Market Value Maximization through Strategic Delegation

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Abstract

In this paper, we develop a model of strategic delegation in which shareholders maintain an objective of market value maximization (MVM) of the firm's assets as measured by a capital asset pricing model (CAPM). Optimal delegation requires that managers maximize a linear combination of expected profits and firm values. An interesting feature of this model is that optimal delegation contracts of the MVM objective mitigate competition relative to standard price and quantity duopoly outcomes. In the MVM model, the delegation encourages managers to control systematic risk, which leads to greater market coordination, higher profits, and higher stock values. Impacts of degree of product differentiation on delegation under price and quantity competitions are also explored extensively. Our findings show that concerns about identifying the mode of competition are overstated in the literature.

Keywords: Market Value Maximization, Strategic Delegation, Quantity Competition, Price Competition, Product Differentiation.

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1 Introduction

The vast majority of industrial organizational theory is constructed on a rather simplistic premise that a firm's first priority is to maximize profits. In practice, however, firm managers driven by equity-based incentive packages are more apt to focus equally on multiple objectives, including profitability, stability of profits, and creating conditions to foster strong anticipated growth of profits. When viewed through the objective of equity market valuation, different market outcomes, interpretations, and policy-relevant factors begin to emerge.

This paper investigates the firm owner's delegation strategies and manager's pricing and production decisions building from a model of equity value maximization in an imperfect differentiated product market. Beginning with Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987), most studies of strategic delegation assume that the manager's objective function is based on a linear combination of profits and product revenue (or its variant, sales), derived from an owner's objective to maximize profits in an oligopoly setting. By providing an incentive package, the owners can manipulate the manager's behavior, commit to strategies in the product market, and gain a strategic advantage.

The strategic delegation literature has since branched out significantly to cover various topics of interest, for example, merger incentive (Ziss, 2001; Gonzalez-Maestre and Lopez-Cunat, 2001; Banal-Estanol and Ottaviani, 2006), multiproduct firm incentive (Barcena-Ruiz and Espinosa, 1999; Moner-Colonques et al., 2004), wage bargaining (Szymanski, 1994; Conlin and Furusawa, 2000), and relative performance measure (Fumas, 1992; Aggarwal and Samwick, 1999; Miller and Pazgal, 2001, 2005). While the delegation literature has focused primarily on the equilibrium incentives through the internal payment scheme, it has provided little guidance as to the proper

specification of the firm's objective function.

In this paper, we develop a model of strategic delegation in which shareholders maintain an objective of market value maximization (MVM) of the firm's assets as measured by a capital asset pricing model (CAPM). In this context, it requires that managers maximize a linear combination of expected profits and firm values. An interesting feature of this model is that optimal delegation contracts of the MVM objective mitigate competition in both price and quantity games compared with the results obtained from the stylized profit maximization objective. In the MVM model, the delegation encourages managers to control systematic risk, which leads to greater market coordination, higher profits, and higher stock values.

The above result is different from what is generally agreed that the delegation strategy depends critically on knowing the mode of competition in the product market. As Fershtman and Judd (1987) among others point out, the owners in the delegation studies are essentially joint Stackelberg leaders and thus obtain the prisoner's dilemma outcome in the sense that they would be better off not engaging in quantity games involving delegation of a market share incentive. On the other hand, in the case of strategic complements, the delegated incentives work to increase the owner's expected profits. Similar conclusions can be drawn from, for example, Eaton and Grossman (1986) in a strategic trade context, Brander and Lewis (1986) and Showalter (1995) in the model of strategic capital structure. Departing from Singh and Vives (1984), Stiegert and Wang (2006) explore the diminishing differences between outcomes in quantity and price competitions through advertising. Miller and Pazgal (2001, 2005) illustrate the equivalence of price and quantity competitions by a relative performance measure, in which managers are given an incentive scheme based on a weighted sum of the firm's own profit and its rival's profit. However, knowing the rival's profit is difficult in practice. Instead of the relative performance approach, by

combining own MVM and profit-maximizing objectives, this paper demonstrates that the disparity of equilibrium outcomes in quantity and price competitions is smaller with delegation than without delegation in the MVM model and the differences under different modes of competition are far less than those derived from the profit-maximizing objective only. Given that the products are differentiated, moreover, impacts of degree of product differentiation on delegation and shareholders' strategic motives in price and quantity games are explored extensively. We show that the mentioned differences can be ignored when the products are sufficiently differentiated.

The remainder of the paper is organized as follows. Section 2 examines a model of a two-stage game in which the shareholders decide manager's combined objective of firm's anticipated profit and CAPM-styled value of equity in the first period, and managers compete in either quantity or price in the second period. We compare quantity-setting and price-setting equilibrium outcomes in three different scenarios, including general MVM, typical MVM, and profit maximization objectives. Concluding remarks and suggestions for future research are offered in section 3.

2   The Model

A model built on the concept of asset value maximization necessarily involves a framework for dealing with uncertainty. We assume a financial market characterized by the Sharpe-Lintner equilibrium. That is,

$$E(\tilde{r}_i) = r + \beta_i \left[E(\tilde{r}_m) - r \right], \tag{1}$$

where r is the risk-free interest rate, $E(\tilde{r}_i)$ and $E(\tilde{r}_m)$ are expected rates of return of asset i and market portfolio, respectively, while β_i is systematic risk or market risk defined by $Cov(\tilde{r}_i, \tilde{r}_m)/Var(\tilde{r}_m)$. The firm i's market value can be obtained by $V_i = \tilde{\pi}_i/(1 + \tilde{r}_i)$, where $\tilde{\pi}_i$ is the stochastic cash flow of net earnings.

The objective function of MVM firm can be easily derived. Because $\widetilde{\pi}_i = (1+\tilde{r}_i)V_i$,

$$
\begin{aligned}
\frac{E(\widetilde{\pi}_i)}{V_i} &= 1 + E(\tilde{r}_i) = 1 + r + \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{Var(\tilde{r}_m)}[E(\tilde{r}_m) - r] \\
&= 1 + r + \left[\frac{E(\tilde{r}_m) - r}{Var(\tilde{r}_m)}\right]\frac{Cov(\widetilde{\pi}_i, \tilde{r}_m)}{V_i}.
\end{aligned}
\tag{2}
$$

Rearranging (2) yields firm i's "market value" objective function:

$$
V_i = \frac{1}{1+r}\left[E(\widetilde{\pi}_i) - \lambda Cov(\widetilde{\pi}_i, \widetilde{r}_m)\right],
\tag{3}
$$

where λ is the equilibrium shadow price of market risk reduction, defined by $[E(\widetilde{r}_m) - r]/\sigma_m^2$ and $\sigma_m^2 = Var(\tilde{r}_m)$.

