} < 0$, $\mathbf{a}_i$

has to be negative ($\mathbf{a}_i < 0$).

Thus, if $\mathbf{b}_i > 0$, then $\mathbf{a}_i < 0$ regardless of the sign of the disequilibrium term u_{t-1}.

Case 2: $\mathbf{b}_i < 0$

If $u_{t-1} > 0$, then $y_{it-1} < \dfrac{-1}{\mathbf{b}_i}(\mathbf{b}_1 y_{1t-1} + \mathbf{b}_2 y_{2t-1} + ... + \mathbf{b}_{i-1} y_{i-1t-1} + \mathbf{b}_{i+1} y_{i+1t-1} + ... + \mathbf{b}_n y_{nt-1})$

and we expect $\mathbf{a}_i u_{t-1} > 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} > 0$, $\mathbf{a}_i$

has to be positive ($\mathbf{a}_i > 0$).

If $u_{t-1} < 0$, then $y_{it-1} > \dfrac{-1}{\mathbf{b}_i}(\mathbf{b}_1 y_{1t-1} + \mathbf{b}_2 y_{2t-1} + ... + \mathbf{b}_{i-1} y_{i-1t-1} + \mathbf{b}_{i+1} y_{i+1t-1} + ... + \mathbf{b}_n y_{nt-1})$

and we expect $\mathbf{a}_i u_{t-1} < 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} < 0$, $\mathbf{a}_i$ has

to be positive ($\mathbf{a}_i > 0$).

Thus, if $\mathbf{b}_i < 0$, then $\mathbf{a}_i > 0$ regardless of the sign of the disequilibrium term, u_{t-1}.

We can, therefore, see that the long-run equilibrium holds if the sign of a_i is the opposite of b_i. We define this as the "correct sign" of the adjustment coefficient when $r = 1$. Since the b vector is invariant to temporal aggregation we can determine the expected sign of a_i from that of b_i under any level of temporal aggregation. This will be termed as the "sign rule".

The problem, however, is that the long-run equilibrium may hold even if the sign is wrong. To see this, from the n equations in (4) obtain $u_t = r u_{t-1} + e_t$, where $r = 1 + a'b$, $a = (a_1, ..., a_n)'$, $b = (b_1, ..., b_n)'$ and e_t is white noise. Co-integration requires $|r| < 1$ which implies $-2 < a'b < 0$. Note that r measures the degree of co-integration: $|r| \to 0$ implies a higher degree of co-integration and $|r| \to 1$ implies a lower degree. Now consider that the variables in the model are arranged such that the first n_1 b coefficients are positive and the second n_2 b coefficients are negative. Let b_1 and b_2 represent these vectors and let the corresponding a vectors be a_1 and a_2 respectively. Assume that a_1 is correctly signed with a negative sign and a_2 is wrongly signed with a negative sign too. Given the inequality $-2 < a_1'b_1 + a_2'b_2 < 0$, if $|a_1'b_1| > |a_2'b_2|$ we get $|r| < 1$ even with the wrong sign. In other words, if the adjustment towards equilibrium is dominated by the adjustment coefficients with the correct sign co-integration continues to hold. However, the wrong sign lowers the degree of co-integration (increases the absolute value of r). How frequent the co-integration with the wrong sign is an empirical question. As we shall see later the verification of this would not be easy with temporally aggregated data.

2.2 Bivariate Case

The sign rule seems to apply even when $r > 1$ provided that the r cointegrating vectors do not appear jointly in an equation. However, when they enter an equation jointly the sign rule does not seem apply. Since the results are not very clear at this stage, in the following analysis we confine to the case $r = 1$ and use a bivariate system to obtain some analytical results under temporal aggregation. We assume that the non-aggregate process is in the form of (3) with the resulting two equations written as[7]

$$\Delta y_{1t} = \boldsymbol{a}_1 u_{t-1} + e_{1t}, \tag{5}$$

$$\Delta y_{2t} = \boldsymbol{a}_2 u_{t-1} + e_{2t}, \tag{5'}$$

where $u_t = \boldsymbol{b}_1 y_{1t} + \boldsymbol{b}_2 y_{2t}$. We assume that

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim iidN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{s}_1^2 & 0 \\ 0 & \boldsymbol{s}_2^2 \end{pmatrix} \right).$$

The zero contemporaneous covariance between the two error terms underlies the assumption that the observation frequency coincides with the causal lag. As before $u_t = \boldsymbol{r} u_{t-1} + \boldsymbol{e}_t$, where $\boldsymbol{r} = 1 + \boldsymbol{a}_1 \boldsymbol{b}_1 + \boldsymbol{a}_2 \boldsymbol{b}_2$ and $\boldsymbol{e}_t = \boldsymbol{b}_1 e_{1t} + \boldsymbol{b}_2 e_{2t}$ with zero mean and variance $\boldsymbol{s}_e^2 = \boldsymbol{b}_1^2 \boldsymbol{s}_1^2 + \boldsymbol{b}_2^2 \boldsymbol{s}_2^2$.

Let $w_{1t} = \Delta y_{1t}$ and $w_{2t} = \Delta y_{2t}$. For $i=1,2$, the variances and covariances of the non-aggregate process in (5) and (5′) can be written as

$$\boldsymbol{g}_u(k) = E(u_t u_{t-k}) = E(u_t u_{t+k}) = \boldsymbol{r}^k \boldsymbol{s}_e^2 / (1 - \boldsymbol{r}^2) \quad \forall k \tag{6}$$

$$E(e_{it} u_{t+k}) = \begin{cases} 0 & \text{if} \quad k < 0 \\ \boldsymbol{r}^k \boldsymbol{b}_i \boldsymbol{s}_i^2 & \text{if} \quad k \geq 0 \end{cases} \tag{7}$$

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[7] Mamingi (1996) used this in his Monte Carlo study.

$$g_{ii}^{w}(k) = E(w_{it} w_{it-k}) = \begin{cases} a_i^2 g_u(k) + s_i^2 & if \quad k = 0 \\ a_i^2 g_u(k) + a_i r^{k-1} b_i s_i^2 & if \quad k > 0 \end{cases} \tag{8}$$

$$g_{ii}^{w}(-k) = g_{ii}^{w}(k)$$

$$g_{iu}(k) = E(w_{it} u_{t-k}) = \begin{cases} a_i \dfrac{r^{k-1}}{1-r} s_e^2 & \forall k > 0 \\ a_i \dfrac{r^{-k+1}}{1-r} s_e^2 + r^{-k} b_i s_i^2 & \forall k \le 0. \end{cases} \tag{9}$$

Let Y_{1t} and Y_{2t} $(t = 1,2,..,N; T = mN)$ be the m-period non-overlapping aggregates of

y_{1t} and y_{2t} respectively and let $W_{1t} = DY_{1t}$ and $W_{2t} = DY_{2t}$. We now consider estimating

the following aggregated process:

$$W_{1t} = a_1^* U_{t-1} + E_{1t} \tag{10}$$

$$W_{2t} = a_2^* U_{t-1} + E_{2t} \tag{10'}$$

where $U_t = \displaystyle\sum_{j=m(t-1)}^{mt-1} u_j$ and E_{it} represent non-overlapping sums of the error process.[8]

The OLS estimates $\hat{a}_i^*$, $p\lim \hat{a}_i^*$ and the t statistics are given by:

$$\hat{a}_i^* = \frac{\sum W_{it} U_{t-1}}{\sum U_{t-1}^2}, \quad p\lim \hat{a}_i^* = \frac{g_{iU}(1)}{g_U(0)} \tag{11}$$

$$\hat{t}_i^* = \frac{\hat{a}_i^*}{[\text{var}(\hat{a}_i^*)]^{1/2}} \tag{12}$$

where

$$\text{var}(\hat{a}_i^*) = \frac{s_{E_i}^2}{\sum U_t^2} = \frac{s_{E_i}^2}{N g_U(0)}, \quad s_{E_i}^2 = \text{var}(E_i) = g_{ii}^W(0) + \hat{a}_i^{*2} g_U(0) - 2\hat{a}_i^* g_{iU}(1). \tag{13}$$

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[8] In addition to temporal aggregation we also examined systematic sampling. In general, systematic sampling does not lead to serious distortions in the adjustment coefficients. In the interest of space we present the results for temporal aggregation only.

