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A Note on Multi-Issue Bargaining with a Finite Set of Alternatives

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Abstract: We study two bilateral multi-issue bargaining procedures with complete information and endogenous agenda, where each issue is associated with a finite set of alternatives. In both procedures, when bargaining frictions are small, we find a large multiplicity of equilibrium agreements, including ones with arbitrarily long delays. Thus, this paper extends the previous results of van Damme et al. (1990) and Muthoo (1991) for the single-issue case to multi-issue cases. Furthermore, we show that in the first procedure (issue-by-issue bargaining), the stationary subgame perfect equilibria alone may support a large multiplicity of inefficient agreements. Confronting a recent study, this implies that it is not necessary to appeal to "strictly controversial" issues in a bargaining problem in order to find multiplicity and delay in agreements.

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1 Introduction

Rubinstein's (1982) theory predicts a unique subgame perfect equilibrium in his alternating offer model, where the pie is divisible arbitrarily. Van Damme et al. (1990) and Muthoo (1991) investigated the implication of the finiteness of alternatives, interacting with small bargaining frictions, and obtained a large multiplicity of equilibrium agreements. In this note, we show how the multiplicity result extends to multi-issue bargaining procedures.

We consider two bilateral multi-issue bargaining procedures with complete information, where each issue is associated with a finite set of alternatives. The first procedure is the issue-by-issue procedure studied by In and Serrano (2000). In this procedure, which we refer to as restricted agenda bargaining, parties bargain using alternating offers according to the following rules. The proposer makes an offer on one and only one of the remaining issues, but he can choose which issue to propose on. The responder either accepts or rejects. If he accepts, that issue is considered settled and he proposes on any one and only one of the remaining issues, and so on. If he rejects, there is a probability of breakdown of negotiations, while with the rest of probability the rejector makes a counterproposal on any one and only one of the pending issues, and so on. The game ends either with breakdown of negotiations or when all issues have been settled. The agenda is therefore endogenous because each proposer can choose which issue comes to the negotiation table on each round. The second procedure is the one studied by Inderst (2000) and In and Serrano (2003). In this procedure, which we call unrestricted agenda bargaining, each proposer can make an offer on any subset of the remaining issues. With this only modification, the rules are the same as in the first procedure.

In both procedures, when bargaining frictions are small, we find a large multiplicity of equilibrium agreements, including ones with arbitrarily long delays. Furthermore, we show that in the first procedure (issue-by-issue bargaining), the stationary subgame perfect equilibria alone may support a large multiplicity of inefficient agreements.

Confronting a recent study by Inderst (2000), the results for the second procedure imply that it is not necessary to appeal to "strictly controversial" issues in a bargaining problem in order to find multiplicity and delay in agreements.

2 Bargaining Procedures and Payoffs

We consider two multi-issue bargaining procedures. Two players 1 and 2 are bargaining over how to split l issues or "pies," where $l < \infty$. The set of alternatives for each issue is finite. Since $l < \infty$, the set of alternatives for all issues is also finite. Let L be the set of issues $\{1, 2, \ldots, l\}$. The size of each issue has been normalized to 1. Let X_k denote the set of all alternatives for the k-th issue in terms of player 1's share. Similarly, let X_S denote the set of all alternatives for the issues in $S \subseteq L$. Note that we use the same term "alternative" both for an element of X_k and for an element of X_S. Starting at time $t = 0$, bargaining takes place over potentially infinitely-many discrete periods according to one of the following bargaining procedures. In even periods player 1 makes an offer, and in odd periods player 2 makes an offer.

For $i = 1, 2$ and $S \subseteq L$, the **restricted agenda bargaining procedure** $\gamma_i^S(\delta)$ induces an infinite horizon game of perfect information and it is defined recursively.

- The first move corresponds to player i, who makes a proposal: he chooses one and only one of the pending issues $k \in S$ and offers the split $(x_k, 1 - x_k)$, where $x_k \in X_k$.
- Player j can then either accept or reject this proposal.
  - If the proposal is accepted, issue k is split accordingly and the procedure $\gamma_j^{S \setminus \{k\}}(\delta)$ is followed.
  - If the proposal is rejected, the procedure $\gamma_j^S(\delta)$ is followed with probability δ $(0 \leq \delta < 1)$, whereas negotiations break down with probability $1 - \delta$. In the latter case, both players receive a zero share from those issues on which there was no agreement.

  The negotiations end either with the breakdown outcome, or when the procedure $\gamma_i^{\emptyset}(\delta)$ must be followed for $i = 1, 2$.