The model in this study is essentially a two-stage sequential duopoly game. In the first period, the owners (shareholders) of each firm delegate the product market decision to managers by properly arranging a linear combination of the firm's anticipated profit and the CAPM-styled value of equity. In the second period, the manager of each firm decides the quantity to produce or the price to charge in the product market. Following the setting of strategic delegation, like Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987), the objective function facing manager i is given by

$$
M_i = (1 - \theta_i)E\Pi_i + \theta_i V_i,
\tag{4}
$$

where θ_i is an incentive parameter chosen by shareholders in firm i, $E\Pi_i = E(\widetilde{\pi}_i)/(1+r)$, and V_i is defined in (3).[1] By rearranging (4), we have

$$
M_i = \frac{1}{1+r}\left[E(\widetilde{\pi}_i) - \theta_i\lambda Cov(\widetilde{\pi}_i, \widetilde{r}_m)\right].
\tag{5}
$$

Note that if $\theta_i = 1$, we observe a full incorporation of the CAPM-styled financial objectives. It turns out that equation (1) facing the managers in the general MVM

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[1]In general, the manager's objective function should be $A + B \times M_i$, where A and B are constant and $B > 0$. Both A and B are irrelevant to the product market decisions.

framework can be rewritten as

$$E(\tilde{r}_i) = r + \theta_i \beta_i \left[E(\tilde{r}_m) - r \right], \tag{6}$$

where θ_i may be interpreted as to how the market risk is actually priced by shareholders in firm i relative to the standard CAPM framework in which $\theta_i = 1$.

Assume that each firm faces uncertain demand and the same constant marginal cost (c) is known with certainty. Both firms' revenues are subject to a random shock that neither can observe when the strategic variables are chosen. As a result, firm i's total revenue is given by

$$\tilde{R}_i = p_i X_i (1 + \tilde{e}), E(\tilde{e}) = 0, Var(\tilde{e}) = \sigma_e^2, \tag{7}$$

where p_i is the price and the random variable $\tilde{e}$ is an idiosyncratic shock on the revenue of firm i. Without loss of generality, it is assumed to have mean of zero. σ_e is the standard deviation of the shock. It is further assumed for every demand curve that the support of the noise is small enough so that negative revenue never occurs. Thus, the expected net earnings is

$$E(\tilde{\pi}_i) = E\left[p_i X_i (1 + \tilde{e}) \right] - X_i c = p_i X_i - X_i c.$$

Moreover, because $Cov(\tilde{\pi}_i, \tilde{r}_m) = Cov(\tilde{e}, \tilde{r}_m) p_i X_i$,

$$V_i = \frac{p_i X_i (1 - \lambda Cov(\tilde{e}, \tilde{r}_m)) - X_i c}{1 + r} = \frac{\phi p_i X_i - X_i c}{1 + r} = \frac{\phi X_i (p_i - d)}{1 + r}, \tag{8}$$

where certainty equivalent $\phi = 1 - \lambda Cov(\tilde{e}, \tilde{r}_m) = 1 - \lambda \rho \sigma_e \sigma_m$ and ρ is the correlation coefficient between the revenue shock and the return on market portfolio. In general, $\phi \in [0, 1]$ and $d = c/\phi$ adjusted marginal cost, provided that $\phi \neq 0$.[2] To keep the model as concise as possible, we focus on the positive ρ.

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[2]We may incorporate cost uncertainty in equation (8) by defining ψ as a certainty equivalent parameter on the cost side. For a strictly convex cost function, $\psi > 1$. After redefining $d = c\psi/\phi$, equation (8) still holds true as long as two sources of uncertainty are independent of each other. The new adjusted marginal cost is higher than the original one. However, because we assume the cost function is linear, we need not adjust the cost uncertainty.

By a parallel logic,

$$M_i = \frac{\phi_i X_i (p_i - d_i)}{1 + r}, \text{ where } \phi_i = 1 - \theta_i \lambda \rho \sigma_e \sigma_m, \text{ and } d_i = c/\phi_i. \quad (9)$$

To simplify the analysis and rule out possible counter-intuitive results, we focus on the case that $\theta_i < 1/(\lambda \rho \sigma_e \sigma_m)$ and $\lambda \rho \sigma_e \sigma_m \neq 0$.

Suppose further that firms face a linear inverse demand function[3] given by (10) in the product market.

$$p_i = \alpha - b X_i - \gamma X_j, \ b \geq \gamma \geq 0, \ i, j = 1, 2, \ i \neq j. \quad (10)$$

$\gamma \geq 0$ implies directly the case of substitutes and $b \geq \gamma$ implies that the own effect (b) is at least as large as the cross effect (γ). In addition, we assume $\alpha > d$.

We also define $\delta = \gamma/b$ to model the degree of (horizontal) product differentiation.[4] The more differentiated the products ($\delta \downarrow$), the smaller the effect of change in quantity (price) of brand j on the price (quantity) of brand i. Note that by assumption $0 \leq \delta \leq 1$. Therefore, (10) can be rewritten as

$$p_i = \alpha - b(X_i + \delta X_j), \ 0 \leq \delta \leq 1, \ i, j = 1, 2, \ i \neq j. \quad (11)$$

Before we proceed further analyses, lemma 1 is very useful to determine the range of θ.

Lemma 1. (a) $d_i \gtreqless d \Longleftrightarrow \theta_i \gtreqless 1$ and (b) $d_i \gtreqless c \Longleftrightarrow \theta_i \gtreqless 0$.

Proof. By definition, $d_i = d \Longleftrightarrow \theta_i = 1$ and $d_i = c \Longleftrightarrow \theta_i = 0$. By assumptions $\theta_i < 1/(\lambda \rho \sigma_e \sigma_m)$ and $\phi = 1 - \lambda \rho \sigma_e \sigma_m \neq 0$, with monotonicity that

$$\frac{\partial d_i}{\partial \theta_i} = \frac{\partial}{\partial \theta_i} \frac{c}{1 - \theta_i \lambda \rho \sigma_e \sigma_m} = \frac{\lambda \rho \sigma_e \sigma_m c}{(1 - \theta_i \lambda \rho \sigma_e \sigma_m)^2} > 0,$$

lemma 1 is proved. □

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[3]See also Dixit (1979), Singh and Vives (1984), and Vives (1999).
[4]Note that the definition here differs from the common setting seen in, for example, Shy (1995).

