Ordered Response Models for Sovereign Debt Ratings[*]

António Afonso, [$] [#] *Pedro Gomes,* [+] *and Philipp Rother* [#]

[$] UECE – Research Unit on Complexity in Economics; Department of Economics, ISEG/TULisbon - Technical University of Lisbon, R. Miguel Lupi 20, 1249-078 Lisbon, Portugal.
[#] European Central Bank, Directorate General Economics, Kaiserstraße 29, D-60311 Frankfurt am Main, Germany
[+] London School of Economics & Political Science.

Abstract

Using ordered logit and probit plus random effects ordered probit approaches, we study the determinants of sovereign debt ratings. We found that the last procedure is the best for panel data as it takes into account the additional cross-section error.

JEL: C25; E44; F30; G15
Keywords: ordered probit; ordered logit; random effects ordered probit; sovereign rating

---

1. Introduction

Sovereign debt ratings are forward-looking qualitative measures of the probability of default, given in the form of a code. As they are a qualitative ordinal measure, the most suitable approach to understand their determinants is an ordered response framework (see for example Hu et al., 2002; Bissoondoyal-Bheenick, 2005). However, this framework is not optimal in that its properties are only valid asymptotically, so if we estimate the determinants of the ratings using a cross-section of countries, we would have too few observations. Therefore, it is imperative to try to maximize the number of observations by using a panel data set. This poses its own difficulties as there is a country-specific error which makes the generalization of ordered probit and ordered logit to panel data is not completely straightforward.

We compare three possible estimation procedures suitable for panel data: ordered probit and ordered logit with a robust variance-covariance matrix, and random effects ordered probit. Although the three procedures are valid, the latter should be considered the best one for panel data as it considers the existence of an additional normally distributed cross-section error. In order to solve the possible problem of correlation of the errors and the regressors, we model the country-specific error, which in practical terms implies adding time averages of the explanatory variables as additional time-invariant regressors. Moreover, our panel data set includes information on rating notations for two of the main rating agencies (Standard & Poor's and Moody's), covering 66 countries between 1996 and 2005.

2. Methodology

The setting is the following. Each rating agency makes a continuous evaluation of a country's credit worthiness, embodied in an unobserved latent variable R^*

$$R_{it}^* = \beta X_{it} + \lambda Z_i + a_i + \mu_{it} . \tag{1}$$

This latent variable has a linear form and depends on X_{it}, which is a vector containing time-varying explanatory variables, and Z_i, a vector of time-invariant variables.

In (1) the index i ($i=1,\dots,N$) denotes the country, the index t ($t=1,\dots,T$) indicates the period, and a_i stands for the country-specific error. Additionally, it is assumed that the disturbances μ_{it} are independent across countries and across time. To deal with possible correlation between the variables in X_{it} we model the error term a_i, as described in Wooldridge (2002) and used by Hajivassiliou and Ioannides (2006). The idea is to express explicitly the correlation between the error and the regressors, stating that the expected value of the country-specific error is a linear combination of the time averages of the regressors $\overline{X}_i$:

$$E(a_i \mid X_{it}, Z_i) = \eta \overline{X}_i . \tag{2}$$

If we modify our initial equation (1) with $a_i = \eta \overline{X}_t + \varepsilon_i$ we obtain

$$R_{it}^* = \beta X_{it} + \lambda Z_i + \eta \overline{X}_i + \varepsilon_i + \mu_{it} , \tag{3}$$

3

where ε_i is an error term by definition uncorrelated with the regressors. In practical terms, we eliminate the problem by including a time average of the explanatory variables as additional time-invariant regressors. We can write our full model as[1]

$$R_{it}^* = \beta(X_{it} - \overline{X}_i) + (\eta + \beta)\overline{X}_i + \lambda Z_i + \varepsilon_i + \mu_{it}. \tag{4}$$

Because there is a limited number of rating categories, the rating agencies will have several cut-off points that draw up the boundaries of each rating category. The final rating will then be given by[2]

$$R_{it} = \begin{cases} AAA & if & R_{it}^* > c_{16} \\ AA+ & if & c_{16} > R_{it}^* > c_{15} \\ AA & if & c_{15} > R_{it}^* > c_{14} \\ \vdots & & \\ < CCC+ & if & c_1 > R_{it}^* \end{cases} \tag{5}$$

The parameters of equation (4) and (5), notably $\beta,\ \eta,\ \lambda$ and the cut-off points c_1 to c_{16} are estimated using maximum likelihood. Since we are working in a panel data setting, the generalization of ordered probit and ordered logit is not straightforward, because instead of having one error term, we now have two. Wooldridge (2002) describes two approaches that can be followed to estimate this model. One "quick and dirty" possibility is to assume we only have one error term that is serially correlated within countries. Under that assumption, one can either do the normal ordered probit

---

[1] By estimating this specification one can interpret β as the short-run impact of the variable on the rating, while $(\beta + \eta)$ gives the long-run effect of a change in the variable on the rating.