For $i = 1, 2$ and $S \subseteq L$, the **unrestricted agenda bargaining procedure** $\Gamma_i^S(\delta)$ induces an infinite horizon game of perfect information and it is defined recursively.

- The first move corresponds to player i, who makes a proposal: he chooses an arbitrary subset of the pending issues $T \subseteq S, T \neq \emptyset$ and offers the split $(x_T, 1 - x_T)$, where $x_T \in X_T$.[1]
- Player j can then either accept or reject this entire proposal (he is not allowed to accept the proposal only for a strict subset of T and reject the rest).
  - If the proposal is accepted, issues T are split accordingly and the procedure $\Gamma_j^{S\setminus T}(\delta)$ is followed.
  - If the proposal is rejected, the procedure $\Gamma_j^S(\delta)$ is followed with probability δ $(0 \leq \delta < 1)$, whereas negotiations break down with probability $1 - \delta$. In the latter case, both players receive a zero share from those issues on which there was no agreement.

The negotiations end either with the breakdown outcome, or when the procedure $\Gamma_i^{\emptyset}(\delta)$ must be followed for $i = 1, 2$.

We shall refer to the procedure $\gamma_i^L(\delta)$ and $\Gamma_i^L(\delta)$ simply as $\gamma_i(\delta)$ and $\Gamma_i(\delta)$. Note our use of the word "procedure" to refer to $\gamma_i^S(\delta)$ or $\Gamma_i^S(\delta)$, as opposed to the word "game," which should be employed to refer to $\gamma_i^S(\delta, x_{-S})$ or $\Gamma_i^S(\delta, x_{-S})$. That is, the shares x_{-S} agreed prior to the game in question do not affect the rules of negotiation, but they do affect the players' payoffs.

Let $x_L = (x_1, x_2, \ldots, x_l)$ and $t_L = (t_1, t_2, \ldots, t_l)$. The outcome (x_L, t_L) means that they reach

- an agreement on x_1 at t_1 if the negotiation has not broken down by t_1,
- an agreement on x_2 at t_2 if the negotiation has not broken down by t_2,
- $\cdots$, and
- an agreement on x_l at t_l if the negotiation has not broken down by t_l.

[1] We use the following notation for the comparison of two arbitrary vectors x and y: $x \geq y$ means $x_k \geq y_k$ for all k; $x > y$ means $x \geq y$ and $x \neq y$; and $x \gg y$ means $x_k > y_k$ for all k. Also, $x_T = (x_k)_{k \in T}$, while 0_T and 1_T are vectors with $|T|$ zeros and ones, respectively. However, we write 0 and 1 instead of 0_T and 1_T when it is clear.

If any element of t_L is infinite, say $t_k = \infty$, then it means that they disagree perpetually on the $k-$th issue. In this case, x_k is immaterial and we make $x_k = 0$. Therefore, $((0, 0, \ldots, 0), (\infty, \infty, \ldots, \infty))$ means that they disagree perpetually on all issues. We call an outcome (x_L, t_L) an **immediate agreement** if no rejection occurs on any issue. Therefore, an outcome (x_L, t_L) is an immediate agreement if $\min\{t_1, t_2, \ldots, t_l\} = 0$ and there is no skip of an integer number in $(t_1, t_2, \ldots, t_l)$. Similarly, (x_S, t_S) is an **immediate agreement** in the subgame $\gamma_i^S(\delta, x_{-S})$ or $\Gamma_i^S(\delta, x_{-S})$ if no rejection occurs on any issue in S.

Both players are von Neumann-Morgenstern expected utility maximizers. We make the following assumptions on the utility functions $u_1(x_1, \ldots, x_l)$ and $u_2(1 - x_1, \ldots, 1 - x_l)$:[2]

- A0 (normalization):

$$u_i(0_L) = 0 \quad \text{for } i = 1, 2.$$

- A1 (interior strong monotonicity):

$$\begin{aligned} u_i(x) > u_i(y) &\qquad \text{if} \quad x > y \text{ and } x \gg 0, \text{ and} \\ u_i(x) \geq u_i(y) &\qquad \text{if} \quad x > y \text{ and some element of } x \text{ is zero.} \end{aligned}$$

Assumption A1 is so weak an assumption that most well-known utility functions including Cobb-Douglas, linear, and quasi-linear utility functions satisfy it. We shall sometimes use a stronger assumption, which linear utility functions satisfy:

- A1′ (strong monotonicity):

$$u_i(x) > u_i(y) \qquad \text{if} \quad x > y.$$

In this paper, we employ subgame perfect equilibrium as our solution concept, which we also refer to as an SPE or simply as an equilibrium.