By the one-to-one mapping for θ_i and d_i, Lemma 1 allows us to focus on the adjusted marginal cost d_i. We may easily characterize the properties of incentive parameter θ_i via a simple transformation from d_i.

The product markets are assumed to be under both price and quantity competitions. The two-stage game is solved by backward induction. We first explore the quantity competition.

Quantity Competition

Under quantity competition, managers in both firms choose quantity strategies simultaneously, taking each other's strategy as a given. In the second period, manager i faces the objective function

$$M_i = \frac{\phi_i}{1+r} X_i(p_i - d_i) = \frac{\phi_i}{1+r} X_i \left[\alpha - b(X_i + \delta X_j) - d_i \right]. \tag{12}$$

Taking a derivative with respect to X_i and rearranging yield

$$2X_i + \delta X_j = \frac{\alpha - d_i}{b}. \tag{13}$$

Equation (13) implicitly defines manager i's reaction function. Impacts of an increase in d_i under quantity competition are depicted in Figure 1(a). As we can see, MVM delegation strategies mitigate competition relative to the traditional Cournot benchmark. Similarly, we can get manager j's reaction function. Solving for optimal quantity and price yields

$$X_i^c = \frac{\alpha(2 - \delta) - 2d_i + \delta d_j}{b(2 + \delta)(2 - \delta)}, \quad p_i^c = \frac{\alpha(2 - \delta) + (2 - \delta^2)d_i + \delta d_j}{(2 + \delta)(2 - \delta)}. \tag{14}$$

Therefore, the value of the firm facing the shareholders in quantity competition becomes

$$
\begin{aligned}
V_i^c &= \frac{\phi X_i^c (p_i^c - d)}{1 + r} \tag{15} \\
&= \frac{\phi}{1+r} \left[\frac{\alpha(2 - \delta) - 2d_i + \delta d_j}{b(2 + \delta)(2 - \delta)} \right] \left[\frac{\alpha(2 - \delta) + (2 - \delta^2)d_i + \delta d_j}{(2 + \delta)(2 - \delta)} - d \right].
\end{aligned}
$$

The reaction function[5] for shareholders in firm i is

$$4(2 - \delta^2)d_i + \delta^3 d_j = (2 - \delta)\left[2(2 + \delta)d - \alpha\delta^2\right]. \tag{16}$$

We may write equilibrium d_i in quantity competition as

$$d^c = \frac{2(2 + \delta)d - \alpha\delta^2}{4 + 2\delta - \delta^2} = d - \frac{(\alpha - d)\delta^2}{4 + 2\delta - \delta^2}. \tag{17}$$

Therefore, equilibrium quantity, price, and firm value in quantity competition can be given by

$$\begin{aligned}
X^c &= \frac{2(\alpha - d)}{b(4 + 2\delta - \delta^2)}, \quad p^c = \frac{(2 - \delta^2)\alpha + 2(1 + \delta)d}{4 + 2\delta - \delta^2}, \\
V^c &= \frac{\phi(\alpha - d)^2}{b(1 + r)} \frac{2(2 - \delta^2)}{(4 + 2\delta - \delta^2)^2}.
\end{aligned}$$

We are interested in the range of θ in the current quantity competition. As a result, proposition 1 follows.

Proposition 1. (a) $\theta^c \leq 1$.

(b) $\theta^c \geq 0$ if

$$c \geq \frac{\alpha\phi\delta^2}{2(2 + \delta)(1 - \phi) + \phi\delta^2} \equiv \underline{c}. \tag{18}$$

Proof. (a) By subtracting d from d^c, we get

$$d^c - d = \frac{-(\alpha - d)\delta^2}{4 + 2\delta - \delta^2} \leq 0,$$

because $\alpha > d$ and $0 \leq \delta \leq 1$. Since $d^c \leq d$, by lemma 1(a), $\theta^c \leq 1$.

(b) To get $\theta^c \geq 0$, we need $d^c \geq c$ by lemma 1(b). Thus,

$$\begin{aligned}
d^c - c &= d^c - d + (1 - \phi)d = \frac{-(\alpha - d)\delta^2}{4 + 2\delta - \delta^2} + (1 - \phi)d \\
&= \frac{1}{4 + 2\delta - \delta^2}\left[-\delta^2\alpha + \frac{2(2 + \delta)(1 - \phi) + \phi\delta^2}{\phi}c\right] \\
&\geq 0, \text{ by condition in (18).}
\end{aligned}$$

[5]The choice variable should be θ_i. However, as proved in lemma 1, d_i is increasing monotonically in θ_i as long as $\theta_i \neq 1/(\lambda\rho\sigma_e\sigma_m)$.

□

Proposition 1 demonstrates that managers are given incentive contracts emphasizing both firm values and expected profits in the quantity competition, and the incentive parameter depends on the marginal cost-demand intercept ratio (c/α). By proposition 1(b), the minimal cost $\underline{c}$ for $\theta^c \geq 0$ depends on the degree of product differentiation provided that all other parameters stay the same. As shown in Figure 2, the more differentiated the product $(\delta \downarrow)$, the less minimal cost $(\underline{c} \downarrow)$ required; i.e., $\partial \underline{c}/\partial \delta \geq 0$. For the cases that $c < \underline{c}$, we have $\theta^c < 0$ and the shareholders do not appreciate firm's market value; instead, they direct more emphasis toward the profits. If production costs are too low, shareholders may not care about the risk component $\lambda Cov(\widetilde{\pi}_i, \widetilde{r}_m)$, the difference between the expected profit and firm's market value. That is, the cost of risk is negligibly small in this case. When the products are less differentiated $(\delta \uparrow)$ and market is more competitive, the managers are stimulated to be more aggressive by being assigned a small or even negative θ. For a given production cost, therefore, the managers may have a positive (negative) θ when the product is more (less) differentiated. We will assume that (18) holds throughout the paper and therefore $0 \leq \theta^c \leq 1$.

Impacts of degree of product differentiation on optimal delegation are examined in the following proposition.

Proposition 2. $\partial \theta^c/\partial \delta \leq 0$, $\partial d^c/\partial \delta \leq 0$, $\partial X^c/\partial \delta \leq 0$, $\partial p^c/\partial \delta \leq 0$, and $\partial V^c/\partial \delta \leq 0$.