[2] We grouped the ratings in 17 categories, by assigning linearly a value of 17 to the best rating, AAA, a value of 2 to B- and a value of 1 to all observations below B-. If we used a specific number for each existing rating notch, it might be hard to efficiently estimate the threshold points between CCC+ and CCC, CCC and CCC- and so on, given that the bottom rating categories have very few observations.

estimation, using a robust variance-covariance matrix estimator to account for the serial correlation, or alternatively we can assume a logistic distribution. The second possibility is to use a random effects ordered probit model, which considers both errors ε_i and μ_{it} to be normally distributed, and accordingly maximizes the log-likelihood. Of the two approaches, the second is the best one, although a drawback the quite cumbersome calculations involved. In what follows, we use the procedure created for STATA by Rabe-Hesketh et al. (2000) and substantially improved by Frechette (2001a, 2001b).

3. Estimation results

We identify the following relevant determinants of sovereign ratings: GDP per capita, real GDP growth, inflation, unemployment, government debt, the fiscal balance, government effectiveness, external debt, foreign reserves, the current account balance, default history, regional dummies and a European Union dummy. Fiscal and external stock and flow variables are used as GDP ratios. The variables of inflation, unemployment, GDP growth, the fiscal balance and the current account entered as a three-year average, reflecting the rating agencies' approach of removing the effects of the business cycle when deciding on a sovereign rating. "Government effectiveness" is a World Bank indicator that measures the quality of public service delivery. The external debt variable was taken from the World Bank and is only available for non-industrial countries, so for industrial countries the value 0 has been used, which is equivalent to using a multiplicative dummy. As for the dummy variable for the European Union, the variable enters with two leads. Default history is assessed by a dummy if the country has defaulted since 1980. We also included a dummy for industrialised countries and another for Latin America and Caribbean countries. Regarding the ratings data, we use the sovereign foreign currency rating attributed by

the two main rating agencies between 1996 and 2005. Data sources comprise the rating agencies and the International Monetary Fund, the World Bank and the Inter-American Development Bank for the explanatory variables.

Table 1 reports the estimation results. In the random effects ordered probit, more variables show up as significant: seven for Moody's, and nine for S&P. This is because the standard deviations are considerably smaller in these methods, in comparison to the other two approachs. The signs of the coefficients are consistent across the estimations.

[Insert Table 1 here]

We evaluate the performance of the three models by focusing on two elements: the prediction for the rating of each individual observation in the sample, and the prediction of movements in the ratings through time. Table 2 presents an overall summary of the prediction errors.

[Insert Table 2 here]

For Moody's we see that the three models are quite similar in predicting the level of rating. Roughly, 45 per cent of the observations are predicted correctly, 80 per cent are predicted within a notch and 95 per cent within 2 notches. For S&P the models perform quite similarly in correctly predicting the rating, but with a higher percentage of predictions within a notch.

Let us now turn to how the models perform in predicting changes in ratings. Table 3 presents the total number of sample upgrades (downgrades), the predicted number of upgrades (downgrades) and the number of upgrades (downgrades) that were correctly predicted by the three models.

[Insert Table 3 here]

Generally, the models correctly predict half of both upgrades and downgrades in the period they actually occurred, and a third more in the following period. The most noticeable difference between the models is not the number of corrected predicted changes, but the total number of predicted changes. In fact, the ordered probit and ordered logit predict more changes than the random effects ordered probit. For instance, for Moody's the first two models predict around 120 upgrades, whereas the random effects ordered probit only predicts 95, the actual number being 58.

4. Conclusion

We have compared three procedures to estimate the determinants of sovereign ratings under an ordered response framework: ordered probit, ordered logit and random effects ordered probit. Of the three, the most efficient method is the random effects ordered probit estimation is the more efficient method, since a considerable number of variables show up as significant that are not picked up using the other two methods. Even though in terms of predicting the ratings, all three methods show similar performance in anticipating changes in ratings, nevertheless the random effects ordered probit slightly outperforms the other two specifications.