[2] We use utility functions for our models, but they can be easily modified to preference-ordering based models, as is Muthoo's (1991) model.

3 Results

We define a **restriction of X_S with respect to x^*_S** as $\{x_S \in X_S : x_{S\setminus\{k\}} = x^*_{S\setminus\{k\}}$ for some $k \in S\}$, and denote it by $X_S|x^*_S$. Figure 1 illustrates X_S and $X_S|x^*_S$ in the utility space. In it, $L = S = \{1, 2\}$, and the utility functions used are $u_1(x_1, x_2) = x_1^{\frac{1}{5}} x_2^{\frac{4}{5}}$ and $u_2(1-x_1, 1-x_2) = (1-x_1)^{\frac{1}{2}}(1-x_2)^{\frac{1}{2}}$. Each issue is associated with possible divisions of a pie into shares in one-tenth, and $x^*_S = (\frac{7}{10}, \frac{3}{10})$.

We say that an alternative $x_S \in X_S$ is an **efficient alternative of X_S given x_{-S}** if there does not exist $x'_S \in X_S$ such that $u_1(x'_S, x_{-S}) \geq u_1(x_S, x_{-S})$ and $u_2(1-x'_S, 1-x_{-S}) \geq u_2(1-x_S, 1-x_{-S})$ with at least one strict inequality. We say that an alternative $x_S \in X_S$ is an **interior alternative of X_S** if $0_S \ll x_S \ll 1$. In Figure 1, x^*_S is an interior alternative of X_S and an efficient alternative of $X_S|x^*_S$, but not an efficient alternative of X_S.

Lemma 1 *Let u_1 and u_2 satisfy Assumptions A0 and A1. If $x^*_L \in X_L$ is an efficient or interior alternative of X_L, then for any $S \subseteq L$, x^*_S is an efficient alternative of $X_S|x^*_S$ given x^*_{-S}.*

Proof. Let x^*_L be an efficient alternative of X_L. Then for any $S \subseteq L$, x^*_S is an efficient alternative of X_S given x^*_{-S}. This is true because $\{y_L \in X_L : y_{-S} = x^*_{-S}\}$ is just a subset of X_L. Now, for any $S \subseteq L$, x^*_S is an efficient alternative of $X_S|x^*_S$ given x^*_{-S}. This is true because $\{y_L \in X_L : y_S \in X_S|x^*_S, y_{-S} = x^*_{-S}\}$ is again a subset of $\{y_L \in X_L : y_{-S} = x^*_{-S}\}$.

Now let x^*_L be an interior alternative of X_L. Then by Assumption A1, for any $S \subseteq L$ and for any $k \in S$, x^*_S is an efficient alternative of $\{y_S \in X_S : y_{S\setminus\{k\}} = x^*_{S\setminus\{k\}}\}$ given x^*_{-S}. Note that $X_S|x^*_S = \cup_{k\in S}\{y_S \in X_S : y_{S\setminus\{k\}} = x^*_{S\setminus\{k\}}\}$. Therefore, x^*_S is an efficient alternative of $X_S|x^*_S$ given x^*_{-S}. ■

Proposition 1 *Consider the restricted agenda bargaining procedure $\gamma_i(\delta)$ for $\delta < 1$ large enough. Let u_1 and u_2 satisfy Assumptions A0 and A1. Then, an immediate agreement on any efficient or interior alternative of X_L in any ordering of the issues can be supported by an SPE.*

Proof. Let x^*_L be an efficient or interior alternative of X_L. Let $\delta < 1$ be large enough so that for any $S \subseteq L$,

