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**Intangible Capital and International
Income Differences**

by

Aamir Rafique Hashmi

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Intangible Capital and International Income Differences

Aamir Rafique Hashmi*†

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Abstract

I add intangible capital to a variant of the neoclassical growth model and study the implications for cross-country income differences. I calibrate the parameters associated with intangible capital by using new estimates of investment in intangibles by Corrado et al. (2006). When intangible capital is added to the model, the TFP elasticity of output increases from 2.14 to 2.64. This finding implies that the addition of intangible capital increases the ability of the neoclassical growth model to explain international income differences by more than a factor of two.

Key words: International Income Differences; Intangible Capital;

JEL Classification Codes: O33; O41; O47

*Departments of Economics, National University of Singapore (Email: aamir@nus.edu.sg, Ph:+65-6516-6547)

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1 Introduction

Some intermediate goods continue to be useful in production after the first period of their use. Examples include computer software, output of research and development (R&D) activity, advertisement, management's time spent on promoting the business and expenditure on training of workers and managers.¹ Since most of the intermediate goods in question are intangible and their benefit extends beyond the first period of their use, I shall call the expenditure on these goods *investment in intangibles* and the accumulated value of this investment, after appropriate depreciation, *intangible capital*.² The expenditure on these intermediate goods ought to be treated as investment, but it is not [Corrado et al. (2006)]. In the National Income and Product Accounts (NIPA) of the US, expenditure on these goods is treated as expenditure on intermediate goods and hence not included in the gross domestic product (GDP).³ This is a measurement error.

In this paper, I study the implications of correcting this measurement error for international income differences. I write a one-sector neoclassical growth model that is very similar to the models in Mankiw et al. (1992) and Chari et al. (1996) except that it also includes intangible capital. In this respect the model is similar to the one in Parente & Prescott (1994). I then ask: how much more of the international income variation can the model explain when it is augmented with intangible capital?

The answer to this question depends crucially on values of the parameters associated with intangible capital. In order to calibrate these parameters one needs, among other targets, an estimate of the size of intangible investment relative to the GDP. Until recently, no credible estimate of this investment was available. Earlier studies by Parente & Prescott (1994) and Prescott (1998) speculated that the size of this investment was around 40% and 32% of the GDP. In a recent study, Corrado et al. (2006) (from here on CHS) provide estimates of intangible investment in the US economy for the postwar period. To my knowledge this is the first study that provides scientific estimates of intangible investment at the macro level. They also provide estimates of the depreciation rate of intangible capital. The contribution of the present study is to use these estimates to pin down the parameters associated with intangible capital. The

¹Some of these items are clearly *services* but for the simplicity of exposition I shall call them *goods*.

²Human capital is also intangible. However, in this paper I shall distinguish between human capital and intangible capital. The reason is that many authors have studied the implications for international income differences of adding human capital to the models of growth. In this paper, I want to study the implications for income differences when intangible capital, as defined above, is added to a growth model that already includes human capital.

³The only exception is the expenditure on computer software that has been treated as investment since 1997.

main finding is that the addition of intangible capital more than doubles the ability of the neoclassical growth model to explain international income differences.

It is hardly surprising that the addition of intangible capital to the neoclassical growth model adds to the model's ability to explain international variation in income. This is because a higher share of reproducible factors in output leads to a higher elasticity of output with respect to TFP. What is surprising is the fact that even a much smaller, relative to the earlier studies, estimate of intangible investment, can more than double the model's ability to generate differences in income.

A higher investment in intangible capital implies a greater share for it in the output. This in turn implies that more of the cross-country variation in output is due to factor accumulation and less due to differences in total factor productivity (TFP) or the efficiency with which these factors are used. This last observation relates this paper to what may be called the 'neoclassical revival debate'. In this debate, one group of economists, most prominent among them are Mankiw et al. (1992), argues that an extended version of the neoclassical growth model can explain most of the variation in cross-country output. The other group argues that factor accumulation cannot explain most of the international variation in output and other factors, summed up under the heading of TFP, play a more important role. Important papers in this tradition include Klenow & Rodriguez-Clare (1997), Hall & Jones (1999) and, more recently, Hulten & Isaksson (2007). In this paper I take an intermediate position. On the one hand, I argue that the neoclassical model can explain a lot more variation in cross-country output than is possible without intangible capital in the model. On the other hand, I acknowledge that even with intangible capital in the model, there is some variation in output that the model cannot explain and hence attributes to differences in TFP.