Acknowledgements

References

Bissoondoyal-Bheenick, E. (2005). An analysis of the determinants of sovereign ratings. *Global Finance Journal* 15 (3), 251-280.

Frechette, G. (2001a). sg158: Random-effects ordered probit, *Stata Technical Bulletin* 59, 23-27. Reprinted in *Stata Technical Bulletin Reprints* 10, 261-266.

Frechette, G. (2001b). sg158.1: Update to random-effects ordered probit, *Stata Technical Bulletin* 61, 12. Reprinted in *Stata Technical Bulletin Reprints* 10, 266-267.

Hajivassiliou, V. and Ioannides, Y. (2006). Unemployment and liquidity constraints. *Journal of Applied Econometrics* (forthcoming).

Hu, Y.-T.; Kiesel, R. and Perraudin, W. (2002). The estimation of transition matrices for sovereign credit ratings. *Journal of Banking & Finance* 26 (7), 1383-1406.

Rabe-Hesketh, S.; Pickles, A. and Taylor, C. (2000). sg120: Generalized linear latent and mixed models. *Stata Technical Bulletin* 53, 47-57. Reprinted in *Stata Technical Bulletin Reprints* 9, 293-307.

Wooldridge, J. (2002). *Econometric Analysis of Cross Section and Panel Data*. MIT Press.

Table 1 – Estimation results

	Moody's			S&P		
	Ord. Probit	Ord. Logit	RE Ord. Probit	Ord. Probit	Ord. Logit	RE Ord. Probit
GDP per capita	1.940***	3.688***	3.422***	1.716***	3.119***	3.246***
	(0.4290)	(0.8852)	(0.3640)	(0.4284)	(0.8303)	(0.3598)
GDP per capita Avg.	0.418	0.713	0.478***	0.252	0.525	1.117***
	(0.2977)	(0.6203)	(0.1743)	(0.3346)	(0.6944)	(0.1851)
GDP growth	2.977	2.563	6.464**	2.613	1.833	5.979*
	(5.1545)	(10.5702)	(3.1390)	(3.6591)	(7.5128)	(3.0989)
GDP growth Avg.	-0.382	-1.633	-9.387**	-8.511	-12.451	-8.430*
	(7.9649)	(17.0063)	(4.6024)	(8.4668)	(18.4477)	(4.6987)
Unemployment	-0.066	-0.119	0.016	-0.020	-0.027	0.152***
	(0.0462)	(0.0940)	(0.0325)	(0.0364)	(0.0680)	(0.0333)
Unemployment Avg.	-0.049*	-0.095*	-0.078***	-0.038	-0.079	0.002
	(0.0263)	(0.0504)	(0.0178)	(0.0273)	(0.0490)	(0.0161)
Inflation	-0.402***	-0.667**	-0.199	-0.515***	-0.942	-0.353**
	(0.1563)	(0.2836)	(0.1408)	(0.2149)	(0.8981)	(0.1398)
Inflation Avg.	-0.464***	-0.748***	-0.623***	-0.621***	-1.106	-0.532***
	(0.1472)	(0.2797)	(0.1553)	(0.2308)	(0.8771)	(0.1559)
Gov Debt	-0.010	-0.012	-0.030***	-0.024***	-0.042**	-0.085***
	(0.0099)	(0.0221)	(0.0066)	(0.0086)	(0.0173)	(0.0071)
Gov Debt Avg.	-0.018***	-0.034***	-0.026***	-0.014**	-0.026*	-0.027***
	(0.0061)	(0.0132)	(0.0036)	(0.0064)	(0.0145)	(0.0031)
Gov Balance	6.727	14.104	13.898***	5.617	12.625	10.187***
	(4.6787)	(9.7663)	(3.7162)	(4.0955)	(8.8900)	(3.3166)
Gov Balance Avg.	3.843	5.498	6.757*	4.001	6.893	8.873**
	(7.1680)	(16.3911)	(3.6739)	(8.1905)	(19.7903)	(3.6894)
Gov Effectiveness	0.293	0.670	0.223	0.220	0.445	0.707**
	(0.2963)	(0.5809)	(0.3464)	(0.2723)	(0.5641)	(0.3392)
Gov Effectiveness Avg.	1.781***	3.086***	3.679***	2.185***	3.803***	4.606***
	(0.3265)	(0.6670)	(0.2734)	(0.3877)	(0.7869)	(0.2825)
External Debt	-0.010***	-0.019***	-0.004**	-0.003	-0.005	-0.002
	(0.0025)	(0.0048)	(0.0016)	(0.0023)	(0.0049)	(0.0021)
External Debt Avg.	-0.005***	-0.009***	-0.004***	-0.006**	-0.008	-0.008***
	(0.0019)	(0.0036)	(0.0013)	(0.0024)	(0.0053)	(0.0012)
Current Account	-8.477***	-15.468**	-8.570***	-7.094**	-13.004**	-4.899**
	(3.2849)	(6.5507)	(2.3683)	(3.0914)	(6.0111)	(2.4020)
Current Account Avg.	4.085	7.693	5.240**	5.939	9.905	18.390***
	(3.5886)	(6.6209)	(2.3725)	(3.9311)	(7.8865)	(2.5493)
Reserves	1.879***	3.262***	2.246***	0.716	1.551	0.205
	(0.5373)	(1.1355)	(0.5141)	(0.5411)	(1.1398)	(0.4914)
Reserves Avg.	0.833	1.469	0.416	1.449	1.963	3.365***
	(0.9723)	(2.1427)	(0.4745)	(1.0258)	(2.1484)	(0.4847)
Def 1	-1.119***	-1.953***	-3.101***	-0.923**	-1.803**	-1.789***
	(0.3238)	(0.6529)	(0.2546)	(0.3994)	(0.9177)	(0.2224)
EU	1.146***	2.097**	2.197***	0.914**	1.428*	0.324
	(0.3792)	(0.8217)	(0.2430)	(0.4475)	(0.8240)	(0.2084)
IND	1.547**	2.912*	3.554***	2.088***	4.196***	3.923***
	(0.7050)	(1.5431)	(0.4609)	(0.6473)	(1.3509)	(0.4799)
LAC	-0.830**	-1.533**	-1.766***	-0.621*	-1.243*	-1.485***
	(0.3696)	(0.7548)	(0.2495)	(0.3616)	(0.7134)	(0.2326)
Log-Likelihood	-700.56	-706.84	-566.33	-743.24	-740.17	-514.46
Observations	551	551	551	564	564	564
Countries	66	66	66	65	65	65