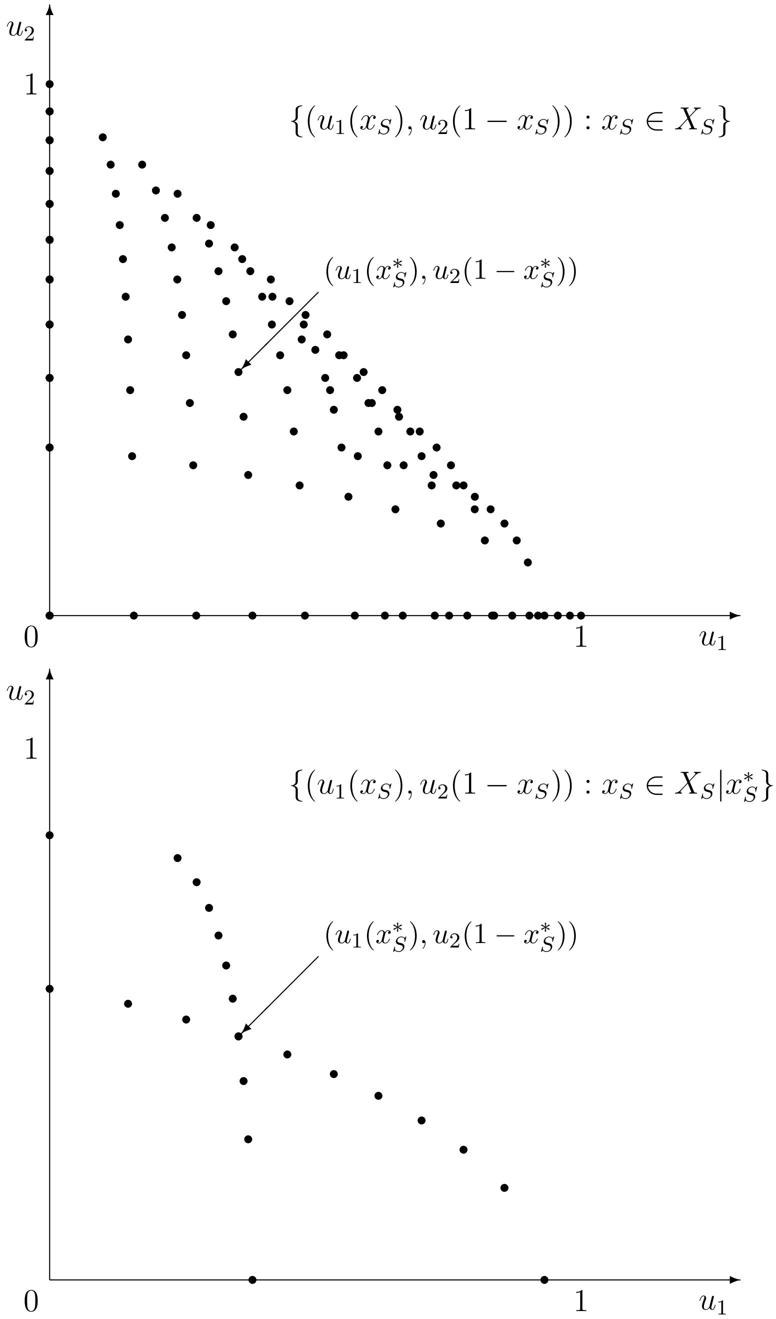

Figure 1: X_S AND $X_S|x_S^*$ IN UTILITY SPACE

- there does not exist any alternative $y_S \in X_S|x_S^*, y_S \neq x_S^*$ such that

$$\begin{aligned} u_1(y_S, x_{-S}^*) &> \delta u_1(x_S^*, x_{-S}^*) \\ \text{and} \quad u_2(1-y_S, 1-x_{-S}^*) &\geq u_2(1-x_S^*, 1-x_{-S}^*), \end{aligned}$$

- and there does not exist any alternative $z_S \in X_S|x_S^*, z_S \neq x_S^*$ such that

$$\begin{aligned} u_2(1-z_S, 1-x_{-S}^*) &> \delta u_2(1-x_S^*, 1-x_{-S}^*) \\ \text{and} \quad u_1(z_S, x_{-S}^*) &\geq u_1(x_S^*, x_{-S}^*). \end{aligned}$$

Such $\delta < 1$ always exists since x_S^* is an efficient alternative of $X_S|x_S^*$ given x_{-S}^* (Lemma 1) and $X_S|x_S^*$ is finite.

Let O be the ordering of the issues $1, 2, \ldots, l$ that we want to support. Let $O(S)$ be the first element of S according to the ordering O. Consider the following pair of strategies.

- (i) Whenever the set of pending issues is S and an agreement has been reached on x_{-S}^* (including the case of $S = L$), player 1 chooses the issue $O(S)$ and offers the split of the issue $(x_{O(S)}^*, 1-x_{O(S)}^*)$, and accepts the offer of $(y_k, 1-y_k)$ if and only if $u_1(y_k, x_{S\setminus\{k\}}^*, x_{-S}^*) \geq u_1(x_S^*, x_{-S}^*)$.
- (ii) Whenever the set of pending issues is S and an agreement has been reached on x_{-S}^* (including the case of $S = L$), player 2 chooses the issue $O(S)$ and offers the split of the issue $(x_{O(S)}^*, 1-x_{O(S)}^*)$, and accepts the offer of $(y_k, 1-y_k)$ if and only if $u_2(1-y_k, 1-x_{S\setminus\{k\}}^*, 1-x_{-S}^*) \geq u_2(1-x_S^*, 1-x_{-S}^*)$.
- (iii) In subgames $\gamma_i^S(\delta, x_{-S})$ where x_{-S} is different from x_{-S}^*, any SPE pair of strategies would do.