2 The Model

Consider a one-sector neoclassical growth model with three types of capital: physical (K), intangible (Z) and human (H). Time is discrete. The aggregate production function is given by

$$Y_t = A_t K_t^{\theta_k} Z_t^{\theta_z} [(1 - u_{ht} - u_{zt}) H_t]^{\theta_h} L_t^{1 - \theta_k - \theta_z - \theta_h}, \quad (1)$$

where Y_t is output, A_t is total factor productivity (TFP), $1 - u_{ht} - u_{zt}$ is the fraction of human capital used in production. I assume that TFP grows exogenously at rate γ and all per capita

variables grow in the steady state at rate g , which is defined as:

$$g = (1 + \gamma)^{\frac{1}{1-\theta_k-\theta_z-\theta_h}} - 1. \quad (2)$$

From this point on, I shall focus on quantities that are stationary in the steady state. Let $y_t \equiv Y_t/[(1+g)(1+n)]^t$, where n is the population growth rate. Let k_t , z_t and h_t be defined in the same manner. Let $a \equiv A_t/[1+\gamma]^t$. With these new variables, the production function becomes

$$y_t = ak_t^{\theta_k} z_t^{\theta_z} [(1 - u_{ht} - u_{zt})h_t]^{\theta_h}. \quad (3)$$

I next specify laws of motion for the three state variables: k , z and h . The law of motion for physical capital is standard and given by

$$(1+g)(1+n)k_{t+1} = (1-\delta_k)k_t + x_{kt}, \quad (4)$$

where δ_k is the depreciation rate and x_k is the investment in physical capital.

There are two popular approaches to model the accumulation of human capital. According to the first approach, human capital accumulation requires financial investment (see, for example, Mankiw et al. (1992) [equation (9a), p.416] and Chari et al. (1996) [equation (3.3) p.11]). According to the second approach, human capital accumulation is time intensive and hence a fraction of human capital has to be taken out of production and devoted to the accumulation of human capital. Examples of this approach include Lucas (1988) [equation (13), p.19] and Prescott (1998) [p.541]. I combine the two approaches and assume that the accumulation of human capital requires both financial investment as well as time.⁴ The law of motion for human capital is

$$(1+g)(1+n)h_{t+1} = (1-\delta_h)h_t + (u_{ht}h_t)^\psi x_{ht}^\phi, \quad (5)$$

where δ_h is the depreciation rate, u_{ht} is the fraction of human capital devoted to the production of human capital and x_{ht} is the financial investment in the accumulation of human capital.

The law of motion for intangible capital is similar to the one for human capital and is given by

$$(1+g)(1+n)z_{t+1} = (1-\delta_z)z_t + (u_{zt}h_t)^\nu x_z^\eta, \quad (6)$$

where δ_z is the depreciation rate, u_z is the fraction of human capital that is devoted to the

⁴For further details on his human capital technology, see Erosa et al. (2007).

production of intangible capital and x_z is the investment in intangible capital. If I assumed $\nu = 0$ and $\eta = 1$, (6) would be very similar to the law of motion for intangible capital in Parente & Prescott (1994). However, I assume $\nu, \eta > 0$ and do not impose any other restriction on these parameters except the general restriction in (7) below. The inclusion of human capital in the production technology for intangible capital is motivated by the large theoretical and empirical literature that suggests a positive connection between the stock of human capital and technology adoption.⁵ Although human capital is important for producing intangible capital, its inclusion in the law of motion is not critical for the main result. I show in Section 3 that even if the fraction of human capital going into the accumulation of intangible capital is negligible, the main result remains intact.

To ensure overall decreasing returns to accumulable factors, I impose the following restriction on parameters of the model:

$$\frac{1 - \theta_k - \eta\theta_z}{\theta_h + \nu\theta_z} > \frac{\phi}{1 - \psi}, \quad (7)$$

which simplifies to $\theta_k + \theta_h + \theta_z < 1$, if $\phi + \psi = 1$ and $\nu + \eta = 1$.⁶

It is important to note that after the addition of intangible capital, y is no longer the measured output as in the National Income and Product Accounts (NIPA). Instead, it also includes investment in intangible capital. In symbols,

$$y = y_m + x_z, \quad (8)$$

where y_m is the measured output (as in NIPA) and x_z is the investment in intangible capital. I call y the *total output* and y_m the *measured output*.

The total output can be used for either consumption or investment in physical, intangible or human capital. Hence the aggregate resource constraint is

$$c_t = y_t - x_{kt} - x_{ht} - x_{zt}. \quad (9)$$

⁵Here, following Parente & Prescott (1994), I interpret intangible capital as technology capital. Notable papers that relate technology adoption to human capital include Nelson & Phelps (1966), Benhabib & Spiegel (1994), Caselli & Coleman (2001), Chander & Thangavelu (2004), Comin & Hobijn (2004), Benhabib & Spiegel (2005) and Beaudry et al. (2006). Keller (2004), in his survey of the literature on international technology diffusion, lists human capital as one of the most important determinants of technology diffusion.