Using Proposition A.1 in Appendix that establishes the relationship between covariances of the aggregated and the non-aggregate processes we get the following relations, again for $i=1,2$:

$$\boldsymbol{g}_U(k) = (1 + L + ... + L^{m-1})^2 \boldsymbol{g}_u(mk + (m-1)) \quad \forall k. \tag{14}$$

$$\boldsymbol{g}_{iU}(k) = (1 + L + ... + L^{m-1})^3 \boldsymbol{g}_{iu}(mk + (m-1)) \quad \forall k \tag{15}$$

$$\boldsymbol{g}_{ii}^W(k) = (1 + L + ... + L^{m-1})^4 \boldsymbol{g}_{ii}^w(mk + 2(m-1)). \tag{16}$$

These expressions provide the link between the parameter estimates and the t-statistics of the aggregated process and the parameters of the non-aggregate process in order to derive a quantitative evaluation of the impact of temporal aggregation.

3. Distortions

There are three cases of interest with regard to Granger causality in the non-aggregate process: (i) no causality, (ii) unidirectional causality and (iii) mutual causality or feedback. The first case clears through without a problem. If the two series are not related in the non-aggregate process then $\boldsymbol{a}_i = 0$, $i = 1,2$, and from (9), $\boldsymbol{g}_{iu}(k) = 0$, $\forall k > 0$. Further, from (15) $\boldsymbol{g}_{iU}(k) = 0$, $\forall k > 0$ and from (11) $\hat{\boldsymbol{a}}_i^* = 0$. Thus, if there is no Granger causality between the series in the non-aggregate process then there will be no Granger causality between them in the aggregated process. In fact, this is valid for the short run dynamics as well (Gulasekaran, 2003).

3.1 Unidirectional Causality in the non-aggregate process

To evaluate this case we set $a_2 = 0$ so that Granger causality runs from y_{2t} to y_{1t} and use the normalized co-integrating vector $(1, b_2)$. We consider $m=3$ and $m=12$ to represent aggregating monthly data to quarterly and annual figures.[9] To assess the impact of the degree of co-integration we consider values of r in the range -0.95 to 0.95. This is the same as setting $a_1 (= r - 1)$ within the range –1.95 to –0.05. We also vary the values of b_2 within the range -20 to 20 to see whether the magnitude of b_2 plays any role in creating distortions. For the computation of the t statistics we consider three combinations of m and N given in Tables 2 and 4.[10]

Co-integration implies that at least one of the adjustment coefficients has to be non-zero. As expected, $\hat{a}_1^*$ remains negative and highly statistically significant regardless the level of aggregation and the sample size (Tables 1 and 2). The magnitude of b_2 seems to matter when a_1 is very small.

The interesting case is $\hat{a}_2^*$ which is expected to be statistically insignificant. The results for $p \lim \hat{a}_2^*$ are given in Table 3 and the t-statistics in Table 4. Table 3 shows that the limiting values of $\hat{a}_2^*$ are not zero, though small in magnitude for certain cases especially when m=3. The magnitude of both a_1 and b_2 play a role in the creation of a non-zero $\hat{a}_2^*$. Nevertheless Table 4 shows that the impact of the magnitude of b_2

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[9] Since monthly data are the aggregates of daily or hourly data we have to set m to very large values which render our analytical expressions unmanageable.

[10] These sample sizes are chosen to be compatible with the Monte Carlo experiments in Lahiri and Mamingi (1995), Choi and Chung (1995) and Mamingi (1996). Since we compute t statistics using the limiting values of the parameter estimates we conducted a limited number of Monte Carlo experiments (with 10,000 replications) to assess the validity of our theoretical results for the sample sizes considered. The Monte Carlo results are the same as our theoretical results.

disappears from the t statistics. However, when a_1 is close to -1 (high degree of co-integration) the t statistics are highly significant regardless the level of aggregation and the sample size. An increase in m or N renders more statistically significant t statistics. These results concur with the Monte Carlo results in Mamingi (1996).

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Tables 1-4

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A comparison with the results in Gulasekaran and Abeysinghe (2002) for a non-cointegrated VAR indicates that the distortionary effects of temporal aggregation are much stronger on the adjustment coefficients than on the stationary dynamics. This means that if weak exogeneity tests are used to impose a causal ordering on the contemporaneous relations they are more likely to go wrong with temporally aggregated data. The most important observation, however, is that the sign of $\hat{a}_2^*$ is the same as that of b_2 though we expect the opposite. This is a clear indication that a distortion may have taken place.

Another point to note is that temporal aggregation appears to increases the degree of cointegration, $\hat{r}^* = 1 + \hat{a}_1^* b_1 + \hat{a}_2^* b_2$. For example, when a_1=-0.25, r=0.75 but $\hat{r}^*$ = 0.56 for m=3 and $\hat{r}^*$=0.19 for m=12 regardless the magnitude of b_2 (b_1=1).[11] As it was noted earlier the wrong sign lowers the degree of cointegration. In this case, however, temporal aggregation seems to offset this effect and produces a high degree of cointegration even with the wrong sign.

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[11] Note that we need more decimal places in Tables 1 and 3 to obtain precise values of $\hat{r}^*$.

3.2 Mutual Causality in the non-aggregate process

In this case both a_1 and a_2 take non-zero values, therefore, the focus of our computations is to see whether temporal aggregation renders one of them zero in the aggregated process leading to misleading inference on causal direction. It is worth noting that even if we correctly find that they are non-zero, this is not going to help us in assigning a causal order to the contemporaneous link. We will have to look for a third variable to solve the identification problem.

Since $a_2 = (r - 1 - a_1)/b_2$ the computational setting in this case is a lot more involved than the previous one. We computed a large number of tables using various combinations of the parameter values. To conserve space we present only one table and summarize the results. In all computations we fixed a_1 to the range -0.95 to -0.05. Overall $\hat{a}_1^*$ emerges with the correct sign and remains statistically significant. Insignificant values occur only when $|a_1|$ is small. Table 5 presents the t statistics for $\hat{a}_2^*$ under one set of parameter configuration. In this table a_2 is positive and varies from cell to cell but takes on smaller values towards the top-left corner. What the table shows is that temporal aggregation may render small a_2s either with statistically insignificant $\hat{a}_2^*$s or with statistically significant $\hat{a}_2^*$s with a wrong (negative) sign. The latter effect magnifies as temporal aggregation increases. In general, distorted inference do not occur when both a_1 and a_2 are large in magnitude.

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Table 5

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4. How to Test for Granger causality with temporally aggregated data?

Although temporal aggregation tends to distort the adjustment coefficients, the sign rule established in Section 2 and the computations in Section 3 show that we may still be able to reach the correct conclusion about the causal direction based on the sign of the adjustment coefficients. To repeat the sign rule, if $b_i > 0$, $a_i < 0$ and if $b_i < 0$, $a_i > 0$.

The results on unidirectional causality are clear-cut. The non-zero adjustment coefficient remains highly significant regardless the level of temporal aggregation and carries the correct sign. An adjustment coefficient with the wrong sign clearly indicates a causal distortion of the underlying zero coefficient. However, the presence of mutual causation makes the inference harder because of the possibility that temporal aggregation may erase the feedback loop and creates a unidirectional relation. Our results nevertheless show that a strong feedback relation does not get distorted by temporal aggregation. Furthermore, when a_1 is reasonably large $\hat{a}_1^*$ always carries the correct sign and remains statistically significant. Therefore, a proper normalization (a selection of the dependent variable) with the help of non-sample information should make the inference easier. Wrong sign on the other coefficients is an indicator of causal distortion. Unfortunately we face an ambiguity here. Our results show that $\hat{a}_2^*$ may take the wrong sign either because a_2 is very small or zero or because a_2 genuinely carries a wrong sign.[12] Although the latter case may only be a theoretical possibility, in practice, with temporally aggregated data we will not be able to differentiate between these

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[12] Note that, as shown in Section 3, when a_2 takes the wrong sign its magnitude has to be smaller than that of a_1 to preserve co-integration.

possibilities. We have to rely on non-sample information to solve this identification problem. We can, therefore, formulate the following rule as a guide.

First, determine the expected sign of the adjustment coefficients from the estimated co-integrating vector. If the estimated adjustment coefficient appears with the correct sign and is statistically significant then it reflects the underlying causal direction in the non-aggregate form. If the coefficient appears with the wrong sign then a causal distortion may have occurred and if such a conclusion is supported by non-sample information then we may treat it as resulting from a zero or near zero coefficient in the non-aggregate form.

5. Some Monte Carlo Results

An upshot of the above analysis is that afterall we may be better off with highly temporally aggregated data for causality testing. The trend has been to move towards more and more disaggregated data but with no promising outcome on Granger causality inference. Annual data, on the other hand, are free from the effects of seasonal adjustment and may well fit into a VAR(1) framework. Since co-integration has to render at least one non-zero adjustment coefficient with the correct sign, the sign distortions on the other coefficients, perhaps combined with non-sample information, would guide us in establishing the causal direction.