Notes: The standard deviations are in parentheses. *, **, *** - statistically significant at the 10, 5, and 1 per cent. When estimating (4) the variables enter the estimation as differences from the country average. An additional regressor for each variable, which represents the time-average within a country, is represented with Avg.

Table 2 – Summary of prediction errors

	Estimation Procedure	Obs.	Prediction error (notches)											% Correctly predicted	% Within 1 notch *	% Within 2 notches **
			5	4	3	2	1	0	-1	-2	-3	-4	-5			
Moody's	Ordered Probit	551	1	5	12	39	92	**259**	88	50	5	0	0	47.0%	79.7%	95.8%
	Ordered Logit	551	5	2	13	35	98	**259**	90	45	4	0	0	47.0%	81.1%	95.6%
	RE Ordered Probit	551	0	8	20	43	104	**258**	74	28	15	1	0	46.8%	79.1%	92.0%
S&P	Ordered Probit	564	0	4	18	25	110	**251**	117	28	11	0	0	44.5%	84.8%	94.1%
	Ordered Logit	564	0	6	18	17	115	**257**	116	26	9	0	0	45.6%	86.5%	94.1%
	RE Ordered Probit	564	1	3	19	38	111	**204**	147	35	6	0	0	36.2%	81.9%	94.9%

Notes: * prediction error within +/- 1 notch. ** prediction error within +/- 2 notches.

Table 3 – Upgrades and downgrades prediction

		Sample Upgrades	Predicted Upgrades	Upgrades correctly predicted at time		Sample Downgrades	Predicted Downgrades	Downgrades correctly predicted at time	
				t	t+1			t	t+1
Moody's	Ordered Probit	58	119	25	18	34	70	17	10
	Ordered Logit	58	122	25	21	34	73	17	11
	RE Ordered Probit	58	95	25	16	34	61	16	8
S&P	Ordered Probit	79	105	34	16	41	72	18	16
	Ordered Logit	79	105	38	20	41	71	20	14
	RE Ordered Probit	79	102	35	20	41	66	18	12