It is clear that if this pair of strategies are played, the immediate agreement on x_L^* in the ordering of the issues O will follow. This ordering O is the agenda at the equilibrium. We claim that there is no profitable deviation from the strategies specified in (i) and (ii). Since x_S^* is an efficient alternative of $X_S|x_S^*$ given x_{-S}^* (Lemma 1), the proposer cannot deviate profitably. It is not profitable for the responder to reject an offer that he is supposed to accept according to the above strategy. Finally, the conditions on δ guarantee that it is not profitable for the responder to accept an offer that he is supposed to reject according to the above strategy. ■

With a stronger assumption A1′ replacing A1, we can say something more about the multiplicity result.

Proposition 2 *Consider the restricted agenda bargaining procedure $\gamma_i(\delta)$ for $\delta < 1$ large enough. Let u_1 and u_2 satisfy Assumptions A0 and A1′. Then, an immediate agreement on any alternative of X_L in any ordering of the issues can be supported by a stationary SPE.*

Proof. Let x_L^* be an element of X_L. Let $\delta < 1$ be large enough so that for any $S \subseteq L$ and for any $x_{-S} \in X_{-S}$,

- there does not exist any alternative $y_S \in X_S | x_S^*, y_S \neq x_S^*$ such that

$$\begin{aligned} u_1(y_S, x_{-S}) &> \delta u_1(x_S^*, x_{-S}) \\ \text{and} \quad u_2(1 - y_S, 1 - x_{-S}) &\geq u_2(1 - x_S^*, 1 - x_{-S}), \end{aligned}$$

- and there does not exist any alternative $z_S \in X_S | x_S^*, z_S \neq x_S^*$ such that

$$\begin{aligned} u_2(1 - z_S, 1 - x_{-S}) &> \delta u_2(1 - x_S^*, 1 - x_{-S}) \\ \text{and} \quad u_1(z_S, x_{-S}) &\geq u_1(x_S^*, x_{-S}). \end{aligned}$$

Although these conditions for δ are stronger than those in the proof of Proposition 1, such $\delta < 1$ always exists given Assumption A1′ (strong monotonicity).

Note also that given Assumption A1′, x_S^* is an efficient alternative of $X_S | x_S^*$ given x_{-S}. Therefore, an immediate agreement on x_L^* in any ordering of the issues O can be supported by the following stationary SPE.

- (i) Whenever the set of pending issues is S, player 1 chooses the issue $O(S)$ and offers the split of the issue $(x_{O(S)}^*, 1 - x_{O(S)}^*)$, and accepts the offer of $(y_k, 1 - y_k)$ if and only if $u_1(y_k, x_{S\setminus\{k\}}^*, x_{-S}) \geq u_1(x_S^*, x_{-S})$.

- (ii) Whenever the set of pending issues is S, player 2 chooses the issue $O(S)$ and offers the split of the issue $(x_{O(S)}^*, 1 - x_{O(S)}^*)$, and accepts the offer of $(y_k, 1 - y_k)$ if and only if $u_2(1 - y_k, 1 - x_{S\setminus\{k\}}^*, 1 - x_{-S}) \geq u_2(1 - x_S^*, 1 - x_{-S})$.

■

In this case, therefore, the stationary subgame perfect equilibria alone support a large multiplicity of inefficient agreements. In fact, any alternative can be agreed immediately at a stationary subgame perfect equilibrium.

It is now standard to construct equilibria with delay from multiple equilibria (see Fernandez and Glazer (1991), Avery and Zemsky (1994), Busch and Wen (1995), and In and Serrano (2000) for related constructions).