To shed further light on the VAR order and the sign distortion we conducted a Monte Carlo experiment with a VAR(2) process which in ECM format is:

$$\begin{pmatrix} \boldsymbol{D}y_{1t} \\ \boldsymbol{D}y_{2t} \end{pmatrix} = \begin{pmatrix} -0.25 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} 0.5 & 0.5 \\ \boldsymbol{f}_{21} & 0.25 \end{pmatrix} \begin{pmatrix} \boldsymbol{D}y_{1t-1} \\ \boldsymbol{D}y_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}. \tag{17}$$

In this process $a_2 = 0$, therefore y_2 is weakly exogenous for a_1 and b vector. However, it is a feedback system if $f_{21} \neq 0$. In the experiment we set f_{21} to two values (0, 0.25). Summary results based on N(0, I) errors and 2000 replications are given in Tables 6 and 7. To see the large sample effect we set the effective sample size (N) to 480 at each level of aggregation.

As for the VAR order selection, SBC tends to choose VAR(1) more often as m increases. However, AIC tends to be profligate. This reflects AIC's tendency to pick longer lags in large samples. The promising observation, however, is that the sign distortion on $\hat{a}_2^*$ remains unchanged regardless the VAR order and whether f_{21} is zero or not. This indicates that we can apply the sign rule even with higher order VAR models.

The tables also show the creation of contemporaneous correlations between the error processes that result from the shrinkage of the VAR order towards unity. It should be noted that a co-integrated VAR(1) process in the non-aggregate form does not create contemporaneous correlations with temporal aggregation. The contemporaneous correlation is such a case is a clear indication of omitted variables. In other words, contemporaneous correlation in an aggregated VAR process may be due to both temporal aggregation and omitted variables.

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Tables 6 and 7

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6. Applications

6.1 Exchange rate, direct vs. cross

The relationship between the three major exchange rates, US$, Deutsch Mark (DM) and Japanese Yen, provides a good illustration of how the sign rule works under unidirectional causality. Theoretically the direct Yen/DM rate should be the same as the cross rate derived from US$/DM and US$/Yen rates. Any deviations will open up arbitrage opportunities for profiteering. However, some deviations may still be observed when transaction costs are higher than the potential profits. Therefore, log(Yen/DM) – log(US$/DM) + log(US$/Yen) forms a co-integrating relation with the co-integrating vector (1, -1, 1). Figure 1 shows the deviations of the daily direct rate from the cross rate. As can be expected these deviations are very small and center around zero. Somewhat surprisingly, though, they show some heteroscedastic behavior. Both AIC and SBC pick a VAR(1) for the three rates (Yen/DM, US$/DM, US$/Yen) all in logarithms. The residual correlation matrix is not diagonal that reflects the systematic sampling of the daily rates. Although the residuals are free from serial correlation, both normality and heteroscadasticity tests fail. We ignore this and proceed to estimate the adjustment coefficients by imposing the above co-integrating vector.[13] The results (based on Johansen ML method) are reported in Table 8. The results under both daily rates and systematically sampled weekly rates show that only a_1 is non-zero. This is also what we expect apriori. However, under temporally averaged weekly rates both a_1 and a_3 turn out to be non-zero. Nevertheless, the wrong sign of a_3 provides the warning sign. Combined with non-sample information that only a_1 could be non-zero we could safely

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[13] The estimated cointegrating vector virtually coincide with (1, -1, 1) vector under both systematic sampling and temporal aggregation.

conclude that non-zero a_3 is a result of temporal averaging. Constraining both a_2 and a_3 to zero also brings the estimate of a_1 closer to unity.

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Figure 1 and Table 8

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6.2 Stock Market and Car Quota Premium in Singapore

This is an interesting example because one variable is available in non-aggregate form. To curb the car population, the Singapore government implemented a car quota system in August 1990. To buy a new car the buyers first have to buy a piece of paper called the certificate of entitlement. The price of this paper, known as the quota premium (QP), is decided through a monthly bidding process. The monthly data of QP are not contaminated by any form of aggregation or systematic sampling.

A key determinant of QP of luxury cars is the performance of the stock market, captured by the stock price index compiled by the Stock Exchange of Singapore (Lai, 2001). Monthly data over 1990M8-1999M4 show that these two variables (in logarithms) are cointegrated and their relationship is well represented by a VAR(1) process with causality running from stock price to QP. Stock prices (in log) follow a random walk. Moreover, the two error processes are also uncorrelated ($s_{12} = 0$). We write the ECM formulation as:

$$\begin{pmatrix} Dy_t \\ Dx_t \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} 1 & b_2 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}, \tag{18}$$

where $y = ln(QP)$ and $x = ln(Stock\ price\ index)$, $\boldsymbol{a}_2 = 0$. Because of the short data span we temporally averaged data up to six months. The results for m=1 (no aggregation) through m=6 are given in Table 9. The results show that $\hat{\boldsymbol{b}}_2$ remains roughly the same as m increases. Being a cointegrated VAR(1) with $\boldsymbol{s}_{12} = 0$ temporal aggregation does not create contemporaneous correlation between the residual processes (r_{12} remains close to zero). However, the magnitude of $\hat{\boldsymbol{a}}_1^*$ increases steadily and remains highly significant. The magnitude of $\hat{\boldsymbol{a}}_2^*$ also tends to increase though not steadily and becomes statistically significant at the 10% level when m=4 and m=6. If one had only the temporally averaged data (say biannual) the wrong (negative) sign of $\hat{\boldsymbol{a}}_2^*$ provides the warning signal. Combined with the information that (log) stock prices follows a random walk one could safely conclude in this case that causality is unidirectional from stock prices to QP.

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Table 9

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6.3 Tax Revenue and Government Expenditure in the US

Barro's (1979) tax smoothing hypothesis offers an interesting contrast to Granger causality testing. Under the assumption that spending causes taxes Barro's model implies that the income tax rate follows a random walk. If the tax rate truly follows a random walk, the standard Granger-causality test will fail to establish the causal direction embodied in Barro's model. Since taxes following a random walk could be consistent with some other hypotheses, many researchers have tested the tax smoothing hypothesis by testing the other implications of the model, see for example, Sahasakul (1986), Huang

and Lin (1993) and Ghosh (1995). Hoover (2001), however, applied his intervention approach to test for the causal direction between taxes and spending and observed mixed results over different time periods. Hoover used quarterly data from 1947 to 1989 in his analysis. In this section we use annual data (1946-2002) and examine how cointegration and the sign rule shed light on the causal direction between taxes and spending.

The data series used in this section are the following.[14] T = real federal government receipts (nominal series deflated by the GNP deflator, P), G = real federal government expenditure net of interest payments (nominal series deflated by P), Y = real GNP, $y = \ln Y$, Δy =GNP growth rate (%), $\pi = (\Delta \ln P)100$ = inflation rate, $\tau = (T/Y)100$ = income tax rate and g = (G/Y)100 = spending rate. Data plot and ADF tests support the assumption that τ, g, and π are I(1) processes.

The literature usually focuses on the budget surplus as a ratio of GNP, τ-g. Figure 2 plots this data series. Although the tax smoothing hypothesis predicts τ-g to be a stationary series (see Huang and Lin, 1993; Ghosh, 1995) the plot in Figure 2 casts doubts on the stationarity of the series. An ADF regression with two lags of $\Delta(\tau$-g$)_t$ produces a t statistic of -2.62 which is insignificant at the 5% critical value of -2.92. In fact the τ-g series suggests some level shifts: on average a budget surplus in the period 1946-1970, a deficit in the period 1971-1993 and a large surplus in the period 1994-2002.

Although the budget surplus (τ-g) may not necessarily be stationary, τ and g form a strong cointegrating relationship with a different cointegrating vector. An OLS regression of τ on g produces highly stable recursive parameter estimates with some small departures occurring after 1994. Based on average recursive OLS estimates we

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[14] The data series were taken from the same source that Hoover (2001) used, National Income and Product Accounts as reported in CITIBASE.

obtain the cointegrating relation $z_t = \tau_t - 0.25g_t - 13$. This series is plotted in Figure 3. An ADF regression based on one lag of Δz_t produces a t = -4.155 which is significant at the 1% critical value of -3.552.