⁶When I calibrate the parameters, I ignore this restriction and choose the parameters to match the targets. I then check whether the calibrated parameters violate the restriction. For the parameters that I report in this paper, the restriction is never violated.

The social planner chooses the sequence $\{c_t, k_{t+1}, z_{t+1}, h_{t+1}, u_{ht}, u_{zt}\}_{t=0}^{\infty}$, given k_0, z_0 and h_0 , to maximize the present discounted value of utility $u(c_t)$. More specifically the planner's problem is

$$\max_{\{(c_t, k_{t+1}, z_{t+1}, h_{t+1}, u_{ht}, u_{zt})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (10)$$

subject to (3), (4), (5), (6) and (9). I assume CRRA preferences and define the period utility function as

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad (11)$$

where σ is the inverse of the inter-temporal elasticity of substitution.⁷

The steady state equilibrium is a set of allocations $\{c, k, z, h, u_h, u_z\}$ such that, given the constraints, utility is maximized and the steady state variants of (3), (4), (5), (6) and (9) are satisfied.

When the model is solved for its steady state, the steady state level of output is

$$y = ba^\xi, \quad (12)$$

where b is a constant that depends on the parameters of the model and ξ (the TFP elasticity of output) is given by

$$\xi = \frac{1-\psi}{(1-\psi)(1-\theta_k - \eta\theta_z) - \phi(\theta_h + \nu\theta_z)}. \quad (13)$$

I assume that technology and preferences are the same across countries and the only thing that differs is the TFP. Hence b is the same across countries and output of country i relative to that of country j is

$$\frac{y_i}{y_j} = \left(\frac{a_i}{a_j}\right)^\xi. \quad (14)$$

In international income comparisons, ξ is the key parameter. In the next subsection I calibrate the parameters of the model to get some idea about the value of ξ .

2.1 Calibration

I calibrate the parameters of the model such that the steady state of the model is consistent with certain long run features (targets) of the US economy. I report the targets and the calibrated parameters in Table 1 and provide details of the calibration strategy in Appendix A. According

⁷In (10), β is the modified discount factor. Given CRRA preferences, β is equal to $\tilde{\beta}(1+n)(1+g)^{1-\sigma}$, where $\tilde{\beta}$ is the discount factor.

to Heston et al. (2006), from 1950 to 2004, the average population growth rate in the US has been 1.17% and per capita consumption growth rate has been 2.34%. Hence I set $n = 0.0117$ and $g = 0.0234$. I choose β such that the implicit real rate of interest is 5%. I choose σ to be equal to 2. This is on the lower side of the range of values used in the literature.⁸ I assume 8% annual depreciation for physical capital. There is no satisfactory way to pin down δ_h (the depreciation rate of human capital). I follow Mankiw et al. (1992) and Chari et al. (1996) and assume that δ_h is equal to δ_k . I shall say more about this parameter when I do sensitivity checks on my results.

I choose θ_k such that the steady state ratio of investment in physical capital (x_k) to measured output (y_m) is 0.2.

The value of ψ depends on the steady state value of u_h i.e. the fraction of time spent accumulating human capital. I assume this fraction to be equal to the ratio of average years of schooling to average life expectancy. The average years of schooling in the US in 2000 were 12.25 [Barro & Lee (2000)] and the life expectancy at birth was 79. This gives $u_h = 0.155$.

Parameters θ_h and ϕ can be jointly identified using a target for investment in human capital as a fraction of GDP (see (17)). I denote this fraction by ι_h . It is clear from (13) that it is the product of θ_h and ϕ that matters for international income differences. However, for the sake of completeness I use a target for skill premium to identify θ_h separately. I define skill premium as the ratio of the combined share of labor and human capital (i.e. $1 - \theta_k - \theta_z$) to the share of labor (i.e. $1 - \theta_h - \theta_k - \theta_z$). The target value of skill premium is a moot point. What makes it even harder to use it as a target is the fact that it has been rising over time [Krusell et al. (2000)]. However, since this target is not going to affect ξ , it is not very important for the question of interest. I use the ratio of average earnings of workers with a high school diploma to the average earnings of workers without high school as my target for the skill premium. According to Diaz-Gimenez et al. (2002) this ratio in the year 1998 was equal to 2.33 (= \$34,211/\$14,705). This target for the skill premium pins down θ_h (see (18)). I then choose ϕ to match investment in human capital as a fraction of GDP. According to Haveman & Wolfe (1995) this fraction is 12.7%.⁹ Hence I set $\iota_h = 0.127$.

There are four parameters related to intangible capital: δ_z , θ_z , η and ν . I use two targets in CHS and the combined share of labor and human capital in output as the third target to pin

⁸See Ljungqvist & Sargent (2004), p.426 for a discussion on the value of σ .