Proof. By $\partial d_i/\partial \theta_i > 0$, for $\partial \theta^c/\partial \delta \leq 0$, we only need to show $\partial d^c/\partial \delta \leq 0$. Thus,

$$
\begin{aligned}
\frac{\partial d^c}{\partial \delta} &= \frac{-2(\alpha - d)(4 + \delta)\delta}{(4 + 2\delta - \delta^2)^2} \leq 0, \\
\frac{\partial X^c}{\partial \delta} &= \frac{-4(\alpha - d)(1 - \delta)}{b(4 + 2\delta - \delta^2)^2} \leq 0,
\end{aligned}
$$

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$$\frac{\partial p^c}{\partial \delta} = \frac{-2(\alpha - d)(2 + 2\delta + \delta^2)}{(4 + 2\delta - \delta^2)^2} \leq 0,$$

$$\frac{\partial V^c}{\partial \delta} = \frac{-4\phi(\alpha - d)^2}{b(1+r)} \frac{(4 + \delta^3)}{(4 + 2\delta - \delta^2)^3} \leq 0.$$

□

By rewriting (11), $p_i = \alpha - b(X_i + \delta X_j) = (\alpha - b\delta X_j) - bX_i$. It implies that as the product becomes more differentiated ($\delta \downarrow$), the residual demand facing firm i increases, which leads to more emphasis on MVM objective, more output, higher price, and higher firm value. Proposition 2 is straightforward because the product market is less competitive when the product is more differentiated.

Let us define equilibrium quantity, price, and firm value under stylized equilibrium market value maximization ($\theta = 1$) and profit maximization ($\theta = 0$) to be X_m^c, p_m^c, V_m^c and X_p^c, p_p^c, V_p^c, respectively. We have

$$X_m^c = \frac{\alpha - d}{b(2 + \delta)}, \ p_m^c = \frac{\alpha + (1+\delta)d}{2 + \delta}, V_m^c = \frac{\phi(\alpha - d)^2}{b(1+r)} \frac{1}{(2 + \delta)^2}; \quad (19)$$

$$X_p^c = \frac{\alpha - c}{b(2 + \delta)}, \ p_p^c = \frac{\alpha + (1+\delta)c}{2 + \delta}, \quad (20)$$

$$V_p^c = \frac{(\alpha - c)}{b(1+r)} \frac{\{\phi\alpha - [(2 - \phi) + (1 - \phi)\delta]\,c\}}{(2 + \delta)^2}.$$

Now, holding the differentiation parameter constant, comparing equilibrium quantity, price, and firm value in three different scenarios leads to proposition 3.

Proposition 3. Suppose that (18) holds, $X_m^c \leq X^c \leq X_p^c$, $p_m^c \geq p^c \geq p_p^c$, and $V_m^c \geq V^c \geq V_p^c$.

Proof. Let us define equilibrium outcomes in quantity competition as

$$X_e^c = X_e^c(d_e^c) = \frac{\alpha - d_e^c}{b(2 + \delta)}, \ p_e^c = p_e^c(d_e^c) = \frac{\alpha + (1+\delta)d_e^c}{2 + \delta},$$

$$V_e^c = V_e^c(d_e^c) = \frac{\phi}{1+r} \frac{\alpha - d_e^c}{b(2 + \delta)} \left[\frac{\alpha + (1+\delta)d_e^c}{2 + \delta} - d \right],$$

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where $d_e^c = d^c, d$, and c for $\theta = \theta^c, 1$, and 0, respectively. For quantity and price, it is easy to see $\partial X_e^c / \partial d_e^c < 0$ and $\partial p_e^c / \partial d_e^c > 0$. By proposition 1, we have $d \geq d^c \geq c$. Therefore, $X_m^c \leq X^c \leq X_p^c$ and $p_m^c \geq p^c \geq p_p^c$. For firm value,

$$\frac{\partial V_e^c}{\partial d_e^c} = \frac{\phi \left[\delta(\alpha - d) - 2(1 + \delta)(d_e^c - d)\right]}{b(1 + r)(2 + \delta)^2}.$$

Therefore,

$$\frac{\partial V_e^c}{\partial d_e^c} > 0, \text{ for } d_e^c \leq d + \frac{\delta(\alpha - d)}{2(1 + \delta)} \equiv \overline{d}_e^c.$$

Because $d \geq d^c \geq c$, we have $\overline{d}_e^c \geq d \geq d^c \geq c$. This completes the proof. □

Proposition 3 shows that when shareholders maximize firm's market values, optimal delegation mitigates product market competition in a quantity-setting game compared with the results obtained from the stylized profit maximization objective; that is, $X^c \leq X_p^c$, $p^c \geq p_p^c$, and $V^c \geq V_p^c$. Another interesting feature of the model emerges in the comparison of stylized MVM ($\theta = 1$) with optimal delegation ($\theta = \theta^c$). Here, optimal delegation leads to a more competitive outcome. The story here parallels that of Fershtman and Judd (1987) (and also Eaton and Grossman, 1986), which we discuss in the concluding comments of this section.

Price Competition

Turning now to competition in price space, the corresponding demand function is

$$X_i = \frac{1}{b(1 - \delta^2)} \left[\alpha(1 - \delta) - p_i + \delta p_j\right]. \tag{21}$$

In the second period, manager i maximizes the objective function

$$M_i = \frac{\phi_i}{1 + r} X_i (p_i - d_i) = \frac{\phi_i}{1 + r} \frac{1}{b(1 - \delta^2)} \left[\alpha(1 - \delta) - p_i + \delta p_j\right] (p_i - d_i). \tag{22}$$

As a result, manager i's reaction function in price competition is

$$2p_i - \delta p_j = \alpha(1 - \delta) + d_i. \tag{23}$$

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Impacts of an increase in d_i on reaction functions are depicted in Figure 1(b). Solving for optimal price and quantity yields

$$p_i^b = \frac{\alpha(1-\delta)(2+\delta) + 2d_i + \delta d_j}{(2+\delta)(2-\delta)}, \quad X_i^b = \frac{\alpha(1-\delta)(2+\delta) - (2-\delta^2)d_i + \delta d_j}{b(1-\delta^2)(2+\delta)(2-\delta)}. \quad (24)$$