We use the following specification to examine the adjustment coefficients α_1 and α_2, α_1 is expected to be negative and α_2 positive. Following Hoover (2001) we use Δy_t and $\Delta \pi_t$ to remove non-policy effects from $\Delta \tau$ and Δg.[15]

$$\Delta \tau_t = \delta_0 + \delta_1 \Delta \tau_{t-1} + \delta_2 \Delta g_{t-1} + \delta_3 \Delta y_t + \delta_4 \Delta \pi_t + \alpha_1 z_{t-1} + \varepsilon_{1t} \tag{19a}$$

$$\Delta g_t = \lambda_0 + \lambda_1 \Delta \tau_{t-1} + \lambda_2 \Delta g_{t-1} + \lambda_3 \Delta y_t + \alpha_2 z_{t-1} + \varepsilon_{2t} \tag{19b}$$

FIML estimation of (19a) and (19b) produces results very similar to OLS estimates because the two error processes are empirically uncorrelated. We, therefore, proceed with OLS estimation. Figures 4 and 5 plot the recursive estimates from the two regressions. Figure 4 shows that the parameter estimates of the tax equation become unstable after 1994, the period of high budget surplus.[16] Nevertheless, the adjustment coefficient estimate $\hat{\alpha}_1$ has the correct sign and is statistically significant. The parameter estimates of the spending equation shown in Figure 5 are more stable even during the high-budget-surplus period. But only the GNP growth rate and the constant term are statistically significant. After dropping the insignificant variables we obtain the following estimates over the period 1946-1994.[17]

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[15] Hoover (2001) uses taxes and spending as a ratio of potential GNP to obtain the tax rate and the spending rate. He then regresses the tax rate on GNP-gap and inflation rate and the spending rate on the GNP-gap and uses the residuals from these regressions to study the causal direction. We also tried this approach but the measurement errors in potential GNP seem to cause parameter instabilities.

[16] The analysis after 1994 seems to require additional variables in the model.

[17] The estimated equations pass the diagnostic tests available in PCGive except that the spending equation shows a mild heteroscadasticity. These results are not reported for brevity.

$$\Delta\tau_t = -0.06 + 0.42\Delta\tau_{t-1} + 0.16\Delta g_{t-1} + 0.09\Delta y_t + 0.12\Delta\pi_t - 0.65z_{t-1}$$

$$\quad\quad (-0.37)\ (3.51) \quad\quad (2.28) \quad\quad (2.44) \quad\quad (2.77) \quad (-4.34) \quad\quad\quad (20a)$$

$$R^2 = 0.71,\ \hat{\sigma}_1 = 0.50$$

$$\Delta g_t = 0.52 - 0.14\Delta y_t + 0.19z_{t-1}$$

$$\quad\quad (2.37)\ (-2.97) \quad\quad (1.22) \quad\quad\quad\quad\quad\quad\quad\quad\quad (20b)$$

$$R^2 = 0.32,\ \hat{\sigma}_2 = 0.68$$

The adjustment coefficient in (20a) has the correct sign and highly significant. The adjustment coefficient in (20b) is statistically insignificant. These results show that causality (in levels) runs from spending to taxes that concurs with Barro's assumption. We have to note, however, that the recursive estimates of the adjustment coefficient in (20b), though statistically insignificant, are highly stable and bear the correct sign. Our previous results on the sign distortion indicate that a pure unidirectional relation produces an $\hat{\alpha}_2 < 0$ after temporal aggregation. Therefore, the correct sign of $\hat{\alpha}_2$ in (20b) seems to have resulted from a mild feedback system in the non-aggregate process. It should also be noted that (20a) clearly rejects a major implication of the tax smoothing hypothesis that the tax rate (adjusted for the effect of Δy_t and $\Delta\pi_t$) is a random walk.

7. Concluding Remarks

Invariance of cointegrating relationships to temporal aggregation offers a promising path for Granger causality testing. In this paper we propose using the error correction formulation to infer the direction of causality between cointegrated variables. Temporal aggregation distorts both the short-run coefficients and the adjustment coefficients in an error correction model. Fortunately, unlike the short-run coefficients, the distortions on the adjustment coefficients occur with a predictable sign-distortion. Based on these

findings we propose a sign-rule for making causal inferences from temporally aggregated data.

Causal inference based on cointegration should be referred to as causality in levels as opposed to causality in differences found in the short-run coefficients (Hendry and Mizon, 1998). As we discussed in the text, causality found in the short-run coefficients, regardless the distortions due to temporal aggregation, is less useful for policy analyses because policy variables in general show feedback effects. Causality in levels, however, plays an important role in policy evaluations because of its connection to super-exogeniety.

Appendix: The relationship between covariances of aggregate and non-aggregate processes

Temporal aggregation involves the construction of non-overlapping sums that can easily be obtained by defining the overlapping sum $(1 + L + ... + L^{m-1})y_t$ and then systematically sampling this variable at every mth interval to obtain the aggregated variable $Y_t = (1 + L + ... + L^{m-1})y_{mt}$, ($t = 1,2,...,N; T=mN$). Let $w_t = (1-L)^d y_t$ and $W_t = (1-L)^d Y_t$. The following result extends the univariate case considered by Stram and Wei (1986). (See Gulasekaran (2003) for further details.)

Proposition A.1

The covariance between the temporally aggregated series W_{it} and W_{jt-k} can be expressed in terms of the covariance between the non-aggregate series w_{it} and w_{jt-k} as, for $k \geq 0$:

$$g_{ij}^W(k) = (1 + L + L^2 + ... + L^{m-1})^{d_i + d_j + 2} g_{ij}^w(mk + (d_j + 1)(m-1)) \tag{A.1}$$

$$g_{ii}^W(k) = (1 + L + L^2 + ... + L^{m-1})^{2(d_i+1)} g_{ii}^w(mk + (d_i + 1)(m-1)) \tag{A.2}$$

where $g_{ij}^W(k) = Cov(W_{it}, W_{jt-k})$, $g_{ij}^w(k) = Cov(w_{it}, w_{jt-k})$, L operates on the index of $g_{ij}(k)$ such that $Lg_{ij}(k) = g_{ij}(k-1)$, $g_{ij}^W(k) = g_{ji}^W(-k)$ and d_i and d_j are integers representing orders of differencing applied to ith and jth series respectively.

Proof

Note that $(1-L)Y_t = Y_t - Y_{t-1} = (1+L+...+L^{m-1})y_{mt} - (1+L+...+L^{m-1})y_{m(t-1)} =$

$(1-L^m)(1+L+...+L^{m-1})y_{mt}$. Therefore, $W_t = (1-L)^d Y_t = (1-L^m)^d (1+L+...+L^{m-1})y_{mt}$

$= (1+L+...+L^{m-1})^{d+1} w_{mt}$. Let $F = L^{-1}$ be the lead operator such that $Fz_t = z_{t+1}$ and let c_s

and e_s be the coefficient of L^s of the polynomial $(1+L+\cdots+L^{m-1})^{d_i+1}$ and

$(1+L+\cdots+L^{m-1})^{d_j+1}$ respectively.

$\boldsymbol{g}_{ij}^W(k) = E[W_{it}W_{jt-k}]$

$= E[(1+L+\cdots+L^{m-1})^{d_i+1}w_{imt}(1+L+\cdots+L^{m-1})^{d_j+1}w_{jm(t-k)}]$

$= E[(c_0 w_{imt} + c_1 w_{imt-1} + \cdots c_{(d_i+1)(m-1)}w_{imt-(d_i+1)(m-1)})$

$\quad (e_0 w_{jmt-mk} + e_1 w_{jmt-mk-1} + \cdots e_{(d_j+1)(m-1)}w_{jmt-mk-(d_{ji}+1)(m-1)})$

$= c_0[e_0\boldsymbol{g}_{ij}^w(mk) + e_1\boldsymbol{g}_{ij}^w(mk+1) + \cdots + e_{(d_j+1)(m-1)}\boldsymbol{g}_{ij}^w(mk+(d_j+1)(m-1))]$

$\quad + c_1[e_0\boldsymbol{g}_{ij}^w(mk-1) + e_1\boldsymbol{g}_{ij}^w(mk) + \cdots + e_{(d_j+1)(m-1)}\boldsymbol{g}_{ij}^w(mk+(d_j+1)(m-1)-1)]$

$\quad + \cdots\cdots$

$\quad + c_{(d_i+1)(m-1)}[e_0\boldsymbol{g}_{ij}^w(mk-(d_i+1)(m-1)) + e_1\boldsymbol{g}_{ij}^w(mk-(d_i+1)(m-1)-1)\cdots +$

$e_{(d_j+1)(m-1)}\boldsymbol{g}_{ij}^w(mk-(d_i+1)(m-1)+(d_j+1)(m-1))]$

$= c_0[(1+F+\cdots F^{m-1})^{d_j+1}\boldsymbol{g}_{ij}^w(mk)] + c_1[(1+F+\cdots F^{m-1})^{d_j+1}\boldsymbol{g}_{ij}^w(mk-1)] + \cdots +$

$\quad + c_{(d_i+1)(m-1)}[(1+F+\cdots F^{m-1})^{d_j+1}\boldsymbol{g}_{ij}^w(mk-(d_i+1)(m-1)]$

$= (1+F+\cdots+F^{m-1})^{(d_j+1)}[c_0\boldsymbol{g}_{ij}^w(mk) + c_1\boldsymbol{g}_{ij}^w(mk-1) + \cdots + c_{(d_i+1)(m-1)}\boldsymbol{g}_{ij}^w(mk-(d_j+1)(m-1))]$

$= (1+F+\cdots+F^{m-1})^{(d_j+1)}(1+L+\cdots+L^{m-1})^{(d_i+1)}\boldsymbol{g}_{ij}^w(mk)$

$= F^{(d_j+1)(m-1)}(1+L+\cdots+L^{m-1})^{d_i+d_j+2}\boldsymbol{g}_{ij}^w(mk)$

$= (1+L+\cdots+L^{m-1})^{d_i+d_j+2}\boldsymbol{g}_{ij}^w(mk+(d_j+1)(m-1)).$

References

Barro, R.J. (1979), "On the determinants of public debt", *Journal of Political Economy*, **87**, 940-971.