Proposition 3 *Consider the restricted agenda bargaining procedure $\gamma_i(\delta)$ for $\delta < 1$ large enough. Let u_1 and u_2 satisfy Assumptions A0 and A1′. Then, any outcome (x_L, t_L) including outcomes involving perpetual disagreement on one or more issues can be supported by a nonstationary SPE.*

Proof. Note that for any $S \subseteq L$ and for any $x_{-S} \in X_{-S}$, 1_S and 0_S are efficient alternatives of X_S given x_{-S} because of Assumption A1′. Therefore, by Proposition 1, the immediate agreements on 1_S and 0_S are supported by an SPE in a subgame $\gamma_i^S(\delta, x_{-S})$ for $\delta < 1$ large enough. Using these extreme equilibria as rewards and punishments in case of deviations, we can support any outcome (x_L, t_L) by a nonstationary SPE.

Let $L = S_1 \cup S_2 \cup \cdots \cup S_n$ with $S_1, S_2, \ldots, S_n$ mutually disjoint, and let (x_L, t_L) be as follows:

- After T_1-period of delay (T_1 rejections by both players together), they agree immediately (without any rejection) on $(x_{S_1}, 1-x_{S_1})$, in a certain ordering of the issues in S_1.
- Then, after T_2-period of delay, they agree immediately on $(x_{S_2}, 1-x_{S_2})$, in a certain ordering of the issues in S_2.
- $\cdots$
- Then, after T_{n-1}-period of delay, they agree immediately on $(x_{S_{n-1}}, 1-x_{S_{n-1}})$, in a certain ordering of the issues in S_{n-1}.
- Then, they disagree perpetually on the issues in S_n.

This outcome is supported by the following nonstationary SPE.

- (i) During the periods of delay or perpetual disagreement, player 1 offers (1,0) for any remaining issue and player 2 rejects it, and player 2 offers (0,1) for any remaining issue and player 1 rejects it.

- (ii) During the periods of immediate agreement on $S_1, S_2, \ldots, S_{n-1}$, the proposer chooses an issue according to the ordering that we want to support and offers the exact split that we want to support, and the responder accepts it.

- (iii) If player 1 deviates from the above rule (i) or (ii) and makes a different offer, then player 2 accepts the offer if and only if the issue is the last remaining issue and the split offered is (0,1). Otherwise, player 2 rejects the offer, and both players start to play the SPE supporting the immediate agreement on 0_T specified in Proposition 1 (T is the set of the remaining issues at that time). If player 1 deviates from the above rule (i) and accepts the offer (0,1) for any issue which is not the last issue, then the SPE supporting the immediate agreement on 0_T specified in Proposition 1 is played (T is the set of the remaining issues at that time). If player 1 deviates from the above rule (ii) and rejects the offer that he was supposed to accept, then the SPE supporting the immediate agreement on 0_T specified in Proposition 1 is played (T is the set of the remaining issues at that time).

- (iv) If player 2 deviates from the above rule (i) or (ii) and makes a different offer, then player 1 accepts the offer if and only if the issue is the last remaining issue and the split offered is (1,0). Otherwise, player 1 rejects the offer, and both players start to play the SPE supporting the immediate agreement on 1_T specified in Proposition 1 (T is the set of the remaining issues at that time). If player 2 deviates from the above rule (i) and accepts the offer (1,0) for any issue which is not the last issue, then the SPE supporting the immediate agreement on 1_T specified in Proposition 1 is played (T is the set of the remaining issues at that time). If player 1 deviates from the above rule (ii) and rejects the offer that he was supposed to accept, then the SPE supporting the immediate agreement on 1_T specified in Proposition 1 is played (T is the set of the remaining issues at that time).

■

Even outcomes involving perpetual disagreement on one or more issues can be supported by an SPE once the issues are only finitely divisible, which contrasts with In and Serrano's (2000) result that perpetual disagreement cannot be supported by an SPE in the restricted agenda multi-issue bargaining when the issues are arbitrarily divisible.

The following two propositions show the multiplicity and delay results for the unrestricted agenda bargaining procedure $\Gamma_i(\delta)$. In this procedure, we call an outcome (x_L, t_L) a **bundled agreement** if $t_1 = t_2 = \cdots = t_l$, and an **unbundled agreement** otherwise.

Proposition 4 *Consider the unrestricted agenda bargaining procedure $\Gamma_i(\delta)$ for $\delta < 1$ large enough. Let u_1 and u_2 satisfy Assumptions A0 and A1. Then, an immediate and bundled agreement on any efficient alternative of X_L can be supported by an SPE.*

Proof. Let x^*_L be an efficient alternative of X_L. Let $\delta < 1$ be large enough so that for any $S \subseteq L$,

- there does not exist any alternative $y_S \in X_S$ such that

$$\delta u_1(x^*_S, x^*_{-S}) < u_1(y_S, x^*_{-S}) < u_1(x^*_S, x^*_{-S}),$$

- and there does not exist any alternative $z_S \in X_S$ such that

$$\delta u_2(1 - x^*_S, 1 - x^*_{-S}) < u_2(1 - z_S, 1 - x^*_{-S}) < u_2(1 - x^*_S, 1 - x^*_{-S}).$$

Such $\delta < 1$ always exists because of the finiteness of X_S.