In the first period, the shareholders' objective function is

$$
\begin{aligned}
V_i^b &= \frac{\phi X_i^b(p_i^b - d)}{1+r} \\
&= \frac{\phi}{1+r}\left[\frac{\alpha(1-\delta)(2+\delta) - (2-\delta^2)d_i + \delta d_j}{b(1-\delta^2)(2+\delta)(2-\delta)}\right]\left[\frac{\alpha(1-\delta)(2+\delta) + 2d_i + \delta d_j}{(2+\delta)(2-\delta)} - d\right].
\end{aligned}
\tag{25}
$$

The reaction function for shareholders in firm i is

$$4(2-\delta^2)d_i - \delta^3 d_j = \alpha(1-\delta)(2+\delta)\delta^2 + (2-\delta^2)(4-\delta^2)d. \quad (26)$$

We may write equilibrium d_i in price competition as

$$d^b = \frac{\alpha(1-\delta)\delta^2 + (2-\delta^2)(2-\delta)d}{4 - 2\delta - \delta^2} = d + \frac{(\alpha - d)(1-\delta)\delta^2}{4 - 2\delta - \delta^2}. \quad (27)$$

Therefore, equilibrium quantity, price, and firm value in price competition are given by

$$
\begin{aligned}
X^b &= \frac{(2-\delta^2)(\alpha - d)}{b(1+\delta)(4 - 2\delta - \delta^2)}, \quad p^b = \frac{2(1-\delta)\alpha + (2-\delta^2)d}{4 - 2\delta - \delta^2}, \\
V^b &= \frac{\phi(\alpha - d)^2}{b(1+r)}\frac{2(1-\delta)(2-\delta^2)}{(1+\delta)(4 - 2\delta - \delta^2)^2}.
\end{aligned}
$$

From (27) we have proposition 4.

Proposition 4. $\theta^b \geq 1$.

Proof. By (27) and assumptions that $\alpha > d$ and $0 \leq \delta \leq 1$, we have

$$d^b - d = \frac{(\alpha - d)(1-\delta)\delta^2}{4 - 2\delta - \delta^2} \geq 0.$$

Together with lemma 1(a), $\theta^b \geq 1$. $\qquad\square$

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Under price competition, those managers pursuing profit maximization are penalized ($1 - \theta^b \leq 0$). The overcompensation for firm's market value ($\theta^b \geq 1$) can be interpreted as shareholders tax on the manager through the risk-bearing. The tax disciplines the manager and prevents him from being too aggressive in his pricing strategy (Fershtman and Judd, 1987). From (6) and proposition 4, furthermore, shareholders overprice the market risk and mitigate price competition in the product market by setting a large θ.

Similar to proposition 2, we have proposition 5.

Proposition 5. $\partial p^b / \partial \delta \leq 0$ and $\partial V^b / \partial \delta \leq 0$.

Proof.

$$
\begin{aligned}
\frac{\partial p^b}{\partial \delta} &= \frac{-2(\alpha - d)[1 + (1-\delta)^2]}{\left(4 - 2\delta - \delta^2\right)^2} \leq 0, \\
\frac{\partial V^b}{\partial \delta} &= -\frac{4\phi(\alpha - d)^2}{b(1+r)} \frac{\left\{(1-\delta)[3 + (1-\delta^2)^2] + \delta^3\right\}}{(1+\delta)^2 \left(4 - 2\delta - \delta^2\right)^3} \leq 0.
\end{aligned}
$$

□

Proposition 5 shows that the product market is less competitive when the product is more differentiated ($\delta \downarrow$). Unlike $\partial X^c / \partial \delta \leq 0$, $\partial X^b / \partial \delta$ is negative with small δ, but positive with large δ.[6] This implies that under price competition, managers will seek to produce less as products become more differentiated in a relatively homogeneous product market (large δ). When products are sufficiently differentiated to begin with, managers will react to greater differentiating by increasing output and raising price. While we have no conclusion on $\partial d^b / \partial \delta$ (or $\partial \theta^b / \partial \delta$), the difference ($d^b - d$ or $\theta^b - 1$), which can be interpreted as shareholders' strategic motives, varying with δ has some

---

[6]This U-shaped relationship of X^b and δ can be seen by examining firm i's residual demand in equation (21) as well.

interesting implications. We will investigate more on this feature under quantity and price competitions in proposition 7.

Let us define equilibrium quantity, price, and firm value under stylized equilibrium market value maximization ($\theta = 1$) and profit maximization ($\theta = 0$) to be X_m^b, p_m^b, V_m^b and X_p^b, p_p^b, V_p^b, respectively, under price competition. We have

$$
\begin{aligned}
X_m^b &= \frac{\alpha - d}{b(1+\delta)(2-\delta)}, p_m^b = \frac{\alpha(1-\delta)+d}{2-\delta}, V_m^b = \frac{\phi(\alpha-d)^2}{b(1+r)}\frac{1-\delta}{(1+\delta)(2-\delta)^2}; \quad (28) \\
X_p^b &= \frac{\alpha - c}{b(1+\delta)(2-\delta)}, p_p^b = \frac{\alpha(1-\delta)+c}{2-\delta}, \quad (29) \\
V_p^b &= \frac{(\alpha - c)}{b(1+r)}\frac{[\phi(1-\delta)\alpha - (2-\phi-\delta)c]}{(1+\delta)(2-\delta)^2}.
\end{aligned}
$$

By comparing equilibrium quantity, price, and firm value in three different scenarios, we have proposition 6.

Proposition 6. $X^b \leq X_m^b \leq X_p^b$, $p^b \geq p_m^b \geq p_p^b$, and $V^b \geq V_m^b \geq V_p^b$.