Choi, I., and B.S. Chung, (1995), "Sampling Frequency and the Power Tests for the Unit Root: A Simulation Study," *Economics Letters*, **49**, 131-136

Demiralp, S. and Hoover, K.D. (2003), "Searching for the causal structure of a vector autoregression", University of California, Davis Working Paper No. 03-03.

Engle, R.F., D.F. Hendry, and J.F. Richard (1983), "Exogenity," *Econometrica*, **50**, 227-304.

Ericsson, N.R., J.S.Irons, and R.W. Tryon, (2001), "Output and inflation in the long run," *Journal of Applied Econometrics*, **16**, 241-253.

Franses, P. H., and H. P. Boswijk (1996) "Temporal Aggregation in a Periodically Integrated Autoregressive Process," *Statistics and Probability Letters*, **30**, 235-240.

Ghosh, A. (1995), "Intertemporal tax-smoothing and the government budget surplus: Canada and the United States", *Journal of Money Credit and Banking*, **27**, 1033-1045.

Granger, C.W.J (1998), "Some Recent Developments in a Concept of Causality," *Journal of Econometrics*, **39**, 199-211.

Granger, C.W.J. (1990), "Aggregation of Time-Series Variables: A Survey," in *Barker, T. and Pesaran M.H. (eds.), "Disaggregation in Econometric Modelling,"* London: Routledge.

Granger, C.W.J. and P.R. Siklos, (1995), "Systematic Sampling, Temporal Aggregation, Seasonal Adjustment, and Cointegration. Theory and Evidence," *Journal of Econometrics*, **66**,357-369.

Granger, C.W.J. and N.R. Swanson, (1992), "Impulse Response Functions Based on a Causal Approach to Residual Orthogonalization in Vector Autoregressions," *W.P. # 92-50, University of California at San Diego.*

Gulasekaran, R. and T. Abeysinghe, (2002) "The distortionary effects of temporal aggregation on Granger causality," *Working Paper, National University of Singapore.*

Gulasekaran, R. (2003) "Effects of Temporal Aggregation and Systematic sampling Model Dynamics and Causal Inference," *Unpublished PhD Dissertation, National University of Singapore.*

Hendry, D.F., and G. Mizon (1980), "An Empirical Application and Monte Carlo Analysis of Tests of Dynamic Specification," *Review of Economic Studies,* **47**, 21-46.

Hendry, D.F. (1995), *Dynamic Econometrics*, Oxford University Press, Oxford.

Hendry, D.F. and Mizon, G.E. (1998), "Exogeneity, causality, and co-breaking in economic policy analysis of small econometric model of money in the UK", *Empirical Economics*, 23, 267-294.

Hoover, K. (2001), *Causality in Macroeconomis,* Cambridge University Press, Cambridge.

Huang, C.H. and Lin, K.S. (1993), "Deficits, government expenditures, and tax smoothing in the United States: 1929-1988", *Journal of Monetary Economics,* **31**, 317-339.

Johansen S. (1995), *Likelihood-based Inference in Cointegrated Vector Autoregressive Models,* Oxford Univ Press.

Lai, W.K. (2001) *Whither the car prices? A case study*, Unpublished B.Soc.Sci. Thesis, National University of Singapore.

Lahiri, K., and N. Mamingi (1995), "Testing for Co-integration: Power Versus Frequency of Observation– Another View," *Economics Letters*, **49**, 121-124.

Mamingi, N. (1996), "Aggregation over time, error correction models and Granger causality: A Monte Carlo investigation," *Economics Letters*, **52**, 7-14

Marcellino, M. (1999), "Some Consequences of Temporal Aggregation in Empirical Analysis," *Journal of Business and Economic Statistics*, Vol. 17**, 1**, 129-136.

Mosconi, R. and Giannini, C. (1992), "Non-causality in cointegrated systems: Representation, estimation and testing", *Oxford Bulletin of Economics and Statistics*, 54, 399-417.

Pagan, A. (1989), "Twenty Years After: Econometrics, 1966-1986," *Contributions to operations research and economics: The twentieth anniversary of CORE*, 319-383.

Pearl, J. (2000), Causality: Models, reasoning, and inference, *Cambridge University Press.*

Pierse, R.G. and .J. Snell, (1995), "Temporal Aggregation and the Power of Tests for a Unit Root," *Journal of Econometrics*, **65**, 333-345.

Sahasakul, C. (1986), "The evidence on optimal taxation over time", *Journal of Monetary Economcis,* **18**, 251-275.

Swanson, N.R. and Granger, C.W.J. (1997), "Impulse response functions based on a causal approach to residual orthogonalization in vector autoregressions", *Journal of the American Statistical Association*, 92(437), 357-367.

Wei, W.W.S. (1990), "Time Series Analysis: Univariate and Multivariate Methods," *Addison-Wesley, California.*

Table 1: Unidirectional Causality: $p\lim \hat{a}_1^*$ when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	-1.95	-1.75	-1.5	-1.25	-1.0	-0.75	-0.5	-0.25	-0.05
m=3									
-20	-2.06	-2.31	-2.43	-2.29	-2.00	-1.63	-1.20	-0.67	-0.15
-10	-2.05	-2.31	-2.43	-2.28	-1.99	-1.63	-1.19	-0.67	-0.15
-8	-2.05	-2.30	-2.42	-2.27	-1.98	-1.62	-1.19	-0.66	-0.15
-6	-2.05	-2.29	-2.40	-2.26	-1.97	-1.61	-1.19	-0.66	-0.15
-4	-2.04	-2.26	-2.36	-2.22	-1.94	-1.59	-1.17	-0.65	-0.14
-2	-2.02	-2.14	-2.19	-2.05	-1.80	-1.49	-1.10	-0.62	-0.14
-1	-1.96	-1.88	-1.81	-1.68	-1.50	-1.26	-0.96	-0.55	-0.12
m=12									
-20	-7.76	-9.18	-8.88	-7.81	-6.49	-5.05	-3.56	-2.01	-0.53
-10	-7.71	-9.12	-8.82	-7.76	-6.45	-5.02	-3.54	-2.00	-0.53
-8	-7.67	-9.08	-8.78	-7.72	-6.42	-5.00	-3.52	-2.00	-0.53
-6	-7.59	-8.99	-8.69	-7.64	-6.35	-4.95	-3.49	-1.98	-0.53
-4	-7.38	-8.73	-8.44	-7.43	-6.18	-4.82	-3.41	-1.94	-0.52
-2	-6.44	-7.58	-7.33	-6.47	-5.40	-4.24	-3.04	-1.77	-0.49
-1	-4.44	-5.15	-4.98	-4.42	-3.75	-3.02	-2.25	-1.41	-0.43

These values are the same for $b_2 > 0$.