⁹This includes private as well as public expenditure on children aged 0-18. For details see Table 1 in Haveman & Wolfe (1995).

down the first three of these parameters. My target for the fourth parameter is the fraction of time spent on accumulation of intangible capital (i.e. u_z). Unfortunately no estimate of this target is available. Instead, I try three different values of this target and compare the results.

The first parameter, δ_z , is the depreciation rate of intangible capital. Little is known about it and based on whatever limited information is available, CHS make certain assumptions about the depreciation rate of various components of intangible capital. I use their *estimates* of depreciation rates of the various components of intangible capital and compute a weighted average, where the weight of each component is its share in intangible investment. This gives a depreciation rate of 34%.

The other two parameters, θ_z and η , can be jointly identified by choosing a target for investment in intangible capital as a fraction of measured output (see (19)). I denote this fraction by ι_z . Here I closely follow CHS. Their definition of investment is based on the idea that “any use of resources that reduces current consumption in order to increase it in the future qualifies as an investment”. They distinguish between tangible and intangible investments. In the tangible category they include the usual investments in structures, tools and machinery. For intangibles, they identify three main categories of investment. The first category is *computerized investment* and consists mainly of computer software. The second category is *innovative property*, which is divided into two subcategories. The first subcategory is *scientific R&D* and consists of National Science Foundation’s industrial R&D series. The second subcategory is *non-scientific R&D*, which includes revenues of non-scientific commercial R&D industry, spending for new product development by financial services and insurance firms and cost of development of new product by the entertainment industry. The third category is *economic competencies*. This is also divided into two subcategories. The first subcategory is *brand equity* and consists of a fraction of the advertisement expenditure. The second subcategory is *firm specific resources* and includes a fraction of the cost of employer-provided worker training and management time devoted to enhancing the productivity of the firm.¹⁰ According to the estimates in CHS, average investment in intangibles was 15.7% of the measured output during the period from 2000 to 2003.¹¹

In order to separately identify θ_z and η , I use the combined share of human capital and labor

¹⁰For further details see CHS.

¹¹This estimate, is much lower than 40% or 32% assumed by Parente & Prescott (1994) and Prescott (1998). However, according to CHS, investment in intangible capital has been increasing over time. Hence this estimate cannot be considered a long term observation about the US economy. In the section on sensitivity analysis, I examine the sensitivity of my conclusions to the choice of this target.

in measured output as the second target. In the context of a standard neoclassical model, it is common to assume that the share of physical capital in measured output is around one-third and the remaining two-third is shared by labor and human capital.¹² This is further supported by the finding in Gollin (2002) that the labor share of income is between 65% and 80% in most of the countries. For calibration results in Table 1, I assume a combined share of labor and human capital in output of 65%.¹³ It is important to note that the choice of this target does not affect the value of ξ in (13) because θ_z and η appear as a product in that equation.

The fourth parameter related to intangible capital is ν . It maps into our target for u_z . We do not have any reliable estimate of the time spent in adopting new technology. However, it is most likely to be a small fraction of the total time allocated to production. In the following analysis I assume that five percent of the working time is spent on adoption of new technology (i.e. $u_z = 0.05$). In Section 3 below, I try other values of this target and show that the main result is not sensitive to the value of u_z assumed here.

2.2 International Income Differences

The implications of the model for international income differences depend on the value of ξ . I first assume that there is no investment in intangible capital i.e. $x_z = u_z = 0$. When the model is calibrated for this special case the value of ξ is 2.14. This implies that in order to explain a fortyfold difference in output between the rich and the poor countries, TFP in the former must be 5.62 times higher than that in the latter ($5.62^{2.14} = 40$).¹⁴ In other words, in the absence of intangible capital, the model can magnify a TFP ratio of 5.62 to an output ratio of 40. Here it is instructive to compare the results of the model with some earlier studies. The parameter estimates in Mankiw et al. (1992) imply a value of ξ equal to 2.44. The calibration in Erosa et al. (2007) implies a value of ξ equal to 2.77. The value of ξ from my calibration is lower than what these other studies found. This can be taken care of by assuming a lower depreciation rate for human capital (see the discussion on δ_h in Section 3 below). For example, if I assumed 4% depreciation for human capital, the value of ξ would be equal to 2.44, the same as implied

¹²See, for example, Mankiw et al. (1992).

¹³This is the combined share of human capital and labor out of measured output. If we used total output instead of the measured output, the share would be around 56%.