Consider the following pair of strategies.

- (i) Whenever the set of pending issues is S and an agreement has been reached on x^*_{-S} (including the case of $S = L$), player 1 bundles all the remaining issues and offers $(x^*_S, 1 - x^*_S)$, and accepts the offer of $(y_T, 1 - y_T)$ if and only if $u_1(y_T, x^*_{S\setminus T}, x^*_{-S}) \geq u_1(x^*_S, x^*_{-S})$.
- (ii) Whenever the set of pending issues is S and an agreement has been reached on x^*_{-S} (including the case of $S = L$), player 2 bundles all the remaining issues and offers $(x^*_S, 1 - x^*_S)$, and accepts the offer of $(y_T, 1 - y_T)$ if and only if $u_2(1 - y_T, 1 - x^*_{S\setminus T}, 1 - x^*_{-S}) \geq u_2(1 - x^*_S, 1 - x^*_{-S})$.
- (iii) In subgames $\Gamma^S_i(\delta, x_{-S})$ where x_{-S} is different from x^*_{-S}, any SPE pair of strategies would do.

It is clear that if this pair of strategies are played, the immediate agreement on x_L^* will follow. We claim that there is no profitable deviation from the strategies specified in (i) and (ii). Since x_S^* is an efficient alternative of X_S given x_{-S}^*, the proposer cannot deviate profitably. It is not profitable for the responder to reject an offer that he is supposed to accept according to the above strategy. Finally, the conditions on δ guarantee that it is not profitable for the responder to accept an offer that he is supposed to reject according to the above strategy. ■

Remark: It can be also shown that an immediate and unbundled agreement on any efficient alternative of X_L can be supported by an SPE. In such an equilibrium, they agree sequentially on disjoint subsets of L without any rejection of an offer.

Proposition 5 *Consider the unrestricted agenda bargaining procedure $\Gamma_i(\delta)$ for $\delta < 1$ large enough. Let u_1 and u_2 satisfy Assumptions A0 and $A1'$. Then, any outcome (x_L, t_L) including outcomes involving perpetual disagreement on one or more issues can be supported by a nonstationary SPE.*

Proof. It can be proven by a similar logic as in Proposition 3. ■

Propositions 4 and 5 imply that once the finiteness of alternatives is assumed, it is not necessary to appeal to "strictly controversial" issues in a bargaining problem in order to find multiplicity and delay in agreements as in Inderst (2000). In the example provided by Inderst (2000), the force that makes the multiplicity and delay is not the introduction of "strictly controversial" issues, but the finiteness of alternatives. We provide an example to show that it is possible to have a unique SPE payoff with the introduction of strictly controversial and arbitrarily divisible issues. Let $u_1(x_1, x_2, x_3) = x_1 + x_2 + x_3$ and $u_2(1 - x_1, 1 - x_2, 1 - x_3) = (1 - x_1) - \frac{1}{2}(1 - x_2) - \frac{1}{4}(1 - x_3)$, where $0 \leq x_k \leq 1$ for $k = 1, 2, 3$. Issue 1 is a mutually beneficial issue and issues 2 and 3 are strictly controversial issues. Note that the set of alternatives is not finite any more. The equation for the Pareto frontier of the utility possibility set $\{(u_1(x_1, x_2, x_3), u_2(1 - x_1, 1 - x_2, 1 - x_3)) : \ x_k \in [0, 1], \ k = 1, 2, 3\}$ is:

$$u_2 = \begin{cases} -\frac{1}{4}u_1 + 1, & u_1 \in [0, 1] \\ -\frac{1}{2}u_1 + \frac{5}{4}, & u_1 \in [1, 2] \\ -u_1 + \frac{9}{4}, & u_1 \in [2, 3]. \end{cases}$$