Table 2: Unidirectional Causality: $t(\hat{a}_1^*)$ when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	-1.95	-1.75	-1.5	-1.25	-1.0	-0.75	-0.5	-0.25	-0.05
Panel 1: T=150, m=3, N=50									
-20	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.9	-18.2	-31.2
-10	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.7	-17.5	-20.5
-8	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.6	-17.1	-17.2
-6	-13.7	-8.0	-7.8	-8.4	-9.2	-10.4	-12.5	-16.2	-13.4
-4	-13.8	-8.0	-7.8	-8.3	-9.1	-10.2	-12.0	-14.3	-9.3
-2	-14.4	-7.9	-7.5	-7.9	-8.6	-9.4	-10.3	-9.9	-4.9
-1	-16.4	-8.1	-7.1	-7.2	-7.7	-8.1	-7.9	-6.3	-2.8
Panel 2: T=600, m=12, N=50									
-20	-4.5	-5.4	-5.8	-6.0	-6.3	-6.5	-6.9	-8.2	-16.2
-10	-4.4	-5.3	-5.8	-6.0	-6.2	-6.4	-6.8	-8.1	-15.6
-8	-4.4	-5.3	-5.8	-6.0	-6.2	-6.4	-6.8	-8.1	-15.1
-6	-4.4	-5.3	-5.7	-6.0	-6.1	-6.4	-6.8	-8.0	-14.4
-4	-4.4	-5.1	-5.6	-5.8	-6.0	-6.3	-6.7	-7.9	-12.7
-2	-4.0	-4.7	-5.1	-5.3	-5.5	-5.8	-6.2	-7.4	-8.7
-1	-3.4	-3.8	-4.1	-4.3	-4.5	-4.8	-5.3	-6.5	-5.4
Panel 3: T=600, m=3, N=200									
-20	-27.2	-16.0	-15.8	-16.9	-18.4	-21.0	-25.7	-36.3	-62.4
-10	-27.3	-16.0	-15.8	-16.9	-18.4	-21.0	-25.5	-35.0	-41.0
-8	-27.3	-16.0	-15.8	-16.8	-18.4	-20.9	-25.3	-34.1	-34.3
-6	-27.4	-16.0	-15.7	-16.8	-18.3	-20.8	-24.9	-32.3	-26.8
-4	-27.6	-16.0	-15.5	-16.5	-18.1	-20.3	-24.0	-28.5	-18.6
-2	-28.8	-15.9	-15.0	-15.8	-17.1	-18.9	-20.7	-19.8	-9.8
-1	-32.8	-16.2	-14.2	-14.5	-15.3	-16.1	-15.8	-12.5	-5.6

These values are the same for $b_2 > 0$.

Table 3: Unidirectional Causality: $p \lim \hat{\boldsymbol{a}}_2^*$ when $\boldsymbol{a}_2 = 0$, $\boldsymbol{b}_1 = 1$ and $\boldsymbol{r} = 1 + \boldsymbol{a}_1$

$\boldsymbol{a}_1$ across/ $\boldsymbol{b}_2$ down	-1.95	-1.75	-1.5	-1.25	-1.0	-0.75	-0.5	-0.25	-0.05
m=3									
-20	-0.01	-0.04	-0.06	-0.06	-0.05	-0.04	-0.02	-0.01	-0.00
-10	-0.02	-0.09	-0.12	-0.12	-0.10	-0.07	-0.05	-0.02	-0.00
-8	-0.02	-0.11	-0.15	-0.15	-0.12	-0.09	-0.06	-0.03	-0.01
-6	-0.03	-0.14	-0.20	-0.20	-0.16	-0.12	-0.08	-0.04	-0.01
-4	-0.05	-0.21	-0.29	-0.29	-0.24	-0.17	-0.11	-0.05	-0.01
-2	-0.08	-0.35	-0.50	-0.49	-0.40	-0.30	-0.19	-0.09	-0.02
-1	-0.10	-0.44	-0.63	-0.61	-0.50	-0.37	-0.24	-0.12	-0.02
1	0.10	0.44	0.63	0.61	0.50	0.37	0.24	0.12	0.02
2	0.08	0.35	0.50	0.49	0.40	0.30	0.19	0.09	0.02
4	0.05	0.21	0.29	0.29	0.24	0.17	0.11	0.05	0.01
6	0.03	0.14	0.20	0.20	0.16	0.12	0.08	0.04	0.01
8	0.02	0.11	0.15	0.15	0.12	0.09	0.06	0.03	0.01
10	0.02	0.09	0.12	0.12	0.10	0.07	0.05	0.02	0.00
20	0.01	0.04	0.06	0.06	0.05	0.04	0.02	0.01	0.00
m=12									
-20	-0.33	-0.40	-0.39	-0.34	-0.27	-0.20	-0.13	-0.06	-0.01
-10	-0.66	-0.80	-0.78	-0.67	-0.54	-0.40	-0.26	-0.12	-0.02
-8	-0.82	-1.00	-0.97	-0.84	-0.68	-0.50	-0.32	-0.15	-0.03
-6	-1.08	-1.31	-1.27	-1.10	-0.89	-0.66	-0.43	-0.19	-0.03
-4	-1.57	-1.91	-1.85	-1.60	-1.29	-0.96	-0.62	-0.28	-0.05
-2	-2.67	-3.24	-3.14	-2.72	-2.20	-1.63	-1.05	-0.48	-0.08
-1	-3.34	-4.05	-3.93	-3.40	-2.75	-2.04	-1.31	-0.60	-0.10
1	3.34	4.05	3.93	3.40	2.75	2.04	1.31	0.60	0.10
2	2.67	3.24	3.14	2.72	2.20	1.63	1.05	0.48	0.08
4	1.57	1.91	1.85	1.60	1.29	0.96	0.62	0.28	0.05
6	1.08	1.31	1.27	1.10	0.89	0.66	0.43	0.19	0.03
8	0.82	1.00	0.97	0.84	0.68	0.50	0.32	0.15	0.03
10	0.66	0.80	0.78	0.67	0.54	0.40	0.26	0.12	0.02
20	0.33	0.40	0.39	0.34	0.27	0.20	0.13	0.06	0.01

Table 4: Unidirectional Causality: $t(\hat{a}_2^*)$ when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	-1.95	-1.75	-1.5	-1.25	-1.0	-0.75	-0.5	-0.25	-0.05
Panel 1: T=150, m=3, N=50									
-20	-1.1	-2.4	-3.1	-3.2	-3.1	-2.7	-2.2	-1.6	-0.7
-10	-1.1	-2.4	-3.1	-3.2	-3.0	-2.7	-2.2	-1.5	-0.7
-8	-1.1	-2.4	-3.1	-3.2	-3.0	-2.7	-2.2	-1.5	-0.7
-6	-1.0	-2.3	-3.1	-3.2	-3.0	-2.6	-2.2	-1.5	-0.7
-4	-1.0	-2.3	-3.0	-3.1	-3.0	-2.6	-2.1	-1.5	-0.7
-2	-1.0	-2.1	-2.7	-2.9	-2.7	-2.3	-2.0	-1.4	-0.6
-1	-0.7	-1.7	-2.1	-2.2	-2.1	-1.8	-1.5	-1.1	-0.5
1	0.7	1.7	2.1	2.2	2.1	1.8	1.5	1.1	0.5
2	1.0	2.1	2.7	2.9	2.7	2.3	2.0	1.4	0.6
4	1.0	2.3	3.0	3.1	3.0	2.6	2.1	1.5	0.7
6	1.0	2.3	3.1	3.2	3.0	2.6	2.2	1.5	0.7
8	1.1	2.4	3.1	3.2	3.0	2.7	2.2	1.5	0.7
10	1.1	2.4	3.1	3.2	3.0	2.7	2.2	1.5	0.7
20	1.1	2.4	3.1	3.2	3.1	2.7	2.2	1.6	0.7
Panel 2: T=600, m=12, N=50									
-40	-3.7	-4.5	-4.8	-4.9	-4.8	-4.6	-4.1	-3.2	-1.6
-20	-3.7	-4.5	-4.8	-4.9	-4.8	-4.6	-4.1	-3.2	-1.6
-10	-3.6	-4.4	-4.8	-4.8	-4.8	-4.5	-4.1	-3.2	-1.6
-8	-3.6	-4.4	-4.8	-4.8	-4.8	-4.5	-4.1	-3.2	-1.5
-6	-3.6	-4.4	-4.7	-4.8	-4.7	-4.5	-4.1	-3.2	-1.5
-4	-3.5	-4.2	-4.6	-4.6	-4.6	-4.4	-4.0	-3.1	-1.5
-2	-3.2	-3.9	-4.1	-4.1	-4.1	-3.9	-3.6	-2.9	-1.4
-1	-2.4	-2.9	-3.1	-3.1	-3.1	-2.9	-2.7	-2.2	-1.1
1	2.4	2.9	3.1	3.1	3.1	2.9	2.7	2.2	1.1
2	3.2	3.9	4.1	4.1	4.1	3.9	3.6	2.9	1.4
4	3.5	4.2	4.6	4.6	4.6	4.4	4.0	3.1	1.5
6	3.6	4.4	4.7	4.8	4.7	4.5	4.1	3.2	1.5
8	3.6	4.4	4.8	4.8	4.8	4.5	4.1	3.2	1.5
10	3.6	4.4	4.8	4.8	4.8	4.5	4.1	3.2	1.6
20	3.7	4.5	4.8	4.9	4.8	4.6	4.1	3.2	1.6
Panel 3: T=600, m=3, N=200									
-20	-2.1	-4.8	-6.3	-6.5	-6.1	-5.4	-4.4	-3.1	-1.3
-10	-2.1	-4.8	-6.3	-6.5	-6.0	-5.4	-4.4	-3.0	-1.3
-8	-2.1	-4.8	-6.3	-6.5	-6.0	-5.4	-4.4	-3.0	-1.3
-6	-2.0	-4.7	-6.1	-6.4	-6.0	-5.3	-4.4	-3.0	-1.3
-4	-2.0	-4.7	-6.0	-6.3	-5.9	-5.1	-4.2	-3.0	-1.3
-2	-1.9	-4.2	-5.5	-5.7	-5.4	-4.7	-3.9	-2.8	-1.2
-1	-1.5	-3.3	-4.2	-4.4	-4.1	-3.7	-3.0	-2.2	-1.0
1	1.5	3.3	4.2	4.4	4.1	3.7	3.0	2.2	1.0
2	1.9	4.2	5.5	5.7	5.4	4.7	3.9	2.8	1.2
4	2.0	4.7	6.0	6.3	5.9	5.1	4.2	3.0	1.3
6	2.0	4.7	6.1	6.4	6.0	5.3	4.4	3.0	1.3
8	2.1	4.8	6.3	6.5	6.0	5.4	4.4	3.0	1.3
10	2.1	4.8	6.3	6.5	6.0	5.4	4.4	3.0	1.3
20	2.1	4.8	6.3	6.5	6.1	5.4	4.4	3.1	1.3