Proof. Let us define equilibrium outcomes in price competition as

$$
\begin{aligned}
X_e^b &= X_e^b(d_e^b) = \frac{\alpha - d_e^b}{b(1+\delta)(2-\delta)}, \quad p_e^b = p_e^b(d_e^b) = \frac{\alpha(1-\delta)+d_e^b}{2-\delta}, \\
V_e^b &= V_e^b(d_e^b) = \frac{\phi}{1+r}\frac{\alpha - d_e^b}{b(1+\delta)(2-\delta)}\left[\frac{\alpha(1-\delta)+d_e^b}{2-\delta} - d\right],
\end{aligned}
$$

where $d_e^b = d^b, d$, and c for $\theta = \theta^b, 1$, and 0, respectively. For quantity and price, it is easy to see $\partial X_e^b/\partial d_e^b < 0$ and $\partial p_e^b/\partial d_e^b > 0$. Thus, from proposition 4 and $d \geq c$, we have $X^b \leq X_m^b \leq X_p^b$ and $p^b \geq p_m^b \geq p_p^b$. For firm value,

$$
\frac{\partial V_e^b}{\partial d_e^b} = \frac{\phi\left[\delta(\alpha - d) - 2(d_e^b - d)\right]}{b(1+r)(1+\delta)(2-\delta)^2}.
$$

Therefore,

$$
\frac{\partial V_e^b}{\partial d_e^b} > 0, \text{ for } d_e^b \leq d + \frac{\delta(\alpha - d)}{2} \equiv \bar{d}_e^b.
$$

Because $d^b \geq d \geq c$, all we need to show is $\overline{d}_e^b \geq d^b$.

$$\overline{d}_e^b - d^b = \frac{\delta(\alpha - d)(2 - \delta)^2}{2(4 - 2\delta - \delta^2)} \geq 0.$$

This completes the proof. □

Proposition 6 shows that similar to the results in quantity competition: when shareholders maximize firm's market values, optimal delegation mitigates product market competition in a price-setting game compared with the results obtained from the stylized profit maximization objective; that is, $X^b \leq X_p^b$, $p^b \geq p_p^b$, and $V^b \geq V_p^b$. Optimal delegation mitigates competition even more than stylized MVM ($\theta = 1$). The shareholders have even higher firm value ($V^b \geq V_m^b$) by delegating product market decision to managers and the delegation gain for shareholders is ($V^b - V_m^b$) because the choice variable, price in the product market is a strategic complement. See Figure 1(b) for more details.

Comparison of Quantity and Price Competitions

In this section we compare strategic motives of shareholders and equilibrium outcomes under quantity and price competitions. In the discussion of proposition 5, we mention that ($d^b - d$) can represent shareholders' strategic motives and this implication can be extended to the case of quantity competition. If there exist no strategic concerns, intuitively, the MVM shareholders will select adjusted marginal cost d by setting $\theta = 1$ for either quantity or price competition. By subtracting d from d^c or d^b, we can measure the intensity of shareholders' strategic motives to manipulate mangers' product market decisions. Proposition 7 examines how shareholders' strategic motives vary with the degree of product differentiation.

Proposition 7. Shareholders have no strategic motives when (1) $\delta = 0$ (monopoly, quantity and price competitions coincide), and (2) $\delta = 1$ (standard Bertrand compe-

tition). However, shareholders have strongest strategic motives when $\delta = 1$ (standard Cournot competition).

Proof. From (17) and (27), we have

$$\Delta^c(\delta) \equiv d^c - d = \frac{-(\alpha - d)\delta^2}{4 + 2\delta - \delta^2}, \qquad \frac{\partial \Delta^c}{\partial \delta} = -\frac{2\delta(4 + \delta)(\alpha - d)}{\left(4 + 2\delta - \delta^2\right)^2}; \tag{30}$$

$$\Delta^b(\delta) \equiv d^b - d = \frac{(\alpha - d)(1 - \delta)\delta^2}{4 - 2\delta - \delta^2}, \quad \frac{\partial \Delta^b}{\partial \delta} = \frac{\delta(8 - 14\delta + 4\delta^2 + \delta^3)(\alpha - d)}{\left(4 - 2\delta - \delta^2\right)^2}. \tag{31}$$

For $\delta \in [0, 1]$, Δ^c is monotonically decreasing in δ and $\Delta^c(0) = 0$ and $\Delta^c(1) = -(\alpha - d)/5$ from (30). From (31), $\Delta^b(0) = \Delta^b(1) = 0$. Note that $\Delta^c(0) = \Delta^b(0) = \Delta^b(1) = 0$ for cases of no strategic motives as $d^c = d$ and $d^b = d$. Moreover, we know that Δ^b and $\partial \Delta^b / \partial \delta$ are continuous and differentiable in domain $[0, 1]$, $\partial \Delta^b(0^+)/\partial \delta > 0$ and $\partial \Delta^b(1)/\partial \delta < 0$. Thus, there exists at least one maximum. Though it is available for an analytical solution to $\max_\delta \Delta^b(\delta)$, $\forall \delta \in [0, 1]$, we only present it numerically for the illustrative purpose. The maximal Δ^b is $0.0731(\alpha - d)$ when $\delta = 0.778$, which can be referred to Figure 3. It turns out that $|\Delta^c(1)| > |\Delta^b(0.778)|$. □

The graphs of $\Delta^c(\delta)$ and $\Delta^b(\delta)$ are depicted in Figure 3. When $\delta = 0$, the product markets are separate, each firm is a monopolist in its own market, and quantity and price decisions coincide. In this case, shareholders have no incentives to act strategically and all they need to do is have managers to maximize firm's market values. The interesting case is that when products are homogeneous ($\delta = 1$), shareholders have opposite strategies under quantity and price competitions. For the case of homogeneous products, we arrive at standard Cournot and Bertrand competitions. The standard Bertrand outcome is essentially a perfect competition, and therefore, shareholders have no incentives to deviate from the typical MVM objective. However, in the standard Cournot competition, we have shown that shareholders have most strategic concerns, in which the difference $(d^c - d)$ is largest.

We further compare the equilibrium outcomes under different modes of competition. By looking at propositions 3 and 6, we have no conclusion on comparisons of equilibrium outcomes between profit-maximizing quantity competition (i.e., X_p^c, p_p^c, V_p^c) and general MVM price competition (i.e., X^b, p^b, V^b) because they depend on the certainty equivalent measure ϕ. For a large ϕ, it is more competitive in the general MVM price game and more likely to get $X_p^c \leq X^b$, $p_p^c \geq p^b$, and $V_p^c \geq V^b$. In what follows, instead, we are interested in the comparison of delegation equilibria under quantity and price competitions.

Proposition 8. $X^c \leq X^b$, $p^c \geq p^b$, and $V^c \geq V^b$.