Then, in the unrestricted agenda bargaining procedure $\Gamma_i(\delta)$ for any $\delta \in [0,1)$, there exists a unique SPE payoff. If $\delta \geq \frac{2}{3}$ for example, the unique SPE payoff in the game $\Gamma_1(\delta)$ is $(\frac{5}{2}\frac{1}{1+\delta}, \frac{5}{4}\frac{\delta}{1+\delta})$ and the unique SPE payoff in the game $\Gamma_2(\delta)$ is $(\frac{5}{2}\frac{\delta}{1+\delta}, \frac{5}{4}\frac{1}{1+\delta})$. The payoff $(\frac{5}{2}\frac{1}{1+\delta}, \frac{5}{4}\frac{\delta}{1+\delta})$ is obtained from $x_1 = 0$, $x_2 = \frac{5}{2}\frac{1}{1+\delta} - 1$, and $x_3 = 1$, and the payoff $(\frac{5}{2}\frac{\delta}{1+\delta}, \frac{5}{4}\frac{1}{1+\delta})$ is obtained from $x_1 = 0$, $x_2 = \frac{5}{2}\frac{\delta}{1+\delta} - 1$, and $x_3 = 1$. As $\delta \to 1$, these payoffs approach $(\frac{5}{4}, \frac{5}{8})$, which can be obtained from $x_1 = 0$, $x_2 = \frac{1}{4}$, and $x_3 = 1$. The equilibrium payoffs in subgames $\Gamma_i^S(\delta, x_{-S})$ are:

$$
(u_1^*(x_{-S}), u_2^*(x_{-S})) =
\begin{cases}
(\frac{1}{1+\delta} + x_2 + x_3, \frac{\delta}{1+\delta} - \frac{1}{2}(1-x_2) - \frac{1}{4}(1-x_3)) & \text{if } S = \{1\} \text{ and } i = 1 \\
(\frac{\delta}{1+\delta} + x_2 + x_3, \frac{1}{1+\delta} - \frac{1}{2}(1-x_2) - \frac{1}{4}(1-x_3)) & \text{if } S = \{1\} \text{ and } i = 2 \\
(x_1 + x_3, (1-x_1) - \frac{1}{4}(1-x_3)) & \text{if } S = \{2\} \\
(x_1 + x_2, (1-x_1) - \frac{1}{2}(1-x_2)) & \text{if } S = \{3\} \\
(\frac{3-2\delta}{2-\delta^2} + x_3, \frac{(4-3\delta)\delta}{2(2-\delta^2)} - \frac{1}{4}(1-x_3)) & \text{if } S = \{1,2\} \text{ and } i = 1 \\
(\frac{(3-2\delta)\delta}{2-\delta^2} + x_3, \frac{4-3\delta}{2(2-\delta^2)} - \frac{1}{4}(1-x_3)) & \text{if } S = \{1,2\} \text{ and } i = 2. \\
(\frac{7-4\delta}{4-\delta^2} + x_2, \frac{(16-7\delta)\delta}{4(4-\delta^2)} - \frac{1}{2}(1-x_2)) & \text{if } S = \{1,3\} \text{ and } i = 1 \\
(\frac{(7-4\delta)\delta}{4-\delta^2} + x_2, \frac{16-7\delta}{4(4-\delta^2)} - \frac{1}{2}(1-x_2)) & \text{if } S = \{1,3\} \text{ and } i = 2. \\
(x_1, (1-x_1)) & \text{if } S = \{2,3\} \\
(\frac{5}{2}\frac{1}{1+\delta}, \frac{5}{4}\frac{\delta}{1+\delta}) & \text{if } S = \{1,2,3\} \text{ and } i = 1 \\
(\frac{5}{2}\frac{\delta}{1+\delta}, \frac{5}{4}\frac{1}{1+\delta}) & \text{if } S = \{1,2,3\} \text{ and } i = 2.
\end{cases}
$$

The SPE strategies for the game $\Gamma_i(\delta)$ follow. In them, let $S \subseteq L$.

- (i) In the initial period of subgames $\Gamma_i^S(\delta, x_{-S})$, player i makes a proposal on all remaining issues. He offers to player j the payoff $u_j^*(x_{-S})$ and asks for himself $u_i^*(x_{-S})$.

- (ii) In the initial period of subgames $\Gamma_i^S(\delta, x_{-S})$, player j accepts a proposal if and only if the utility u that he would get by accepting (assuming he receives the equilibrium SPE payoff to the first proposer as specified above, for the issues still pending) satisfies $u \geq u_j^*(x_{-S})$.

The uniqueness of the SPE payoff can be proved in the same way as in In and Serrano (1993).

It can be shown, however, that in the restricted agenda bargaining procedure $\gamma_i(\delta)$, there is a large multiplicity of equilibrium agreements, including ones with delay for $\delta < 1$ large enough, which is due to the agenda restriction.

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