Table 5: Mutual Causality: $t(\hat{a}_2^*)$ when $b_1 = 1$, $r = 0$ and $a_2 = (r - 1 - a_1)/b_2$

a_1 across/ b_2 down	-0.95	-0.85	-0.75	-0.65	-0.55	-0.45	-0.35	-0.25	-0.15	-0.05
Panel 1: T=150, m=3, N=50										
-20	-2.9	-2.3	-1.8	-1.2	-0.4	0.4	1.5	2.8	4.2	6.0
-10	-2.9	-2.3	-1.8	-1.1	-0.4	0.5	1.6	2.8	4.3	6.0
-8	-2.8	-2.3	-1.8	-1.1	-0.3	0.5	1.6	2.9	4.3	6.1
-6	-2.8	-2.3	-1.7	-1.1	-0.3	0.6	1.6	2.9	4.4	6.1
-4	-2.7	-2.2	-1.6	-1.0	-0.2	0.7	1.8	3.1	4.5	6.1
-2	-2.4	-1.8	-1.2	-0.4	0.4	1.5	2.5	3.7	5.1	6.4
-1	-1.7	-0.9	0.0	1.0	2.1	3.2	4.3	5.4	6.4	7.3
Panel 2: T=600, m=12, N=50										
-20	-4.7	-4.5	-4.4	-4.1	-3.7	-3.3	-2.6	-1.6	0.1	3.6
-10	-4.7	-4.5	-4.3	-4.0	-3.7	-3.2	-2.5	-1.5	0.3	3.3
-8	-4.6	-4.5	-4.2	-4.0	-3.6	-3.1	-2.4	-1.3	0.3	3.2
-6	-4.6	-4.4	-4.2	-3.9	-3.5	-3.0	-2.2	-1.2	0.5	3.0
-4	-4.5	-4.2	-4.0	-3.7	-3.2	-2.6	-1.8	-0.7	0.9	2.9
-2	-4.0	-3.6	-3.2	-2.7	-2.1	-1.2	-0.3	0.8	2.0	3.1
-1	-2.8	-2.2	-1.5	-0.7	0.2	1.2	2.1	2.9	3.6	4.2
Panel 3: T=600, m=3, N=200										
-20	-5.7	-4.7	-3.6	-2.3	-0.8	0.9	3.0	5.6	8.5	12.1
-10	-5.7	-4.7	-3.6	-2.2	-0.8	1.0	3.1	5.6	8.6	12.1
-8	-5.6	-4.7	-3.6	-2.2	-0.7	1.0	3.1	5.7	8.6	12.2
-6	-5.6	-4.6	-3.5	-2.1	-0.7	1.1	3.2	5.8	8.7	12.2
-4	-5.5	-4.5	-3.2	-1.9	-0.3	1.5	3.6	6.1	8.9	12.3
-2	-4.8	-3.7	-2.3	-0.8	0.9	2.9	5.0	7.5	10.2	12.9
-1	-3.4	-1.8	0.0	2.0	4.1	6.3	8.6	10.8	12.8	14.5

For $b_2 > 0$ the table entries are the same with the opposite sign.

Table 6: Monte Carlo Results based on VAR(2): $f_{21}=0$

	AIC choice of VAR order, %			SBC choice of VAR order ,%			% of negative $\hat{a}_2^*$			Average contemporaneous correlation between residuals		
	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)
m=3	0.0	8.3	91.8	0.0	80.9	19.1	91.5	100	94.7	0.620	0.640	0.650
m=12	4.0	82.8	13.3	73.3	26.7	0.0	100	100	100	0.978	0.977	0.977
m=60	6.0	80.0	13.1	79.7	20.4	0.0	100	99.95	99.8	0.999	0.999	0.999

Table 7: Monte Carlo Results based on VAR(2): $f_{21}=0.25$

	AIC choice of VAR order, %			SBC choice of VAR order ,%			% of negative $\hat{a}_2^*$			Average contemporaneous correlation between residuals		
	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)	VAR(1)	VAR(2)	VAR(3)
m=3	0	0	100	0	17.4	82.7	100	100	100	0.850	0.780	0.780
m=12	1.2	87.2	11.7	52.8	47.2	0	100	100	100	0.991	0.991	0.991
m=60	40.9	51.8	7.4	97.8	2.25	0	100	100	100	0.999	0.999	0.999

Table 8. Estimated adjustment coefficients
Cointegrating relation: log(Yen/DM)-log(US\$/DM)+log(US\$/Yen)

Adjustment coefficients	Daily rates	Weekly rates End of period	Weekly rates Average
a_1	-0.917* (0.015)	-0.889* (0.236)	-1.940* (0.202)
a_2	-0.046 (0.025)	-0.116 (0.152)	-0.326 (0.182)
a_3	0.050 (0.037)	-0.254 (0.258)	0.666* (0.301)
Sample size	922	184	184

Note: Numbers in parentheses are standard errors. * indicates the absolute values bigger than 2SE. If a_2 and a_3 are restricted to zero the estimates of a_1 in columns 3 and 4 move closer to minus unity.

Table 9. Estimates for car quota premium and stock price example

	$\hat{a}_1^*$	$\hat{a}_2^*$	$\hat{b}_2$	r_{12}	N
m=1	-0.191 (0.048)	-0.003 (0.007)	-3.71	-0.02	104
m=2	-0.230 (0.062)	-0.016 (0.013)	-3.56	-0.10	51
m=3	-0.342 (0.093)	-0.015 (0.021)	-3.74	-0.02	34
m=4	-0.368 (0.109)	-0.046 (0.026)	-2.79	0.12	25
m=5	-0.483 (0.133)	-0.027 (0.036)	-3.06	-0.04	20
m=6	-0.572 (0.095)	-0.088 (0.044)	-3.25	-0.01	16