¹⁴According to Heston et al. (2006), in the year 2000 the ratio of real GDP of the richest 10% countries to that of the poorest 10% countries was 41.5. In this paper, I round this ratio to the nearest tens and study how big are the TFP differences needed to explain a forty fold difference in real output. Throughout the paper, I shall use the phrase ‘TFP ratio’ to refer to the TFP ratio between the rich and the poor countries required to generate fortyfold difference in outputs. Moreover, the phrase ‘observed income differences’ in the paper would mean the fortyfold income differences.

by the parameter estimates in Mankiw et al. (1992). If I assumed 2.5% depreciation for human capital, the value of ξ would be equal to 2.76, which is almost the same as what Erosa et al. (2007) find. However, it is not the absolute value of ξ that is important for the question that I am trying to answer. What is important is the increase in the value of ξ once intangible capital is added to the model.

I next calibrate the full model using the target values of x_z/y_m and u_z as reported in Table 1. For the full model the value of ξ is 2.64.¹⁵ With this value of ξ we need a TFP ratio of 4.05 to explain a fortyfold difference in output. Recall that this ratio was 5.62 if we assumed zero investment in intangible capital. Another way to look at this difference is the following. In the model with no investment in intangibles, a TFP ratio of 5.62 could generate a fortyfold income difference ($5.62^{2.14} = 40$). In the full model, the same TFP ratio can generate a ninety-fivefold income difference ($5.62^{2.64} = 95$). This is the main finding of the paper that the addition of intangible capital to a standard neoclassical growth model, more than doubles the model's ability to explain cross-country income variation.

3 Sensitivity Analysis

I have shown above that the ability of the neoclassical growth model to explain international income differences is significantly improved when intangible capital is added to the model together with physical and human capital. In this section I study the sensitivity of this conclusion to changes in some of the parameters and targets about which, in my opinion, we have less reliable information than others. My strategy for analysis in this section is the following. I pick a parameter (or target), one at a time, and try two or three different values for it other than the one used in the analysis above. I then study what happens to the comparison between the model without intangible capital and the full model at different values of the parameter or the target.

All the relevant numbers are reported in Table 2. The rows in **bold** show the parameter (or target) values and corresponding values of ξ s and TFP ratios used in the analysis above. These are my preferred values. Although in Table 2 I have reported the values of ξ and the TFP

¹⁵For international income comparisons, we are interested in relative measured output i.e. $\frac{y_{mi}}{y_{mj}}$. However, $\frac{y_{mi}}{y_{mj}} = \frac{y_i}{y_j}$. To see this, note that for country i , $y_{mi} = y_i - x_{zi} = ba_i^\xi - b_{xz}a_i^\xi = (b - b_{xz})a_i^\xi$. Likewise for country j , $y_{mj} = (b - b_{xz})a_j^\xi$. Since I have assumed same preferences and technology across countries, b and b_{xz} are the same in all countries. Hence $\frac{y_{mi}}{y_{mj}} = \left(\frac{a_i}{a_j}\right)^\xi = \frac{y_i}{y_j}$.

ratio, in the sensitivity analysis below I shall focus on just one number: the income difference generated by the full model using the TFP ratio of the model without intangible capital. I shall denote this number by y^R to signify that it is the relative income of a rich country compared to that of a poor country under the full model given that the relative income under the model without intangible capital was 40. y^R is reported in the last column of Table 2. The main result in this paper is that y^R is 95 i.e. if the model without intangible capital can generate 40-fold income differences with a certain TFP ratio, the full model can generate 95-fold income differences with the same TFP ratio. When I change a parameter or a target and y^R remains close to 95 or increases above 95, I shall conclude that my main result is robust to the change in the parameter or the target. However, if as a result of a change in a parameter or a target, y^R falls well below 95, I shall conclude that my main result is sensitive to a change in the parameter or the target.

A quick look at the last column of Table 2 shows that in case of a change in the following five parameters (or targets), y^R either increases or does not change much. The parameters (or targets) are: σ , δ_h , ι_h , u_h and u_z . I conclude that the main conclusion of this paper is not sensitive to the choice of these parameters or targets. However, I would still like to comment further on δ_h , i.e. the depreciation rate of human capital.

There is no reliable estimate of δ_h available. Earlier studies have assumed different depreciation rates for human capital. Lucas (1988), for example, assumed zero depreciation. Mankiw et al. (1992) and Chari et al. (1996) assumed that the depreciation rate of human capital was equal to that of physical capital. I have followed the same assumption in this paper. There is a large literature that tries to measure the value of human capital in an economy.¹⁶ There are especially quite a few studies about the US. However, the results are all over the place. At one extreme, some studies conclude that the value of the stock of human capital is the same or even less than the value of the stock of physical capital. At the other extreme, some studies find the stock of human capital to be twenty times as valuable as the stock of physical capital. In view of such inconclusive evidence it is hard to make any precise statement about the relative size of human capital, which could help us pin down the depreciation parameter. Intuitively, it seems highly unlikely that if in a particular period both u_h and x_h were zero, the aggregate stock of human capital in the economy would fall by 8%. In my opinion, the aggregate human capital of a country depreciates at a much lower rate. If this is the case, it will further strengthen

¹⁶See the survey article by Le et al. (2003).

the conclusion that the addition of intangible capital increases the neoclassical growth model's ability to explain international income variation.