Proof. We directly compute the corresponding differences:

$$
\begin{aligned}
X^c - X^b &= \frac{2(\alpha - d)}{b(4 + 2\delta - \delta^2)} - \frac{(2 - \delta^2)(\alpha - d)}{b(1 + \delta)(4 - 2\delta - \delta^2)} \qquad (32) \\
&= \frac{-\delta^4(\alpha - d)}{b(1 + \delta)(4 + 2\delta - \delta^2)(4 - 2\delta - \delta^2)} \leq 0, \\
p^c - p^b &= \frac{(2 - \delta^2)\alpha + 2(1 + \delta)d}{4 + 2\delta - \delta^2} - \frac{2(1 - \delta)\alpha + (2 - \delta^2)d}{4 - 2\delta - \delta^2} \qquad (33) \\
&= \frac{\delta^4(\alpha - d)}{(4 + 2\delta - \delta^2)(4 - 2\delta - \delta^2)} \geq 0, \\
V^c - V^b &= \frac{\phi(\alpha - d)^2}{b(1 + r)} \frac{2(2 - \delta^2)}{(4 + 2\delta - \delta^2)^2} - \frac{\phi(\alpha - d)^2}{b(1 + r)} \frac{2(1 - \delta)(2 - \delta^2)}{(1 + \delta)(4 - 2\delta - \delta^2)^2} \qquad (34) \\
&= \frac{\phi(\alpha - d)^2}{b(1 + r)} \frac{4(2 - \delta^2)\delta^5}{(1 + \delta)(4 + 2\delta - \delta^2)^2(4 - 2\delta - \delta^2)^2} \geq 0.
\end{aligned}
$$

(32)-(34) complete the proof. $\qquad\square$

Proposition 8 shows that the delegation price competition is more competitive than the delegation quantity competition, though the former is less competitive than the stylized MVM game from proposition 6. Together propositions 3 and 6 with 8, we have $X_m^c \leq X^c \leq X^b \leq X_m^b$, $p_m^c \geq p^c \geq p^b \geq p_m^b$, and $V_m^c \geq V^c \geq V^b \geq V_m^b$. The differences of equilibria between quantity and price competitions are smaller

under optimal delegation. From proposition 8, we also notice that the differences decrease rapidly when the products are more differentiated and these differences are not important if the products are sufficiently differentiated.

In Fershtman and Judd (1987), the shareholders' objective is to maximize profits leading to a delegation strategy of a linear combination of profits and revenues. The result is that under quantity competition the firm is actually worse off relative the standard Cournot outcome, while it is better off under price competition. Their finding ushered in the critique about the sensitivity of these models to the mode of conduct. However, in the current MVM framework, where managers maximize a linear combination of firm values and expected profits, the firm is better off under both quantity and price competitions relative to the results with the profit maximization objective. We further show that the differences of delegation equilibrium outcomes under different modes of competition can be ignored when the products become more differentiated. It suggests that concerns about identifying the mode of competition have been overstated in the literature.

3   Concluding Remarks

In this paper, we developed a model of equity value maximization that allows shareholders to manipulate manager's incentives by strategically arranging manager's objective scheme in a duopoly framework. In a two-stage setting, the shareholders decide manager's combined objective of firm's anticipated profits and CAPM-styled firm values in the first stage, and managers compete in either quantity or price in the second stage. This study makes several interesting findings.

First, according to equation (6), incentive parameter θ represents how the market risk is actually priced by shareholders relative to the standard CAPM framework. In

propositions 1 and 4, optimal delegation contracts of the MVM objective imply that the market risk is less priced ($0 \leq \theta^c \leq 1$) under the quantity competition, but over priced ($\theta^b \geq 1$) under the price competition. The shareholders encourage managers to control market risk through delegation.

Second, impacts of degree of product differentiation on optimal delegation are examined in propositions 2 and 5. In general, the product market is less competitive when the product is more differentiated. While the product differentiation has monotonic influences on the incentive parameter, quantity produced, price charged, and firm value in quantity competition; however, it only has the same effects on price and firm value in price competition. The impacts of differentiation on incentive parameter and quantity are complicated under price competition.

Third, propositions 3 and 6 compare equilibrium quantity, price, and firm value in three different scenarios, including MVM delegation, stylized CAPM ($\theta = 1$), and profit maximization ($\theta = 0$) for both quantity and price games. The results indicate that strategic delegation of the MVM objective mitigates competition in both price and quantity games compared with the results obtained from the typical profit maximization objective. By manipulating the incentive parameter and allowing managers to control the market risk, the optimal delegation leads to greater market coordination, higher profits, and higher stock values.

Fourth, we explore the differences between delegation equilibrium outcomes under quantity and price competitions. The intensity of shareholders' strategic motives to manipulate mangers' product market decisions is explored in proposition 7. While there exist no strategic motives in the monopoly case, where quantity and price decisions coincide; an interesting case occurs when products are homogeneous. In the homogeneous product market, shareholders have no incentives to deviate from the typical MVM objective in the standard Bertrand competition, but have strongest

incentives in the standard Cournot competition. In addition, we also demonstrate that the delegation price competition is more competitive than the delegation quantity competition in proposition 8. The disparity of equilibrium outcomes in quantity and price games is smaller with delegation than without delegation. We further show that the mentioned difference can be ignored when the products are sufficiently differentiated. It implies that delegating optimally is more important than knowing the modes of competition in the product market.

There are some interesting extensions for future research on strategic delegation in the MVM framework. Without delegation, Stiegert and Wang (2006) explore the endogenous determination of degree of product differentiation through advertising. It is worth working on the delegation case. Furthermore, the theories presented in this paper can be tested empirically. An application of strategic delegation under price competition in the MVM framework is the focus of a companion study on the U.S. margarine and butter markets (Wang, Stiegert, and Dhar, 2006).

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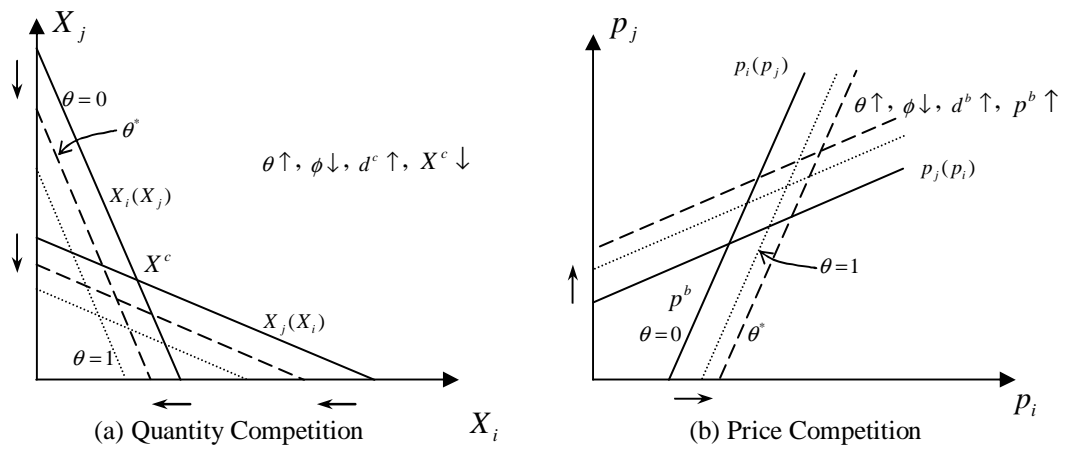
(a) Quantity Competition