r_{12} is the contemporaneous correlation of residuals. N is the effective sample size. The numbers in parentheses are standard errors.
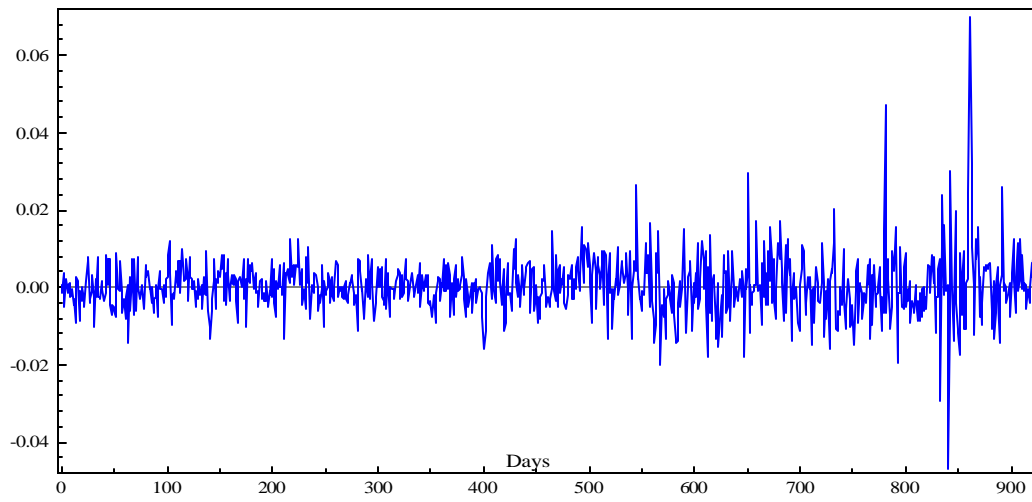
Figure 1. Deviations of logarithms of daily Yen/DM direct rate from the cross rate
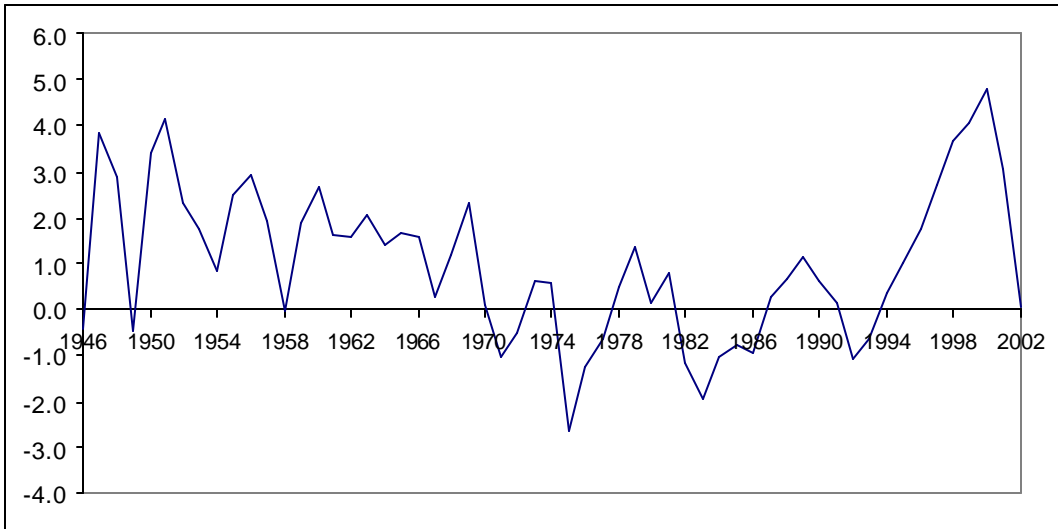(July 3, 1995 – Dec 31, 1998)
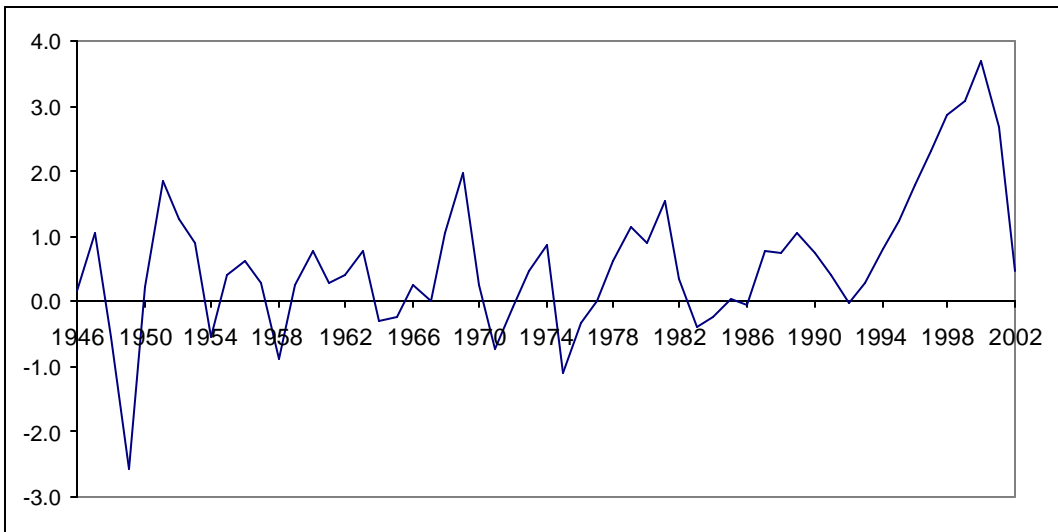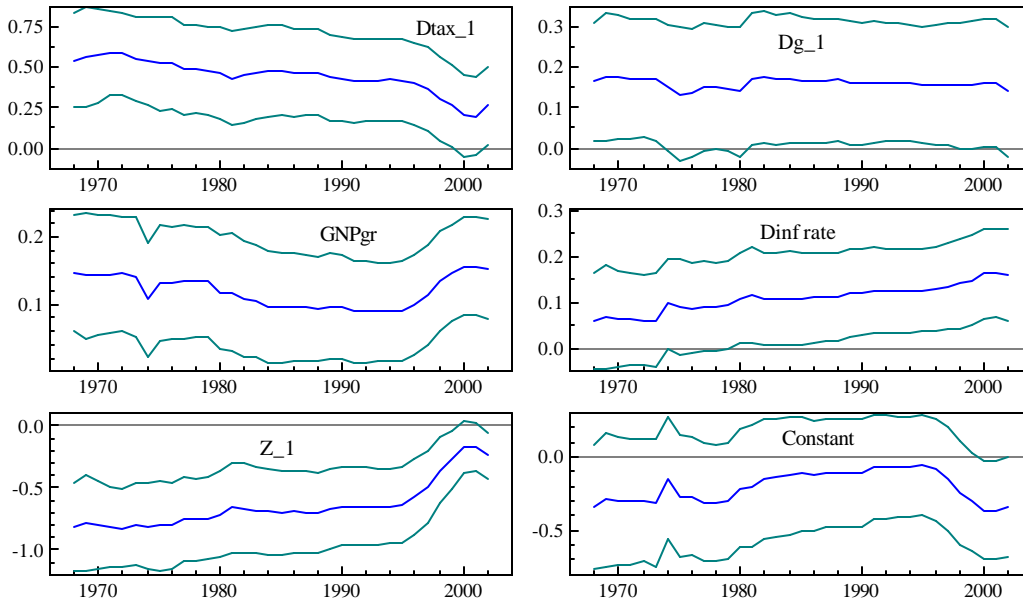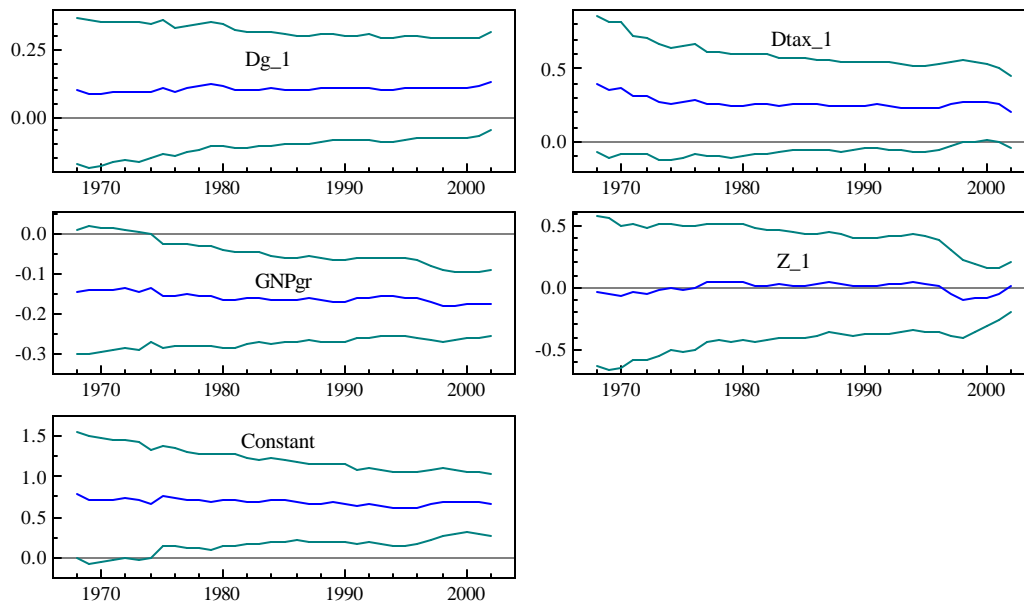
Figure 2. Budget surplus as a ratio of GNP (%)

Figure 3. Cointegrating relation between tax rate and spending rate

Note: Outer lines show the 2SE confidence bands. The error correction term is z_{t-1}.

Figure 4. OLS recursive estimates of the tax equation.

Note: Outer lines show the 2SE confidence bands. The error correction term is z_{t-1}.

Figure 5. OLS recursive estimates of the spending equation.