There is one parameter (δ_z) and one target (ι_z) in Table 2, which, when changed, can adversely affect the main conclusion of the paper. I comment on each separately. First, I comment on δ_z (the depreciation rate of intangible capital). In the analysis above, I use $\delta_z = 0.34$, based on the estimates in CHS. To arrive at these estimates they use empirical evidence and some educated guesses. If the actual depreciation rate of intangible capital is less than 34%, I am fine because my results are further strengthened.¹⁷ However, if the actual rate is more than 34% my results are somewhat weakened. For example, if I assume full depreciation of intangible capital then $y^R = 79$, which is still almost twice as large as 40. Hence the ability of the neoclassical model still almost doubles. But this leads to another issue. If the depreciation rate of intangible capital is 100%, there is no difference between intangible capital and intermediate goods. If that is the case then why add just a fraction of intermediate goods to the model. Why not add *all* the intermediate goods. This issue is important and needs further comment. I return to it in Section 4 below.

I now comment on the effects of a change in my target for ι_z , the ratio of investment in intangibles to the measured income, on y^R . This target pins down η and θ_z jointly. Following the estimates in CHS, I chose $\iota_z = 0.157$. This is based on their estimates of investment in intangible capital in the US during the period 2000-2003. However, according to CHS, this investment has been rising over time and if we compute the average for the post-WWII period, it is close to 0.10. It is instructive to see how the model fares when a lower target for ι_z is chosen. When ι_z is lowered from 0.157 to 0.10, y^R declines from 95 to 69. Hence the improvement in the model's ability to explain income differences when intangible capital is added to it, depends crucially on the size of investment in intangible capital. This is hardly surprising. In fact, the main point of the paper is that this investment is not too small and hence by excluding intangible capital from the analysis we omit some of the variation in output that the neoclassical model is capable of explaining. Also note that my target value for ι_z is much lower than what Parente & Prescott (1994) and Prescott (1998) assumed. If I assumed $\iota_z = 0.4$, as Parente & Prescott (1994) did, y^R would shoot up to 436.

¹⁷The implicit depreciation rate of intangible capital in Parente & Prescott (1994) is close to 0.03. If I use this depreciation rate, i.e. $\delta_z = 0.03$, the value of ξ increases to 3.85 and y^R jumps to 770.

4 Concluding Remarks

In this paper I construct a variant of the neoclassical growth model to study its implications for international income differences. The model features intangible capital in addition to physical and human capital. I use recent estimates of investment in intangible capital to pin down some key parameters of the model. The main finding is that the addition of intangible capital, to an otherwise standard neoclassical growth model, more than doubles the model's ability to account for cross-country variation in income. Specifically the same TFP ratio that generates a fortyfold income difference in the model without intangible capital can generate a ninety-fivefold difference in income with intangible capital in the model. This result is robust to different parameterizations of the model. However, there are at least two caveats that must be noted.

The first caveat is the following. In a general sense the paper generates this result by adding a fraction of intermediate goods to the neoclassical growth model. The fraction that is added consists of the intermediate goods that do not depreciate away completely in the production process. Here a relevant question is: what if all the intermediate goods produced in the economy are added to the model? This would be like assuming an aggregate production function similar to the one in Romer (1990). In fact this is the main idea behind a recent paper by Jones (2008). The answer, as elaborated by Jones (2008), is that the inclusion of all intermediate goods will increase the value of ξ significantly and the ability of the neoclassical model to generate realistic income differences from very small differences in TFP will improve tremendously.

To study the implications of Jones (2008) for the results of the present paper, I have written down a stripped down version of Jones (2008) in Appendix B. In this model total output is produced by using intermediate goods in addition to the three types of capital, i.e., physical, human and intangible. Recall that 'intangible capital' consists of those intermediate goods that have a less than 100% depreciation rate. 'Intermediate goods' in this model are those that have a depreciation rate of 100%. First, assume that both financial and time investment in intangible capital is zero i.e. $x_z = u_z = 0$. This, in effect, amounts to excluding intangible capital from the model. My calibration results show that the value of ξ for this model would be 4.48. This is huge compared to any thing that we have seen in the literature so far. With ξ this big, we need a TFP ratio of just 2.28 to generate fortyfold income differences. Next, I add intangible capital to the model by allowing both x_z and u_z to be positive. When I recalibrate the model,

using the same targets for x_z and u_z as in Section 2 above, I get a value of ξ equal to 4.91. This is bigger than 4.48 but the difference is not as big as between the values of ξ in the model without intangible capital and the full model. Another way to compare the two versions (the one without intangible capital and the other with intangible capital) of the model in Appendix B is to use the same TFP ratio. If we use a TFP ratio of 2.28, the version without intangible capital can generate fortyfold income differences ($2.28^{4.48} = 40$). With the same TFP ratio the version with intangible capital generates fifty-sevenfold income differences ($2.28^{4.91} = 57$). Hence the addition of intangible capital to a model that already features intermediate goods does not improve the model's ability to generate international income differences by as much as the same addition to a model without intermediate goods does.