X_j

$\theta = 0$

θ^*

$X_i(X_j)$

X^c

$X_j(X_i)$

$\theta = 1$

$\theta\uparrow,\ \phi\downarrow,\ d^c\uparrow,\ X^c\downarrow$

X_i

(b) Price Competition

p_j

$p_i(p_j)$

$\theta\uparrow,\ \phi\downarrow,\ d^b\uparrow,\ p^b\uparrow$

$p_j(p_i)$

$\theta = 1$

p^b

$\theta = 0$

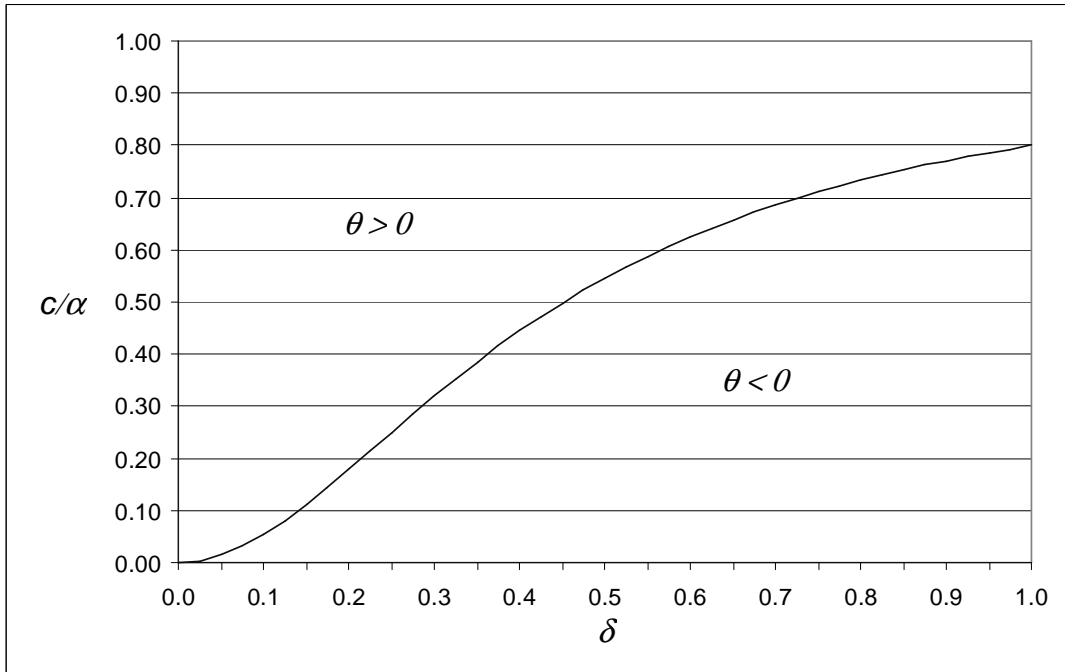θ^*

p_i

Figure 1: Effects of an Increase in θ

Figure 2: Marginal Cost-Demand Intercept Ratio (c/α) vs. Degree of Product
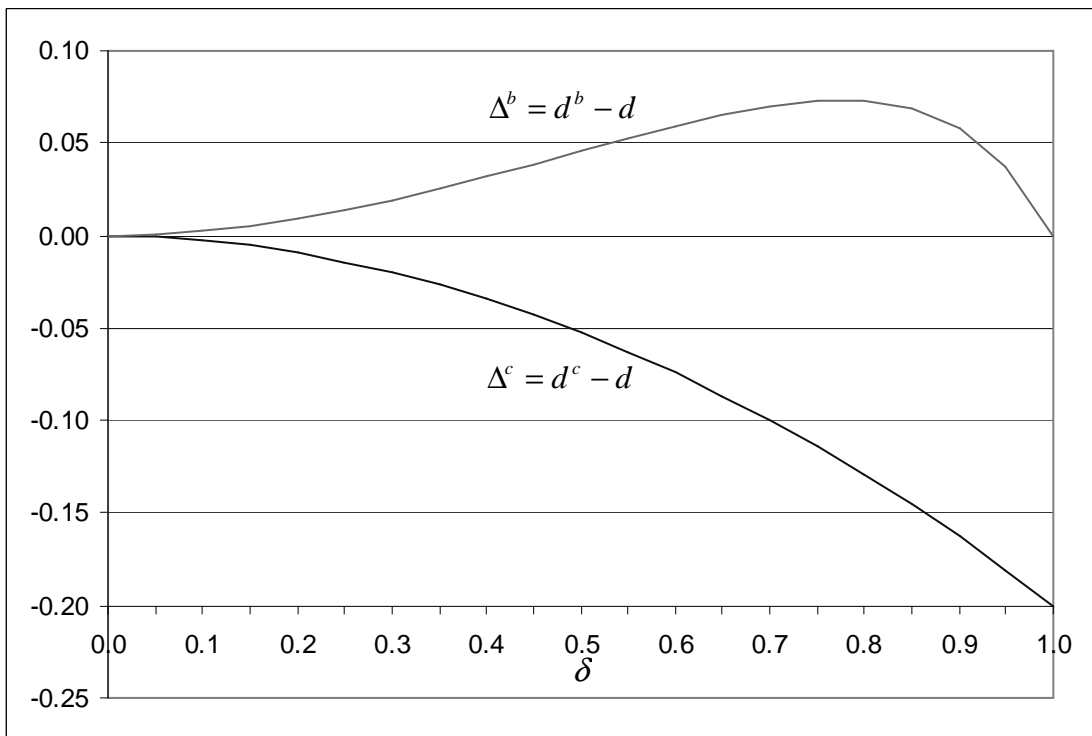Differentiation (δ) under Quantity Competition

Figure 3: Strategic Motives of Shareholders under Quantity and Price Competitions