The second caveat is about out-of-steady-state dynamics of the model. Given that the combined share of the three types of capital (i.e. $\theta_h + \theta_k + \theta_z$) is 0.76 in the full model, the model exhibits slow transition to steady state. Slow in the sense that it cannot explain growth miracles. However, the convergence rate implied by the model is more in line with the estimates in Barro & Sala-i-Martin (1992), who require a share of capital parameter of around 0.8 to explain the observed rates of convergence.

Nevertheless, the paper clearly shows that the neoclassical growth model can explain a large fraction of cross-country variation in income. It also shows that factor accumulation is more important than previously thought and investment in software, R&D, product promotion etc. is as important as investment in physical and human capital.

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A Calibration Strategy

Let

$$\begin{aligned} D_{1i}(n, g, \beta, \delta_i) &= \beta[(1+g)(1+n) - (1-\delta_i)] \\ D_{2i}(n, g, \beta, \delta_i) &= [(1+g)(1+n) - \beta(1-\delta_i)], \end{aligned}$$

where $i = \{h, k, z\}$. Also let $\iota_i \equiv x_i^{SS}/y_m^{SS}$, where SS in the superscript refers to steady-state values. I now describe the calibration strategy in some detail.

The targets of population growth rate, per capita consumption growth rate and depreciation of physical capital match one-to-one with parameters n , g and δ_k . Parameter σ is chosen from the empirical literature. Parameter β , the modified discount rate, depends on n , g , σ and $\tilde{\beta}$, where $\tilde{\beta} = 1/(1+r)$ and r is the target real interest rate. Specifically, $\beta = \tilde{\beta}[(1+g)(1+n)]^{1-\sigma}$.

I pick θ_k to match the target for ι_k . From the steady-state solution of the model,

$$\theta_k = \frac{\iota_k}{1 + \iota_z} \frac{D_{2k}}{D_{1k}}, \quad (15)$$

because

$$\frac{x_k^{SS}}{y^{SS}} = \frac{x_k^{SS}/y_m^{SS}}{y^{SS}/y_m^{SS}} = \frac{\iota_k}{1 + \iota_z}.$$

I pick ψ to match the steady-state target for u_h , the fraction of time spent on accumulating human capital. From the steady-state of the model,

$$\psi = u_h^{SS} \frac{D_{2h}}{D_{1h}}. \quad (16)$$

Once I have determined ψ , I pick ϕ and θ_h jointly to match the steady-state target for x_h/y_m . The steady-state of the model gives,

$$\phi\theta_h = \frac{\iota_h}{1 + \iota_z} \frac{D_{2h}}{D_{1h}} (1 - u_h - u_z). \quad (17)$$

It is important to point out that for the question of interest, it is the product $\phi\theta_h$ that matters. However, in order to identify ϕ and θ_h separately, I pick θ_h to match some empirical estimate of the skill premium (SP). To do so, I define the SP as the ratio of the combined share of

human capital and labor to the share of labor in output, i.e.

$$SP = \frac{1 - \theta_k - \theta_z}{1 - \theta_k - \theta_h - \theta_z}.$$

This gives,

$$\theta_h = (1 - \theta_k - \theta_z) \left(1 - \frac{1}{SP}\right). \quad (18)$$

I pick δ_z to match the target depreciation rate of intangible capital. Parameters η and θ_z are picked jointly to match the target investment in intangible capital. The steady state solution of the model gives

$$\eta\theta_z = \frac{\iota_z}{1 + \iota_z} \frac{D_{2z}}{D_{1z}}. \quad (19)$$

Once again, it is important to note that it is the product $\eta\theta_z$ that matters for the question of interest. However, the two parameters can separately be identified using as target the combined share of human capital and labor in the measured output. This share is defined as

$$\text{SHC} = (1 - \theta_k - \theta_z)(1 + \iota_z), \quad (20)$$

where SHC is the combined share of human capital and labor in the measured output. (20) gives the following value for θ_z .

$$\theta_z = 1 - \theta_k - \frac{\text{SHC}}{1 + \iota_z}. \quad (21)$$

The last parameter is ν . To pin it down my target is the fraction of time spent accumulating intangible capital (u_z). Given this target the following expression solves for ν .

$$\nu = \frac{u_z}{1 - u_h - u_z} \frac{\theta_h}{\theta_z} \frac{D_{2z}}{D_{1z}}.$$

B A Model with Intermediate Goods

Consider the following variant of the model in the main text. The production function is given by

$$y_t = a \left(k_t^{\theta_k} [(1 - u_{ht} - u_{zt}) h_t]^{\theta_h} \right)^{1-\mu} \left(z_t^{\theta_z} m_t^{1-\theta_z} \right)^\mu, \quad (22)$$

where m denotes the intermediate good component of the final good used in production. Intermediate goods depreciate fully in production. Hence the law of motion for m is given by

$$(1 + g)(1 + n)m_{t+1} = x_{mt}, \quad (23)$$

where x_{mt} is investment in intermediate goods. In words, this is the part of the final output that is used as intermediate input in the production of final output in the next period. The laws of motion for other state variables, namely h , k and z are the same as in the model of the main text. The steady-state output is

$$y = ba^\xi, \quad (24)$$

where ξ is given by

$$\xi = \frac{1 - \psi}{(1 - \psi)[1 - \theta_k(1 - \mu) - \mu(1 - \theta_z) - \eta\mu\theta_z] - \phi[\theta_h(1 - \mu) + \nu\mu\theta_z]}. \quad (25)$$

The only new parameter in this model is μ . Following Jones (2008), I set it equal to 0.5. I then ask the following question: if all the investment in intermediate goods is treated as x_m , as had been done in the NIPA until recently, what would be the value of ξ ? The answer is: 4.48. With this value of ξ , we need a TFP ratio of 2.28 to explain forty-fold income differences. I next ask, if some part of the investment in intermediate goods is treated as x_z , as suggested by CHS, what would be the value of ξ ? The answer is: 4.91 (based on $x_z/y_m = 0.157$). With this value of ξ , the required TFP ratio is 2.12.

Table 1: Calibrated Parameter Values and Targets

Parameter	Value	Target	Target Value
g	0.02	Growth in p.c. consumption	2.34%
n	0.01	Growth in population	1.17%
β	0.94	Real interest rate	5%
σ	2.00	Empirical literature	-
δ_k	0.08	Empirical literature	-
δ_h	0.08	Same as δ_k	-
θ_k	0.27	x_k/y_m	0.20
θ_h	0.32	Skill premium	2.33
ψ	0.24	u_h	0.155
ϕ	0.42	x_h/y_m	0.127
δ_z	0.34	Estimates in Corrado et al. (2006)	-
θ_z	0.17	Combined share of L and H in Y_m	0.65
η	0.94	x_z/y_m	0.157
ν	0.14	u_z	0.05
ξ	2.64		

Table 2: Sensitivity Analysis

Sensitivity Parameter or Target	Model w/o z		Full Model		y^R
	ξ_0	$TFPR_0$	ξ_F	$TFPR_F$	
$\sigma =$					
2.0	2.14	5.62	2.64	4.05	95
2.5	2.35	4.79	2.97	3.46	105
3.0	2.63	4.06	3.42	2.94	121
$\delta_h =$					
0	6.61	1.75	9.73	1.46	232
0.04	2.44	4.53	3.05	3.35	100
0.08	2.14	5.62	2.64	4.05	95
0.16	1.97	6.53	2.41	4.62	92
$\delta_z =$					
1.00	2.14	5.62	2.53	4.30	79
0.34	2.14	5.62	2.64	4.05	95
0.17	2.14	5.62	2.79	3.75	124
0.08	2.14	5.62	3.10	3.29	211
$\iota_h =$					
0.127	2.14	5.62	2.64	4.05	95
0.086	1.85	7.31	2.26	5.10	90
0.054	1.68	8.97	2.04	6.10	88
$\iota_z^* =$					
0.400	2.14	5.62	3.52	2.85	436
0.157	2.14	5.62	2.64	4.05	95
0.100	2.14	5.62	2.45	4.50	69
$u_h =$					
0.100	2.09	5.82	2.58	4.18	94
0.155	2.14	5.62	2.64	4.05	95
0.200	2.18	5.43	2.70	3.93	96
$u_z^* =$					
0.01	2.14	5.62	2.63	4.08	94
0.05	2.14	5.62	2.64	4.05	95
0.10	2.14	5.62	2.65	4.02	97

* In the case of model without intangible capital, $\iota_z = u_z = 0$.

Note: $\xi_0 = \xi$ in the model without intangible capital

$TFPR_0 =$ TFP ratio in the model without intangible capital

$\xi_F = \xi$ in the full model

$TFPR_F =$ TFP ratio in the full model

$y^R = TFPR_0^{\xi_F}$